Handin 3

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The system

Suppose two systems:

$$x_A(t+1) = \frac{1}{2}x_A(t) - 3,$$

and

$$x_B(t+1) = x_B(t)x_B(t) = x_B(t)^2.$$

Question 1

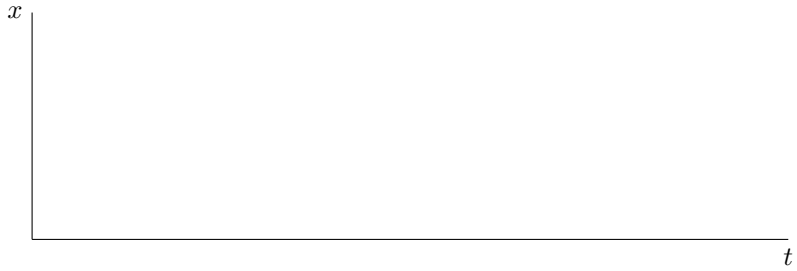
Sketch $x_i(t+1)$ for the two systems in terms of x_i.

Answer: We begin by rewriting x_A in terms of $x_A(0)$,

$$\begin{aligned} x_A(t+3) &= \frac{1}{2}x_A(t+2) - 3 \\ &= \frac{1}{2}\left(\frac{1}{2}x_A(t+1) - 3\right) - 3 \\ &= \frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}x_A(t) - 3\right) - 3\right) - 3, \end{aligned}$$

from this, we find that the recursion formula is given by:

$$x_A(t) = \left(\frac{1}{2}\right)^t x_A(0) - 3\left(2 - \left(\frac{1}{2}\right)^{t-1}\right).$$



The second system, x_B can also be rewritten:

$$\begin{aligned} x_B(t+3) &= x_B(t+2)^2 \\ &= x_B(t+1)^4 \\ &= x_B(t)^6. \end{aligned}$$

Thus, the recursion formula gives:

$$x_B(t) = x_B(0)^{2t}$$

Question 2

How does the value of $\lim_{t\to\infty} x_i$ depend on the initial condition for $x(0)$ for the two systems.

Answer:

Question 3

For the first system, consider some initial condition $x_A(0)$ and a nearby condition, such that $\tilde{x}_a(0) = x_A(0) + \delta x_A(0)$. Calculate the difference in the iterations.

Answer:

Question 4

The largest Lyapunov exponent is the exponential rate at which infinitesimally close initial conditions separate, i.e.

$$\lambda = \lim_{t\to\infty} \frac{1}{t} \ln\left|\frac{\delta x(t)}{\delta x(0)}\right|.$$

What is the Lyapunov exponent of system A? Note that $\delta x(0)$ is truly infinitesimal.

Answer: