THIRD SEMESTER B.E. DEGREE EXAMINATION
CBCS MODEL QUESTION PAPER - 1
DISCRETE MATHEMATICAL STRUCTURES

Time : 3 Hrs. Max. Marks : 100

MODULE - 1

1. a. **Define a tautology and contradiction. Prove that for any propositions p, q, r the compound proposition $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology.** **(06 M)**

Ans: **Tautology:**

A compound proposition which is true for all possible truth values of its components is called a tautology.

Contradiction : A compound proposition which is false for all possible truth values of its components is called a contradiction.

1	2	3	4	5	6	7	8
p	q	r	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \wedge (p \rightarrow r)$	$p \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
0	0	0	1	1	1	1	1
0	0	1	1	0	1	1	1
0	1	0	0	1	0	1	1
0	1	1	0	0	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	0	1	1	1
1	1	0	0	1	0	0	1
1	1	1	0	0	1	1	1

$\therefore [(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology.

1. b. **Write dual, negation, converse, inverse and contrapositive of the statement given below. If Kabir wears brown pant, then he will wear white shirt.** **(06 M)**

Ans: Let P : Kabir wears brown pant

q : He will wear white shirt.

$(p \rightarrow q)^d \Leftrightarrow (\sim p \vee q)^d \Leftrightarrow \sim p \wedge q$

Dual: Kabir is not wearing brown pant and he is wearing white shirt.

Negation: $\sim(p \rightarrow q) \Leftrightarrow \sim(\sim p \vee q) \Leftrightarrow p \wedge \sim q$

Kabir wears brown pant or he does not wear white shirt.

Converse : $q \rightarrow p$

If Kabir wears white shirt then he will wear brown pant.

Inverse : $\sim p \rightarrow \sim q$

If Kabir does not wear brown pant, then he will not wear white shirt.

Contra positive : $\sim q \rightarrow \sim p$

If Kabir does not wear white shirt, then he does not wear brown pant.

1. c. Establish the validity of the following argument:

$$\forall x, \left[p(x) \rightarrow \{ q(x) \wedge r(x) \} \right]$$
$$\forall x, \left[p(x) \rightarrow s(x) \right]$$
$$\overline{\qquad\qquad\qquad\qquad\qquad}$$
$$\therefore \forall x, \left[r(x) \wedge s(x) \right]$$

(08 M)

Ans:

Steps	Description	Reason
1	$\forall x, \{ [p(x) \rightarrow q(x)] \wedge r(x) \}$	Premise
2	$p(a) \rightarrow [q(a) \wedge r(a)]$	Rule of Universal specification
3	$\sim p(a) \vee \{ q(a) \wedge r(a) \}$	$p \rightarrow q \Leftrightarrow \sim p \vee q$
4	$\forall x, [p(x) \wedge s(x)]$	Premise
5	$p(a) \wedge s(a)$	Rule of Universal Specification
6	$p(a)$	Law of Conjunctive Simplification
7	$q(a) \wedge r(a)$	(6) & (2) Law of detachment
8	$r(a)$	Law of Conjunctive Simplification
9	$s(a)$	(5) Rule of Conjunctive Simplification
10	$r(a) \wedge s(a)$	(8), (9)
11	$\forall x, [r(x) \wedge s(x)]$	Rule of Universal Specification

$\therefore$ The given argument is valid.

OR

2. a. Prove that $(\sim p \vee q) \wedge [p \wedge (p \wedge q)] \Leftrightarrow p \wedge q$ without using truth tables. Hence deduce that $(\sim p \wedge q) \vee [p \vee (p \vee q)] \Leftrightarrow p \vee q$ **(06 M)**

Ans: $\Leftrightarrow (\sim p \vee q) \wedge [p \wedge (p \wedge q)] \Leftrightarrow (\sim p \vee q) \wedge (p \wedge q)$ Idempotent law.

$\Leftrightarrow [$ [illegible] $\wedge (p \wedge q)] \vee [q \wedge (p \wedge q)]$. Distributive law

$\Leftrightarrow F_0 \vee (p \wedge q)$. Inverse and Idempotent law.

$(\sim p \vee q) \wedge [p \wedge (p \wedge q)] \Leftrightarrow p \wedge q$, Identity law.

Taking duality both sides,

$(\sim p \wedge q) \vee$ [illegible] $\vee (p \vee q)] \Leftrightarrow p \vee q$

2. b. Give (i) a direct proof (ii) an indirect proof, (iii) Proof by contradiction for the following statement "If n is an odd integer, then n + 9 is an even integer". **(08 M)**

Ans: Proof:

Let p : n is odd, q : n + 9 is even

Given $p \to q$

Direct Proof:

Assume p is true.

$\Rightarrow$ n is odd

$\Rightarrow n = 2k + 1$, $k \in z$

$\Rightarrow n + 9 = 2k + 1 + 9$

$\Rightarrow n + 9 = 2k + 10$

$\Rightarrow n + 9 = 2(k + 5)$

$\Rightarrow n + 9 = 2k_1$, $\quad k_1 = k + 5 \in z$

$\Rightarrow n + 9$ is even $\Rightarrow$ q is true.

$\therefore p \to q$ is true.

(ii) Indirect Proof:

T. P $\sim q \to \sim p$ is true.

Assume $\sim q$ is true.

$\Rightarrow n + 9$ *is not even*

$\Rightarrow n + 9$ *is odd*

$\Rightarrow n + 9 = 2k + 1, k \in z$

$\Rightarrow n = 2k + 1 - 9$

$\Rightarrow n = 2k - 8$

$\Rightarrow n = 2(k - 4)$

$\Rightarrow n = 2k_1$ *Where* $k_1 \in z$

$\Rightarrow$ *n is even* $\Rightarrow$ *n is not odd*

$\Rightarrow \sim P$ *is true*

$\therefore \sim q \to \sim p$ *is true*

i.e., $p \to q$ *is true*

(iii) Proof by Contradiction:

Assume $p \to q$ is False. i.e., p is true and q is false.

Consider q is False.

$\Rightarrow n + 9$ *is odd*

$\Rightarrow n + 9 = 2k + 1, k \in z$

$\Rightarrow n = 2k + 1 - 9$

$\Rightarrow n = 2k - 8$

$\Rightarrow n = 2(k - 4)$ $\quad \Rightarrow n = 2k_1, k_1 \in z$

$\Rightarrow$ *n is even* $\quad \Rightarrow$ *p is false*

Since p cannot be both true and false which is a contradiction. $\therefore$ Our assumption is wrong.

Hence $p \to q$ is true.

2. c. Prove that the following logical equivalence without using truth tables:

$$(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$$

(06 M)

Ans: $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p) \wedge [\neg q \wedge (\neg q \vee r)]$, Commutative Law

$\Leftrightarrow (p \to q) \wedge \neg q$, Absorption law.

$\Leftrightarrow (\neg p \vee q) \wedge \neg q$. $p \to q \Leftrightarrow \neg p \vee q$

$\Leftrightarrow \neg q \wedge (\neg p \vee q)$, Commutative Law

$\Leftrightarrow (\neg q \wedge \neg p) \vee (\neg q \vee q)$, Distributive Law

$\Leftrightarrow \neg(q \vee p) \vee \varphi$

$\Leftrightarrow \neg(q \vee p)$

MODULE - 2

3. a. Prove that $\displaystyle\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}, \forall n \in z^{+}$ **(08 M)**

Ans: $\displaystyle\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}, \forall n \in z^{+}$

Basic Step:

$$S(1) : \frac{1}{1+1} = \frac{1}{1+1}$$

$$= \frac{1}{2} = \frac{1}{2}$$

$\therefore S(1)$ *is true*

The statement S(n) is verified for n = 1.

Induction Step:

We assume that the statement S(n) is true for n = k, Where k is an integer ≥ 1. i.e. we assume that the following statement is true:

$$S(k) = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \ldots\ldots\ldots + \frac{1}{k(k+1)} = \frac{k}{k+1}$$

Using this we find that

$$= \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)} = \frac{(k+1)^2}{(k+1)(k+2)}$$

$$= \frac{k+1}{k+2} = \frac{k+1}{k+1+1}$$

$\therefore$ S(k + 1) is true.

Thus on the basis of the assumption that S(n) is true for n = k ≥ 1, the truthness of S(n) for n = k + 1 is established. Hence the proof.

3. b. Obtain a recurrence definition for the sequence in each of the following:
(i) $a_n = 5n$ (ii) $a_n = 2 - (-1)^n$

(06 M)

Ans. (i) Here $a_1 = 5$, $a_2 = 10$, $a_3 = 15$, $a_4 = 20$, ... We can write these as $a_1 = 5$ and $a_n = a_{n-1} + 5$ for $n \geq 2$. This is a recursive definition of the given sequence.

(ii) Here $a_1 = 3$, $a_2 = 1$, $a_3 = 3$, ...$a_n = 2-(-1)^n$ and

$$a_{n+1} - a_n = -(-1)^{n+1} + (-1)^n$$

$$= (-1)^n \{1 + (-1)^2\} = 2(-1)^n$$

$$a_{n+1} = a_n + 2(-1)^n \text{ for } n \geq 1$$

$$\text{Thus, } a_1 = 3 \text{ and } a_{n+1} = a_n + 2(-1)^n \text{ for } n \geq 1$$

is recursive definition of the given sequence.

3. c. Find the number of proper divisors of 441000.

(06 M)

Ans: For a positive integer n, the divisors of n other than 1 and n are called proper divisors.

$441000 = 2^3 \times 3^2 \times 5^3 \times 7^2$

∴ Every divisor of $n = 441000$ must be of the form $d = 2^p \times 3^q \times 5^r \times 7^s$ where $0 \leq p \leq 3$, $0 \leq q \leq 2$, $0 \leq r \leq 3$, $0 \leq s \leq 2$

Thus for a divisor d, p can be chosen in 4 ways, q in 3 ways, r in 4 ways and s in 3 ways.

According, the number of possible d's is $4 \times 3 \times 4 \times 3 = 144$.

Of these, 1 and n are not proper divisors.

∴ The number of proper divisors of the given number is $= 144 - 2 = 142$.

OR

4. a. Find the coefficient of $x^2 y^2 z^3$ in the expansion of $(x + y + z)^7$

(06 M)

Ans: By Multinomial theorem.

$$\binom{7}{n_1, n_2, n_3}(x)^{n_1}(y)^{n_2}(z)^{n_3}$$

$$\text{For } n_1 = 2, n_2 = 2, n_3 = 3,$$

$$\binom{7}{2, 2, 3}(x)^2(y)^2(z)^3 = \frac{7!}{2!2!3!}x^2y^2z^3$$

$$= 210\, x^2y^2z^3$$

The required coefficient is = 210

4. b. In how many ways can 10 identical pencils be distributed among 5 children in the following cases:

(i) There are no restrictions.

(ii) Each child gets atleast one pencil

(iii) The youngest child gets atleast two pencils.

(06 M)

Ans: (i) The required number is

$C(5 + 10 - 1, 10) = C(14, 10)$

$= \frac{14!}{10!\,4!} = 1001$

(ii) First we distribute one pencil to each child. Then there remain 5 pencils to be distributed. The number of ways of distributing these 5 pencils to 5 children is the required number.

$C(5 + 5 - 1, 5) = C(9, 5)$

$= \frac{9!}{5!\,4!} = 126$

(iii) First we give two pencils to the youngest child. Then there remain 8 pencils to be distributed. The number of ways of distributing these 8 pencils to 5 children is the required number.

$C(5 + 8 - 1, 8) = C(12, 8)$

$= \frac{12!}{8!\,4!} = 495$

4. c. Define the following:

(i) Well ordering principle **(ii) Principle of Mathematical Induction** **(08 M)**

Ans: **(i) Well Ordering Principle:** Every nonempty subset of Z^{+} contains a smallest (least) element

(ii) Principle of Mathematical Induction:

Suppose to prove that a certain statement S(n) is true for all integers $n \geq 1$. The method of proving such a statement on the basis of the induction principle is called the method of Mathematical Induction. This method consists of two steps, called basic step and the induction step.

Basic Step: Verify that the statement S(1) is true, (ii) Verify that S(n) is true for n = 1.

Induction Step: Assuming that S(k) is true, where k is an integer ≥ 1, Show that S(k + 1) is true.

MODULE - 3

5. a. Let A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5, 6}.

i. How many functions are there from A to B?

ii. How many of these are one to one?

iii. How many are onto?

iv. How man}] functions are there from B to A?

v. How many of these are onto?

vi. How many are one to one? **(06 M)**

$\frac{n!}{(n-m)!}$

Ans: i) $|A| = m = 4 \; |B| = n = 6$

Number of functions A to B $= 6^4$

ii) One to one from A to B $\frac{n!}{(n-m)!} = 360$

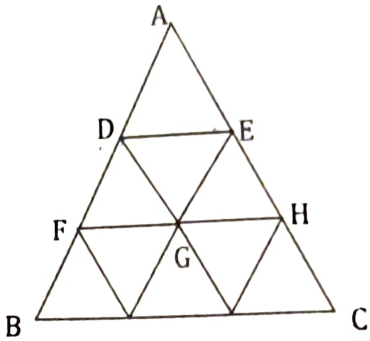
iii) No onto functions from A to B

iv) Functions from B to A = $4^6 = 4096$

v) Onto from B to A = 1560

vi) No one to one function from B to A

5. b. Let ΔABC be an equilateral triangle of side 1. Show that if we select 10 points in the interior. There must be atleast two points whose distance apart is less than 1/3. (06 M)



Ans: In ΔABC, divide each side into three equal parts and form the nine congruet triangles. Shown in the figure.

Let R, be the interior of ΔADE together with the points on segment. DE, excluding DE region R_2, is the interior of ΔDFG together with the points on segments DG. FG excluding DE. Regions R_3, R_4,..............R_9 are defined similarly so that the interior of ΔABC is the union of these nine regions and $R_i \cap R_j = \varphi$ for $i \neq j$. then if 10 points are chosen in the interior of ΔABC, atleast two of these points are in Ri for some $1 \leq i \leq 9$, and these two points are at a distance less than 1/3 from each other.

5. c. Define a binary relation. Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by a Rb if a is a multiple of b. Represent R as a set of ordered pairs. Draw the digraph of R. Write the matrix of R.

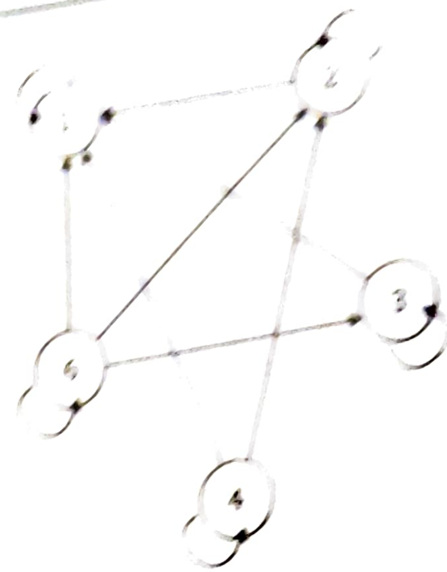
(08 M)

Ans: **Binary Relation:** Let A and B be two sets, then a subset of A × B is called a binary relation or just a relation form A to B. If R is a relation from A to B then R is a set of ordered pairs (a, b) where a ∈ A and b ∈ B.

$$R = \{(a, b) \mid a, b \in A \text{ and } a \text{ is a multiple of } b\}$$

$$R = \left\{\begin{matrix}(1,1), (2,1), (2,2), (3,1), (3,3), \\ (4,1), (4,2), (4,4), (6,1), (6,2), \\ (6,3), (6,6)\end{matrix}\right\}$$

$$M(R) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$



OR

6. a. For a fixed integer $n > 1$. Prove that the relation "Congruent Modulo n" is an equivalence relation on the set of all integers, Z. **(08 M)**

Ans: For a, b $\in$ Z, we say that "a is congruent to be modulo n" written symbolically as $a \equiv b$ (mod n)

(i.e) $a - b$ is a multiple of n. or equivalently $a - b = k_1$ for some $k \in z$.

Let us denote this relation by R so that a R b means $a \equiv b$(mod n).

1. Reflexive:

For every $a \in z$, $a - a = 0$ is a multiple of n.

i.e. $a \equiv a$(mod n).

$\therefore$ R is reflexive.

2. Symmetric:

For all a, b $\in$ z

a R b $\Rightarrow a \equiv b$(mod n).

$\Rightarrow a - b$ is a multiple of n

$\Rightarrow b - a$ is a multiple of n

$\Rightarrow b \equiv a$(mod n).

$\Rightarrow$ b R a

$\therefore$ R is Symmetric.

3. Transitive:

For all a, b, c $\in$ z

a R b and b R c $\Rightarrow a \equiv b$(mod n). and $b \equiv c$(mod n).

$\Rightarrow a - b$ and $b - c$ is a multiple of n.

$\Rightarrow a - b + b - c = a - c$ is a multiple of n.

$\Rightarrow a \equiv c$(mod n).

$\Rightarrow a \, R \, c$

$\therefore$ R is Transitive.

$\therefore$ R is an equivalence relation.

6. b. Draw the Hasse diagram for all positive integer divisors of 72. **(06 M)**

Ans: The set of all positive divisors of 72 is,

$D_{72} = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}$

Relation R is divisibility (a R b if f a divides b) is a partial order on this set.

1 is related to all elements of D_{72}.

2 is related to 2, 4, 6, 8, 12, 18, 24, 36, 72

3 is related to 3, 6, 9, 12, 18, 36, 72

4 is related to 4, 8, 12, 36, 72

6 is related to 6, 12, 18, 24, 36, 72

8 is related to 8, 24, 72

9 is related to 9, 18, 36, 72

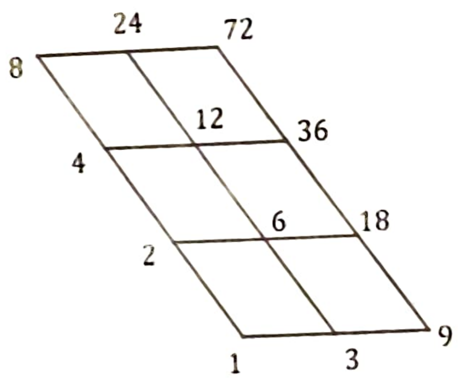
12 is related to 12, 24, 36, 72

18 is related to 18, 36, 72

24 is related to 24, 72

36 is related to 36, 72

72 is related to 72.



c. Prove that a function $f : A \to B$ is invertible iff it is one-to-one and onto. **(06 M)**

Ans: First suppose that f is invertible. There exists a unique function $g : B \to A$ such that $gof = I_A$ and $fog = I_B$.

Take any $a_1, a_2 \in A$. Then $f(a_1) = f(a_2)$

$\Rightarrow g\{f(a_1)\} = g\{f(a_2)\}$

$\Rightarrow (gof)(a_1) = (gof)(a_2)$

$\Rightarrow I_A(a_1) = I_A(a_2)$

$\Rightarrow a_1 = a_2, \therefore f \text{ is } 1-1$

Take any $b \in B$, $b = I_B(b) = (fog)(b)$

$b = f\{g(b)\}$

$\therefore$ b is the image of an element $g(b) \in A$ under f. $\therefore$ f is onto.

$\therefore$ f is 1 – 1 and onto.

Conversely suppose that f is 1 – 1 and onto. Then for each $b \in B$ there is a unique $a \in A$ such that $b = f(a)$. Now, consider the function $g : B \to A$ defined by $g(b) = a$. then,

$$(gof)(a) = g\{f(a)\} = g(b) = a = I_A(a) \text{ and}$$
$$(fog)(b) = f\{g(b)\} = f(a) = b = I_B(b)$$

$\therefore$ f is invertible with g as the inverse

Hence the proof.

MODULE - 4

7. a. Out of 30 students in a hostel. 15 study History and 8 study Economics and 6 study Geography. It is known that 3 students study all these subjects. Show that 7 or more students study none of these subjects. **(08 M)**

Ans: Let S denote the set of all students in the hostel and A_1, A_2, A_3 denote the sets of students. Who study History, Economics and Geography respectively.

Given, $|S| = 30, |A_1| = 15, |A_2| = 8, |A_3| = 6$

$$S_1 = \Sigma|A_i| = 15 + 8 + 6 = 29$$
$$S_3 = |A_1 \cap A_2 \cap A_3| = 3$$
$$|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}| = |S| - \Sigma|A_i| + \Sigma|A_i \cap A_j| - |A_1 \cap A_2 \cap A_3|$$
$$= |S| - s_1 + s_2 - s_3$$
$$= 30 - 29 + s_2 - 3$$
$$|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}| = s_2 - 2 \text{ Where } s_2 = \Sigma|A_i \cap A_j|$$

Since $A_1 \cap A_2 \cap A_3$ is a subset of 

$$d_4 = 4! \times \left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right\}$$

$$= 24 \times \left\{1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right\}$$

$$= 12 - 4 + 1$$

$$d_4 = 9$$

Then nine derangements of 1, 2, 3, 4 are,

2143 2341 2413

3142 3412 3421

4123 4312 4321

7. c. A bank pays a certain % of annual interests on deposits, compounding the interests once in 3 months. If a deposit doubles in 6 years and 6 months. What is the annual % of interest paid by the bank? (06 M)

Ans: Let the annual rate of interest be x%. Then the quarterly rate of interest is,

$$\left(\frac{x}{4}\right)\% = \frac{x}{400}$$

Let P_0 denote the deposit made in Rs and P_n denote that the value of the deposit at the end of the nth quarter. Then,

$$P_{n+1} = P_n + \left(\frac{x}{400}\right)P_n$$

$$P_{n+1} = \left(1 + \frac{x}{400}\right)P_n, \quad n \geq 0 \qquad \text{....(1)}$$

This is the recurrence relation for the problem.

The general solution of the homogeneous relation is,

$$P_n = \left(1 + \frac{x}{400}\right)P_0, \quad For\ n \geq 1 \qquad \text{....(2)}$$

From what is given, we have $P_n = 2P_0$ When n = 26 using this in (2).

$$\left(1 + \frac{x}{400}\right)^{26} = 2$$

$$\log_e\left(1 + \frac{x}{400}\right) = \frac{\log_e^2}{26} = 0.02666$$

$$1 + \frac{x}{400} = e^{0.02666} = 1.027$$

$$x = 400 \times 0.027$$

$$x = 10.8$$

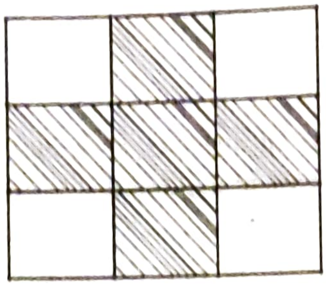
Thus the annual rate of interest paid by the bank is 10.8%.

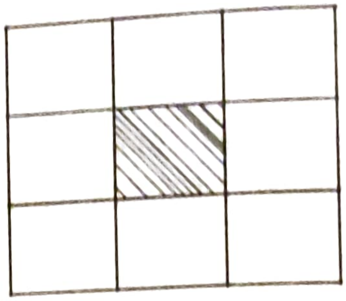
OR

8. a. Find the root polynomial for the 3 × 3 board by using the expansion formula **(06 M)**

Ans:

Let us mark the square which is at the central of the board as *. Then the boards D and E appear as shown below.





For the board D. $r_1 = 4, r_2 = 2, r_3 = r_4 = 0$

$\therefore r(D, x) = 1 + 4x + 2x^2$

For the board E. $r_1 = 8, r_2 = 14, r_3 = 4, r_4 = 0$

$r_5 = r_6 = r_7 = r_8 = 0$

$r(E, x) = 1 + 8x + 14x^2 + 4x^3$

The expansion formula is,

$r(c, x) = xr(D, x) + r(E, x)$

$= x(1 + 4x + 2x^2) + (1 + 8x + 14x^2 + 4x^3)$

$\boxed{r(c, x) = 1 + 9x + 18x^2 + 6x^3}$

8. b. The number of virus affected files in a system is 1000 (to start with) and this number increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(06 M)**

Ans: In the beginning, the number of virus affected files is 1000, Let us denote this by a_0.

Let a_n denote the number of virus affected files after 2n hours. Then the number increases by $a_n \times \frac{250}{100}$ in the next two hours.

Thus, after 2n + 2 hours, the number is ,

$a_{n+1} = a_n + a_n \times \frac{250}{100}$

$= a_n(1 + 2.5)$

$a_{n+1} = a_n(3.5)$

This is the recurrence relation for the number of virus affected files. Solving this relation, we get.

$a_n = (3.5)^n a_0 = 1000 \times (3.5)^n$

This gives the number of virus affected files after 2n hours. From this, we get for n = 12,

$a_{12} = 1000 \times (3.5)^{12}$

$\mathbf{a_{12} = 3379220508}$

8. c. Determine the number of positive integers n such that $1 \leq n \leq 100$ and n is not divisible by 2, 3, or 5 **(08 M)**

Ans: Let S = {1, 2, 3,................, 100}

$|S| = 100$

Let A_1, A_2, A_3 be the subsets of S whose elements are divisible by 2, 3 and 5 respectively.

$$|A_1| = \left\lfloor \frac{100}{2} \right\rfloor = [50] = 50$$

$$|A_2| = \left\lfloor \frac{100}{3} \right\rfloor = [33.333] = 33$$

$$|A_3| = \left\lfloor \frac{100}{5} \right\rfloor = [20] = 20$$

$$|A_1 \cap A_2| = \left\lfloor \frac{100}{6} \right\rfloor = [16.666] = 16 \qquad [\because \text{LCM of 2 and 3 is 6}]$$

$$|A_1 \cap A_3| = \left\lfloor \frac{100}{10} \right\rfloor = [10] = 10 \qquad [\because \text{LCM of 2 and 5 is 10}]$$

$$|A_2 \cap A_2| = \left\lfloor \frac{100}{16} \right\rfloor = [6.666] = 6 \qquad [\because \text{LCM of 3 and 5 is 15}]$$

$$|A_1 \cap A_2 \cap A_3| = \left\lfloor \frac{100}{30} \right\rfloor = [3.333] = 3 \qquad [\because \text{LCM of 2, 3 and 5 is 30}]$$

$$|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}| = |S| - \{|A_1| + |A_2| + |A_3|\} + \{|A_1 \cap A_2| + |A_1 \cap A_3| + |A_2 \cap A_3| - |A_1 \cap A_2 \cap A_3|\}$$

$$= 100 - (50 + 33 + 20) + (16 + 10 + 6) - 3$$

$$|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}| = 26$$

∴ The required number is 26.

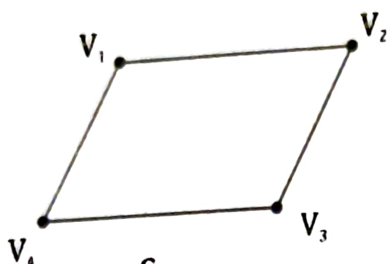
These 26 numbers are 1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 73, 79, 77, 83, 89, 91 and 97.

MODULE - 5

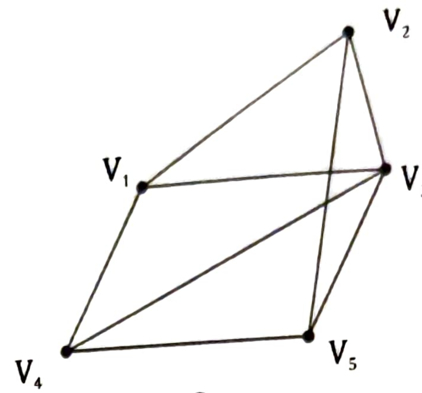
9. a. Define Subgraph. Spanning subgraph. Induced subgraph and complete graph with example. **(08 M)**

Ans: **Subgraph:** Given two graphs G and G_1, we say that G_1 is a subgraph of G if the following conditions hold. (i) all the vertices and all the edges of G_1 are in G. (ii) Each edge of G_2 has the same end vertices in G as in G_1.

Example:



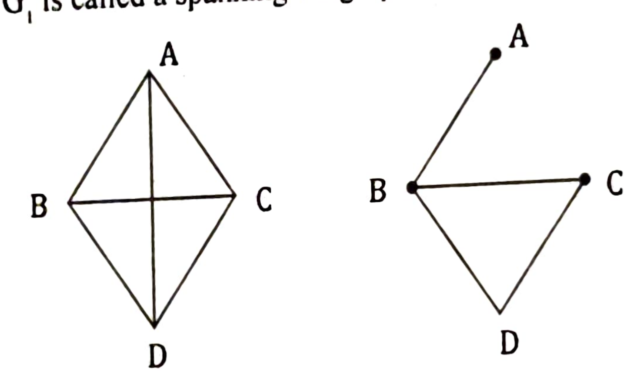
G_1



G

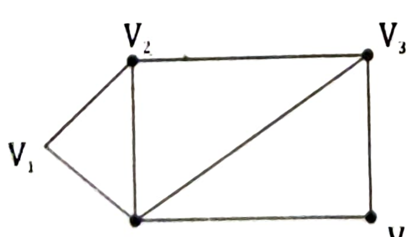
Spanning Subgraph: Given a graph G = (V, E), there is a subgraph $G_1 = (V_1, E_1)$ of G such that $V_i = V$. Then G_1 is called a spanning subgraph of G.

Example:

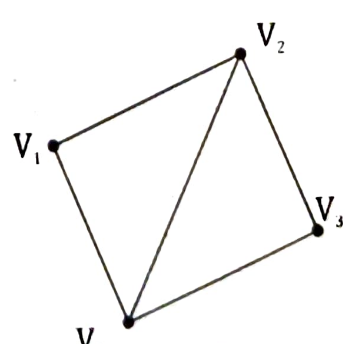


Induced Subgraph: Given a graph G = (V, E), Suppose there is a subgraph $G_1 = (V_1, E_1)$ of G such that every edge { A, B } of G. Where A, B $\in V_1$ is an edge of G_1 also. Then G_1 is called on induced subgraph of G (induced by V_1) and is denoted by $\langle V_1 \rangle$.

Example:



G



G_1

Complete Graph: A Simple graph of order ≥ 2 in which there is an edge between every pair of vertices is called a complete graph.

Example:

k_3

9. b. **Determine the order | V | of the graph G = (V, E) in the following cases:**

(i) G is a cubic graph with 9 edges.

(ii) G is regular with 15 edges.

(iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3. (06 M)

Ans: (i) Suppose the order of G is n. Since G is a Cubic graph. all vertices of G have degree 3 and therefore the sum of the degrees of vertices is 3n. Since G has 9 edges, we should have

$3n = 2 \times 9$ (by handshaking property).

$n = 6$. Thus the order of G is 6.

(ii) Since G is regular. all vertices of G must be of the same degree. say k. If G is of order n. then the sum of the degrees of vertices is kn. Since G has 15 edges, we should have $kn = 2 \times 15$ so that $k_i = 30 / n$. Since k has to be a positive integer. It follows that n must be a divisor of 30. Thus, the possible orders of G are 1, 2, 3, 5, 6, 10, 15 and 30.

(iii) Suppose the order of G is n.

Since two vertices of G are of degree 4 and all others of degree 3, the sum of the degrees of vertices of G is $2 \times 4 + (n-2) \times 3$

Since G has 10 edges, we should have

$2 \times 4 + (n-2) \times 3 = 2 \times 10$

$8 + 3n - 6 = 20$

$n = 6$

9. c. Prove that A connected graph is a tree iff it is minimally connected. (06 M)

Ans: Suppose G is a connected graph which is not a tree. Then G contains a cycle C. The removal of any one edge e from this cycle will not make the graph disconnected. Therefore, G is not minimally connected. Thus if a connected graph is not a tree then it is not minimally connected. This is equivalent to saying that if a connected graph is minimally connected then it is a tree.

Conversely. Suppose G is a connected graph which is not minimally connected. Then there exists an edge e in G such that G - e is connected. There fore . e must be in some cycle in G. This implies that G is not a tree. Thus, if a connected graph is not minimally connected then it is not a tree. This is equivalent to saying that if a connected graph is a tree, then it is minimally connected.

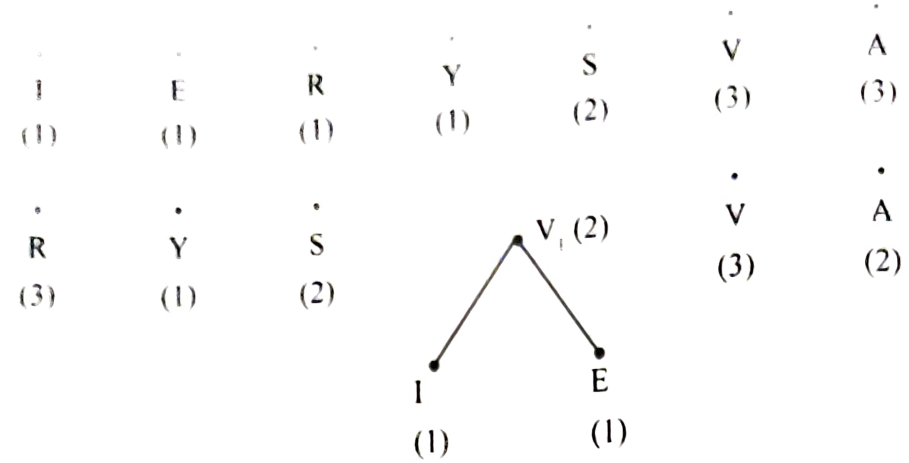
OR

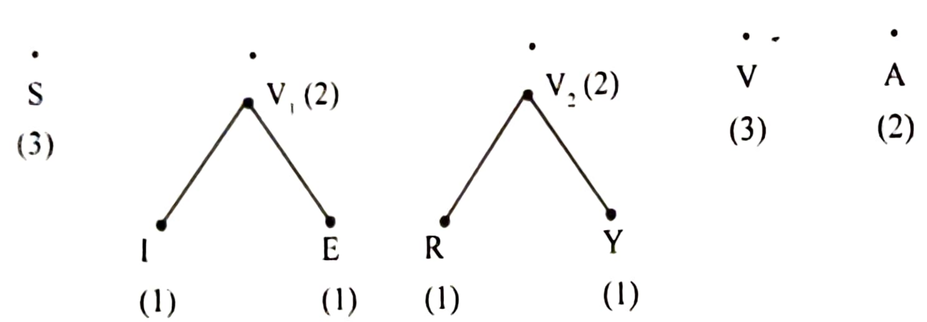
10. a. Obtain the optimal prefix code for the word VISVESVARAYA. Indicate the code. (06 M)

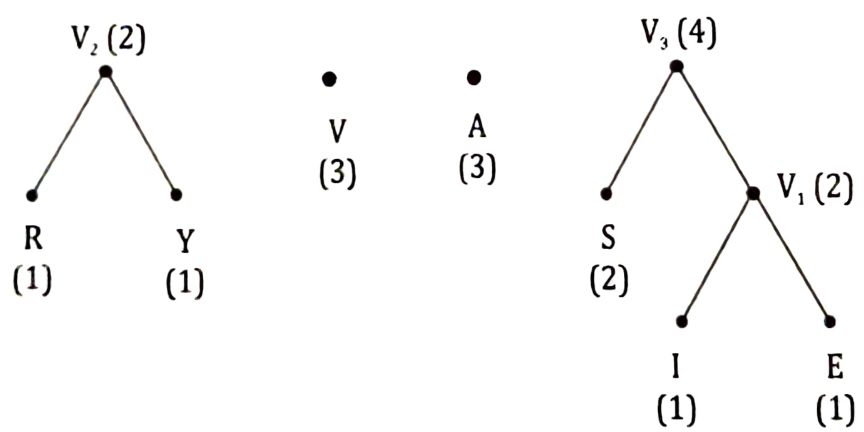
Ans:

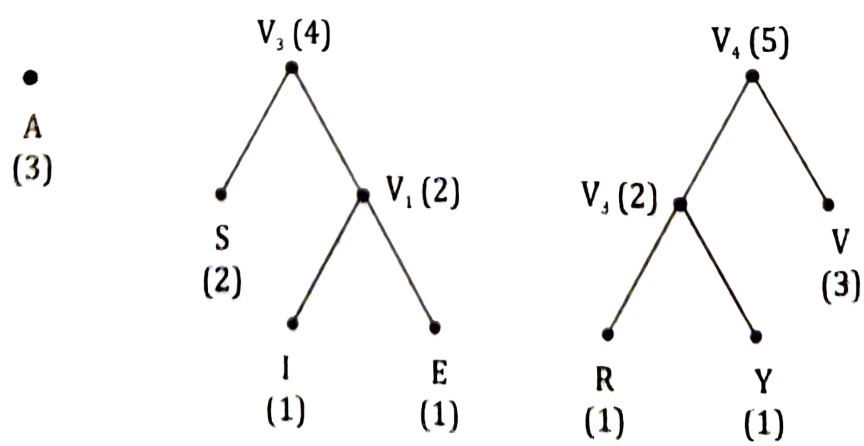
V	I	S	E	A	R	Y
(3)	(1)	(2)	(1)	(3)	(2)	(3)

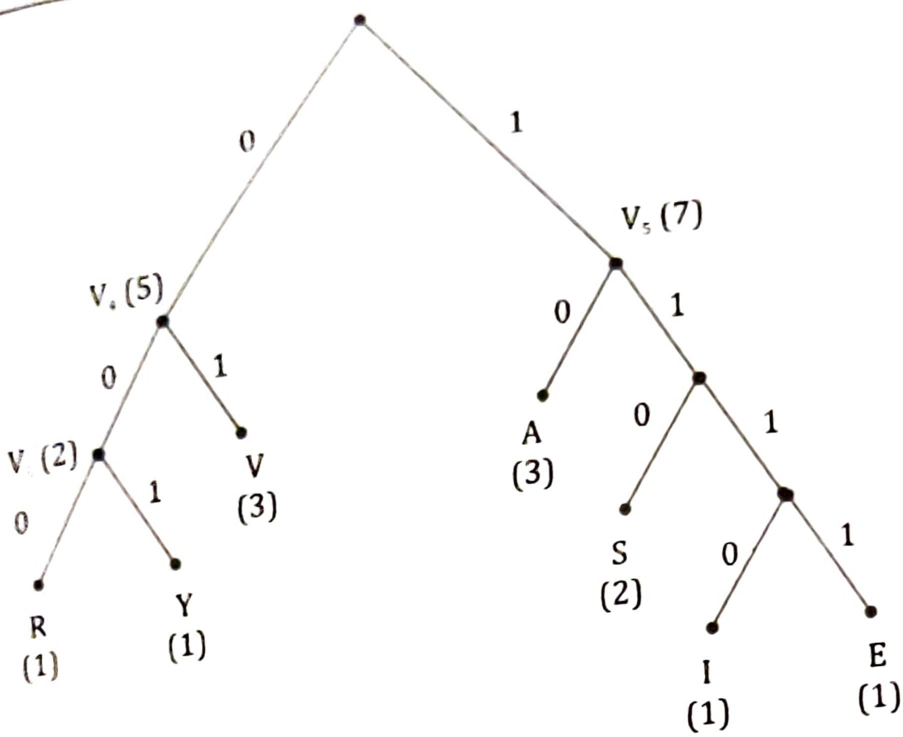
We first arrange the symbols such that their frequencies are in non-decreasing order.


I (1) E (1) R (1) Y (1) S (2) V (3) A (3)
R (3) Y (1) S (2) V₁ (2) I (1) E (1) V (3) A (2)


S (3) V₁ (2) I (1) E (1) V₂ (2) R (1) Y (1) V (3) A (2)


V₂ (2) R (1) Y (1) V (3) A (3) V₃ (4) S (2) V₁ (2) I (1) E (1)


A (3) V₃ (4) S (2) V₁ (2) I (1) E (1) V₄ (5) V₃ (2) R (1) Y (1) V (3)



V : 01 I : 1110 S : 110 E : 1111 A : 10 R: 000 Y : 001.

VISVESVARAYA : 0111101100111111100110000100110

10. b. For the following graph determine: **(08 M)**

(i) A walk from b to d that is not a trail.

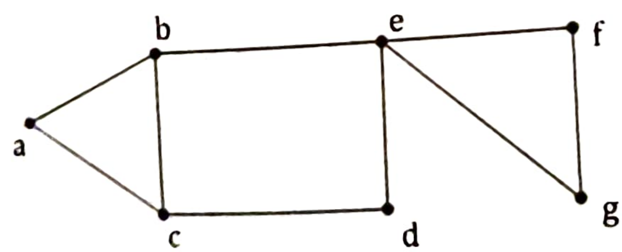
(ii) A b – d trail that is not a path.

(iii) A path form b and d.

(iv) A closed walk from b to b that is not a circuit.

(v) A circuit form b to b that is not a cycle.

(vi) A cycle form b to b.



Ans: **(i)** {b, e}, {e, f}, {f, g}, {g, e}, {e, b}, {b, c}, {c, d} is a walk but not a trail because the edge {b, e} is repeated.

(ii) {b, e}, {e, f}, {f, g}, {g, e}, {e, d} is a trail but not a path since vertex e is repeated.

(iii) {b, c}, {c, d} is a path since no vertex is repeated and no edge is repeated.

(iv) {b, e}, {e, f}, {f, g}, {g, e}, {e, b} is a closed walk (starting and ending at b) but is not a circuit because the edge {b, e} is repeated.

(v) {b, e}, {e, f}, {f, g}, {g, e}, {e, d}, {d, c}, {c, b} is a circuit but not a cycle because the vertex e is repeated.

(vi) {b, e}, {e, d}, {d, c}, {c, a}, {a, b} is a cycle where no vertex and no edge is repeated (and sequence starts and closes at b).

10. c. (i) $T_1 = (V_1, E_1)$ and $T_2 = (V_2, E_2)$ be two trees where $|E_1| = 17$ and $|V_2| = 2|V_1|$. Then find $|V_1|$, $|V_2|$ and $|E_2|$.

(ii) Let $F_2 = (V_2, E_2)$ is a forest with $|V_2| = 62$ and $|E_2| = 51$. How many trees determine F_2.

(iii) Let $F_1 = (V_1, E_1)$ be a forest of 7 trees where $|E_1| = 40$ what is $|V_1|$? **(06 M)**

Ans:

$$|E_1| = 17$$

$$|V_1| = |E_1| + 1$$

$$|V_1| = 17 + 1 = 18$$

$$\textit{But,} \quad |V_2| = 2|V_1|$$

$$|V_2| = 2(18) = 36$$

$$\textit{Then,} \quad |E_2| = |V_2| - 1$$

$$= 36 - 1$$

$$|E_2| = 35$$

(ii) Let F be a forest with k components (trees). If n is the number of vertices and m is the number of edges in F, then n = m + k or

$$|V| = |E| + k$$

$$\textit{Here,} \quad |V_2| = 62, |E_2| = 51$$

$$|V_2| = |E_2| + k$$

$$62 = 51 + k$$

$$k = 11$$

(iii)

$$\textit{Here,}\ k = 7, |E_1| = 40$$

$$|V_1| = |E_1| + k$$

$$|V_1| = 40 + 7$$

$$|V_1| = 47$$

THIRD SEMESTER B.E. DEGREE EXAMINATION
CBCS MODEL QUESTION PAPER - 2
DISCRETE MATHEMATICAL STRUCTURES

Time : 3 Hrs. Max. Marks : 100

MODULE - 1

1. a. Write the following in symbolic form and establish. If the argument is valid : If A gets the supervisor's position and works hard, then he will get a a raise. If he gets a raise, then he will buy a new car. He has not bought a new car. Therefore A did not get the supervisor's position or he did not workhard. (08 M)

Ans: p : A gets the supervisor's position

q : A works hard r : A gets a raise

s : he buys a new car.

$(p \wedge q) \rightarrow r$

$r \rightarrow s$

$\sim s$

$\therefore \sim p \vee \sim q$

Steps	Description	Reason
1	$\sim s$	Premise
2	$r \rightarrow s$	Premise
3	$\sim r$	Step (1) , (2) Modus Tollens
4	$(p \wedge q) \rightarrow r$	Premise
5	$\sim (p \wedge q)$	Steps (3), (4) Modus Tollens
6	$\sim p \wedge \sim q$	De-Morgan's Law

$\therefore$ The given argument is valid.

1. b. Let p, q be primitive statements for which implication $p \rightarrow q$ is false. Determine the truth values of the following

(i) $p \wedge q$ (ii) $\neg p \vee q$ (iii) $p \rightarrow q$ (iv) $\neg q \rightarrow \neg p$ (06 M)

Ans: Given $p \rightarrow q$ is false

$\therefore p = 1, q = 0$

(i)	$p \wedge q$	$= 1 \; 0$	$= 0$
(ii)	$\neg p \vee q$	$= 0 \; 0$	$= 0$
(iii)	$p \rightarrow q$	$= 0 \rightarrow 1$	$= 1$
(iv)	$\neg q \rightarrow \neg p$	$= 1 \rightarrow 0$	$= 0$

1. c. Let p(x), q(x) and r(x) be open statements. Determine whether the following argument a) is valid or not.

b)

$$\begin{array}{l} \forall x, [p(x) \rightarrow q(x)] \\ \forall x, [q(x) \rightarrow r(x)] \\ \hline \therefore \forall x, [p(x) \rightarrow r(x)] \end{array}$$

(06 M)

Ans:

Steps	Description	Reason
1	$\forall x, p(x) \rightarrow q(x)$	Premise
2	$p(c) \rightarrow q(c)$	(1) by the rule of universal specification
3	$\forall x, q(x) \rightarrow r(x)$	Premise
4	$q(c) \rightarrow r(c)$	(3) by the rule of universal specification
5	$p(c) \rightarrow r(c)$	(2), (4) Rule of Syllogism
6	$\forall x, p(x) \rightarrow r(x)$	(5), Rule of Universal Generalization

$\therefore$ The given argument is valid.

2. a. If $p(x) : x \geq 0$, $q(x) : x^2 \geq 0$, $r(x) : x^2 - 3x - 4 = 0$, $s(x) : x^2 - 3 > 0$. Find the truth values of the following:

(i) $\exists x, [p(x) \wedge q(x)]$ (ii) $\forall x, [p(x) \rightarrow q(x)]$

(iii) $\forall x [q(x) \rightarrow s(x)]$ (iv) $\forall x, [r(x) \vee s(x)]$

(v) $\exists x, [p(x) \wedge r(x)]$ (vi) $\forall x, [r(x) \rightarrow p(x)]$

(08 M)

Ans: **(i)** There exists a real number x for which both of p(x) and q(x) are true. For example x = 1.

$\therefore \exists x, p(x) \wedge q(x)$ is true. Truth value is 1.

(ii) For every a real number x, the statement q(x) is true. i.e., q(x) cannot be false for any real x.

$\therefore$ $p(x) \rightarrow q(x)$ cannot be false for any real x.

$\therefore$ □x, $p(x) \rightarrow q(x)$ is true. It is truth value is 1.

(iii) s(x) is false and q(x) is true for x = 1

$\therefore$ $q(x) \rightarrow s(x)$ is false for x = 1. $\therefore$ $q(x) \rightarrow s(x)$ is not always true. $\therefore \forall x, q(x) \rightarrow s(x)$ is false. It is true value is 0.

(iv) $x^2 - 3x - 4 = 0 = (x - 4)(x + 1)$. $\therefore$ r(x) is true only for x = 4 or x = −1, r(x) and s(x) are false for x = 1. $\therefore$ $r(x) \vee s(x)$ is not always true

$\therefore$ □x, $r(x) \vee s(x)$ is false. It is true value is 0.

(v) for x = 4, Both of p(x) and r(x) are true.

$\exists x, p(x) \wedge q(x)$ is true. It is truth value is 1.

(vi) p(x) is false and r(x) is true for x = −1,

$\therefore$ $r(x) \rightarrow p(x)$ is false for x = −1.. $\therefore$ $r(x) \rightarrow p(x)$ is not always true. $\therefore \forall x, r(x) \rightarrow s(x)$ is false. It is truth value is 0.

b. **Negate and simplify each of the following:** **(06 M)**

(i) $\exists x.\left[p(x) \wedge q(x)\right]$ (ii) $\forall x.\left[p(x) \rightarrow \sim q(x)\right]$

(iii) $\forall x.\left[q(x) \vee q(x)\right]$

Ans:

(i) $\sim\left[\exists x.\left[p(x) \vee q(x)\right]\right] \equiv \forall x.\left[\sim\{p(x) \vee q(x)\}\right]$

$\equiv \forall x. \sim p(x) \wedge \sim q(x)$

(ii) $\forall x.\left[p(x) \rightarrow \sim q(x)\right] \equiv \exists x.\{\sim\left[p(x) \wedge \sim q(x)\right]\}$

$\equiv \exists x. \sim\left[p(x) \wedge \sim q(x)\right]$

(iii) $\forall xp\left[q(x) \vee q(x)\right] \equiv \exists x, \{\sim\left[p(x) \rightarrow q(x)\right]\}$

$\equiv \exists x, \sim\left[p(x) \wedge \sim q(x)\right]$

2. c. **Define converse. Inverse and contrapositive of a conditional with truth table. Write down the contrapositive of p → (q → r) with :** **(06 M)**

(i) only one occurrence of the connective →

(ii) no occurrence of the connective →

Ans: Consider the condition p → q. Then

1. q → p is called the converse of p → q
2. ~p → ~q is called the inverse of p → q
3. ~q → ~p is called the contrapositive of p → q.

Truth table for converse, inverse and contrpositive is as follows:

p	q	~p	~q	p → q	q → p	~p → ~q	~q → ~p
0	0	1	1	1	1	1	1
0	1	1	0	1	0	0	1
1	0	0	1	0	1	1	0
1	1	0	0	1	1	1	1

(i) p → (q → r) ⇔ ~(q → r) → ~p, contrapositive

⇔ ~(q ∧ ~ r) → ~p, ~(p → q) ≡ p ∧ ~q

(ii) p → (q → r) ⇔ ~(q → r) → ~p, contrapositive

⇔ (q ∧ ~ r) → ~p, ~(p → q) ≡ p ∧ ~q

⇔ ~(q ∧ ~ r) ∨ ~p, p → q ≡ ~p ∨ q

⇔ ~[(q ∧ ~ r) ∧ p], De-Morgan's Law.

MODULE - 2

3. a. If F0, F1, F2,............ are Fibonacci numbers. Prove that $\sum_{i=0}^{n} F_i^2 = F_n \times F_{n+1}$ for all positive integers n. **(06 M)**

Ans. We first note that,

$$\sum_{i=0}^{1} F_i^2 = F_0^2 + F_1^2 = 0 + 1 = 1 = 1 \times 1 = F_1 \times F_2$$

Because $F_1 = F_2 = 1$ This verifies the required result for n = 1.

Next we assume the result for n = k ≥1; that is, we assume that,

$$\sum_{i=0}^{k} F_i^2 = F_k \times F_{k+1}$$

Consequently $\sum_{i=0}^{k+1} F_i^2 = \sum_{i=0}^{k} F_i^2 + F_{k+1}^2$

$= (F_k \times F_{k+1}) + F_{k+1}^2$

$= F_{k+1} \times (F_k + F_{k+1})$

$\sum_{i=0}^{k} F_{k+1} \times (F_k + F_{k+1})$, *because* $F_{k+2} = F_{k+1} + F_k$. This shows that the required result is true for n = k + 1.

Hence, by mathematical induction, the required result is true for all integers n ≥ 1.

3. b. A sequence $\{a_n\}$ is defined recursively by $a_1 = 4$, $a_n = a_{n-1} + n$ for $n \geq 2$. Find a_n in explicit form. **(06 M)**

Ans: Using the given recursive formula repeatedly, we find that,

$a_n = a_{n-1} + n$

$= [a_{n-2} + (n - 1)] + n$

$a_n = a_{n-3} + (n - 2) + (n - 1) + n$

$a_n = a_{n-4} + (n - 3) + (n - 2) + (n - 1) + n$

- - - - - - - - - - - - - -

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$= a_1 + 2 + 3 + 4 + \ldots\ldots\ldots\ldots + n$

$a_n = a_1 - 1 + 1(1 + 2 + 3 + \ldots\ldots\ldots + n)$

Using $a_1 = 4$ and $1 + 2 + 3 + \ldots\ldots\ldots + n = \frac{1}{2} n(n + 1)$

This becomes

$$a_n = 3 + \frac{1}{2} n(n + 1)$$

This is the explicit formula for a_n.

3. c. A woman has, 11 close relative and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations:

(i) Two particular persons will not attend separately.

(ii) Two particular person will not attend together. (08 M)

Ans. (i) Since two particular person will not attend separately, they should both be invited or not inivited. If both of them are invited, then three more invites are to be selected from the remaining a relatives. This can be done in,

$$C(9,3) = \frac{9!}{6!\,3!} = 84 \textit{ ways}$$

If both of them are not invited, then five invitees are to be selected from 9 relatives. This can be done in,

$$C(9,5) = \frac{9!}{5!\,4!} = 126 \textit{ ways}$$

There fore, the total number of ways in which the invitees can be selected in this case is ,

$84 + 126 = 210$

(ii) Since two particular person (say P_1 and P_2) will not attend together, only one of them can be invited or none of them can be invited. The number of ways of choosing the invitees with P_1 invited is,

$$C(9,4) = \frac{9!}{5!\,4!} = 126 \textit{ ways}$$

Similarly the number of ways of choosing the invitees with P_2 invited is 126.

If both P_1 and P_2 are not invited, the number of ways of choosing the invitees is,

$C(9, 5) = 126$

Thus, the total number of ways in which the invitees can be selected in this case is

$126 + 126 + 126 = 378$.

OR

4. a. Determine the number of integer solution for $x_1 + x_2 + x_3 + x_4 + x_5 < 40$, Where (i) $x_i \geq 0$, $1 \leq i \leq 5$ (ii) $x_i \geq -3$, $1 \leq i \leq 5$. (08 M)

Ans: (i) The number of nonnegative integer solutions of the equation.

$$x_1 + x_2 + x_3 + x_4 + x_5 = 39$$

$$x_1 + x_2 + x_3 + x_4 + x_5 = 39 - x_6$$

Where $39 - x6 \leq 39$ so that x6 is nonnegative integer. Thus the required number is the number of non-negative solutions of the equation, $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 39$,

This number is

$C(6 + 39 - 1, 39) \quad = C(44, 39)$

$$= \frac{44!}{39!5!} = 1,086,008$$

(*ii*) We require, $x_1 \geq -3, x_2 \geq -3, x_3 \geq -3, x_4 \geq -3, x_5 \geq -3$

Let us set $y_1 = x_1 + 3, y_2 = x_2 + 3, y_3 = x_3 + 3, y_4 = x_4 + 3, y_5 = x_5 + 3.$

Given, $x_1 + x_2 + x_3 + x_4 + x_5 = 39$

$$y_1 - 3 + y_2 - 3 + y_3 - 3 + y_4 - 3 + y_5 - 3 = 39$$

$$y_1 + y_2 + y_3 + y_4 + y_5 = 39 + 15$$

$$y_1 + y_2 + y_3 + y_4 + y_5 = 54$$

The number of non-negative solutions of this equation is,

$C(5 + 54 - 1, 54) \quad = C(58, 54)$

$$= \frac{48!}{54!4!} = 4, 24270$$

4. b. How many arrangements of the letters MISSISSEPPI have no consecutive S's ? (06 M)

Ans: The number of letters in the given words is 11 of which 4 are I, S, 2 are P and 1M. The number of arrangements of the letters in the given word is

$$= \frac{11!}{4!4!2!1!} = 34, 650$$

If we disregard the S's, the remaining 7 letters can be arranged in

$$= \frac{7!}{4!2!1!} = 105 \text{ ways}$$

In each of these arrangements, there are 8 possible locations for the fours S's. These locations can be chosen in C(8, 4) ways.

(M ↑ I ↑ I ↑ I ↑ P ↑ P ↑ I)

∴ The number of arrangements having no consecutive S's is 105 × C(8, 4) = 7350

4. c. Prove that every positive integer n ≥ 24 can be written as a sum of 5's and / or 7's (06 M)

Ans: Here, we have to prove the statement.

S(n) : n can be written as a sum of 5's and / or 7's is true for all integers n ≥ 24.

Basic Step:

We note that,

24 = (7 + 7) + (5 + 5)

Induction Step: We assume that S(n) is true. for n = k where k ≥ 24. Then,

k = (7 + 7 + ...) + (5 + 5 + ...)

Suppose this representation of k has r number of 7's and s numbers of 5's. Since k ≥ 24, we should have r ≥ 2 and s ≥ 2.

Using this representation of k, we find that

$$k+1 = \left\{ \underbrace{(7+7+\ldots)}_{r} + \underbrace{(5+5+\ldots\ldots\ldots)}_{s} \right\} + 1$$

$$= \underbrace{(7+7+\ldots)}_{r} + (7+7)\underbrace{(5+5+\ldots\ldots\ldots)}_{s} + 1$$

$$k+1 = \underbrace{(7+7+\ldots)}_{(r-2)} + \underbrace{(5+5+\ldots\ldots\ldots)}_{(s+3)}$$

This show that (k + 1) is a sum of 7's and 5's

$\therefore$ s(k + 1) is true.

Hence, by mathematical induction, S(n) is true for all positive integers $n \geq 24$.

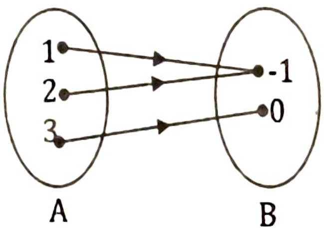
MODULE - 3

5. a.Define the following with one example for each

(i) Function (ii) One-to-One function (iii) Onto function (iv) Reflexive **(08 M)**

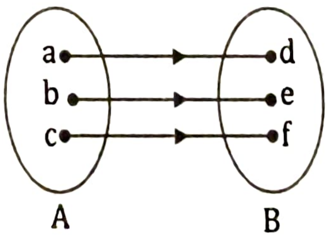
Ans: **(i) Function:** A Function $f : A \to B$ is a relation form $A \to B$ in which elements appear exactly once as the first component of an ordered pair in the relation.

Example:



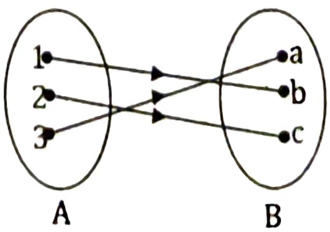
(ii) One-to-One Function: A Function $f : A \to B$ is called One-to-One or injective if each element of B appears atmost once as the image of f.

Example:



(iii) Onto Function:

A function $f : A \to B$ is called onto if $f(A) = B$ for all $b \in B$ there is atleast one $a \in A$ with $f(a) = b$.



(iv). Reflexive:

For every $a \in z$, $a - a = 0$ is a multiple of n.

i.e. $a \equiv a(\text{mod } n)$.

$\therefore$ R is reflexive.

5. b. Let $f : R \to R$, $g : R \to R$ be defined by $f(x) = x^2$ and $g(x) = x + 5$. Determine f o g and g o f. Show that the composition of two functions is not commutative. **(06 M)**

Ans: Let $f : R \to R$ defined by $f(x) = x^2$

$g : R \to R$ defined by $g(x) = x + 5$

$(f \circ g)(x) = f\{g(x)\} = f(x + 5) = (x + 5)^2$

$(g \circ f)(x) = g\{g(x)\} = g(x^2) = (x^2 + 5)^2$

$\therefore \quad (f \circ g)(x) \neq (g \circ f)(x)$

$\therefore$ The composition of two functions is not commutative.

5. c. Let A = {1, 2, 3, 4} and set R be the relation defined by $R = \{(x, y) \mid x, y \in A, x \leq y\}$. Determine whether R is reflexive, symmetric, transitive and Antisymmetric. **(06 M)**

Ans:

$$A \times A = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4), \\ (2,1),(2,2),(2,3),(2,4), \\ (3,1),(3,2),(3,3),(3,4), \\ (4,1),(4,2),(4,3),(4,4) \end{Bmatrix}$$

Given $R = \{(x, y) \in A \times A, x \leq y\}$

$$R = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4), \\ (2,2),(2,3),(2,4),(3,3), \\ (3,4),(4,4) \end{Bmatrix}$$

R is reflexive, because □x ∈ A, (x, x) ∈ R

R is not symmetric, because if (x, y) ∈ R then (y, x) ∈ R □ (x, y) ∈ A × A

R is transitive, since □ x, y , z ∈ A if

(x, y), (y, x) ∈ R ⇒ (x, z) ∈ R

R is Anti symmetric, since □ a, b ∈ A

If a R b and b R a ⇒ a = b

OR

6. a. Find the number of equivalence relations that can be defined on a finite set A with |A| = 6 **(08 M)**

Ans: Since | A | = 6, a partition of A can have atmost 6 cells. Treating the elements of A as objects and cells as containers, we find that the number of partitions having k cells is precisely S(6, k). Since k varies from 1 to 6 , the total number of different partitions of A is.

$$p(6) = S(6,1) + S(6,2) + S(6,3) + S(6,4) + S(6,5) + S(6,6) \quad \ldots\ldots(1)$$

$$S(6,1) = \frac{1}{1!}\sum_{k=0}^{1}(-1)^k \ {}^1C_{1-k}\,(1-k)^6 = 1$$

$$S(6,2) = \frac{1}{2!}\sum_{k=0}^{2}(-1)^k \ {}^2C_{2-k}\,(2-k)^6$$

$$= \frac{1}{2}\left(2^6 - 2\right) = 31$$

$$S(6,3) = \frac{1}{3!}\sum_{k=0}^{3}(-1)^k \ {}^3C_{3-k}\,(3-k)^6$$

$$= \frac{1}{6}\left\{3^6 - \left(3 \times 2^6\right) + \left(3 \times 1^6\right)\right\}$$

$$S(6,3) = 90$$

$$S(6,4) = \frac{1}{4!}\sum_{k=0}^{4}(-1)^k \ {}^4C_{4-k}\,(4-k)^6$$

$$= \frac{1}{24}\left\{4^6 - \left(4 \times 3^6\right) + \left(6 \times 2^6\right) - \left(4 \times 1^6\right)\right\}$$

$$S(6,4) = 65$$

$$S(6,5) = \frac{1}{5!}\sum_{k=0}^{5}(-1)^k \ {}^5C_{5-k}\,(5-k)^6$$

$$= \frac{1}{120}\left\{5^6 - \left(5 \times 4^6\right) + \left(10 \times 3^6\right) - \left(10 \times 2^6\right) + \left(5 \times 1^6\right)\right\}$$

$$S(6,5) = 15$$

$$S(6,6) = 1$$

(1) becomes,

$p(6) = 1 + 31 + 90 + 65 + 15 + 1 = 203$

Since each partition of A corresponds to an equivalence relation on A, it follows that if $|A| = 6$, then 203 equivalence relations can be defined on A.

6. b. Let A = { 1, 2, 3, 4} and Let R be the relation on A defined by x R y iff "x divides y", written x | y.

(a) Write down R as a set of ordered pairs.

(b) Draw the digraph of R.

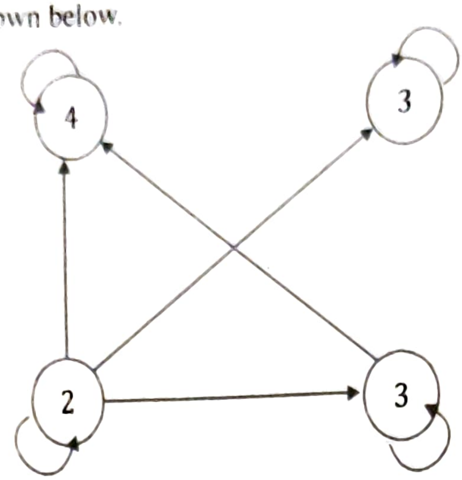
(c) Determine the in-degrees and out-degrees of the vertices in the digraph. **(06 M)**

Ans: **(a)**

$$R = \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), \\ (2,2), (2,4), (3,3), (4,4) \end{Bmatrix}$$

(b)

The digraph of R is as shown below.



(c)

Vertex	1	2	3	4
In-degree	1	2	2	3
Out-degree	4	2	1	1

6. c. Suppose A, B, C $\leq$ Z $\times$ Z with A = { (x, y) / y = 5x – 1} B = {(x, y) / y = 6x} c= {(x, y) / 3x – y = – 7}

Find (i) $A \cap B$ (ii) $B \cap C$ (iii) $\overline{\overline{A} \cup \overline{C}}$ (iv) $\overline{B} \cup \overline{C}$ **(06 M)**

Ans:
(i)

$(x, y) \in A \cap B \quad \Leftrightarrow (x, y) \in A \text{ and } (x, y) \in B$

$\Leftrightarrow y = 5x - 1 \text{ and } y = 6x$

$\Leftrightarrow 5x - 1 = y = 6x$

$\Leftrightarrow x = -1, y = -6$

$\therefore \quad A \cap B = \{-1, -6\}$

(ii)

$(x, y) \in B \cap C \quad \Leftrightarrow (x, y) \in B \text{ and } (x, y) \in C$

$\Leftrightarrow y = 6x \text{ and } 3x - y = -7$

$\Leftrightarrow 6x = 3x + 7$

$\Leftrightarrow 3x = 7, \text{ i.e., } x = \frac{7}{3} \notin Z$

$\therefore \quad B \cap C = \varphi$

$\Leftrightarrow (x, y) \in A$ and $(x, y) \in C$

$\Leftrightarrow y = 5x - 1$ and $3x - y = -7$

$\Leftrightarrow 5x - 1 = y = 3x + 7$

$\Leftrightarrow x = 4, y = 19$

$\overline{\bar{A} \cup \bar{C}} = A \cap C = \{(4, 19)\}$

(iv)

We note that $\bar{B} \cup \bar{C} = \overline{B \cap C}$. It has been seen in (ii) above that $B \cap C = \varphi$

$\bar{B} \cup \bar{C} = \overline{B \cap C} = \bar{\varphi} = Z \times Z$ (*Universal Set*)

MODULE - 4

7. a. There are n pairs of children gloves in a box. Each pair is of different color. Suppose the right gloves are distributed at random to 'n' children and there after the left gloves are also distributed to them at random. Find the probability that

(i) no child gets a matching pair.

(ii) every child gets a matching pair.

(iii) Exactly one child gets a matching pair.

(iv) atleast two children get matching pair. (08 M)

Ans: Any one distribution of n right gloves to n children determines a set of n places for the n pairs of gloves. Let us take these as the natural places for the pairs of gloves. The left gloves can be distributed to n children in n! ways.

(i) The event of no child getting a matching pair occurs if the distribution of the left gloves is a derangement. The number of derangement is d_n. $\therefore$ The required probability is,

$$P = \frac{d_n}{n!} = 1 - \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots\ldots + (-1)^n \frac{1}{n!}$$

(ii) The event of every child getting a matching pair occurs in only one distribution of the left gloves.

The required probability is, $P_2 = \frac{1}{n!}$

(iii) The event of exactly one child getting a matching pair occurs when only one left glove is in the natural place and all others are in wrong places. The number of such distribution is d

The required probability is,

$$P = \frac{d_{n-1}}{n} = \frac{1}{n}\left\{1 - \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots + (-1)^{n-1}\frac{1}{(n-1)!}\right\}$$

(iv) The event of atleast 2 children getting a matching pair occurs if the vent of no child or one child getting a matching pair does not occurs.

The probability is, $P_4 = 1 - (P_2 + P_3)$

7. b. Determine in how many ways can the letters in the word ARRANGEMENT be arranged so that there are exactly two pairs of consecutive letters. **(06 M)**

Ans: Out of the total 11 letters in ARRANGEMENT. There are two A's , 2 R's, 2 E's, 2 N's and 1G, 1T, 1M.

$$|S| = \frac{11!}{2!2!2!2!1!1!1!} = \frac{11!}{(2!)^4}$$

$$|S| = 2,494,800$$

Let A_1, A_2, A_3, A_4 be the set of permutations in which A's, E's, N's and R's appears in pairs respectively.

$$|A_i| = \frac{10!}{2!2!2!1!1!1!1!} = \frac{10!}{(2!)^3}$$

$$|A_i| = 453,600, \text{ for } i = 1, 2, 3, 4$$

$$|A_i \cap A_j| = \frac{9!}{2!2!(1!)^5} = \frac{9!}{(2!)^2} = 90720 \text{ for } i \neq j$$

$$|A_i \cap A_j \cap A_k| = \frac{8!}{2!} = 20,160$$

$$|A_i \cap A_j \cap A_k \cap A_p| = 7! = 5040$$

$$S_1 = 4C_1 \times |A_i| = 4C_1 \times 453600 = 1,814,400$$

$$S_1 = 4C_2 \times |A_i| = 4C_2 \times 90720 = 544,320$$

$$S_1 = 4C_2 \times |A_i \cap A_j| = 4C_3 \times 20,160 = 80,640$$

$$S_1 = 4C_2 \times |A_i \cap A_j \cap A_k \cap A_p| = 4C_4 \times 5040 = 5040$$

The number of elements in S that satisfy exactly m of the n conditions $(0 \le m \le n)$ is

$$E_m = S_m - \binom{m+1}{1} S_{m+1} + \binom{m+2}{1} S_{m+2} + \dots\dots\dots + (-1)^{n-m} \binom{n}{n-m} S_n$$

Put $m = 2$

$$E_2 = S_2 - 1\binom{3}{1} S_3 + \binom{4}{2} S_4$$

$$E_2 = 544,320 - 3(80,640) + 6(5040) = 332,640$$

7. c. A bank pays a certain % of annual interests on deposits, compounding the interests once in 3 months. If a deposit doubles in 6 years and 6 months. What is the annual % of interest paid by the bank? **(06 M)**

Ans: Let the annual rate of interest be x%. Then the quarterly rate of interest is.

$$\left(\frac{x}{4}\right)\% = \frac{x}{400}$$

Let P_0 denote the deposit made in Rs and P_n denote that the value of the deposit at the end of the nth quarter. Then.

$$P_{n+1} = P_n + \left(\frac{x}{400}\right) P_n$$

$$P_{n+1} = \left(1 + \frac{x}{400}\right) P_n, \quad n \geq 0 \qquad \ldots (1)$$

This is the recurrence relation for the problem.

The general solution of the homogeneous relation is,

$$P_n = \left(1 + \frac{x}{400}\right)^n P_0, \quad For\ n \geq 1 \qquad \ldots (2)$$

From what is given, we have $P_n = 2P_0$ When n = 26 using this in (2).

$$\left(1 + \frac{x}{400}\right)^{26} = 2$$

$$\log_e \left(1 + \frac{x}{400}\right) = \frac{\log_e^2}{26} = 0.02666$$

$$1 + \frac{x}{400} = e^{0.02666} = 1.027$$

$$x = 400 \times 0.027$$

$$x = 10.8$$

Thus the annual rate of interest paid by the bank is 10.8%.

OR

8. a. In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns spin, game, path or net occurs? (08 M)

Ans: The specified letters are 26 in number. Let S denote the set of all permutations of these letters without repetition.

$|S| = 26!$

Let A_1 be the subset of S which contains the pattern sping.

$|A_1|$ = Number of permutations of distinct objects consisting of the single pattern spin and the 22 letters not present in this pattern.

$|A_1| = (1 + 22)! = 23!$

Similarly. A_2, A_3, A_4 are the subsets of S which respectively contain the patterns game, path, net.

$|A_2| = 23!$ $\qquad |A_3| = 23!,$ $\qquad |A_4| = (1 + 23!) = 24!$

$|A_1 \cap A_2|$ = Number of permutation of distinct object consisting of the two patterns spin and game and the 18 letters not present in these two patterns.

$|A_1 \cap A_2| = (2 + 18)! = 20!$

$|A_1 \cap A_3| = 0$ Because, no permutation of distinct objects can contains both the patterns spin and path.

Likewise, $|A_1 \cap A_3| = 0$, $|A_2 \cap A_4| = 0$, $|A_3 \cap A_4| = 0$, $|A_1 \cap A_2 \cap A_3| = 0$, $|A_1 \cap A_2 \cap A_4| = 0$, $|A_1 \cap A_3 \cap A_4| = 0$, $|A_2 \cap A_3 \cap A_4| = 0$, $|A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5| = 0$

$|A_1 \cap A_4|$ = Number of permutations of distinct objects consisting of the patterns spind and net, and the letters not present in these patterns.

= Number of permutations of distinct objects consisting of the single pattern spinet, and the 20 letters not present in this pattern

$= (1 + 20)!$

$|A_1 \cap A_4| = 21!$

The required number of permutation is given by.

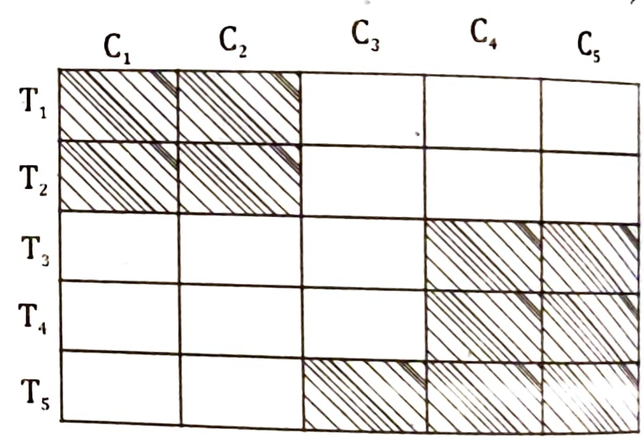
$$|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4}| = |S| - \Sigma|A_i| + \Sigma|A_i \cap A_j| - \Sigma|A_i \cap A_j \cap A_k| + |A_1 \cap A_2 \cap A_3 \cap A_4|$$

$$= 26! - (23! + 23! + 23! + 24! + 24!) + (20! + 0 + 0 + 0 + 0 + 21!) - (0 + 0 + 0 + 0) + 0$$

$$|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4}| = 26! - \{3 \times (23)! + 24!\} + (20! + 21!)$$

8. b. Five Teacher1 T_1, T_2, T_3, T_4, T_5 are to be made class teachers for five classes C_1, C_2, C_3, C_4, C_5 one teachers for each class. T_1 and T_2 do not wish to become the class teachers for C_1 or C_2, T_3 and T_4 for C_4 or C_5 and T_5 for C_3 or C_4 or C_5. In how many ways can the teacher be assigned the works. **(06 M)**

Ans: The situation can be represented by the board shown below in which the rows respectively represent the teachers. T_1, T_2, T_3, T_4, T_5 and the columns respectively represent the classes C_1, C_2, C_3, C_4, C_5 and the shaded squares together represent the forbidden places involved.



For the board C made up of the shaded squares in the above figure, the root polynomial is,

$$r(C, x) = r(C_1, x) \times r(C_2, x)$$

$$= (1 + 4x + x^2) \times (1 + 7x + 10x^2 + 2x^3)$$

$$= 1 + 11x + 40x^2 + 56x^3 + 28x^4 + 4x^5$$

Here $r_1 = 11$, $r_2 = 40$, $r_3 = 56$, $r_4 = 28$, $r_5 = 4$

$S_0 = 5! = 120$, $S_1 = (5 - 1)! \times r_1 = 264$

$S_2 = (5 - 2)! \times r_2 = 240$, $S_3 = (5 - 3)! \times r_3 = 112$

$S_4 = (5 - 4)! \times r_4 = 28$, $S_5 = (5 - 2)! \times r_5 = 4$

$\therefore$ The number of ways in which the work can be assigned is,

$$S_0 - S_1 + S_2 - S_3 + S_4 - S_5 = 120 - 264 + 240 - 112 + 28 - 4$$

$$= 8$$

8. c. Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$ for $n \geq 3$ given that $a_0 = -1$ and $a_1 = 8$ **(06 M)**

Ans. The coefficients of a_n, a_{n-1}, a_{n-2} are $C_n = 1$, C_{n-1} and $C_{n-2} = -6$ respectively.

The characteristic equation is,

$k^2 + k - 6 = 0$

$(k + 3)(k - 2) = 0$

The roots are $k_1 = -3$ and $k_2 = 2$ which are real and distinct.

The general solution is,

$a_n = A \times (-3)^n + B \times 2^n$ ....(1)

Where A and B are arbitrary constants.

From this solution, we get $a_0 = A + B$,

$a_1 = -3A + 2B$, using the given values of a_0 and a_1 these becomes.

$-1 = A + B$, $\quad 8 = -3A + 2B$

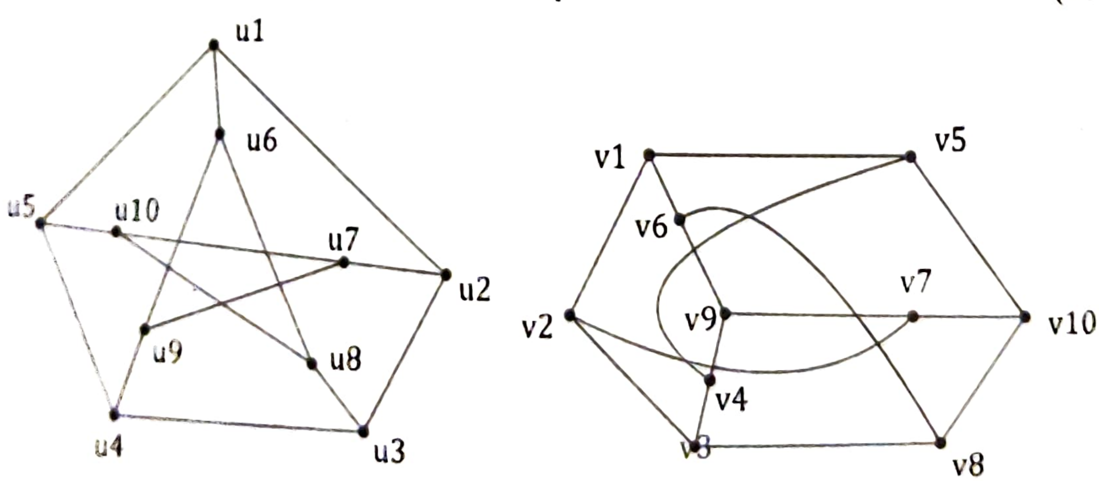
Solving these, we get $A = -2$ and $B = 1$

Putting these in (1), we get,

$a_n = -2 \times (-3)^n + 2^n$

MODULE - 5

9. a. Show that the following graphs are isomorphic. **(06 M)**



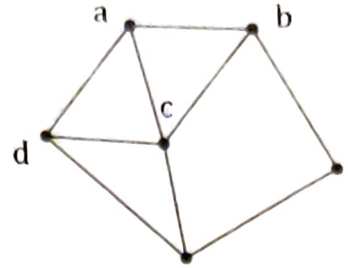
Ans: We first note that each of the two graphs is 3-regular and has 10 vertices. Consider the one-ton-one corresponding between the vertices as shown below:

$u_i \to v_i$ for $i = 1, 2, 3, \ldots\ldots, 10$

The above mentioned corresponding yields one-to-one corresponding between the edges in the two graphs with the property that adjacent vertices in the first graph correspond to the adjacent vertices in the second graph and vice-versa.

$\therefore$ The two graphs are isomorphic.

9. b. How many different paths of length 2 are there in the undirected graph G shown below: **(06 M)**



The number of paths of length 2 that pass through the vertex a is the number of pairs of edges incident on a. Since 3 edges are incident on a, this number is ${}^3C_2 = 3$.

Similarly, the number of paths of length 2 that pass through the vertices b, c, d, e, f are respectively.

${}^3C_2 = 3,\ {}^4C_2 = 6,\ {}^3C_2 = 3,\ {}^2C_2 = 1,\ {}^3C_2 = 3.$

$\therefore$ The total number of paths of length 2 in the given graph is $3 + 3 + 6 + 3 + 1 + 3 = 19$.

9. c. Prove that in every tree $T = (V, E)$, $|E| = |V| - 1$ **(08 M)**

Ans: (i) Let $T = (V, E)$ is a tree with n - vertices and e-edges.

We prove by mathematical induction on n.

Step I:

For $n = 1, 2, 3$

$n = 1$

$e = 0$

$e = n - 1$

$n = 2$

$e = 1$

$e = n - 1$

$n = 3$

$e = 2$

$e = n - 1$

$\therefore$ Result is true for $n = 1, 2, 3$ vertices.

Step II:

Assume that the result is true for $n \le k$

Step III:

Let $T = (V, E)$ be a true with $n = k + 1$ vertices.

Let $x = \{a, b\}$ be any edge in tree T. Clearly $T - x$ is a disconnected graph have exactly two components say T_1, and T_2.

Let T_1 has n_1 vertices and T_2 has n_2 vertices clearly $n_1 + n_2 = k + 1$

Since T contains no cycles, the graph T_1 and T_2 do not have any cycles.

$\therefore T_1$ and T_2 are tress, $n_1 \le k, n_2 \le k$

$\therefore$ By Step II, the result is true for both T1 and T2.

Number of edges in $T_1 = n_1 - 1$

Number of edges in $T_2 = n_2 - 1$

Number of edges in $T - x = n_1 - 1 + n_2 - 1$

$= n_1 + n_2 - 2$

$= k + 1 - 2$

$T - x = k - 2$

$\therefore$ Number of edges in $T = k - 1 + 1$

$= (k + 1) - 1$

$= n - 1$

$\therefore$ Result is true for $n = k + 1$

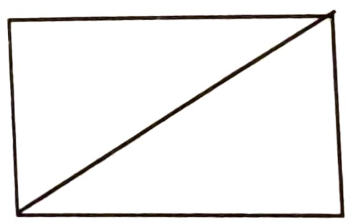
$\therefore$ By principle of mathematical inductions, the result is true for all trees.

i.e. $|E| = |V| - 1$

OR

10. a. Define : (i) Spanning tree. (ii) Binary rooted tree.

Find all the non isomorphic spanning trees of the graph shown below: (08 M)

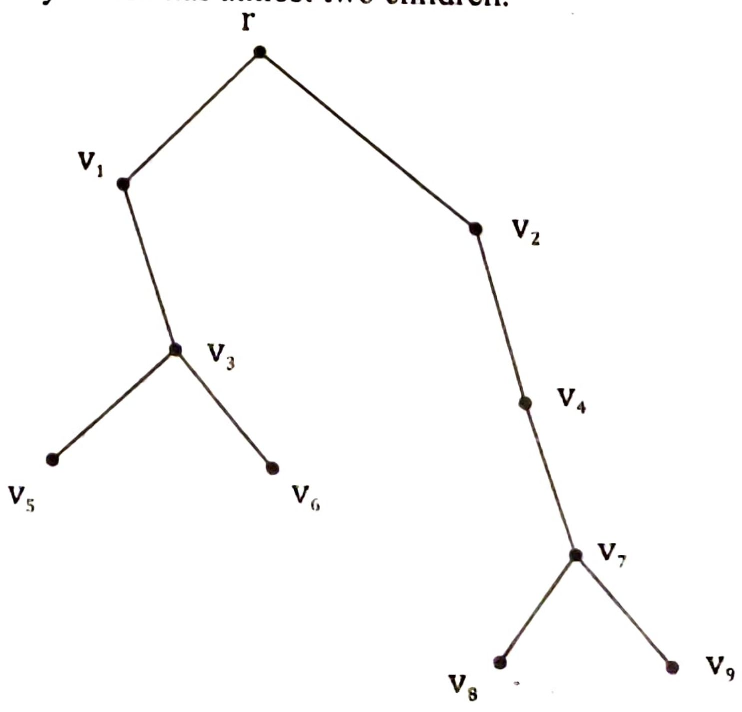


Ans: (i) Spanning Tree:

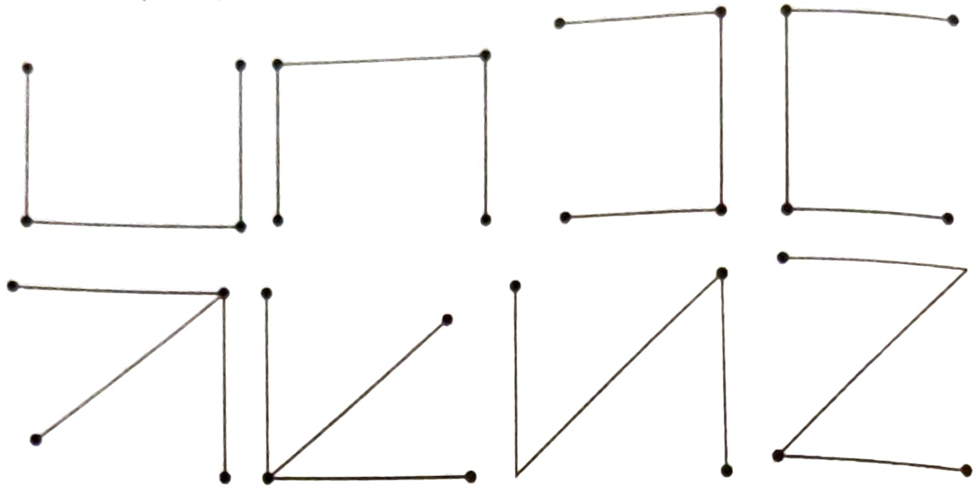
Let G be a connected graph. A subgraph T of G is called a spanning tree of G, if (i) T is a tree (ii) T contains all vertices of G.

