Exercise 63

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Question

We wish to show the that the following expression

$$F_i = \epsilon_{ijk} \left(\partial_j B_k\right) |_0 \int x_m J_j(x) d^3x, \tag{1}$$

is equivalent with the following expression

$$F_i = \partial_i (B_k m_k).$$

Answer

We're given that the following holds for any arbitrary vector V_n,

$$V_n \int x_n J_j d^3x = -\epsilon_{jrs} V_r m_s,$$

which implies that we can rewrite eq (1) to the following

$$\begin{aligned}
F_i &= -\epsilon_{ijk}\epsilon_{jrs}(\partial_r B_k)|_0 m_s \\
F_i &= \epsilon_{ikj}\epsilon_{jrs}(\partial_r B_k)|_0 m_s \\
&= (\delta_{ir}\delta_{ks} - \delta_{is}\delta_{kr})\,(\partial_r B_k)|_0 m_s \\
&= (\delta_{ir}\delta_{ks} - \delta_{is}\delta_{kr})\left(\partial_r(B_k m_s) - \partial_r(m_s)B_k\right) \\
&= \partial_i(B_k m_k) - \partial_k(m_i)B_k \\
&= \partial_i(B_k m_k).
\end{aligned}$$

And thus we've shown that the two expressions are equivalent.