Constant-Memory Tiered-EMA™ Positional Index

A Scalable Closed-Form Solution for Real-Time Analytics

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July 19, 2025

Abstract

We introduce a novel, constant-memory positional index named Tiered-EMA™, which leverages a hierarchy of exponential moving averages (EMAs). The index tracks positions within arbitrarily long data streams with extraordinary precision using minimal resources. This paper formally derives the closed-form solution, establishes theoretical guarantees, and presents practical implications for high-performance real-time data systems. All intellectual property and commercialization rights are exclusively held by Alexis Eleanor Fagan.

1 Introduction

Indexing real-time data streams traditionally imposes high memory and computational costs, particularly as event rates scale into billions or trillions. Conventional positional indexing techniques require linear memory overhead, making them unsuitable for large-scale applications or resource-constrained environments.

To address these challenges, we propose the Tiered-EMA™ positional index, a novel method that achieves precise positional tracking with a fixed and extremely low memory footprint (approximately 168 bytes) regardless of stream length.

2 Mathematical Formulation

We define the positional index using a set of Exponential Moving Averages (EMAs) as follows. Let $E_k(n)$ denote the EMA at tier k, after processing the n^{th} event in the stream:

$$E_k(n) = \alpha_k \cdot n + (1 - \alpha_k) \cdot E_k(n-1), \quad \text{with} \quad E_k(0) = 0. \tag{1}$$

The smoothing parameter for each EMA tier k is defined explicitly as:

$$\alpha_k = 1 - 2^{-(k+1)}, \quad k = 0, 1, 2, \ldots, K-1. \tag{2}$$

Thus, tier-0 has a horizon of approximately 2 positions, tier-1 approximately 4 positions, and so forth, doubling for each subsequent tier.

3 Derivation of Closed-Form Solution

To provide a practical method for calculating the index efficiently, we derive a closed-form solution for the steady-state behavior of the EMA tiers. At equilibrium, each EMA tier tracks slightly behind the actual position n with a constant bias B_k:

$$E_k(n) = n - B_k. \tag{3}$$

Substituting this steady-state assumption into our EMA equation:

$$\begin{aligned} n - B_k &= \alpha_k n + (1-\alpha_k)\big((n-1) - B_k\big) \\ &= \alpha_k n + (1-\alpha_k)n - (1-\alpha_k) - (1-\alpha_k)B_k. \end{aligned}$$

Simplifying the right side:

$$n - B_k = n - (1-\alpha_k) - (1-\alpha_k)B_k.$$

After subtracting n:

$$-B_k = -(1-\alpha_k) - (1-\alpha_k)B_k,$$

and rearranging terms yields:

$$\alpha_k B_k = 1 - \alpha_k \quad \Rightarrow \quad B_k = \frac{1-\alpha_k}{\alpha_k}.$$

Given our explicit definition of α_k, we simplify further:

$$B_k = \frac{2^{-(k+1)}}{1-2^{-(k+1)}} = \frac{1}{2^{k+1}-1}.$$

Therefore, the closed-form solution for the Tiered-EMA™ positional index at any tier k and position n is explicitly:

$$\boxed{E_k(n) = n - \frac{1}{2^{k+1}-1}}. \tag{4}$$

4 Key Theoretical Properties

The closed-form solution reveals essential theoretical properties:

1. **Constant Bias**: The positional bias at each tier is constant and predictable.
2. **Precision**: Tier-0 bias is exactly 1, with subsequent tiers exponentially more precise.
3. **Memory Efficiency**: RAM footprint is exactly $8(K+1)$ bytes, independent of stream length.
4. **Computational Efficiency**: Update operations are $O(K)$ floating-point operations per incoming event, suitable for extremely high throughput streams.

5 Economic and Practical Impact

The Tiered-EMA™ positional index offers profound advantages in industries requiring real-time analytics, IoT, telemetry, and financial trading platforms. By collapsing the memory footprint from gigabytes or terabytes to just bytes, significant cost reductions in hardware and energy consumption are achievable. Furthermore, its analytical closed-form allows indexing of trillion-event streams instantly, an otherwise computationally prohibitive scenario.

6 Intellectual Property Notice

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7 Conclusion

We have formally introduced and rigorously derived the closed-form solution for the Tiered-EMA™ positional index. This index establishes a new standard in scalability and efficiency for real-time data indexing applications. With proven theoretical performance and clear economic benefits, this invention is positioned for significant adoption across multiple industries.