(ii) Binary rooted tree: A rooted tree T is called a binary tree if every vertex of T is of out-degree ≤ 2 i.e, if every vertex has atmost two children.

Example:

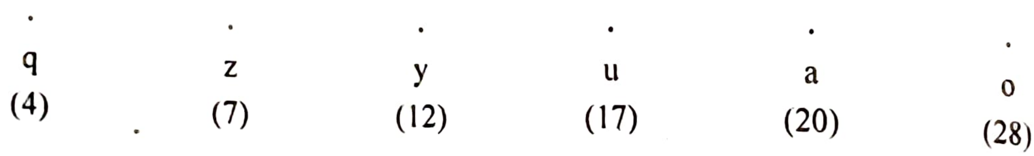


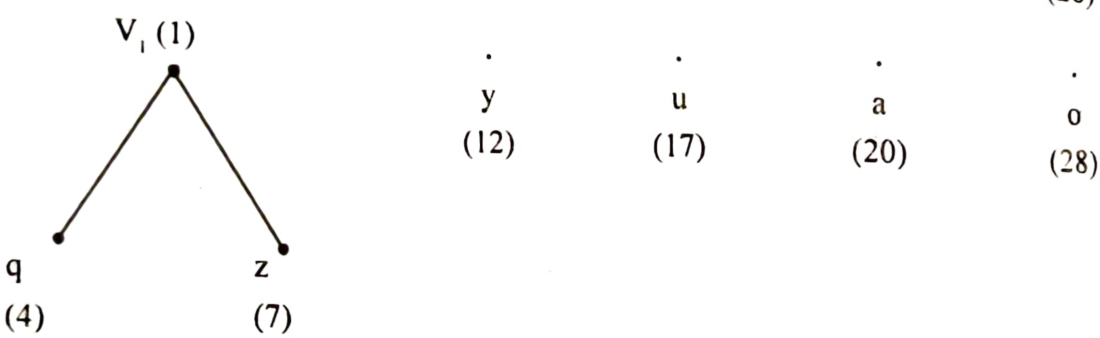
The non isomorphic spanning trees for the given graph:

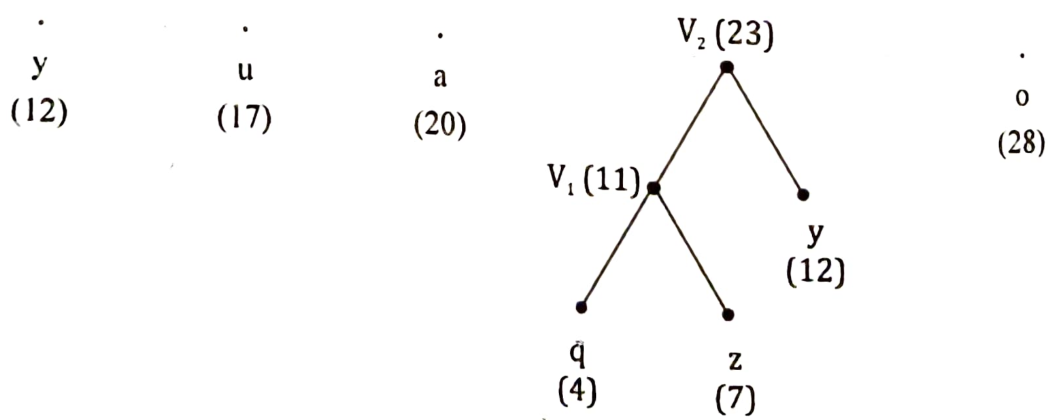


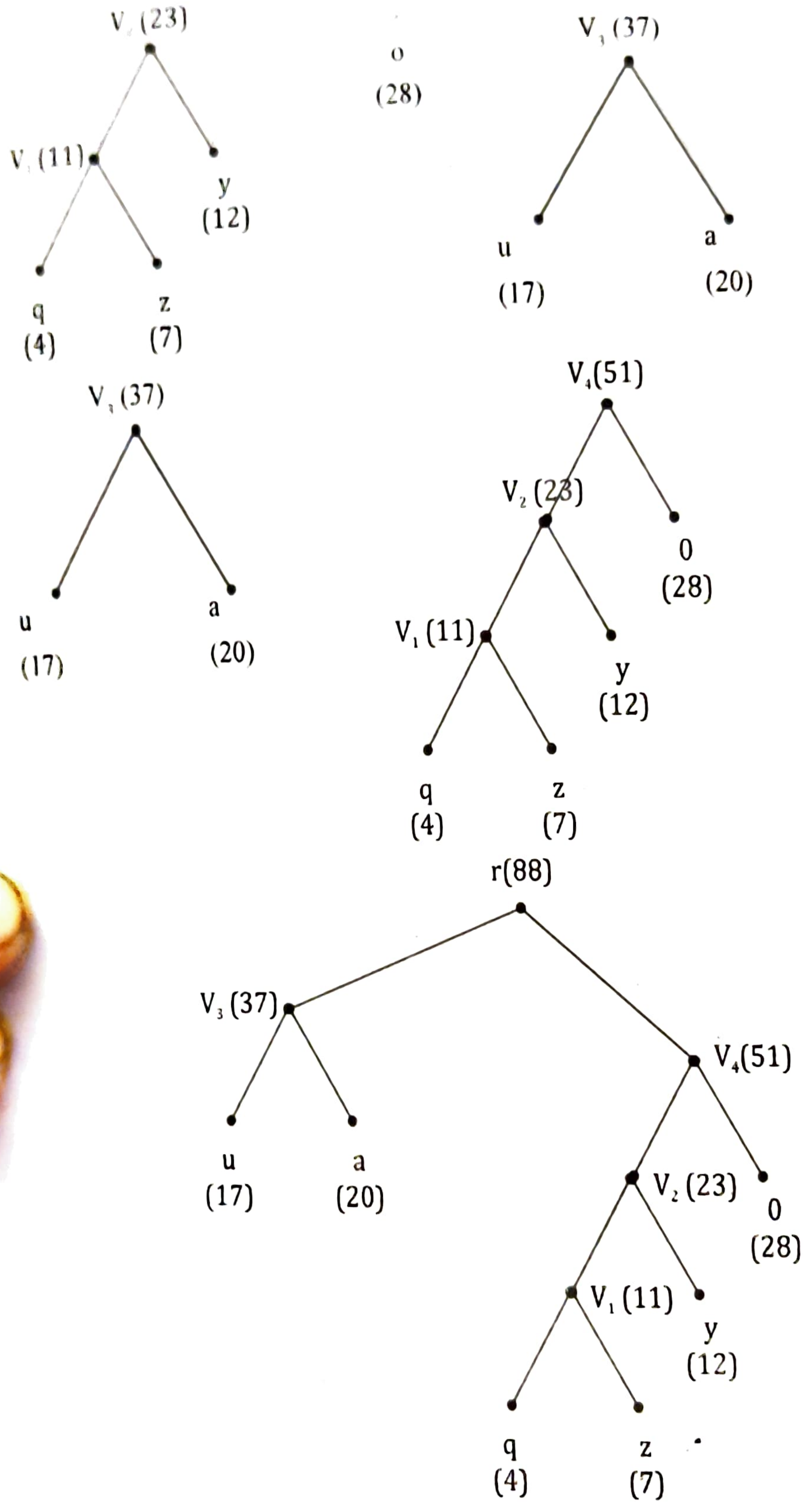
10. b. Construct an optimal prefix code for the symbols a, 0, q, u, y, z that occur with the frequencies 20, 28, 4, 17, 12, 7 respectively. **(06 M)**

Ans:








V (23)
V (11)
y
(12)
q
(4)
z
(7)
o
(28)
V (37)
u
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(20)
V (37)
u
(17)
a
(20)
V4(51)
V2 (23)
0
(28)
V1 (11)
y
(12)
q
(4)
z
(7)
r(88)
V3 (37)
V4(51)
u
(17)
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(20)
V2 (23)
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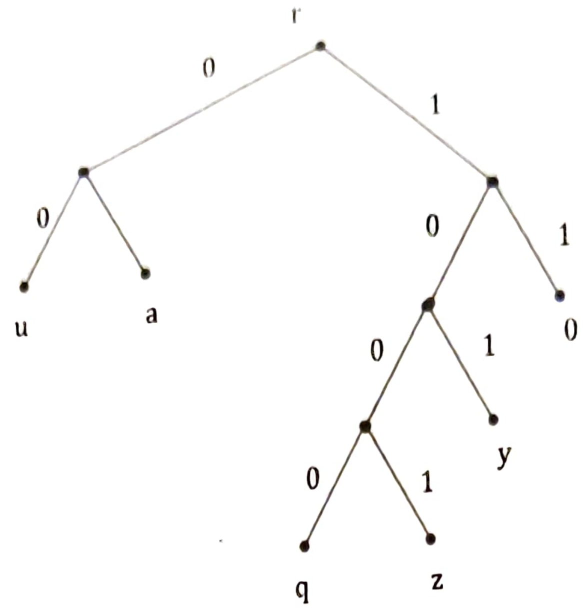
(28)
V1 (11)
y
(12)
q
(4)
z
(7)



a:01, b : 11, q:1000, u: 00, y : 101: z : 1001

10. c. Prove that the undirected graph G = (V, E) has an Eules circuit if and only if G is connected and every vertex in G has even degree. **(06 M)**

Ans: First, suppose that G has an Eules circuit, while tracing this circuit we observe that every time the circuit meets a vertex V it goes through two edges incident on V (with the one through which we enter V and the other through which we depart from V). This is true for all vertices that belong to the circuit. Since the circuit contains all edges. it meets all the vertices atleast once. Therefore, the degree of every vertex is a multiple of two . (i.e. every vertex is of even degree).

Conversely, suppose that all the vertices of G are of even degree. Now we construct a circuit starting at an arbitrary vertex V and going through the edges of G such that no edge is traced more than once. Since every vertex is of even degree, we can depart from every vertex other than V. In this way, we obtain a circuit q having V as the initial and final vertex.

If this circuit contains all the edges in G, then the circuit is an Euler circuit. If not let us consider the subgraph H obtained by removing form G all edges that belong to q. the degrees of vertices in this subgraph are also even. Since G is connected, the circuit q and the subgraph. H must have atleast one vertex, say V^1 in common. Staring from V^1 . we can construct a circuit q^1 in H as was ond in G. The two circuits q and q^1 together constitute a circuit which starts and edges at the vertex V and has more edges than q. If this circuit contains all the edges in G, then the circuit is an Euler circuit. Otherwise, we repeat the process until we get a circuit that starts form V and ends at V and which contains all edges in G. In this way, we obtain an Euler circuit in G.

Third Semester B.E. Degree Examination, CBCS - Dec 2016/ Jan 2017

Discrete Mathematical Structures

Time: 3 hrs Max. Marks: 80

Note : *Answer any FIVE full questions, selecting ONE full question from each module.*

Module - 1

1. a. **Let p, q and r be propositions having truth values 0, 0 and 1 respectively. Find the truth values of the following compound proposition**
i)$(p \wedge q) \rightarrow r$ ii) $p \rightarrow (q \wedge r)$ iii) $p \wedge (r \rightarrow q)$ iv) $p \rightarrow (q \rightarrow (-r))$ (04 Marks)

Ans. i. $(p \wedge q) \rightarrow r$
Since $p \wedge q$ is false and r is true $(p \wedge q) \rightarrow r$ in true. Thus the truth value of $(p \wedge q) \rightarrow r$ in 1

ii) $p \rightarrow (q \wedge r)$
Since q is false and r is true $q \wedge r$ is false. Also p is false. Therefore $p \rightarrow (q \wedge r)$ is true. Thus, the truth value of $p \rightarrow (q \wedge r)$ is 1.

iii. $p \wedge (r \rightarrow q)$
Since r is true and q is false, $r \rightarrow q$ is false. Also, p is false. Hence $p \wedge (r \rightarrow q)$ is 0.

iv) $p \rightarrow (q \rightarrow (\sim r))$
Since r is true , ~r is false, since q is false $q \rightarrow (\sim r))$ is true, also , p is false, therefore $p \rightarrow (q \rightarrow (\sim r))$ is true.
Thus the truth value of $p \rightarrow (q \rightarrow (\sim r))$ is 1.

b. **Define tautology. Prove that for any propositions p, q, r the compound proposition**
$[(p \vee q) \wedge \{(p \rightarrow r) \wedge (q \rightarrow r)\}] \rightarrow r$ is tautology. (04 Marks)

Tautology : A compound proposition which is true for all possible truth values of its components is called tautolog's.
$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a tautolog. The following truth table proves the required result.

p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$	$(p \vee q) \wedge \{(p \rightarrow r) \wedge (q \rightarrow r)\}$	$(p \vee q) \wedge \{(p \rightarrow r \wedge (q - r)\} \rightarrow r$
0	0	0	0	1	1	1	0	1
0	0	1	0	1	1	1	0	1
0	1	0	1	1	0	0	0	1
0	1	1	1	1	1	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

c. Establish the validity of the following argument

$\forall x.[p(x) \vee q(x)]$

$\exists x. \neg p(x)$

$\forall x.[\neg q(x) \vee r(x)]$

$\forall x.[s(x) \rightarrow \neg r(x)]$

$\therefore \exists x \neg s(x)$

(04 Marks)

Ans.

1.$\forall x.p(x) v q(x)$ (premise)

2.$\sim p(x) \rightarrow q(x)$ $(p \rightarrow q \Leftrightarrow \sim p v q)$

3.$\sim p(a) \rightarrow q(a)$ [(2) universal specification]

4.$\forall x \sim q(x) v r(x)$ (Premise)

5.$\forall x q(x) \rightarrow r(x)$ $[p \rightarrow q \Leftrightarrow \sim p v q]$

6.$q(a) \rightarrow r(a)$ [(5) universal specification]

7.$\sim p(a) \rightarrow r(a)$ [(3) & (6) syllogirsm]

8.$\exists x, \sim p(x)$ (premise)

9.$\sim p(a)$ [7 universal specification]

10.$r(a)$ [(6) & (8) modus pones]

11.$\forall x s(x) \rightarrow \sim r(x)$ [premise]

12.$\forall(x), r(x) \rightarrow \sim (S(x))$

13.$r(a) \rightarrow \sim S(a)$

14.$\sim S(a)$

15.$\exists x, \sim S(x)$

d. Give i) direct proof and ii) proof by contradiction for the following statement. "If 'n' is an odd integer, then n+9 is an even integer". (04 Marks)

Ans. i. Direct proof : Assume that n is an odd integer $n = 2k + 1$ for some integer k

$(n + 9 = 2k + 1 + 9)$

$= 2k + 10$

$= 2(k + 5)$

=> n + 9 is even

ii. Proof by contraction :

Assume that given statement false

i.e. $P \rightarrow q$ is true, q is false n is odd and n + 9 is not even

=> n + 9 is odd

$n + 9 = 2k + 1$

$n = 2k - 8$

$= 2(k - 4)$

=> n is even

This construction the assumption that n is odd. Hence the given statement must true.

OR

2. a. Define dual of a logical statement. Verify the principle of duality for the following logical equivalence $[\sim (p \wedge q) \to \sim p \vee (\sim p \vee q)] \Leftrightarrow (\sim p \vee q)$.(04 Marks)

Ans. Dual of a logical statement :

Let S be a statement cantaining no. other logical connectives other than ^, v , J. The dual of S denotes S^d is a statement.

$\sim(p \wedge q) \to \sim p \vee (\sim p \vee q) \Rightarrow (\sim p \vee q)$

The given logical equivalence is $u \Leftrightarrow v$ where

$u = \sim(p \wedge q) \to (\sim p \vee(\sim p \vee q))$ and $v = \sim p \vee q$

We note that

$u \Leftrightarrow \sim\sim(p \wedge q) \vee (\sim p \vee(\sim p \vee q))$

$\Leftrightarrow (p \wedge q) \vee (\sim p \vee(\sim p \vee q))$

$\therefore u^d \Leftrightarrow (p \vee q) \wedge (\sim p \wedge (\sim p \wedge q))$

$\Leftrightarrow (p \vee q) \wedge (\sim p \wedge q)$

$\Leftrightarrow [p \wedge (\sim p \wedge q)] \vee [q \wedge (\sim p \wedge q)]$

$\Leftrightarrow (F_o \wedge q) \vee (q \wedge \sim p)$

$\Leftrightarrow F_o \vee (q \wedge \sim p)$

$\Leftrightarrow q \wedge \sim P$

Also $v^d \Leftrightarrow \sim p \wedge q \Leftrightarrow q \wedge \sim p$

$u^d \Leftrightarrow v^d$

This verifies the principle of duality for the given logical equivalence.

b. Prove the following by using laws of logic

i) $P \to (q \to r) \Leftrightarrow (P \wedge q) \to r$

ii) $[\sim p \wedge (\sim q \vee r)] \vee [(q \wedge r) \vee (p \wedge q)] \Leftrightarrow r$. (04 Marks)

Ans. i. $P \to (q \to r) \Leftrightarrow (P \wedge q) \to r$

$p \to (q \to r) \Leftrightarrow [\sim p \vee (q \to r)]$

$\Leftrightarrow \sim p \vee (\sim q \vee r)$

$\Leftrightarrow (\sim p \vee \sim q) \vee r$

$\Leftrightarrow \sim (p \wedge q) \vee r$

$\Leftrightarrow (p \wedge q) \to r$

ii) $[\sim p \wedge (\sim q \vee r)] \vee [(q \wedge r) \vee (p \wedge q)] \Leftrightarrow r$

$[\sim p \wedge (\sim q \wedge r)]$

$\Leftrightarrow (\sim p \wedge \sim q) \wedge r$ [Associative]

$\Leftrightarrow [\sim p \wedge \sim q] \wedge r$ [Demorgan's]

$\Leftrightarrow r \wedge [\sim (p \vee q)]$ [Commutative]

$(q \wedge r) \vee (p \wedge r)$

$\Leftrightarrow (r \wedge q) \vee (r \wedge p)$ [Commutative]

$\Leftrightarrow r \wedge (q \vee p)$ [Distributive]

$\Leftrightarrow r \wedge (p \vee q)$

$\Leftrightarrow r \wedge (p \vee q)$

$\therefore [\sim p \wedge (\sim q \wedge r)] \vee (q \wedge r) \vee (p \wedge r)$

$\Leftrightarrow \{r \wedge [\sim (p \vee q)]\} \vee \{r \wedge (p \wedge q)\}$

$\Leftrightarrow r \wedge \{\sim (p \vee q)\} \vee (p \wedge q)$

$\Leftrightarrow r \wedge$ To because $[\sim (p \vee q)] \vee (p \vee q)$ is always true.

$\Leftrightarrow r$

c. Establish the validity of the following argument using the rules of inference: **(04 Marks)**

$[p \wedge (p \to q) \wedge (s \vee t) \wedge (r \to \sim q)] \to (s \vee t)$

Ans. The given argument reads

p

$p \to q$

$S \vee r$

$\underline{r \to \sim q}$

$\therefore S \vee t$

We note that

$(S \vee r) \wedge (r \to \sim q) \Leftrightarrow (\sim S \to r) \wedge (r \to \sim q)$

$\Leftrightarrow (\sim S \to \sim q)$ using rule of syllosism

$\Leftrightarrow q \to S$ using contrapositive

$\therefore (p \to q) \wedge (S \vee r) \wedge (r \to \sim q) \Leftrightarrow (p \to q) \wedge (q \to S)$

$\Leftrightarrow (p \to S)$ using rule of syllogism

Consequently

$p \wedge (p \to q) \wedge (S \vee r) \wedge (r \to \sim q) \Leftrightarrow p \wedge (p \to S)$

$\Leftrightarrow S$ by the rule of detachement

$\Leftrightarrow S \vee t$ by the rule of dijunative amplification

This show that the given argument is valid.

d. Define i) open sentence ii) quantifiers. For the following statements, the universe comprises all non-zero integers. Determine the truth values of each statement:

i) $\exists x, \exists y(xy=1)$ ii) $\exists x, \forall y(xy=1)$ iii) $\forall x, \exists y(xy=1)$. **(04 Marks)**

Ans. i) **Open statement :**

A sentence is said to be an open statement if

i. It contains one or more variable

ii. It is not a statements (preposition)

iii. It becomes a statement when the variable in it are replaced by certain allowable values.

ii) Quantifiers :

"For some x" " for all x" represents the quantity.

The statement containing these words are called quantifiers.

i. $\exists x, \exists y[xy=1]$: True(Take $x=1, y=1$)

ii. $\exists x, \forall y[xy=1]$:(For a specified x, $xy=1$ for every y is not true)

iii. False(For $x=2$ there is no integer y such that $xy=1$)

Module-2

3. a. By mathematical induction, prove that **(05 Marks)**

$$1^2+3^2+5^2.......+(2n-1)^2=\frac{n(2n+1)(2n-1)}{3}$$

Ans. Let S(n) denote the given statement

Basic step we note that S(1) is statement

$$1^2=\frac{1}{3}\times 1\times 3$$

Which clearly true

Inuction step: We assume that S(n) is true for $n=k$ where $k\geq 1$ then

$$1^2+3^2+5^2+....(2k-1)^2=\frac{1}{3}k(2k-1)(2k+1)$$

Adding $(2k+1)^2$ to both sides of this we obtain

$$1^2+3^2+5^2+.....+(2k-1)^2+(2k+1)^2$$

$$=\frac{1}{3}k(2k-1)(2k+1)+(2k+1)^2$$

$$=\frac{1}{3}(2k+1)\{k(2k-1)+3(2k+1)\}$$

$$=\frac{1}{3}(2k+1)(2k^2+5k+3)$$

$$=\frac{1}{3}(2k+1)(k+1)(2k+3)$$

This is precisely the statement S(k+1). Thus the statement S(k+1) is true whenever the statement S(k) where $k\geq 1$ is true.

Hence, by mathematical induction. It follows that S(n) is true for all integers $n \geq 1$.

b. For the Fibonacci sequence show that

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

(05 Marks)

Ans. For the Fibnacci sequence $F_0, F_1, F_2, \ldots$

$$\text{P.T } F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

For $n = 0$ and $n = 1$ The required result reads

$$F_0 = \frac{1}{\sqrt{5}}(1-1) = 0 \qquad F_1 = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)\right] = 1$$

which are true

Thus the required result is true for $n = 0$ and $n = 1$

We assume that the result is true for $n = 0, 1, 2, \ldots k$ where $k \geq 1$ then we find that

$$F_{k+1} = F_k + F_{k-1}$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^k - \left(\frac{1-\sqrt{5}}{2}\right)^k\right] + \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1} - \left(\frac{1-\sqrt{5}}{2}\right)^{k+1}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\left\{\frac{1+\sqrt{5}}{2}+1\right\} - \left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\left(\frac{1-\sqrt{5}}{2}+1\right)\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\left(\frac{6+2\sqrt{5}}{4}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\left(\frac{6-2\sqrt{5}}{4}\right)\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\left(\frac{1+\sqrt{5}}{2}\right)^2 - \left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\left(\frac{1-\sqrt{5}}{2}\right)^2\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{k+1}\right]$$

This show that the required result is true for $n = k+1$

Hence by mathematical induction, the result is true for all non-negative integer n.i.S

c. A women has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations : i) There is no restriction on the choice ii) Two particular persons will not attend separately iii) Two particular persons will not attend together. **(06 Marks)**

Ans. i. Since there is no restriction on the choice of invitees five out y can be invited in

$C(11,5) = \frac{11!}{6!5!} = 462$ says

ii. Since two particular persons will not attend separately, they should both be invited or not invited. If both y then are invited, then three more invitees are to be selected from the remaining 9 relatives thin can be done in

$C(9,3) = \frac{9!}{6!3!} = 84$ ways

If both of them are not invited, then five invites are to selected from 9 relatives thin can be done in

$C(9,5) = \frac{9!}{5!4!} = 126$ ways

∴ The total number y ways in which the invitees can be selected in thin case in 84 + 126 = 210.

iii. Since two particular person (say p_1, p_2) will not attend together only one of them can be invited or none of them can be invited. The number of ways y choosing the invites with p_1 invited is

$C(9,4) = \frac{9!}{5!4!} = 126$

Similarly the number of ways of choosing the invites with p_2 invited is 126. If both p_1 and p_2 are not invited.

The number of ways of choosing the invited is

$C(9,5) = 126$

Thus the total number of ways in which the inviter can be selected in this case is

$126 + 126 + 126 = 378$

OR

4. a. Prove that every positive integer n > 24 can be written as a sum of 5's and /or 7's. **(04 Marks)**

Ans. We have P.T the statement

S(n) : n can written an a sum of 5's and/ or 7's is true for all integers $n \geq 24$

We note that

$24 = (7 + 7) + (5 + 5)$

Induction step : We assume that S(n) is true for n = l where $k \geq 24$ then

$k = (7 + 7 + ...) + (5 + 5 + ...)$

Suppose this representation of k has r number of 7's an s number of 5's

Since $k \geq 24$ we should have $r \geq 2$ and $S \geq 2$

Using this representation of k we find that

$$\begin{aligned} k + 1 &= \{(7 + 7 + ...) + (5 + 5 + ...)\} + 1 \\ &= (7 + 7 + ...) + (7 + 7) + (5 + 5 + ...) + 1 \\ &= (7 + 7 + ...) + (5 + 5 + ...) \end{aligned}$$

This show that (k + 1) is a sum of 7's and 5's thus S(k + 1) is true.

b. Find an explicit definition of the sequence defined recursively by $a_1 = 7, a_n = 2a_{n-1}+1$ for $n > 2$. **(04 Marks)**

Ans. By defination of reccursive

We find $a_n = 2a_{n-1} + 1 = 2(2a_{n-2} + 1) + 1$

$$= 2[2(2a_{n-3} + 1) + 1] + 1 = 2^3 a_{n-3} + 2^2 + 2 + 1$$

....

....

$$= 2^{n-1} a_{n-(n-1)} + 2^{n-2} + 2^{n-3} + ... + 2^2 + 2 + 1$$

$$= 2^{n-1} a_1 + \left(1 + 2 + 2^2 + ... + 2^{n-3} + 2^{n-2}\right)$$

Using $a_1 = 7$ and teh standard result

$$1 + a + a^2 + ... + a^{n-1} = \frac{a^{n-1}}{a-1} \text{ for } a > 1$$

$$a_n = 7 \times 2^{n-1} + \left(2^{n-1} - 1\right) = \left(8 \times 2^{n-1}\right) - 1$$

This expression serves as an explicit defination for the given sequence

c. i) How many arrangements are there for all letters in the word SOCIOLOGICAL? ii) In how many of these arrangements A and G are adjacent? In how many of these arrangements all the vowels are adjacent? **(04 Marks)**

Ans. i. The given word has 12 letter of which 3 or 0, 2 each are C,I,L and I each are S, A,G

$\therefore$ The no. of arrangements of these letter is

$$\frac{12!}{3!2!2!2!1!1!} = 99,79,200$$

ii. If in an arrangement A and G are to be adjacent we treat A and G together as a single letter say x so that we have 3 no. of 0's 2 each of C,I,L and one each of S and X to talking to 11 letter these can be arranged in

d. Find the coefficient of i) xy in the expansion of (2x - 3y) ii) a b c d in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. **(04 Marks)**

Ans. The general term in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$

$$\binom{16}{n_1, n_2, n_4, n_5} (a)^{n_1} (2b)^{n_2} (-3c)^{n_3} (2d)^{n_4} (5)^{n_5}$$

for $n_1 = 2, n_2 = 3; n_3 = 2, n_4 = 5$ and $n_5 = 16 - (2 + 3 + 2 + 5) = 4$

$$\binom{16}{2,3,2,5} a^2 (2b)^3 (-3c)^2 (2d)^5 (5)^4 = \binom{16}{2,3,2,5,4} \times 2^2 \times (-3)^2 \times 2^5 \times 5^4 \times a^2 b^3 c^2 d^5$$

$\therefore$ The coefficient of $a^2 b^3 c^2 d^5$ is $2^8 \times 3^2 \times 5^4 \times \frac{16!}{2!3!2!5!4!} = 3 \times 2^5 \times 5^3 \times \frac{16!}{(a!)^2}$

Module-3

5\. a. **Let a function f : R—>R be defined by $f(x) = x^2 + 1$. Find the images of $A_1=\{2, 3\}$, $A_2 = \{-2, 0, 3\}$, $A_3 = (0, 1)$ and $A_4 = [-6, 3]$.** **(04 Marks)**

Ans. Let

Images of subset A_1

$f(2) = 5$. $f(3) = 10$. $f(A) = \{5,10\}$

Images of subset A_2.

$f(-2) = 5$. $f(0) = 1$. $f(3) = 10$, $f(A) = \{5,1,10\}$

Images y subset A_3,

$A_3 = \{x \in R/0<x<1\}$

$f(A_3) = \{f(x) \mid 0 < x < A\} = \{x^2 + 1 \mid 0 < x < 1\}$

Images of subset A_4

$f(A_4) = \{f(x) \mid -6 \le x \le 3\}$

$= \{x^2 + 1 \mid -6 \le x \le 3\}$

b. **ABC is an equilateral triangle whose sides are of length one cm each. If we select 5 points inside the triangle, prove that at least two of these points are such that the distance between them is less than *1/2* cm.** **(04 Marks)**

Ans. Consider the triangle DEF formed by the mid point of the sides BC, CA & AB of the given triangle. Then the triangle ABC. Then the triangle ABC is partitioned into four small equilateral triangle. Each of which has sides equal to 1/2 cm. Treating each of these four partions as a pigens. We find points chosen inside the triangle as pigeons, we find by using the pigeon principle that the atleast on portion must contain two or more point evidently. the distance between such point is less than 1/2 cm.

c. **Let f, g, h be functions from z to z defined by $f(x) = x - 1$, $g(x) = 3x$**

$$\text{and } h(x) = \begin{cases} 0 & \text{if x is even} \\ 1 & \text{if x is added} \end{cases}$$

Determine (fo(g o h))(x) and ((f o g)oh) (x) and verify that fo (g o h) = (f o g)o h. **(04 Marks)**

Ans. We have $(goh)(x) = g\{h(x)\} = 3\,h(x)$

$\therefore fo\,(g \circ h)(x) = f\{(g \circ h)(x)\}$

$= f\{3h(x)\} = 3h(x) - 1$

$$= \begin{cases} -1 & \text{if x is even} \\ 2 & \text{if x is odd} \end{cases} \quad ...(1)$$

On the other hand

$$(fog)(x) = f\{g(x)\} = g(x) - 1 = 3x - 1$$

$$\therefore \{(fog)oh\}(x) = (fog)h(x)$$

$$= 3h(x) - 1$$

$$\text{Eqn}(1)\&(2)\text{ it followes that } = \begin{cases} -1 & \text{if x is even} \\ 2 & \text{if x is odd} \end{cases}$$

$$f_\circ(goh) = (fog)oh$$

d. For A = {a, b, c, d, e} the Hasse diagram for the Poset (A, R) is as shown in Fig Q5(d). Determine the relation matrix for R and Construct the digraph for R (04 Marks)

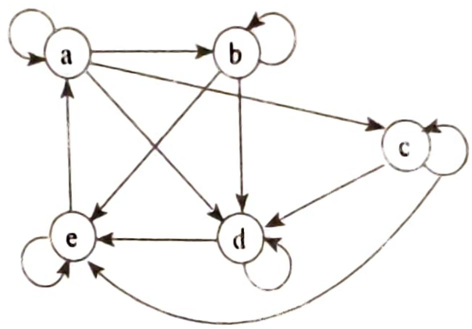
Ans. By examining the given have diagram, we note that

R = {(a,a), (a,b), (a,c), (a,d), (a,e), (b,b), (b,d), (b,c),(c,c),(c,d),(c,e),(d,d),(d,e),(e,e)}

a. The matrix of R is as shown below

$$M(R) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

b. The diagram of R and shown below



OR

6. a. Let A = {1,2, 3} and B = {2, 4, 5}. Determine the i) Number of binary relations on A.

ii)Number of relations from A to B that contain (1, 2) and (1, 5) iii) Number of relations from A, B that contain exactly five ordered pairs iv) Number of binary relations on A that contains at least seven ordered pairs. (04 Marks)

Ans. i. No. of binary relation of A is $2^{mm} = 2^{3^2} = 2^9 = 512$

ii. $2^7 = 128$ no. of relation from A $\rightarrow$ B that contain the element (1,2) & (1,5)

iii. A x B contains 9 ordered pairs the no. of relation from A to B that contain exactly 5 ordered pairs. The no. of choosing 5. Ordered pairs from ordered pairs is $9C_5$ = 126.

iv. Similarly the no. of binary relation on A that contain atleast 7 elements.

$9C_7 + 9C_8 + 9C_9 = 46$

b. Let A = B = R be the set of the real numbers, the functions f : A $\rightarrow$ B and g: B $\rightarrow$ A be defined by $f(x) = 2x^3 - 1, \forall x \in A; g(y) = \left\{\frac{1}{2}(y+1)\right\}^{1/3} \forall y \in B$. **Show that each off and g is the inverse of the other.** (04 Marks)

Ans. We find that, for any $x \in A$

$$(gof) = g(f(x)) = g(y) = \left\{\tfrac{1}{2}(y+1)\right\}^{1/3}, \text{where } y = f(x)$$

$$= \left\{\frac{1}{2}(2x^3 - 1 + n)\right\}^{1/3}, \text{because } y = f(x) = 2x^3 - 1$$

$$= x$$

Thus $gof = I_0$

Next, for any $y \in B$

$$(fog)(y) = f(g(y)) = f\left[\left\{\tfrac{1}{2}(y+1)\right\}^{1/3}\right]$$

$$= 2\left[\left\{\tfrac{1}{2}(y+1)^{1/3}\right\}\right]^3 - 1$$

$$= 2\left[\tfrac{1}{2}(y+1)\right] - 1 = y$$

Thus $foy = I_B$

Accordingly each of f and g is an invertible function, and farther more each is the inverse of the other.

c. Define a relation R on A x A by (x_1, y_1) R (x_2, v_2) iff $x_1+y_1 = x_2+y_2$, where A ={1, 2, 3, 4, 5}.
i) Verify that R is an equivalence relation on A x A.
ii) Determine the equivalence classes [(1,3)] and [(2, 4)]. **(04 Marks)**

Ans. i. For all $(x,y) \in A \times A$, , we have $x + y = x + y$; that is (x,y) R (x,y).
Therefore, R is reflexive, Next, take any $(x_1, y_1), (x_2, y_2) \in A \times A$ & Suppose that $(x_1, y_1) R (x_2, y_2)$ Then $x_1 + y_1 = x_2 + y_2$. This gives $x_2 + y_2 = x_1 + y_1$, which means fact $(x_2, y_2) R (x_1, y_1)$. Therefore R is $x_2 + y_2 = x_1 + y_1$ which mean fact $(x_2, y_2) R (x_1 y_1)$. Therefore R is Symmetric
Next, take any $(x_1, y_1), (x_2, y_2), (x_3, y_3) A \times A$ & suppose that $(x_1, y_1) R (x_2, y_2)$ & $(x_2, y_2) R (x_3, y_3)$. Thus $x_1 + y_1 = x_2 + y_2$ & $x_3 + y_3$. This gives $x_1 + y_1 = x_3 + y_3$; that is $(x_1, y_1) R (x_3 y_3)$. Therefore R = u fransitive
Thus R is reflexive, symmetric and transitive. Therefore, R is an equivalence relation.
ii. We note that

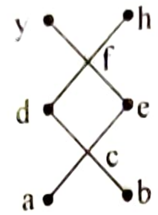
$$[(1,3)] = \{(x,y) \in A \times A (x,y) R (1,3)\}$$

$$= \{(x,y) \in A \times A \quad x + y = 1 + 3\}$$

$$= (1,3), (2,2), (3,1), \text{because } A = \{1,2,3,4,5\}$$

Similarly, $[(2,4)] = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

d. **Consider the Hasse diagram of a POSET (A, R) given in Fig Q6(d). If B = {c, d, e} find ail upper bounds, lower bounds, the least upper bound and the greatest lower bound of B.** **(04 Marks)**



Ans. By examining the given Hasse diagram, we note the following

i. All of c, d,c which are in B are related to f,g,h. Therefore f,g,h are upper bounds of B.

ii. The elements a.b. and c are related to all of c,d,e which are in B. Therefore, a,b and c are lower bounds of B.

iii. The upper bound f of B is related to the other upper bounds g and h of B. Therefore, f is the LUB of B.

iv. The lower bounds a & b of B are related to the lower bound C of B. Tehrefore, C is the GLB of B.

Module-4

7. a. **Determine the number of positive integers n such that 1< n < 100 and n is not divisible by 2, 3, or 5.** **(04 Marks)**

Ans. Let S = {1,2,3,....100}, Then |S| = 100. Let A_1 , A_2 , A_3 be the subjects of S whose elements are divisible by 2,3 5 respectively. Then we have to find

$|\overline{A}_1, \cap \overline{A}_2 \cap \overline{A}_3|$

We note that

$|A_1|$ = No.of elements in S that are divisible by 2

$= [100/2] = [50] = 50$

$|A_2|$ = No.of elements in S that are divisible by 3

$= [100/3] = [33.333] = 33$

$|A_3|$ = No.of elements in S that are divisible by 5

$= [100/5] = [20] = 20$

$|A_1 \cap A_2|$ = No.of elements in S that are divisible by 2 & 3

$= [100/6] = [16.606] = 16$

$|A_1 \cap A_3|$ = No.of elements in S that are divisible by 3 & 5

$= [100/15] = [6.666] = 6$

$|A_1 \cap A_3 \cap A_3|$ = No.of elements in S that are divisible by 2,3 & 5 $= [100/30] = [3.333] = 3$

Now the principle of inclusion exclusion gives

$$|\overline{A}_1 \cap \overline{A}_2 \cap \overline{A}_3| = |S| - \{|A_1| + |A_2| + |A_3|\} + \{|A_1 \cap A_2| + |A_1 \cap A_3| + |A_2 \cap A_3|\} - |A_1 \cap A_2 \cap A$$

$$= 100 - (80 + 33 + 20) + (16 + 10 + 6) - 3 = 26$$

b. In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? (04 Marks)

Ans. Let S denote the set of all permutation of the 26 letter. Then $|s| = 26!$. Let A be the set of all permutation in which CAR appear. This word, CAR conists of three letters which form a single block the set A, therefore consider of all permutations which contain this single block and the 23 demaining letters. Therefore, $|A_1| = 24!$ Similarly, if A_2, A_3, A_4 are the sets of all permutation which contain DOG, PUN and BYTE respectively, we have

$|A_2| = 24!, |A_3| = 24!, |A_4| = 23!$

Likewise, we find that

$$|A_1 \cap A_2| = |A_1 \cap A_3| = |A_2 \cap A_3| = (26-6+2)! = 22!$$

$$|A_1 \cap A_4| = |A_2 \cap A_4| = |A_3 \cap A_4| = (26-7+2) = 21!$$

$$|A_1 \cap A_2 \cap A_3| = (26-9+3)! = 20!$$

$$|A_1 \cap A_2 \cap A_4| = |A_1 \cap A_3 \cap A_4| = |A_2 \cap A_3 \cap A_4| = (26-10+3)! = 19!$$

$$|A_1 \cap A_2 \cap A_3 \cap A_4| = (26-13+4)! = 17!$$

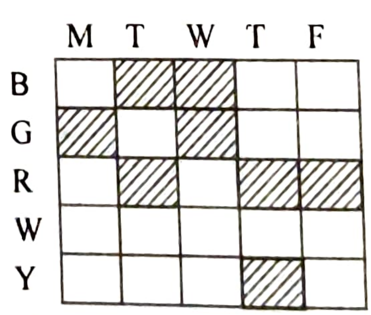
Therefore, the required number of permutation is given by

$$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \bar{A}_4| = |S| - \Sigma|A_i| + \Sigma|A_i \cap A_j| - \Sigma|A_i \cap A_j \cap A_k|$$

$$+ |A_1 \cap A_2 \cap A_3 \cap A_4| = 26! - (3 \times 24! \times 23!) + (3 \times 22! + 3 \times 21!) = (20! + 3 \times 19! + 17!)$$

c. A girl student has Sarees of 5 different colors, blue, green red, white and yellow. On Monday she does not wear green, on Tuesdays blue or red, on Wednesday blue or green, on Thursday red or yellow; on Friday red. In how many ways can she dress without repeating a color during a week (from Monday to Friday)? (04 Marks)

Ans. The situation here can be represented by the board shown below in which the rows respectively represent the B,G,R,w,Y and the columns respectively represent Mondays through Fridays, and the shaded sqceaves together represent the constraints on the colours worn.



For the board C made up of the shaded squares in the above figure, the rock polynomial is

$r(c,x) = 1 + 8x + 20x^2 + 17x^3 + 4x^4$

Thus, here $r_1 = 8, r_2 = 20, r_3 = 17, r_4 = 4$

Thus here $r_1 = 8, r_2 = 20, r_3 = 17, r_4 = 4$

Consequence

$S_0 = S! = 120$, $S_1 = (5-1)!\, X r_1 = 192$, $S_2\ (S-2)! \times r_2 = 120$

$S_3 = (5-3)! \times r_3 = 34$, $S_4 = (5-4)! \times r_4 = u$

Therefore, the number of way of dressing is

$S_0 - S_1 + S_2 - S_3 + S_4 = 120\text{-}192\text{+}120\text{-}34\text{+}4 = 18$

d. The number of affected tiles in a system J 000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(04 Marks)**

Ans. In the beginning, the number of virus affected files is 1000. Let m denote this by a_0.

Let a_n denote the number of virus affected files after 2n hours. Then the number increase by an x 250/100 in the next two hours. Thus, after 2n + 2 hours, the number is

$$a_{n+1} = a_n + a_n \times \frac{250}{100}$$

$$= a_n(1+2.5) = a_n(3.5)$$

This is the recurrence relation for the numbers of virus affect files. Solving this relation, we get

$a_n = (3.5)^n a_0 = 1000 \times (3.5)^n$

This gives the number of virus affected files after 2n hours. From this, we get (for n = 12)

$a_{12} = 1000 \times (3.5)^{12} = 3379220508$

This is the number of virus affected files after one day.

OR

8. a. In how many ways can one arrange the letters in the word CORRESPONDENTS so that

i)There is no pair of consecutive identical letters?

ii) There are exactly two pairs of consecutive identical letters? **(06 Marks)**

Ans. In the word correspondents, there occur one each of C,P,D and T and two each of O,R,E,S,N if S is the set of all permutations of tuese the letters , we have

$$|S| = \frac{14!}{(2!)^5}$$

Let A_1, A_2, A_3, A_4, A_5 be the sets of permutation in which 0's, R's, E's, S's, N's appear in pairs, respectively. Then

$$|A_i| = \frac{13!}{(2!)^4} \text{ for } i = 1,2,3,4,5$$

$$|A_i \cap A_j| = \frac{12!}{(2!)^3}, |A_i \cap A_j \cap A_k| = \frac{11!}{(2!)^{21}}$$

$$|A_i \cap A_j \cap A_{15} \cap A_p| = \frac{10!}{(2!)}, |A_1 \cap A_2 \cap \ldots \cap A_5| = 9!$$

From these, we get

$$S_0 = N = |S| = \frac{14!}{(2!)5}, S_1 = C(5,1)\frac{13!}{(2!)^4}, S_2 = C(5,2)\times\frac{12!}{(2!)^3}$$

$$S_3 = C(5,3)\times\frac{11!}{(2!)^2}, S_4 = C(5,4)\times\frac{10!}{2!}, S_5 = \nabla C(5,5)\times 9!$$

Accordingly, the number of permutations where there is no pair of consecutive identical letters is

$$E_0 = S_0 - \binom{1}{1}S_1 + \binom{2}{2}S_2 - \binom{3}{3}S_3 + \binom{4}{4}S_4 - \binom{5}{5}S_5$$

$$= \frac{14!}{(2!)^5} - \binom{5}{1}\times\frac{13!}{(2!)^4} + \binom{5}{2}\times\frac{12!}{(2!)^3} = \binom{5}{3}\times\frac{11!}{(2!)^2} + \binom{5}{4}\times\frac{10!}{2!} - \binom{5}{5}\times 9!$$

$$E_2 = S_2 - \binom{3}{1}S_3 + \binom{4}{2}S_4 - \binom{5}{3}S_5$$

$$= \binom{5}{2}\times\frac{12!}{(2!)^3} - \binom{3}{1}\binom{5}{3}\times\frac{11!}{(2!)^2} + \binom{4}{2}\times\frac{10!}{2!} - \binom{5}{5}\times 9!$$

$$E_0 = S_2 - \binom{3}{1}S_3 + \binom{4}{2}S_4 - \binom{5}{3}S_5$$

$$= \binom{5}{2}\times\frac{12!}{(2!)^3} - \binom{3}{1}\binom{5}{3}\times\frac{11!}{(2!)^2} + \binom{4}{2}\binom{5}{4}\times\frac{10!}{2!} - \binom{5}{3}\binom{5}{5}\times 9!$$

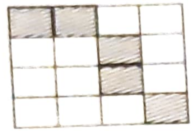
Lartily, the no. of permutation which there are at least 3 pair of consecutive identical letters is

$$L_3 = S_3 - \binom{3}{2}S_4 + \binom{4}{2}S_5$$

$$= \binom{5}{3}\times\frac{11!}{(2!)^2} - \binom{3}{2}\binom{5}{4}\times\frac{10!}{2!} + \binom{4}{2}\binom{5}{5}\times 9!$$

b. An apple, a banana, a mango and an orange are to be distributed to four boys Bi, B2, B3, and B4. The boys Bi and B_2 do not wish to have apple, the boy, B3 does not want banana or mango and B4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? (05 Marks)

Ans. The situation can be described by the board shown in fig in which the rows respectively representatively represent apple, banna, mango and orange and teh columns represent the boys B1, B2, B3 B4 resectively. Also the shaded squares together represent the for bidden placed i the distribution.



Let the consider the board C consisting of the shaded squares in Fig. We note that c is formed by the mutually disjoint boards C1, C2, C3 shown in fig.

As such the book polynomial for C is

$r(c,x) = r(x1) \times r(c2,x) \times r(S,x)$

By inspection we find that

$r(c_1,x) = 1+2x, r(c_2,x) = 1+2x, r(c_3,x) = 1+x$

Accordingly we have

$r(c,x) = (1+2x) < (1+x) = 1+5x+5x^2+4x^3$

Thus for c, $r_1 = 5\, n_2 = 8, r_3 = 4$

$S_0 = 4! = 24; S_1 = (4-1)! \times r_1 = 30$

$S_2 = (4-2)! \times r_2 = 16, r_3 = (4-3)! \times r_3 = 4$

$\overline{N} = S_0 - S_1 + S_2 - S_3 = 24 - 30 + 16 - 4 = 6$

This in the number of ways of distributing the fruits, under the given constrains.

c. Solve the recurrence relation $a_n = 3a_n.i - 2a_n\text{-}2$ for $n > 2$ given that ai = 5 and a_2 = 3. (05 Marks)

Ans. Given : $a_1 = 5$ & $a_2 = 3$

The co-efficient an, an-1 and an-2 are respectively

$C_n = 1, C_{n-1} = -3$ and $C_{n-2} = 2$

$\therefore k^2 - 3k + 2 = 0$ or $(k-2)(k-1) = 0$

Whose roots $k_1 = 2, k_2 = 1$

$a_n = A \times 2^n + B \times 1^n$

Wheel A and B use arbitary constat

$a_1 = 5$ & $a_2 = 3$ in then weget

$5 = 2A + B$

$3 = 44 + B$

solve $A = -1$ $B = 7$

$\therefore a_n = -2^n + 7$

Module-5

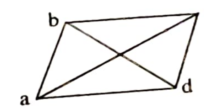
9. a. **Define:**
i)Bipartite graph ii)Complete bipartite graph iii) Regular graph iv)Connected graph with an example. (04 Marks)

Ans. **i. Bipartile graph :**
A graph G(V.E) is called bupartile graph if the vertex set V can be partitioned into disjoint set V1 and V2 such that each edge in G has one vertex in V1 and other vertex in V2.

ii. Complete Bipartile Graph :
A bipartite graph $G(V_1,V_2,E)$ is called complete bipartile graph if these is an edge between every vertex in V_1 and every vertex in V_2.

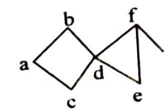
iii. Regular graph :
A graph in which every vertex is of same degree is called a regular graph
If deg (v) = k then it is called k - regular graph

Eg :

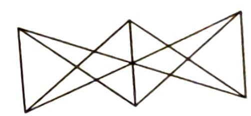
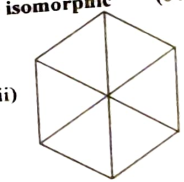


iv. Connected graph :
If there is a path between every pair of vertices. then te graph is called connected graph.

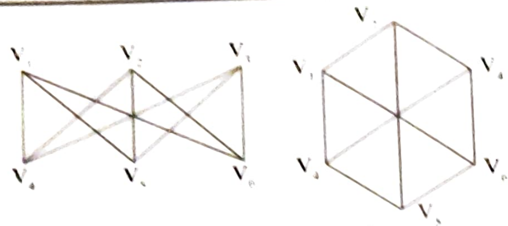
Eg :



b. Define isomorphism. Verify the two graphs are isomorphic (04 Marks)

i)  ii) 

Ans. Let $G1(V_1E_1)$ & $G_2(V_2,E_2)$ be two graphs
A function $f: V_1 - V_2$ or f L $V(G_1) - V(G_2)$ is called a isomorphism if
i. f is one - to - one and onto i.e. f is one t one correspondence.
ii. $\forall$ a b $\in$ v {a,b} $\in E_1$ if {f(a),f(b)} $\in E_2$ when such a fun exists g_G and G_2 are caled inomorphism gaph.



Both the graph have 6 vertices each of degree 3 & 9 edges bearing the edges in the 2 graph in mind consider the correspondence between the edges as show below

$\{U_1, U_4\} \rightarrow \{V_1, V_2\}$

$\{U_1, U_5\} \rightarrow \{V_1, V_5\}$

$\{U_1, U_6\} \rightarrow \{V_1, V_6\}$

$\{U_2, U_4\} \rightarrow \{V_4, V_3\}$

$\{U_2, U_4\} \rightarrow \{V_4, V_2\}$

$\{U_2, U_6\} \rightarrow \{V_4, V_6\}$

$\{U_2, U_6\} \rightarrow \{V_5, V_6\}$

$\{U_3, U_4\} \rightarrow \{V_5, V_6\}$

$\{U_3, U_4\} \rightarrow \{V_3, V_2\}$

$\{U_3, U_5\} \rightarrow \{V_3, V_3\}$

These yield correspondence between the vertices

$U_1 \leftrightarrow V_1$

$U_2 \leftrightarrow V_4$

$U_3 \leftrightarrow V_5$

$U_4 \leftrightarrow V_2$

$U_5 \leftrightarrow V_3$

$U_6 \leftrightarrow V_6$

Above correspondence between the edge and the vertices are one - to - one correspondence and that these presence the adjancy of vertices.

c. Show that a tree with n vertices has n-1 edges. (04 Marks)

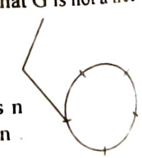
Ans. Let G be connected graph with n vertices and n - 1 edges, assume that G is not a free Contains a cycle say C

Let e be an edge in C

The graph G will not become disconnected if e is deleted

Thus G - e is a connected graph but an the other hand G-e has n verticis, n - 2 efges it cannot be connected this in a contradiction.

This means that G must be free.



d. Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code. (04 Marks)

Ans. R - 1 I - 1

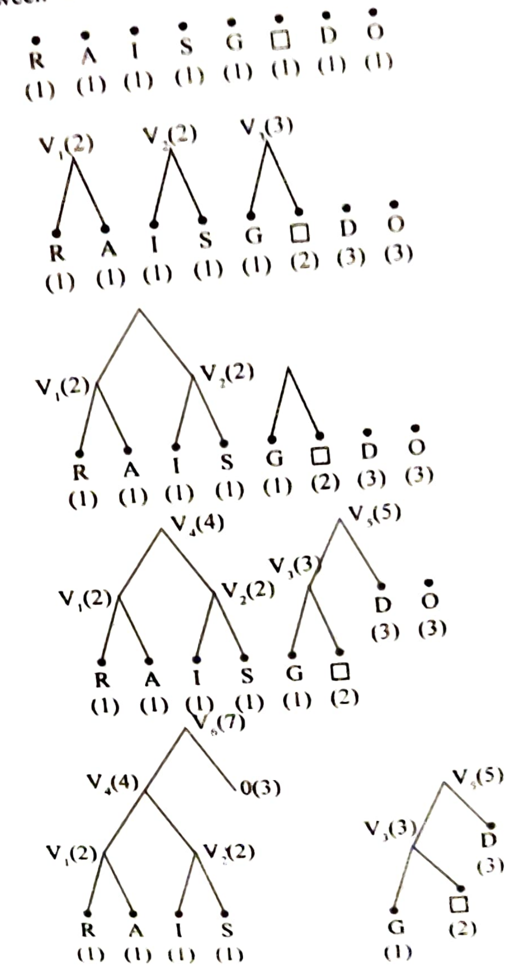
O - 3 S - 1

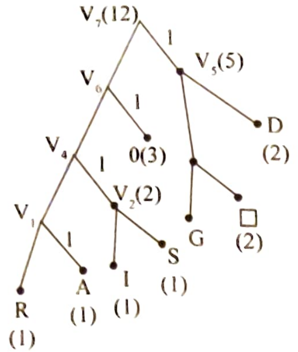
A - 1 G - 1

D - 2

□ - 2

□ Space between "Road and is once " and is and good in once"





R – 0 0 0 0	0 – 0 1
A – 0 0 0 1	G – 100
I – 0 0 1 0	□ - 101
S – 0 0 1 1	D – 11

OR

10. a. Determine the order |V| of the graph G = (V, E) in
i) G is a cubic graph with 9 edges
ii) G is regular with 15 edges
iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.
(04 Marks)

Ans. i) Suppose the order of G is n

G is a cubic graph all the vertices of G have degree 3 and therefore the sum of degree of vertices is

$3 \times n = 2.9$

$n = 18/3 = 6$

The order of G is 6

ii) Since G is regular all the verticies of G must be of same degree k(say)

If G is of order n then the sum of degree of vertices in kn. Since G has 15 edges

We should have kn = 2.15

k = 30/n

k has to be +ve integer if follower that n must be a divisor of 30. Thus the possible order of G are 1,2,3,5,6,10,15 and 30

iii. Suppose the order fo G in n

The verticies of G are of degree 4 and all other of degree 3. the sum of the degree of verticies of G is

$2.4 + (n - 2)3$

G has 10 edges we should have

$2.4 + (n - 2)3 = 2(10)$

$\Rightarrow n = 6$

$\therefore$ Order of G is 6

b. Prove that in a graph
i) The sum of the degrees of all the vertices is an even number and is equal to twice the number of edges in the graph.
ii) The number of vertices of odd degrees is even. **(04 Marks)**

Ans. i) In an alternative form this property reaches follows

Fo a graph G = (V.E)

$$\sum_{r \in V} \deg(V) = 2|E|$$

This property is obvious from the fact that while counting the degree of vertices each edges is counted twice.

This aforescid property is pocelarly called the hand shaking property because essentially statesthat it several people shake hands, then the total number of hands shaken must be even because just two hands are involved in each handshake.

ii) Consider a graph with n vertices, suppose k of the these vertices are of odd degree so that the remaining n - k vertices are if even degree, denote the vertices with odd degree by V1, V2,....V4 and the vertices with even degree by Vk+1 , Vk+2...Vn then theorem of the degree of the vertices is

$$\sum_{i=1}^{n} \deg(vi) = \sum_{i=1}^{k} \deg(Vi) + \sum_{i=k+1}^{n} \deg(vi) \quad ...(1)$$

In view of the hand shaking property the sum on the left hand side of the above expression is equal to twice. The number of edges in the graph as such this sum is even. Further, the second sum in the right hand side sum of the degree of vertices with even degree such this sum is also even.

$\therefore$ The first sum in the right hand side must also be even that is

$\deg(V_1) + \deg(V_2) + ... + \deg(V_k) =$ Even ....(2)

But each of $\deg(V_1)$, $\deg(V_2)$ .... $\deg(V_k)$ in odd

$\therefore$ The no. of y terms in teh left hand side of (2) must be even that in k is even.

c. Discuss the solution of Konigsberg bridge problem. **(04 Marks)**

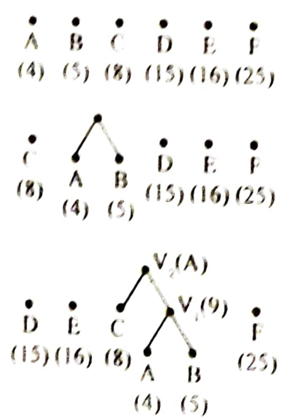
Ans. In the 18th century city named konigsberg in east prussia. There flowed a river named pregal river which divided the city into 4 puts two of these pure were the blanks of a river and two were islands. These parts were connected with each other through seven brides as shown in the fig.

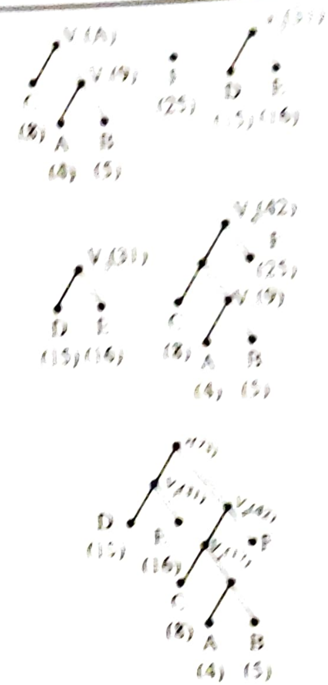
The citizens of the city seemed to have pose the following problem
By starting at any of 4 land can we return to that area after crossing each of the 7 bridges exactly once
This problem is known as konigsbers bride problem this problem remained unsolved for several year solution is given by euler
Denote the land areas of the city by A,B,P, where A,B are blanks of the river P and Q are the islands
Construct a graph by treating the land area as 4 vertices and the 7 bridges connected Then as 7 edges

In this graph
d(A) = d(B) = d(Q) = 3
deg (P) = 5
which are not even
The graph does not have an eulerian circuit. This ream that does not exist a closed walk that contains all the edges exactly once. It is not possible to walk over each of the 7 bridges exactly once and return to the statics point
Also be proved if all vertices are of degree then it has euler circuit in it

d. Define optimal tree and construct an optimal tree for a given set of weights {4, 15, 25, 5, 8, 16}. Hence find the weight of the optimal tree. **(04 Marks)**

Ans. A three in thin set which carrier the minimum weight is called an optimal tree for the weight
Let {4,15,25,5,8,16}





WT $= (4 \times 4) + (5 \times 3) + (15 \times 2) + (16 \times 2) + (25 \times 2) = 172$

Third Semester B.E. Degree Examination, CBCS - June / July 2017

Discrete Mathematical Structures

Time: 3 hrs Max. Marks: 80

Note : *Answer any FIVE full questions, selecting one full question from each module.*

Module - 1

1. a. Define the following with an example for each

i) Proposition **ii) Tautology**

iii) Contradiction **iv) Dual of statement.** **(06 Marks)**

Ans. i. Proposition : A proposition is a statement which can be decided either true or false but not both.

eg : Bangalore is in Karnataka T

Three is a prime no T

7 is divisible by 3 F

ii. Tautology : A compound proposition which is true for all possible truth values of its components is called a tautology.

Ex : $[(p \to q) \wedge (q \to r)] \to (p \to r)$

iii. Contradiction : A compound proposition which is false for all possible truth table values of its components is called a contradiction.

Ex : $(p \veebar q) \wedge (p \leftrightarrow q)$ is a contradiction

$(p \veebar q) \wedge (p \to q)$ is a contradiction

iv. Dual of statement

Suppose u is a compound proposition that contains To and Fo as components we replace each occurance of To by Fo and Fo resply. Then resulting compound proposition is called dual of U and denoted by u^d.

ex : u : $p \wedge (q \vee \neg r) \vee (S \wedge To)$

$p \vee (q \wedge \neg r) \wedge (S \vee Fo)$

b. Establish the validity of the following argument using rules of inference. If the band could not play rock music or the refreshments were not served on time, then the new year party could have been cancelled and Alica would have been angry. If the party were cancelled, then refunds would have to be made. No refunds were made, therefore the band could play rock music. **(05 Marks)**

Ans. p – The band could play rock music.

q – The refreshments were delivered on time.

r – The new year's party was concelled

S – Alicker was angry

t –Refunds had to be made.

Proof by contradiction is the right argument valid or invalid

Conclusion p be false

→ p is true

→ q is true

→ S is true

→ r is false

→ r is true

→ Contradiction

Argument is valid

c. Determine the truth value of the following statements if the universe comprises all nonzero integers :

i) $\exists x \exists y[xy = 2]$

ii) $\exists x \forall y.[xy = 2]$

iii) $\forall x \exists y[xy = 2]$

iv) $\exists x \exists y[(3x + y = 8) \wedge (2x - y) = 7]$

v) $\exists x \exists y[(4x + 2y = 3) \wedge (x - y = 1)$ **(05 Marks)**

Ans. i. False$(x = 2, y = 2)$

ii. False (Far a specified x, xy = 2 for x is not true)

iii. False equation 3x + y = 8 and 2x - 7 = 7 donot have common integer solution.

iv. Fals do not have common integer solution.

OR

2. a. Find the possible truth values for p, q and r if

i) $p \to (q \vee r)$ – FALSE **ii) $p \wedge (q \to r)$ – TRUE.** **(05 Marks)**

Ans. i. $p \to (q \vee r) \to$ False

$p \to (q \vee r)$ can be false only when p is true and $q \vee r$ is false also $q \vee r$ is false only when both q and r are false. Hence the truth values of p,q,r are 1,0,0 respectively.

ii. $p \wedge (q \to r)$ is true

$p \wedge (q \to r)$ can be true only when p is true and $q \to r$ is true. Also, $q \to r$ can be true when (a) r is true and q is true or false & (b) r is false and q is false. Hence the possible truth value of p,q,r are shown in the following

p	q	r
1	1	1
1	0	1
1	0	0

b. Show that $(p \wedge (p \to q)) \to q$ is independent of its components. **(06 Marks)**

Ans. $(p \wedge (p \to q) \to q$ is independent of its component

p	q	p → q	r = p ^ (p → q)	r → q
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	1	1	1

c. **Give a direct proof for each of the following :**

i) For all integers k and *l*, if k and *l* are both even, then k + *l* is even

ii) For all integers k and *l*, if k and *l* are both even, then k * *l* is even. (05 Marks)

Ans. i) For all integers k and l, if k and l are both were, tehn k +l here the contrapositive of the given statemetn is "For all integer k and l if one k l is odd and the other in even, then k + l is odd.

We how prove this contrapositive

For any integer k and l, assume that one of k is in odd and teh other is even. Suppose k odd an dl is een tehn k = 2m + 1 and l = 2n for some integers m and n. consequently k + l = (2m + 1) + 2n = 2(m + n) + 1 is odd, we find that k +l is odd. This proves the contrapositive.

This proof of the contrapositive server as an indirect proof of the given statement.

ii. For all integers k and l, if k and l are both even, then k x l is even.

Module-2

3. a. **Prove by mathematical induction, for every positive integer 8 divides $5^n + 2.3^{n-1} + 1$.** **(06 Marks)**

Ans. **Basic step :**

We note that

$A_1 = 5^1 + 2.3^0 + 1 = 8$

Thus for n = 1, the number An is a multiple of 8 induction step : Assume that An is a multiple of 8 for A

$n = k \geq 1$

Using the given defination of An. We find that

$$A_{k+1} - A_k = \left(5^{k+1} + 2.3^k + 1\right) - 5^k + 2.3^{k-1} + 1$$

$$= (5-1)5^k + 2(3-1)3^{k-1}$$

$$= 4\left(5^k + 3^{k-1}\right)$$

Since 5 & 3 are odd, 5th and 3^{k-1} are also odd. Conequently $5^k + 3^{k-1}$ is wen. Hence $4(5^k + 3^{k-1})$ is multiple of 8. That is $(A_{k+1} - A_k)$ is a multiple of 8. Since A_k is a multiple of 8 by assumption, if follows that A_{k+1} is also a multiple of 8. Thus A_n in a multiple of 8 for n = k +1 is A_n is a multiple of 8 for n = k.

This complete the proof of the required result, by induction.

b. **Assuming PASCAL language is case insensitive, an identifier consists of a single letter followed by upto seven symbols which may be letters or digits (26 letters, 10 digits). There are 36 reserved words. How many distinct identifiers are possible in this version of PASCAL?** **(05 Marks)**

Ans. The given word has 6 letter all of which are distinct. The required number of strings in the same as the number of permutation of these size letter 7 at the time. This number and 36 reserved wards.

$$^{36}C = \frac{36!}{(36-7)!} = \frac{36}{2a} = 8347680$$

c. **Find the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$.** **(05 Marks)**

Ans. The general term in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$

$$\binom{16}{n_1, n_2, n_3, n_4, n_5}(a)^{n_1}(2b)^{n_2}(-3c)^{n_3}(2d)^{n_4}(5)^{n_5}$$

For $n_1 = 2, n_2 = 3, n_3 = 2, n_4 = 5 \,\&\, n_5 = 16-(2+3+2+5) = 4$ this becomes

$$\binom{16}{2,3,2,5,4}a^2(2b)^3(-3c)^2(2d)^5\,5^4 = \binom{16}{2,3,2,5,4} \times 2^3 \times (-3)^2 \times 2^5 \times 5^4$$

$$\times a^2b^3c^2d^5$$

Therefore, the coefficient of $a^2b^3c^2d^5$ is

$$2^8 \times 3^2 \times 5^4 \times \frac{16!}{2!3!2!5!4!} = 3 \times 2^5 \times 5^3 \times \frac{16!}{(a!)^2}$$

OR

4. a. **Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$.** **(05 Marks)**

Ans. Here we have to prove that the statement

S(n) : un < (n² - 7)

is true for all positive integers n > no, where $n_o = 6$

Basic step : we observe that

S(6) : (4 x 6) < (6² – 7)

is true. Thus S(n) is true for $n = n_o = 6$

Induction Step : We assume that S(n) is true for n = k where $k \geq 6$.

Then

$4(k + 1) = 4k + 4$

$< (k^2 - 7) + 2k + 1$ because when $k \geq 6$ we have $2k + 1 \geq 13 > 4$

$= (k + 1)^2 - 7$

This shows that S(k+1) is true

By mathematical induction, it now follower that S(n) is tree for all positive integer n ≥ 6. This proves the required result.

b. **Lucas numbers are defined recursively as $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 2$. If Fi' are fibonacci numbers and Li' are the Lucas numbers, prove that $L_n = F_{n-1} + F_{n+1}$ for all positive integers n.** **(05 Marks)**

Ans. Using the defination given we find that

$L_2 = L_1 + L_0 = 1 + 2 = 3$ $\quad$ $L_3 = L_2 + L_1 = 3 + 1 = 4$

$L_4 = L_3 + L_2 = 4 + 3 = 7$ $\quad$ $L_5 = L_4 + L_3 = 7 + 4 = 11$

$L_6 = L_5 + L_4 = 11 + 7 = 18$ $\quad$ $L_9 = L_8 + L_7 = 47 + 29 = 76$

$L_8 = L_7 + L_6 = 29 + 18 = 47$ $\quad$ $L_7 = L_6 + L_5 = 18 + 1 = 29$

$L_{10} = L_9 + L_8 = 76 + 47 = 123$

$F_{n-1} + F_{n+1}$

(i) For $n = 1$

$L_1 = 1 = 0 + 1$

$= F_0 + F_2$

$= F_{1-1} + F_{1+1}$

$\therefore S(n)$ is true for $n = 1$

(ii) Assume that $S(n)$ is true

$n = 1, 2, 3 \ldots (k-1)k$

(iii) For $n = k + 1$

$L_{k+1} = L_k + L_{k-1}$

$= \left[F_{(k-1)+1} + F_{(k-1)-1}\right] + \left[F_{k-1} + F_{k+1}\right]$

$= F_{k-2} + F_k + F_{k-1} + F_{k+1}$

$= (F_{k-2} + F_{k-1}) + (F_k + F_{k+1})$

$= F_k + F_{k+2}$

$L_{k+1} = F_{(k+1)-1} + F_{k+1+1}$

$\therefore S(n)$ is true for $n = k + 1$

By principle of mathematical induction

$\therefore S(n)$ is true for $n \geq 1$

c. Find the number of distinct terms in the expansion of $(w+x+y+z)^{12}$. (06 Marks)

Ans. We have by the multinomial theorem.

$$\sum_{n_i} \binom{n}{n1, n2, \ldots nk} x_1^{n_1} \cdot x_2^{n_2} \ldots x_4^{nk} = (x_1 + x_2 + \ldots + x_t)$$

Takies $x_1 = 1, x_2 = 1 \ldots x_k = 1$ in this we get

$$\sum_{n_i} \binom{n}{n_1, n_2, \ldots n_k} = (1 + 1 + \ldots + 1)^n = k^n$$

The sum in the left hand side of the above expression is the sum of the co-efficient in the expansion of $(x1 + x2 + \ldots + xk)^n$ this sum is equal to k^n as shown . If follows from the result. That the sum of the coefficient in the expansion of $(w+x+y+z)^{12}$ is $(1+1+1+1)^{12} = 4^{12}$

Module-3

5. a. Let A= {a, b, c, d} and B= {1,2,3,4,5, 6}.

i) How many functions are there from A to B? How many of these are one-to-one? How many are onto?

ii) How many functions are there from B to A? How many of these are one-to-one? How many are onto? (06 Marks)

Ans. Here $|A| = m = 4$ & $|B| = n = 6$. Therefore

a. The number of functions possible from A to B is $n^m = 6^4 = 1296$

The number of one - to one function possible from A to B is $\dfrac{n!}{(n-m)!} = \dfrac{6!}{2!} = 360$

There is no onto funciton from A to B

b. The number of functions possible from B to A is $m^n = 4^6 = 4096$

There is no one to one function from B to A

The number or onto function from B to A is

$$P(6,4) = \sum_{k=0}^{4} (-1)^k \left({}^4C_{4-k}\right)(4-k)^6$$

$$= 4^6 - 4 \times 3^6 + 2 \times 2^6 - 4 = 1560$$

b. Prove that if f : A → B, g : B → C are invertible functions, then g of: A → C is invertible and $(gof)^{-1} = f^{\prime} og^{-1}$. (06 Marks)

Ans. Proof : Since f and g are invertible funcitons, then both one - to one and onto. Therefore gof is invertible. Consequently g of both one - to - one and onto. Therefore gof is invertible. Now the inverse f^{-1} of f is a function from B to A and the inverse g^{-1} of g is a function from C to B. Therefore, if $h = f^{-1} \circ g^{-1}$ then h is a function from C to A.

We find that

$(gof)\, oh = (gof)\, 0\, (f^{-1} og^{-1}) og^{-1} = go(fo\, f^{-1}) og^{-1} = go\, I_B\, og^{-1}$

$= gog^{-1} = I_c$

And $ho(gof) = (f^{-1} og^{-1})\, o\, (gof)$

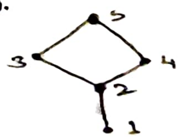
$= f^{-1} o(g^{-1} og) of = f^{-1}\, oIB\, of$

$= f^{-1}\, of = I_A$

The above expression show that h is the inverse of gof : that is $h = (gof)^{-1}$. Thus, $(gof)^{-1} = h = f^{-1}\ og^{-1}$

This complete the proof of the theorem

c. For the Hasse diagram, given in Fig. Q5(c), write i) maximal ii) minimal iii) greatest and iv) least element (s). (04 Marks)



Ans. An element $a \in A$ is called a maximal element of A if there exerts no element $x = a$ in A such that a R x.

An element $a \in A$ is called a minimal element of A if there exists no element $x \neq a$ in A such that x R a.

An element $a \in A$ is called a greatest element of A if xRa for all $x \in A$
An element $a \in A$ is called a least element of A if aRx for all $x \in A$
In the poret represented by the following diagram, is a maximal as well as a greatest element and 1 is 0 minimal as well as least element.

OR

6. a. Let f, g : $z^+ \to z^+$, where for all $x \in z^+$, f (x) = x +1 and g(x) = max {1, x −1}.
i) What is the range of f?
ii) Is fa onto function?
iii) Is f one-to-one?
iv) What is the range of g?
v) Is g an onto function? **(05 Marks)**

Ans. Let f,g : z+ → z+ f(x) = x + 1 g(x) = Max {1,x−1}
f(x) = g(x)
x + 1 = Max {1,x+1}
a. The subset the smaller of all elementation of A units foundation is called range
∴ f is ray of 1
b. If f in one-to one function
c. The range function subset of images of all element of an g
d. If g is a onto function.

b. If f : A→B and B_1, $B_2 \le B$, then prove the following :
i) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$ ii) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$
iii) $f^{-1}(\overline{B_1}) = \overline{f^{-1}(B_1)}$ **(06 Marks)**

Ans.

i. $f^{-1}(B_1 \cup B_2) = \{A \in x \mid f(x) \in B_1 \cup B_2\}$

∴ For any $x \in A$

$\Rightarrow x \in f^{-1}(B_1 \cup B_2)$

$\Rightarrow f(x) \in B_1 \cup B_2$

$\Rightarrow f(x) \in B_1$ or $f(x) \in B_2$

$\Rightarrow x \in f^{-1}(B_1)$ or $x \in f^{-1}(B_2)$

$\Rightarrow x \in f^{-1}(B_1) \cup f^{-1}(B_2)$

$\therefore f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$

ii. Similarly for any $x \in A$

$\Rightarrow x \in f^{-1}(B_1 \cap B_2)$

$\Rightarrow f(x) \in (B_1 \cap B_2)$

$\Rightarrow f(x) \in B_1$ & $f(x) \in B_2$

$\Rightarrow x \in f^{-1}(B_1)$ & $x \in f^{-1}(B_2)$

$\Rightarrow x \in f^{-1}(B_1) \cap f^{-1}(B_2)$

$\therefore f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cap f^{-1}(B_1)$

iii. For any $x \in A$

$x \in f^{-1}(\overline{B_1}) \Rightarrow f(x) \in \overline{B_1}$

$f(x) \notin B_1$

$x \in \overline{f^{-1}(B_1)}$

$x \in f^{-1}(B_1)$

$f^{-1}(c) = \overline{f^{-1}(c)}$

c. Let A = {1, 2, 3, 4}, R = {(1, 3), (1, 1), (3, 1), (1, 2), (3, 3), (4, 4)}be the relation on A. Determine whether the relation R is reflexive, irreflexive, symmetric, antisymmetric or transitive. **(05 Marks)**

Ans. $\forall a \in A$, a R a => R is reflexive
We observe that the given relation R in reflexing and transitive. Further R does not certain ordered pair of the form (a,b) and (b,a) with b = a. therefore R in antisymmetric.
As such R in a partial on A.
Contained defination of symmetric machine.

Module-4

7. a. Determine the number of positive integers n where $1 \le n \le 100$ and n is not divisible by 2, 3 or 5. **(05 Marks)**

Ans. Let S = {1,2,3......1000}
|S| = 100
Let A1, A2, A3 be the subsets of S where elements are divisible by 2,3 or 5 reply
We need to find $|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}|$

$|A_1| = 100/2 = 50$

$|A_2| = 100/3 = 33.33 = 33$

$|A_3| = 100/5 = 20$

$|A_1 \cap A_2| = 100/6 = 16.66 = 16$

$|A_1 \cap A_3| = 100/10 = 10$

$|A_2 \cap A_3| = 100/15 = 6.66 = 6$

$|A_1 \cap A_2 \cap A_3| = 100/30 = 3.33 = 3$

$|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}| = |S| - \Sigma A_i + \Sigma A_i \cap A_j - |A_1 \cap A_2 \cap A_3|$

$= |S| - (|A_1| + |A_2| + |A_3|) + [|A_1 \cap A_2| + |A_1 \cap A_3| + |A_2 \cap A_3|] - [A_1 \cap A_2 \cap A_3]$

$= 10 - (50 + 33 + 20) + (16 + 10 + 6) - 3$

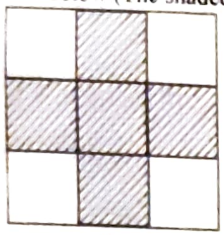
$= 26$

b. Describe the expansion formula for rook polynomials. Find the rook polynomial for 3 × 3 board using the expansion formula. **(05 Marks)**

Ans.

3 x 3 board

Let us mark the square which is at the centre of the board as *, then the board D and E appears as shown below (The shaded parts are the deleted parts)



D

1	2	3
4		5
6	7	8

E

For the board D, we find that r = 4, r2 = 2, r3 = r4 = 0 hence $r(D,x) = 1 + 4x + 2x^2$

$r(E,x) = 1 + 8x + 14x^2 + 4x^3$

Now, the expansion formula gives

$$r(C_{3\times3},x) = x\,rD(x) + r(E,x)$$

$$= x\left(1 + 4x + 2x^2\right) + \left(1 + 8x + 14x^2 + 4x^3\right)$$

$$= 1 + 9x + 18x^2 + 6x^3$$

c. Solve the recurrence relation $b_n = bD_{n-1} - b^2D_{n-2}$, $n \ge 3$ given $D_1 = b > 0$ and $D_2 = 0$. **(06 Marks)**

Ans.

$D_n = bD_{n-1} - b^2D_{n-2}$ for $n \ge 3$

Given $D_1 = b > 0$ and $D_2 = 0$

For the given relation, the characteristic equation is $k^2 - bk + b^2 = 0$, whose roots are

$$k = \frac{b \pm \sqrt{b^2 - 4b^2}}{2} = \frac{b}{2}\left(1 \pm i\sqrt{3}\right)$$

∴, the general solution for D_n is

$$D_n = r^n\left(A\cos n\theta + B\sin n\theta\right) \quad ...(1)$$

where A & B are arbitary constants, and

$$r = \left|\frac{b}{2}\left(1 \pm i\sqrt{3}\right)\right| = \frac{b}{2}\left(\sqrt{1^2 + 3}\right) = b \;\&\; \tan\theta = \frac{\sqrt{3}}{1} = \sqrt{3}$$

So that $\theta = \pi/3$. Thus we have

$$D_n = b^n\left[A\cos\frac{n\pi}{3} + B\sin\frac{n\pi}{3}\right] \quad ...(2)$$

Using the given condition $D_1 = b > 0$ & $D_2 = 0$ in this,

$$b = b\left[A\cos\frac{\pi}{3} + B\sin\frac{\pi}{3}\right] \;\&\; 0 = b^2\left[A\cos\frac{2\pi}{3} + B\sin\frac{2\pi}{3}\right]$$

Which can be rewritten as

$$1 = \frac{1}{2}A + \frac{\sqrt{3}}{2}B \;\&\; 0 = -\frac{1}{2}A + \frac{\sqrt{3}}{2}B$$

Solving these, we get $A = 1$ & $B = 1/\sqrt{3}$. Putting these values of A and B into (2), we get

$$D_n = b^n\left[\cos\frac{n\pi}{3} + \frac{1}{\sqrt{3}}\sin\frac{n\pi}{3}\right]$$

This is the solution of the given relation under the given condition

OR

8. a. In how many ways can we arrange the letters in the CORRESPONDENTS so that;
i) There is no pair of consecutive identical letters?
ii) There are exactly two pairs of consecutive identical letters
iii) There are atleast 3 pairs of consecutive identical letters **(06 Marks)**

Ans. In the word correspondents, there occur one each of C,P,D and T, and two each of O,R,E,S,N. If S is the set of all permutations of these letters, we have

$$|S| = \frac{14!}{(2!)^5}$$

Let $A_1, A_2, A_3, A_4 A_5$ be the sets of permutations in which O's, R's, E's, S's, N's appear in pair, respectively then

$$|A_i| = \frac{13!}{(2!)^4} \text{ for } i = 1,2,3,4,5$$

Also, $|A_i \cap A_j| = \frac{12!}{(2!)^3}$, $|A_i \cap A_j \cap A_k| = \frac{11!}{(2!)^2}$

$|A_i \cap A_j \cap A_k \cap A_p| = \frac{10!}{(2!)}$, $|A_1 \cap A_2 \cap .... \cap A_5| = 9!$

From there, we get

$S_0 = N = |S| = \frac{14!}{(2!)^5}$, $S_1 = C(5,1) \times \frac{13!}{(2!)^4}$, $S_2 = C(5,2) \times \frac{12!}{(2!)^3}$

$X_3 = C(5,3) \times \frac{11!}{(2!)^2} S_4 = C(5,4) \times |S| = \frac{14!}{(2!)^5} \times \frac{10!}{2!}$, $S_5 = C(5,5) \times 8!$

Accordingly, the number of permutations where there is no pair of consecutive identical letter is

$$E_0 = S_0 - \binom{1}{1}S_1 + \binom{2}{2}S_2 - \binom{3}{3}S_3 + \binom{4}{4}S_4 - \binom{5}{5}S_5$$

$$= \frac{14!}{(2!)^5} - \binom{5}{1}\times\frac{13!}{(2!)^4} + \binom{5}{2}\times\frac{12!}{(2!)^3} - \binom{5}{3}\times\frac{11!}{(2!)^2}\times\binom{5}{4}\times\frac{10!}{2!} - \binom{5}{5}\times 9!$$

Next, the number of permutations wher these are exactly two pair of consecutive identical letters is

$$E_2 = S_2 - \binom{3}{1}S_3 + \binom{4}{2}S_4 - \binom{5}{3}S_5$$

$$= \binom{5}{2}\times\frac{12!}{(2!)^3} - \binom{3}{1}\binom{5}{3}\times\frac{11!}{(2!)^2} + \binom{4}{2}\binom{5}{4}\times\frac{10!}{2!} - \binom{5}{3}\binom{5}{5}\times 9!$$

Lastly, the number of permutation where there are at least three pairs of consecutive identical letters is

$$L_3 = S_3 - \binom{3}{2}S_4 + \binom{4}{2}S_5$$

$$= \binom{5}{2}\times\frac{11!}{(2!)^2} - \binom{3}{2}\binom{5}{4}\times\frac{10!}{2!} + \binom{4}{2}\binom{5}{5}\times 9!$$

b. Find the recurrence relation and the initial conditions for the sequence 0, 2, 6, 12, 20, 30, 42. Hence find the general term of the sequence. **(05 Marks)**

Ans. Let the given sequence be (a_r), then we note that

$a_0 = 0, a_1 = 2, a_1 - a_0 = 2$

$a_2 = 6, a_2 - a_1 = 4, a_3 = 12, a_3 - a_2 =$

$a_4 = 20, a_4 - a_3 = 8, a_5 = 30, a_5 - a_4 = 10,$

$a_6 = 42, a_6 - a_5 = 12$ & so on

Evidentlly

$a_n - a_{n-1} = 2n$, or $a_n = a_{n-1} + 2n$, for $n \geq 1$

This is the recurrence relation for the given sequence, with $a_0 = 0$ as teh initial condition.

From this recurrence relation, even note that

$a_n - a_{n-1} = 2n$

$a_{n-1} \quad = 2(n-1)$

$a_{n-2} - a_{n-3} = 2(n-2)$

......

......

$a_3 - a_2 = 2\times 3$

$a_2 - a_1 = 2\times 2$

$a_1 - a_0 = 2\times 1$

Adding all these we get

$a_n - a_0 = 2\{n + (n-1) + (n-2) + \ldots\ldots + 3 + 2 + 1\}$

$= 2 \cdot \frac{n(n+1)}{2} = n(n+1)$

or $a_n = n(n+1) + a_0 = n(n+1) + 0 = n^2 + n$

This is the general term of the given sequence

c. Find the general solution of the equation S(k) + 3S(k - 1). - 4S (k - 2) = 4^k. **(05 Marks)**

Ans. Consider the recurrence relation S(k) - 35(k - 1) - 4S(k - 2) = 4k

The associated homogenous relation is

$S(k) - 3S(k-1) - 4S(k-2) = 0$

The characteristic equation $a^2 - 3a - 4 = 0$

$(a + 1)(a - 4) = 0$

$a = -1, 4$

The roots are $-1, 4$

$\therefore S^h(k) = b1(-1)^k + b_2 4^k$

A function of the form $d4^k$ will not be a particular solution of the non - homogenous relation since it solve the associated homogenous relation. When the RHS involves an exponential function with a base that equals a characteristic root. Then the particular solution will be of the form $d_k 4^k$

Substituting $d_k 4^k$ for S(k)

$d_k 4^k - 3d(k-1)4^{k-1} - 4d(k-2)4^{k-2} = 4^k$

$16dk4^k - 12d(k-1)4^{k-2} - 4d(k-2)4^{k-2} = 4k$

$\therefore 20d = 16 \Rightarrow d = \frac{16}{20} = 0.8$

$\therefore S(k) = b_1(-1)^k + b_2 4^k + 0.8k4^k$

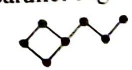
Module - 5

9\. **a. Define the following with an example.**

i) Simple graph **ii) Regular graph** **iii) Subgraph**

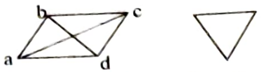
iv) Maximal subgraph **v) Induced subgraph.** **(05 Marks)**

Ans. i. **Simple graph :** A graph with no parallel edges and no loops Rs called simple graph

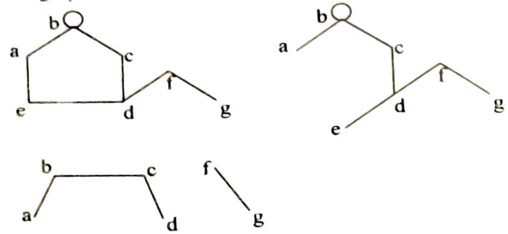


ii. **Regular grap :** A graph in which every vertex is of same degree is called a regular graph.

If deg(v) = k then it is called k - regular graph.

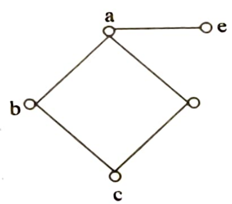


iii. Subgraph : A graph H(w,f) is said to be a subgraph of G(V,E) if w <V and FCE



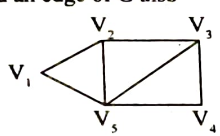
iv) Maximal subgraph

A graph is is defined as a set of nodes and a set of lines that connect the nodes this is sometimes written mathamatically on G = (V,E) or G(V,E). Here is one way to draw a graph.



v. Induced subgraph

Given a graph G(V,E) suppose there is a subgraph G(V,E) of F, such that every edge {A,B} of G were A,B EV1 and an edge of G also



b. Show that there exists no simple graphs corresponding to the following degree sequences

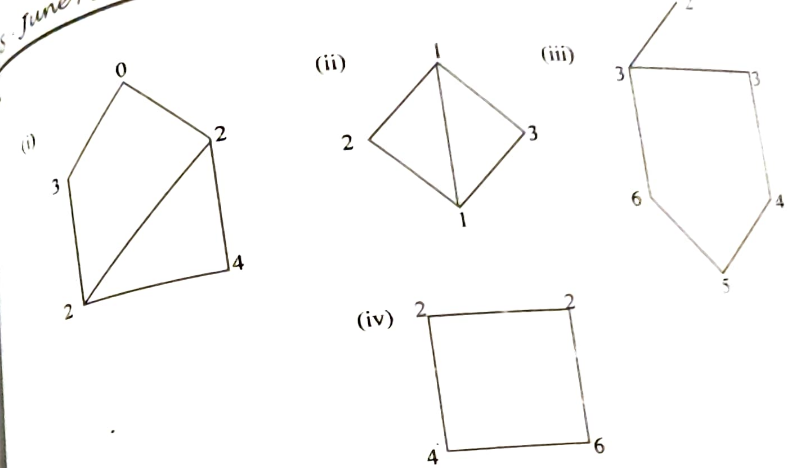
i) 0,2,2,3,4

ii) 1,1,2,3

iii) 2,3,3,4,5,6

iv) 2,2,4,6.

(04 Marks)



c. Let T = (V, E) be a complete m-ary tree with |V| = n. If T has i leaves and i internal vertices, then prove the following :

i) $n = m \cdot i + 1$

ii) $l = (m - 1)\, i + 1$

iii) $i = \dfrac{(l-1)}{(m-1)} = \dfrac{(n-1)}{m}$

(07 Marks)

Ans. We recall that a complex m - aray tree T is a rooted tree in which every internal vertex in of out degree m

We have $n = l + i$

Since the out degree of leaf of T is zero and out degree of every internal vertex of T is m.

Sum of the out - degree of vertices of T = $(1 \times 0) + (i + m) = im$ ....(ii)

Sum of the in - degree of vertices of T = $(1 \times 0) + (n - 1) = n - 1$ ....(iii)

First theorem of Digraph theory

$$n = i.m + 1 \qquad ....(iv)$$

$$i = \frac{n-1}{m} \qquad ....(v)$$

Using (iv) is expression (i) we get

$$im + 1 = l + i$$

$$l = (m-1)i + 1 \qquad ...(vi)$$

$$i = \frac{l-1}{m-1} \qquad ....(vii)$$

Lastly putting expression (vii) in expression (i) we get

$$n = 1 + \frac{l-1}{m-1} \quad \text{or}$$

$$(m-1)n = l(m-1) + l - l = lm - l$$

$$l = \frac{(m-1)n+1}{m}$$

OR

10. a. In the graph shown in Fig. Q10(a). Determine

i) a walk from b to d that is not a trail

ii) b – d trail that is not a path

iii) a path from b to -d

iv) a closed walk from b to b that is not a circuit

v) a circuit from b to b that is not a cycle

vi) a cycle from b to b

(06 Marks)

Ans.

i. $ba_4ea_7fa_9ga_8ea_4ba_3ca_5d$

ii. $ba_3ca_2aa_1ba_4ea_6d$

iii. ba_3ca_5d

iv. $ba_4ea_7fa_9ga_8ea_4b$

v. $ba_3ca_5da_6ea_7fa_9ga_8ea_4$

vi. $ba_1aa_2ca_3b$

b. Determine the order |V| of the graph G = (V, E) in the following cases

i) G is cubic graph with 9 edges

ii) G is regular with 15 edges

iii) G has 10 edges with 2 vertices of degree 4 and all other of degree 3. (06 Marks)

Ans. i. Suppose the order of G is n since G is a cubic graph, all vertices of G have degree 3, and therefore the sum of the degree of vertices is 3n. Since G has a edges we should have 3n = 2 x 9 so that n = 6. Thus, the order of G is 6.

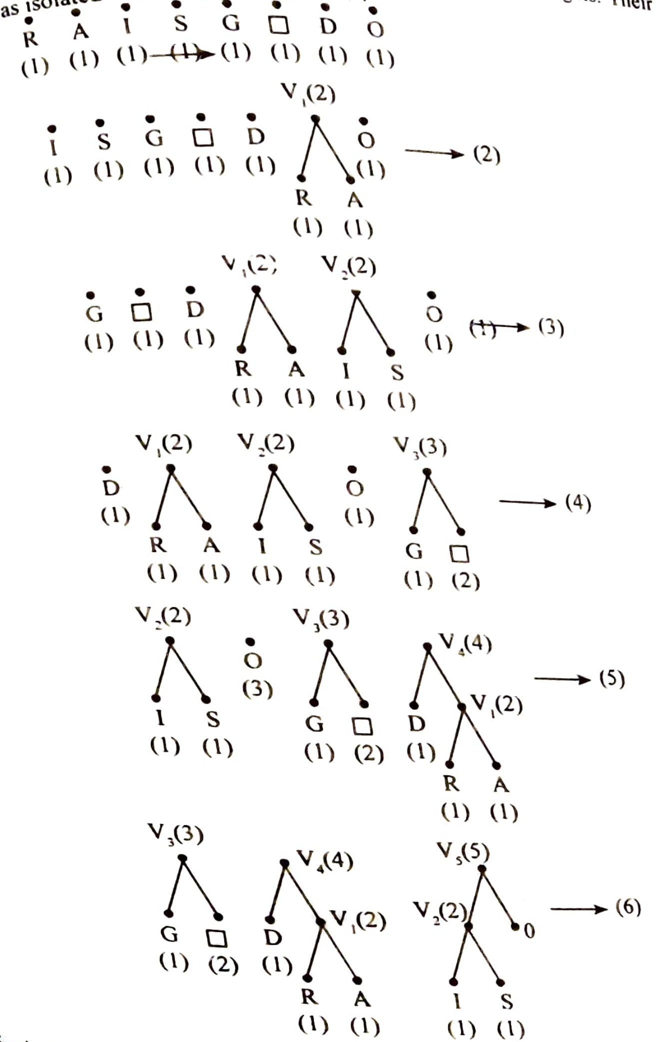
ii. Since G is regular, all vertices of G must be of the same degree, say k . If G is of order h, then sum of the degrees of vertices is kn. Since G has 15 edges, we should have kn = 2 x 15 so that k = 30/n. Since k has to be a positive integer, if follows that n must be advisor of 30. Thus, the possible orders of G are 1,2,3,5,6,10,15 and 30.

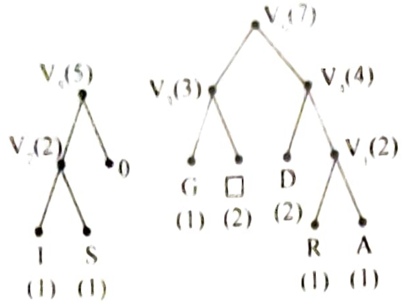
iii. Suppose the order of G is n. Since two vertices of G are of degree 4 and all others are of degree 3, the sum of the degrees of vertices of G is 2 x 4 + (n - 2) x 3. The sum of the degrees of vertices of G is 2 x 4 + (n - 2) x 3. Since G has edges, we should have 2 x 4 + (n - 2) x 3 = 2 x 10. This given n = 6. Thus, the order of G is 6.

Obtain the optimal prefix code for the string ROAD IS GOOD. (04 Marks)

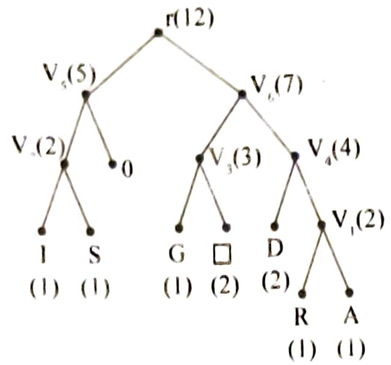
The given menage consits of letters R,O,A,D,I,S,G with frequency 1,3,1,2,1,1,1 respectively. Furthur, there is a blank space occuring twice.

1st we arrange the letters and in the non decreasing order of their weiguls. Their representation as isolated vertices is shown below :

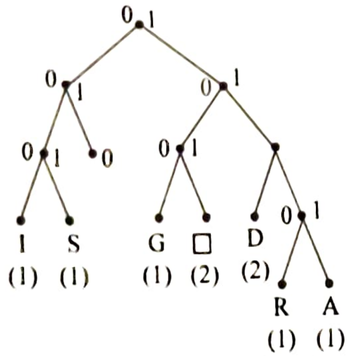




→ (7)



→ (8)



→ (9)

We now construct an optimal free having thes symbols as leaves by using the Huffman's procedures. The graphs obtained in successive steps of the procedure are shown below in Fig (2) and (3) in the order of their occurence The labelled version of the final free is show in fig 9.

The tree shown in Fig 10.37 is the optimal tree that we sought . From tree, we obtain the following optional prefix code for the symbols with which we started.

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note : Answer any FIVE full questions, selecting ONE full question from each module.

Module - 1

1\. a. **Prove that for any three propositions p,q,r[P→(q ^ r)] ⇔ [(p → q)^(p→r)]. Using truth table.** **(05 Marks)**

Ans. Let us prepare the following truth table as

p	q	r	q ^ r	p → (q ^ r)	p → q	p → r	(p → q) ^ (p → r)
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	0	1	1	1	1
1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1
1	0	1	0	0	0	1	0
1	1	0	0	0	1	0	0
1	1	1	1	1	1	1	1

Colums (5) & (8) of the above table show that p → (q ^ r) and (p → q) ^ (p → r) have truth value

∴ They are logically equivalent

b. **Establish the validity of the argument :** **(06 Marks)**

p → q

q → (r ^ s)

¬r v (¬t v u)

<u>p ^ t</u>

∴ u

Ans.

Steps	Reasons
p → q	Premise
q → (r ^ s)	Premise
p → (r ^ s)	Steps (1) & (2) and low of syllogism
p ^ t	Premise
p	Step (4) and rule of conjuctive simplification
r ^ s	Step (3) and (5) and rule of detcutimined
r	Step (6) and rule of conjuctive simplification
¬r v(¬t v u)	Premise

[illegible] ∨ u	Step (8) associative law and margins law
t	Step (4) and rule of conjugative simplification
t	(7) & (10) rule of conjugative
u	(9) & (11) disjuntive syllogism

c. **Prove that for all integers 'k' and 'l', if 'k' and 'l' are both odd, then k + l is even and kl is odd by direct proof.** **(05 Marks)**

Ans. Let $k = 2m + 1$, $l = 2n + 1$ odd integer for semi m and n.

$k + l = 2(m + n + 1)$, $kl = 4mn + 2(m + n) + 1$

We observe $k + l$ is divisible by 2

kl is not divisible by 2

∴ $k + l$ is even, kl is odd

They are arbitrary

OR

2. a. **Determine the truth value of each of the following quantified statements; the universe being the set of all non - zero integers.** **(05 Marks)**

i. $\exists x, \exists y\, [xy = 1]$

ii. $\exists x, \forall y\, [xy = 1]$

iii. $\forall x, \exists y\, [xy = 1]$

iv. $\exists x, \exists y\, [(2x + y = 5) \wedge (x - 3y = -8)]$ **(06 Marks)**

v. $\exists x, \exists y[(3x - y = 17) \wedge (2x + 4y = 3)]$

Ans. i. T($x = 1, y = 1$)

ii. F(For a x $xy = 1$, $\forall y$ is not true)

iii. F($x = 2$ not y such that $xy = 1$)

iv. F $x = 1, y = 3$

v. F equation does not have common solution

b. **Find whether the following arguments are valid or not for which the universe is set of all triangles. In triangle XYZ, there is no pair of angles of equal measure. If the triangle has two sides of equal length, then it isosceles. If the triangle is isosceles, then it has two angles of equal measure. Therefore triangle XYZ has no two sides of equal length.** **(05 Marks)**

Ans. p(x) : x has 2 sides of equal length

q(x) : x is isosceles

r(x) : x has 2 angle of equal measure

c : Triangle xyz

$\neg$ r(c)

$\forall x : p(x) \forall q(x)$

$\underline{\forall x : q(x) \forall r(x)}$

∴ $\neg$ p(0)

Steps	Reasons
$\forall x[p(c) \to q(c)]$	Premise
$p(c) \to q(c)$	(1) q universe
$\forall x[q(x) \to q(x)]$	Premise
$q(c) \to r(c)$	(3) q universe
$p(c) \to r(c)$	(2) & (4) syllogism
$\neg r(c)$	Premise
$\neg p(c)$	(5) & (6)

c. **If a proposition has truth value 1, determine all truth value assignments for the primitive propositions p,r,s for which the truth value of following compound proposition is 1.** **(05 Marks)**

$[q \to \{(\neg p \vee r) \wedge \neg s\}] \wedge \{\neg s \to (\neg r \wedge q)\}$

Ans. $u = q \to \{(\neg p \vee r) \wedge \neg s\}$

$v = \neg s \to (\neg r \wedge q)$

Truth y compound proposition is 1

u & v truth value = 1

$\neg p \vee r$ has truth value 2

$\neg s$ has truth value = 1 s = 0

$\neg r \wedge q$ has truth value 1 ∴ $\neg r = 1$ ∴ r = 0

$\neg p \vee r$ ∴ p = 0 p = q - s = 0

Module - 2

3. a. **Prove by mathematical induction that, for every positive integer n, 5 divides $n^5 - n$.** **(05 Marks)**

Ans. Basic step $i = 1^5 - 1 = 0$

Induction step assumption

i.e., divides $k^5 - k = 5m$

$(k+1)^5 - (k+1) = k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1 - (k+1)$

$= 5(m + k^4 + 2k^3 + 2k^2 + k)$

b. **For the Fibonacci sequence $F_0, F_1, F_2, \ldots$ prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$** **(06 Marks)**

Ans. $F_0 = 0, F_1 = 1.$

Then the result is tree for n = 0, 1

$F_{k+1} = F_k + F_{k-1}$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^k - \left(\frac{1-\sqrt{5}}{2}\right)^k\right] + \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1} - \left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\left(\frac{6+2\sqrt{5}}{4}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\left(\frac{6-2\sqrt{5}}{4}\right)\right]$$

c. **Find the coefficient of :** **(05 Marks)**
i. **x^9y^3 in the expansion $(2x - 3y)^{12}$**
ii. **x^{12} in the expansion $x^3(1 - 2x)^{10}$**

Ans. i. By binomial theorem

$(2x - 3y)^{12} = \sum {}^{12}C_r (2x)^r (-3y)^{12-r}$

Co-efficient $x^9y^3 = -2^{10} \times 3^3 \times 11 \times 10 (r = 9)$

(ii) $(1-2x)^{10} = \sum {}^{10}C_r\, 1^{10-r} (-2x)^r$

Co-efficient $x^{12} = -10 \times 2^9 = -5120 (r = 9)$

OR

4\. a. **By mathematical induction. Prove that, for every positive integer n, the number $A_n = 5^n + 2.3^{n-1} + 1$ is a multiple of 8.** **(05 Marks)**

Ans. Basic step $p_1 = 8$ multiple of 8

Induction step : Assumption. substitution simplification

$A_{k+1} - A_k = 4(5^k + 3^{k-1})$

Since 5 & 3 are odd

5^k & 3^{k-1} is odd. sum is even

b. **How many positive integers 'n' can we form using the digits 3,4,4,5,5,6,7 if we want 'n' to exceed 5,000,000.** **(06 Marks)**

Ans.

$n = x_1x_2x_3x_4x_5x_6x_7$

Then

$x_1 = 5, 6$ or 7

$x_1 = 5 = (6!)$

$= 360$

Similarly

$x_1 = 6 = 180$

$x_1 = 7 = 18$

$\therefore$ Total $= 720$

c. **A certain question paper contains three parts A,B,C with four questions in part A, five questions in part B and six questions in part C. It is required to answer seven questions selecting atleast two questions from each part. In how many ways can a student select his seven questions for answering?** **(05 Marks)**

Ans. Possible ways y answering

Part A	B	C	
2	2	$3 \rightarrow {}^4C_2 \times {}^5C_2 \times {}^6C_3$	$= 1200$
2	3	$2 \rightarrow {}^4C_2 \times {}^5C_3 \times {}^6C_2$	$= 900$
3	2	$2 \rightarrow {}^4C_3 \times {}^5C_2 \times {}^6C_2$	$= 600$
		Total	$= 2700$

Module - 3

5\. a. **Let $f : R \rightarrow R$ be defined by $f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \le 0 \end{cases}$**

(i) Determine $f\left(\frac{5}{3}\right), f^{-1}(3), f^{-1}([-5,5])$

(ii) Also prove that if 30 dicrionaries contain a total of 61,327 pages, then atleast one of the dictionary must have atleast 2045 pages. **(05 Marks)**

Ans.

(i) $f\left(\frac{5}{3}\right) = 3 \times \frac{5}{3} - 5 = 0$

$f^{-1}(3) = \{x \in R / f(x) = 3\} = \left\{\frac{8}{3}, -\frac{2}{3}\right\}$

$f^{-1}(-5, 5) = \left\{x \in R / -\frac{4}{3} \le x \le \frac{10}{3}\right\} = \left\{\frac{-4}{3}, \frac{10}{3}\right\}$

(ii) Pages $\rightarrow$ Pigeon

Dictionaries $\rightarrow$ Pigeon holes we have p = 2044

b. **Prove that if $f : A \rightarrow B$ and $g\ B \rightarrow C$ are invertible function then g of $: A \rightarrow C$ is an invertible function and $(gof)^{-1} = f^{-1} og^{-1}$.** **(06 Marks)**

Ans. gof $\rightarrow$ one - to - one , onto,

gof irreitible, $h = f^{-1}og^{-1}$ $n : c \rightarrow A$

$(gof)oh = gog^{-1} = IC$

$ho(gof) = f^{-1}of = IA$

Let A = {1,2,3,4,5}. Define a relation R on A x A by $(x_1,y_1) R (x_2,y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$

i. Determine whether R is an equilence relation on A x A

ii. Determine equivalence class [(1,2)], [(2,5)] **(05 Marks)**

Ans. i. R is an equivalence relation

(x, y) R (x, y) ... Reflexive

(x, y) R (x, y) ...

... symmetry

Transitive also

(x, y) R (x, y)

(x, y) R (x, y) ... (x, y) R (x, y)

(ii) [(1,2)] = {(1,2),(2,1)}

[(2,5)] = {(2,5)(3,4)(4,3)(5,2)}

OR

6\. a. **Let f and g be functions from R to R define by f(x) = ax + b and g(x) = 1 − x + x². If (g o f)(x) = 9x² − 9x + 3. Determined a,b** **(05 Marks)**

Ans. 9x² − 9x + 3 = (gof)(x)

a²x² + (2ab − a)x + (1 − b + b²)

9 = a², 9 = a − 2ab

3 = 1 − b + b²

a = ±3

a = 3, b = −1

a = −3, b = 2

b. **Let A = {1,2,3,4,6,12}. On A define the relation R by aRb aRb if and only if 'a' divides 'b'**

i. Prove that R is a partial order on A

ii. Draw the Hasse diagram

iii. Write down the matrix of relation. **(06 Marks)**

Ans. R = {(1,1)(1,2)(1,3)(1,4)(1,6)(1,12)(2,2), (2,4), (2,6), (2,12), (3,3), (3,6), (3,12), (4,4), (4,12), (6,6),(6,12), (12,12)}

It is reflexive, symmetric, transitive

It is a partial order

$$M(R) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 6 \\ 12 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 6 & 12 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

c. **Consider the poset whose Hasse diagram is given below. Consider B = {3,4,5}. Refer Fig.Q6(c) . Find :**

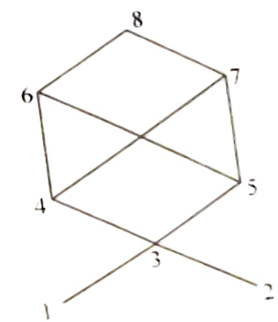
i. All upper bounds of B

ii. All lower bounds of B

iii. The least upper bound of B

iv. The greatest lower bound of B

v. Is this a Lattice? **(05 Marks)**



Ans. Upper bound of B = {6,7,8}

Lower bound = {1,2,3}

LUB {B} = no luB

GLB(B) = 3

It is not a lattice as GLB {1,2} does not exist.

Module - 4

7. a. Out of 30 students in a hostel: 15 study history 8 study economics and 6 study geography. It is known that 3 students study all these subjects. Show that 7 or more students study none of these subjects. **(05 Marks)**

Ans. S denote set of all student in hostel A_1, A_2, A_3 study history Economics, geography

$S_1 = \Sigma|A_i| = 29$

$S_3 = |A_1 \cap A_2 \cap A_3| = 3$

$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3| = |S| - \Sigma|A_i| + \Sigma|A_i \cap A_j|$

$\Sigma|A_1 \cap A_2 \cap A_3|$

$= S_2 - 2$

$S_2 = \Sigma|A_i \cap A_j| \geq 3|A_1 \cap A_2 \cap A_3| = 9$

$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3| \geq 9 - 2 = 7$

b. Five teachers T_1, T_2, T_3, T_4, T_5 are to be made class teachers for five classes C_1, C_2, C_3, C_4, C_5 one teacher for each class. T_1 and T_2 do not wish to become the class teachers for C_1 or C_2, T_3 and T_4 for C_4 or C_5 and T_5 for C_3 or C_4 or C_5. In how many ways can the teachers be assigned work without displeasing any teacher. **(06 Marks)**

Ans.

	C_1	C_2	C_3	C_4	C_5
T_1					
T_2					
T_3					
T_4					
T_5					

$r(c, x) = 1 + 11x + 40x^2 + 56x^3 + 28x^4 + 4x^5$

$r_1 = 11, r_2 = 40, r_3 = 56, r_4 = 28, r_5 = 4$

$S_0 = 5!, S_1 = (5-1)! \times r_1 = 264$

$S_2 = 240$

$S_3 = 112$

$S_4 = 28$

$S_5 = 4$

Now of ways

$S_0 - S_1 + S_2 - S_3 + S_4 - S_5 = 8$

6. a. Solve the recurrence relation $a_{n+1} - 6a_{n-1} + 9a_{n-2} = 0$ form $n \geq 2$. **(05 Marks)**

Ans. Characteristic equation

$x^2 - 6x + 9 = 00$

$x^2 - 6x = 0$

$(x - 3) = 3$

$(x - K_1 \ldots$

$K = (A + B) 3^n$

$a_0 = 5, B = -1$

$A = 5$

$a_n = (5 - n)3^n$

OR

a. Solve the recurrence relation $a_{n-1} 3_{an-1} = 5 \times 3^n$ for $n \geq 1$ given that $a_0 = 2$. **(05 Marks)**

Ans. $a_n = 3a_{n-1} + f(n+1)$ where $f(n) = 5 \times 3^n$

$a_n = 3^n a_0 + 3^{n-1} f(1) + 3^{n-2} f(2) + 3^{n-3} f(3) + \ldots + 3^0 f(n)$

$a_n = 3^n a_0 + 3^{n-1} (5 \times 3) + 3^{n-2} \times 5 \times 3^2$

$a_n = 2 \times 3^n + 3^{n-1} \ldots$

$= 2 \times 3^n + 5 \times n3^n$

$= (2 + 5n)3^n$

b. Let a_n denote the number of n - letter sequences that can be formed using the letters A, B and C such that non terminal A has to be immediately followed by a B. Find the recurrence relation for a_n and solve it. **(06 Marks)**

Ans. $a_n = a_{n-1} + 2a_{n-2}$

$k = 1 \pm \sqrt{2}$

$a_n = A(1+\sqrt{2})^n + B(1-\sqrt{2})^n$

$a_1 = 3, a_2 = 7$

$3 = A(1+\sqrt{2}) + B(1-\sqrt{2})$

$7 = A(1+\sqrt{2}) + B(1-\sqrt{2})^2$

$A + B = 1$ $\quad$ $A - B = \sqrt{2}$ $\quad$ $a_n = \frac{1}{2}\left[(1+\sqrt{2})^{n+1} + (1-\sqrt{2})^{n+1}\right]$

c. **Find the number of permutations of English letters which contain exactly two of the pattern car, dog, pun, byte.** **(05 Marks)**

Ans. S Permutation of 26 letter

$|S| = 26!$ $A_1 \to$ car $|A_1| = 24!$

A_2, A_3, A_4 respectively Dog, Pun, Byte, which

are $|A_2| = 24!$ $|A_3| = 24!$ $|A_4| = 23!$

$|A_1 \cap A_2| = |A_1 \cap A_3| = |A_2 \cap A_3| = 22!$

$|A_1 \cap A_4| = |A_2 \cap A_4| = |A_3 \cap A_4| = 21!$

$|A_1 \cap A_2 \cap A_3| = 20!$

$|A_1 \cap A_2 \cap A_4| = |A_1 \cap A_3 \cap A_4| = |A_2 \cap A_3 \cap A_4| = 19!$

$|A_1 \cap A_2 \cap A_3 \cap A_4| = 17!$

$S_0 = 26!$ $S_1 = 3 \times 24! + 23!$

$S_2 = (3 \times 22!) + (3 \times 21!)$

$S_4 = 17!$

$E_2 = 3 \times (22! + 21!) - 3 \times (20! + 3 \times 19!) + 6 \times 17!$

Module - 5

9. a. **Discuss Konigsberg bridge problem.** **(05 Marks)**

Ans. Is it possible to start from any one banle area and cover all bridges exactly one and reach starting point. No solution no euler circuit. graph.

b. **Let G - G(V,E) be a simple graph with m edges and 'n' vertices. Then prove that:**

(i) $m \le \frac{1}{2} n(n-1)$

(ii) For a complete graph k_n, $m = \frac{1}{2} n(n-1)$ edges

(iii) How many vertices and edges are there for K_4 - and K_{11}

Ans. No. of edges do not exact number of pair of vertices and no. of pair of vertices chosen from n vertices of ${}^nC_2$

For a complete graph exactly ${}^nC_2$ edges

k_4 = Edge = 28

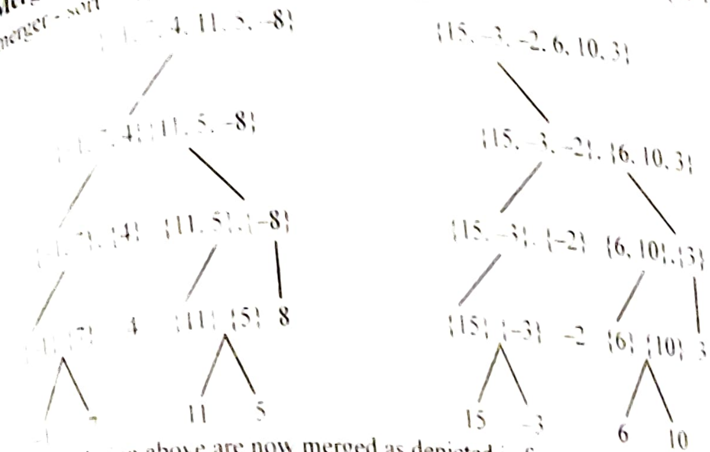
Vertices = 11

k_{11} = 78 vectices

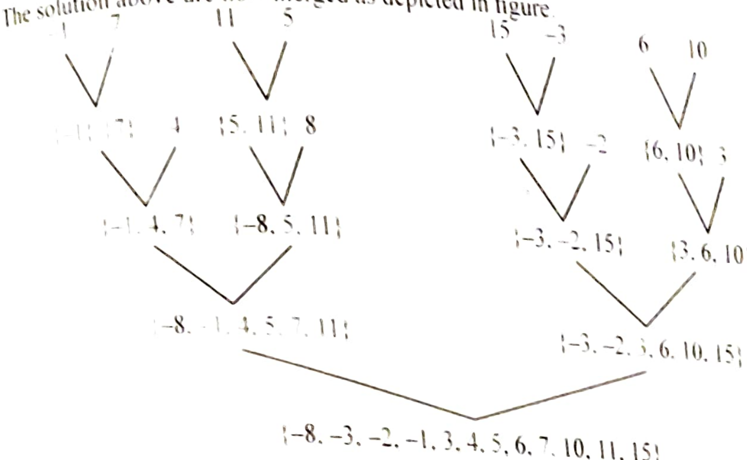
= 77 edges

c. **Merge sort the list –1,7,4,11,5,–8,15,–3,–2,6,10,3.** **(05 Marks)**

Ans. merger - sort –1,7,4,11,5,–8,15,–3,–2,6,10,3



The solution above are now merged as depicted in figure.



OR

10. a. **Prove that a tree with 'n' vertices has n – 1 edges.** **(05 Marks)**

Ans. From the definition of a tree a root comprise indegree zero and all other nodes comprise indegree one. There should be (n-1) incoming area to the (n – 1) non - root nodes. of there is any another arc, this are should be terminating at any of the nodes. If the node is root, after that in indegree will become one and that in contradiction along wit the fact that root all time indegree zero. If the end point of this extra edge in any non - root node after that in indegree will be two which is once again a contradiction.

Therefore there cannot be more arcs.

∴ Hence a tree of a n vertices will have exactly (n – 1) edges

b. Obtain an optimal prefix code for the message LETTER RECEIVED indicate the code and weight. **(06 Marks)**

Ans. [illegible] that we sought. from this tree we obtain the following optimal [illegible] with which we started

V 1011 [illegible] 1010 [illegible] R 011 T 11

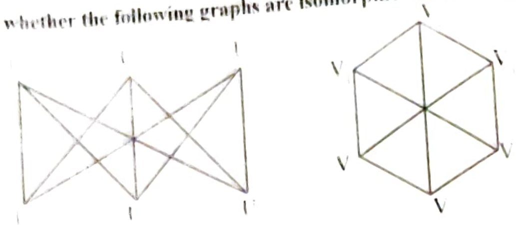
[illegible] 001 [illegible] 010

Accordingly the code for the given message

[illegible]

[illegible]1001110101011000

c. Determine whether the following graphs are isomorphic or not. **(05 Marks)**



Ans.

[illegible]

These yield following correspondence between the vectors

[illegible]

Edges and the certicies are one - to - one correspondance and that there preserve the adjancy of vertices. In view of the existence of these correspondence we infer that the two graph are inomorphic

Third Semester B.E. Degree Examination, CBCS - June/July 2018

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note : *Answer any FIVE full questions, selecting ONE full question from each module.*

Module - 1

1. a. **Prove that for any propositions p,q,r the compound proposition :** **(06 Marks)**
$\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$ is a tautology.

Ans.

p	q	r	$p \to q$	$q \to r$	$p \to r$	$p(q \to r)$	$(p \to q) \to (p \to r)$	$(1) \to (2)$
0	0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

b. **Prove the following logical equivalence using the laws of logie :** **(05 Marks)**
$(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg (q \vee p)$.

Ans. $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)]$

$(p \to q) \wedge [\neg q \wedge (\neg q \vee r)]$

$(p \to q) \wedge \neg q$

$\neg[(p \to q] \to \neg q]$

$\neg[\neg(p \to q) \vee q]$

$\neg[(p \wedge \neg q) \vee q]$

$\neg[(q \vee p) \wedge (q \vee \neg q)]$

$\neg[(q \vee p)]$

c. **Prove the following logical equivalence using the laws of logic :** **(05 Marks)**
$(\neg p \wedge (\neg q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r.$

Ans.

$$[\neg p \wedge (\neg q \wedge r)] \Leftrightarrow (\neg p \wedge \neg q) \wedge r$$

$$\Leftrightarrow [\neg(p \vee q)] \wedge r$$

$$\Leftrightarrow r \wedge [\neg(p \vee q)]$$

$$\text{and}(q \wedge r) \vee (p \wedge r) \Leftrightarrow (r \wedge q) \vee (r \wedge p)$$

$$\Leftrightarrow r \wedge (q \vee p)$$

$$\Leftrightarrow r \wedge (p \vee q)$$

$$[\neg p \wedge (\neg q \wedge r) \vee (q \wedge r) \vee (p \wedge r)]$$

$$\Leftrightarrow \{r \wedge [\neg(p \vee q)]\} \vee \{r \wedge (p \vee q)\}$$

$$\Leftrightarrow r \wedge \{\neg(p \vee q) \vee (p \vee q)\}$$

$$\Leftrightarrow r$$

OR

2. a. Prove the validity of the arguments using rule of inference.

$(\neg p \vee q) \rightarrow (r \wedge s)$
$r \rightarrow t$
$\underline{\neg t}$
$\therefore p$ **(05 Marks)**

Ans.

$$[(\neg p \vee \neg q) \rightarrow (r \wedge s)] \wedge (r \rightarrow t) \wedge (\neg t)$$

$$\Rightarrow [(\neg p \vee \neg q) \rightarrow (r \wedge s)] \wedge \neg r$$

$$\Rightarrow [(\neg p \vee \neg q) \rightarrow (r \wedge s)] \wedge (\neg r \vee \neg s)$$

$$\Rightarrow [(\neg p \vee \neg q) \rightarrow (r \wedge s)] \wedge [\neg(r \wedge s)]$$

$$\Rightarrow \neg(\neg p \vee \neg q)$$

$$\Rightarrow p \wedge q$$

$$\Rightarrow p$$

b. Test validity of the arguments using rule of inference.

$(\neg p \vee q) \rightarrow r$
$r \rightarrow (s \vee t)$
$\neg s \wedge \neg u$
$\underline{\neg u \rightarrow \neg t}$
$\therefore p$ **(05 Marks)**

Ans.

$$[(\neg p \vee q) \rightarrow r] \wedge [r \rightarrow (s \vee t)] \wedge (\neg s \wedge \neg u) \wedge (\neg u \rightarrow)$$

$$\Rightarrow [(\neg p \vee q) \rightarrow (s \vee t)] \wedge \neg s \wedge [\neg u \wedge (\neg u \rightarrow \neg t)]$$

$$\Rightarrow [(\neg p \vee q) \rightarrow (s \vee t)] \wedge [\neg s \wedge \neg t]$$

$$\Rightarrow [(\neg p \vee q) \rightarrow (s \vee t)] \wedge [\neg(s \vee t)]$$

$$\Rightarrow \neg(\neg p \vee q)$$

$$\Rightarrow p \wedge \neg q$$

$$\Rightarrow p$$

c. Find whether the following argument is valid ;
No Engineering student of 1st or 2nd semester studies logic
$\underline{\text{Anil is an engineering student who studies logic}}$
$\therefore$ Anil is not in second semester **(06 Marks)**

Ans. Let set of are engineering students be universe
$p(x)$: x is in 1st sem
$q(x)$: x is in 2nd sem
$r(x)$: x studies logic
a : Anil

$$\forall x\{p(x) \vee q(x) \rightarrow \neg r(x)\}$$

$$\underline{\qquad r(a) \qquad}$$

$$\therefore \neg q(a)$$

$$\forall x\{[p(x) \vee q(x)] \rightarrow r(x)\} \wedge r(a)$$

$$\Rightarrow \{[p(a) \vee q(a)] \rightarrow \neg r(a)\} \wedge r(a)$$

$$\Rightarrow r(a) \wedge \{r(a) \rightarrow \neg[p(a) \vee q(a)]\}$$

$$\Rightarrow \neg[p(a) \vee q(a)]$$

$$\Rightarrow \neg p(a) \wedge \neg q(a)$$

$$\Rightarrow \neg q(a)$$

Valid arguments

Module - 2

3. a. Prove by mathematical induction that : **(05 Marks)**
$1^2 + 3^2 + 5^2 + \ldots + (2n - 1)^2 = 1/3\ n(2n - 1)(2n + 1)$.

Ans. Let $s(n)$ denote the given statement

Basic step $\therefore s(1)\ 1^2 = \frac{1}{3} \times 1 \times 2$ which is true

Induction step:

It is true for $n = k$ where $k \geq 1$

$$1^2 + 3^2 + 5^2 + \ldots + (2x-1)^2 = \frac{1}{3} x (2x-1)(2x+1)$$

Adding $(2x+1)^2$ both sides

$$1^2 + 3^2 + 5^2 + \ldots + (2x-1)^2 + (2x+1)^2$$

$$= \frac{1}{3} x (2x-1)(2x+1) \rightarrow (2x+1)^2$$

$$= \frac{1}{3}(2x+1)\{k(2u-1) + 3(2u+1)\}$$

$$= \frac{1}{3}(2x+1)\{2x^2 + 5x + 3\}$$

$$= \frac{1}{3}(2x+1)(x+1)(2x+3)$$

b. A sequence $\{C_n\}$ is defined recursively by, $C_n = 3C_{n-1} - 2C_{n-2}$ for all $n \geq 3$ with $C_1 = 5$ and $C_2 = 3$ as the initial conditions, show that $C_n = -2^n + 7$. **(06 Marks)**

Ans. From the given recursive relation, we have

For n ... 2C ... 3 ... 2x5 using C ... 5

C ... 3C ... 2C ... 2 ... 7 C ... 3

9 ... 10 ...

So it will be true for k also

Then using the given rewisive relation

$C \ldots 3c(x-1) \ldots (x-1) \ldots 3C \ldots -2C$

$\ldots (\ldots 7) - 2x(-2^{\ldots} + 7)$

$= (-3 \ldots 2^{\ldots} + 21) + (2^{\ldots} - 14)$

$(-3+1)2^k + (21-14) = -2^{k+1} + 7$

c. Determine the coefficient of xyz^2 in the expansion of $(2x - y - z)^4$. **(05 Marks)**

Ans. The general term is given by

$\binom{4}{n_1\, n_2\, n_3}(2x)^{n_1}(-y)^{n_2}(-z)^{n_3}$

By taking $n_1 = 1, n_2 = 1, n_3 = 2$ we have

$\binom{4}{1\,1\,2}(2x)(-y)(-z)^2$

$= \dfrac{4!}{1!1!2!}(-2xyz^2) = -24xyz^2$

OR

4. a. A certain question paper contains two parts A and B, each containing 4 questions. How many different ways a student can answer 5 questions by selecting atleast 2 questions from each part? **(05 Marks)**

Ans. Part A - 4 question , part B - 4 question

Atleast 2 question to be selected from each part to fulfill the requirement of 5 question

i.e. (Part A - 2Q & Part B - 3Q)

OR

(Part A - 3Q & Part B - 2Q)

i.e., $C \times C + C \times C$

24 + 24 = 48 ways

b. Prove by mathematical induction that, for every positive integer n, 5 divides $n^5 - n$. **(06 Marks)**

Ans. Let S(n) be the given statements

Basic ... 5 decides 1 ... 1 this is true

Induction

We assume that S(n) is true for

n ... i.e., 5 divides $k^5 - k$

This means that $k^5 - k$ is multiple of 5

$k^5 - k = 5m$

i.e. $(x+1)^5 - (x+1) = (x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1) \ldots$

$= (x^5 - k) + 5(x^4 + 2x^3 + 2x^2 + k)$

$= 5m + 5(x^4 + 2x^3 + 2x^2 + k)$

$= 5(m + k^4 + 2x^3 + 2x^2 + k)$

c. How many numbers greater than 1000000 can be formed by using the digits 1,2,2,2,4,4,0? **(05 Marks)**

Ans. The given number of digits are 7 in number and 1000000 also has 7 digits.

In order to have the numbers greater than 1000000 the number has to be begin with 1 or 2, or 4

The number of number begins with $1 = \dfrac{6!}{3!\,2!} = 60$

The number of number begins with $2 = \dfrac{6!}{2!\,2!} = 180$

The number of number begins with $4 = \dfrac{6!}{3!} = 120$

By sem rule $= 60 + 180 + 120 = 360$

Module-3

5. a. Let $f: R \to R$ be defined by **(06 Marks)**

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$$

Ans.

$f^{-1}(0) = \{5/3\}$

$f^{-1}(1) = \{2, 0\}$

$f^{-1}(-1) = \{4/3\}$

$f^{-1}(3) = \{\frac{8}{3}, \frac{-2}{3}\}$

$f^{-1}(-3) = \{\frac{-2}{3}\}$

$f^{-1}(-6) = \phi$

b. Evaluate S(5, 4). **(05 Marks)**

Ans. S(5,4)

$$S(m,n) = \frac{1}{n!}\sum_{k=0}^{n}(-1)^k({}^nC_{n-k})(n-4)^m$$

$$S(5,4) = \frac{1}{4!}\sum_{k=0}^{4}(-1)^k({}^4C_{4-k})(4-k)^5$$

$$= \frac{1}{4!}\{4^5 - ({}^4C_3)\times 3^5 + ({}^4C_2)\times 2^5 - ({}^5C_1)\times 1^5\}$$

$$= \frac{1}{4!}\{4^5 - 4\times 3^5 + 6\times 2^5 - 4\} = \frac{240}{4!} = 10$$

c. Let f, g, h be the function form R to R defined by f(x) = x + 2, g(x) = x - 2, h(x) = 3x for ail x ∈ R. Find gof, fog, fof, hog, foh. **(05 Marks)**

Ans. gof(x) = g{f(x)} = g{x+2} = (x+2) − 2 = x

fog(x) = f{g(x)} = f{x-2} = (x-2) + 2 = x

fof(x) = f{f(x)} = f(x+2} = x+2+2 = x+4

hog(x) = h{g(x)} = h{x-2} = 3(x-2) = 3x - 6

foh(x) = f{h(x)} = fh{3x} = 3x + 1 = 3x + 2

OR

6. a. Let 'S' be the set of all non-zero integers and A = S x S on A, define the relation R by (a, b)R(c, d) if and only if ad = bc. Show that 'R' is an equivalence relation. **(06 Marks)**

Ans. First we note that (a,a) R(a,a) because aa = aa

For any a ∈ S ∴ the given R in reflexive on A

Suppose (a,b) R (c)

d then ad = bc and therefore cb = da

Hence (Cd) R (a,b) ∴ R is symmetric on A.

|||ly (a.b) R (c.d) & (c.d) R (e,f) then ad = bc &

cf = de , which yield af = be hence (a,b) R (e,f)

∴ R is transitive on 'A'

∴ 'R' is equilarance relation.

b. Draw the Hasse diagram representing the positive divisors of 36. **(06 Marks)**

Ans. D_{36} = {1,2,3,4,6,9,12,18,36}

∴ the relation R is

1 related to all element of D_{36}

2 related to all element of 2,4,6,12,18,36

3 related to all element of 3,6,9,12,18,36

4 related to all element of 4,12,36

6 related to all element of 6,12,18,36

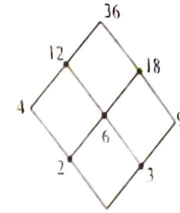
9 related to all element of 9,18,36

12 related to all element of 12 & 36

18 related to all element of 18 & 36

36 related to all element of 36

∴ The hence diagram



c. Let A = {a, b, c, d, e}. Consider the partition P - {{a, b}, {c, d}, {e}} of A. Find the equivalence relation inducing this partition. **(04 Marks)**

Ans. The given portions consists of 3 blocks

{a,b} {c,d} & {e}

Since a,b belongs to same block we have aRa, aRb, bRa,bRb

|||ly Cd belongs to same block we have CRd cR c dRd dRc

|||ly e belongs to same block we have one block so CRe

Then equilance relation is

R = {(aa) (ab) (ba) (bb) (cd) (cc) (dd) (dc) (ee)}

Module-4

7. a. In a survey of 260 college students, the following data were obtained. 64 had taken mathematics course, 94 had taken CS course, 58 had taken EC course, 28 had taken both Mathematics and EC course, 26 had taken both Mathematics and CS course, 22 had taken both CS and EC course, and 14 had taken all three types of course. Determine how many of these students had taken none of the three subjects. **(05 Marks)**

Ans. Let U represents the universe set representing all the students in survey.

|||ly M.C.E represents set q students of mathematics CS, and EC womens

|V| = 260 |M| = 64 |C| = 94 |E| = 58 |Mne| = 26

|C ^ E| = 22 |E ^ M| = 28 |M ^ C ^ E| = 14

$|\bar{M}\cap\bar{C}\cap\bar{E}| = |\overline{M\cup C\cup E}| = ?$

$|M\cup C\cup E| = |M| \rightarrow |C| + |E| - |Mne| - (C^E) - |E\cap M| \rightarrow |m\cap C\cap L|$;2m

$\therefore |M\cup C\cup E| = 154$

$|\overline{M\cup C\cup E}| = |U| - |M\cup C\cup E| = 260 - 154 = 106$; 2m

b. Find the rook polynomial for the 3x3 hoard using expansion formula. **(06 Marks)**

Ans. 3 x 3 board

1	2	3
4	5	6
7	8	9

Divide the board into 'D' & 'E' board

$R(D,x) = 1 + 4x + 2x^2$

$R(E,x) = 1 + 8x + 14x^2 + 4x^3$

Using expansion formula

$R(C,x) = xR(D,x) + R(E,x)$

$= x(1 + 4x + 2x^2) + (1 + 8x + 14x^2 + 4x^3)$

$= 1 + 9x + 18x^2 + 6x^3$

c. **Solve the recurrence relation :**

$a_n + a_{n-1} - 6a_{n-2} = 0$ $n \geq 2$, given $a_0 = -1$ and $a_1 = 8$. **(05 Marks)**

Ans. [illegible]

i.e $k^2 + k - 6 = 0$ or $(k+3)(x-2) = 0$

$k = -3, 2$

General solution $a_n = C_1(-3)^n + C_2 2^n$ — (1)

Consider $a_0 = -1$ $a_1 = 8$

Putting $n = 0$ & $n = 1$ in (1) we have

$a_0 = C_1 + C_2$ or $C_1 + C_2 = -1$ — (2)

$a_1 = -3C_1 + 2C_2$ or $-3C_1 + 2C_2 = 8$ — (3)

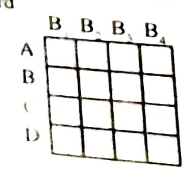
Solving (2) & (3) we get $C_1 = -2, C_2 = 1$

Then (1) became $a_n = (-2)(-3)^n + 2^n$

OR

8\. a. **An apple, a banana, a mango and an orange are to be distributed among 4 boys B_1, B_2, B_3, B_4. The boys B_1 and B_2 do not wish to have an apple, the boy B3 does not want banana or mango and B4 refuses orange. In how many ways the distribution can be made so that no boy is displeased.** **(06 Marks)**

Ans. 4 fruits and 4 boys so 4 x 4 board



Let apply the multiplication theorem

$r(C,x) = r(C_1,x) \times r(C_2,x) \times r(C_3,x)$

$r(C_1,x) = 1 + 2x$ [illegible]

$r(C_2,x) = $ [illegible]

$r(C_3,x) = 1 + x$

$r(C,x) = 1 + 5x + 8x^2 + 4x^3$

$r_1 = 5$ $r_2 = 8$ $r_3 = 4$

$S_k = r_k \times (n-k)!$ where $n = 4$

$S_0 = r_0 \times 4! = 1 \times 24 = 24$

$S_1 = r_1 \times 4! = 5 \times 6 = 30$

$S_2 = r_2 \times 2! = 8 \times 2 = 16$

$S_3 = r_3 \times 1! = 4 \times 1 = 4$

Finally $N = S_0 - S_1 + S_2 - S_3 = 24 - 30 + 16 - 4 = 0$

b. **How many permutation of 1, 2, 3, 4, 5, 6, 7, 8 are not derangements?** **(04 Marks)**

Ans. The no. of permutation of 1.2.3.....8 are 8!

The no. of deragement of 1.2.3.....8 are D8

The required no. of permutation that are not derangement denoted by $\bar{D}_8$ is given by

$\bar{D}_8$ 8! – D8

$\bar{D}_8 = 40320 - 14833 = 25487$

c. **The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day.** **(06 Marks)**

Ans. In the beginning the number of virus affected files is 1000. Let us denote this by a_0 let a_n denote the number of virus affected files after 2n hours.

Then the number increases by $a_n \times \frac{250}{100}$ in the next two hours then after 2n + 2 hours

The number $a_n + 1 = a_n + a_n \times \frac{250}{100}$

$= a_n(1 + 2.5) = a_n 3.5$

This is the rewerrence relation for the number of virus affected solve the relation

$a_n = (3.5)^n a_0 = 1000 \times (3.5)^n$

This given virus affected files after hours

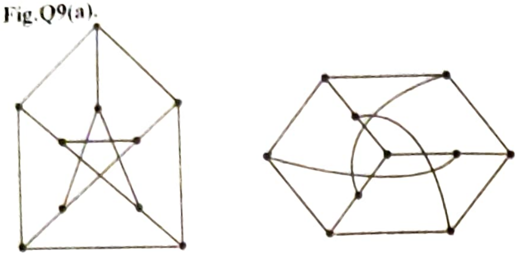
From this we get for (n = 12)

$a_{12} = 1000 \times (3.5)^{12} = 3379220508$

These number of virus affected files after one day

Module-5

9. a. Define isomorphism. Show that the following graph are isomorphic to each other. Refer Fig.Q9(a). **(06 Marks)**



Ans. Definition of isomorphic : One - to - one correspondence between edges and vertices along with adjacency between then should be preserved vertex adjency.

a ⟷ a   along with its edge

b ⟷ b   adjancency should be preserved

c ⟷ c

c ⟷ c   it is isomorphic with each other

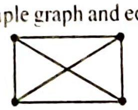
i ⟷ i

b. "A tree with 'n' vertices is having 'n - 1' edges" . Prove the given statement. **(05 Marks)**

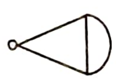
Ans. Proof for the given theorem i.e., a tree with 'n' vertices is having n - 1 edges proof by any method or by taking graph.

c. Define complete graph, general graph and Bipartite graph with example for each. **(05 Marks)**

Ans. Complete graph : Order ≥ 2 & simple graph and edge betwen every pair and vertises.

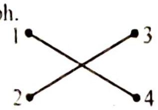


General graph : which may contain loop or parallel edges or multiple edges any one of them.



Biparlite graph : $G(c,W) \to G(V_1 \cup V_2, E)$

There is no edge between v1 to v1 or v2 and there should be edge between v1 to v2 or v2 to v1 then its biporlite graph.



OR

10. a. For a graph with "n" vertices and "m" edges, if "δ" is minimum, "Δ" is maximum of the degree of vertices. Show that: **(05 Marks)**

$\delta \le \frac{2m}{n} \le \Delta.$

Ans. Let $d_1, d_2, d_3, \ldots, d_n$ be the degrees of the vertices of the graph.

$\sum_{i=1}^{n} d_i = 2|E| = 2m$ ...(1)

Next $f = \min(d_1, d_2, d_3, \ldots, d_n)$

$f = \le d_1 \,\&\, \le d_2 \ldots\ldots \le d_n$

$\therefore nf \le \sum_{i=1}^{n} d_i$ or $nf \le 2m$ ...(2)

Next $\Delta = \max(d_1, d_2, d_3, \ldots, d_n)$

$\Delta = \ge d_1, \Delta \ge d_2 \ldots\ldots \Delta \ge d_n$

$\therefore n\Delta \ge \sum_{i=1}^{n} d_i$ or $2m \le n\Delta$ ...(3)

From (2) & (3) we have

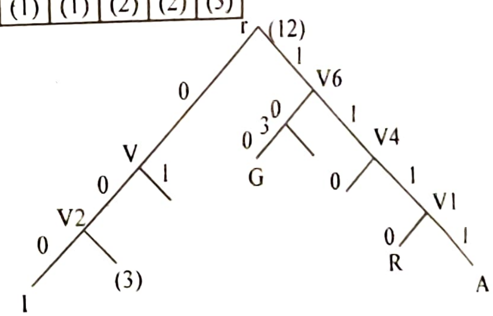
$nf \le 2m \le n\Delta$

$\therefore f \le \frac{2m}{n} \le \Delta - 2m$

b. Obtain the optimal prefix code for the message "ROAD IS GOOD". Indicate the code. **(06 Marks)**

Ans. Totally 7 letters & □ (space)

R	A	I	S	G	□	D	0
(1)	(1)	(1)	(1)	(1)	(2)	(2)	(3)

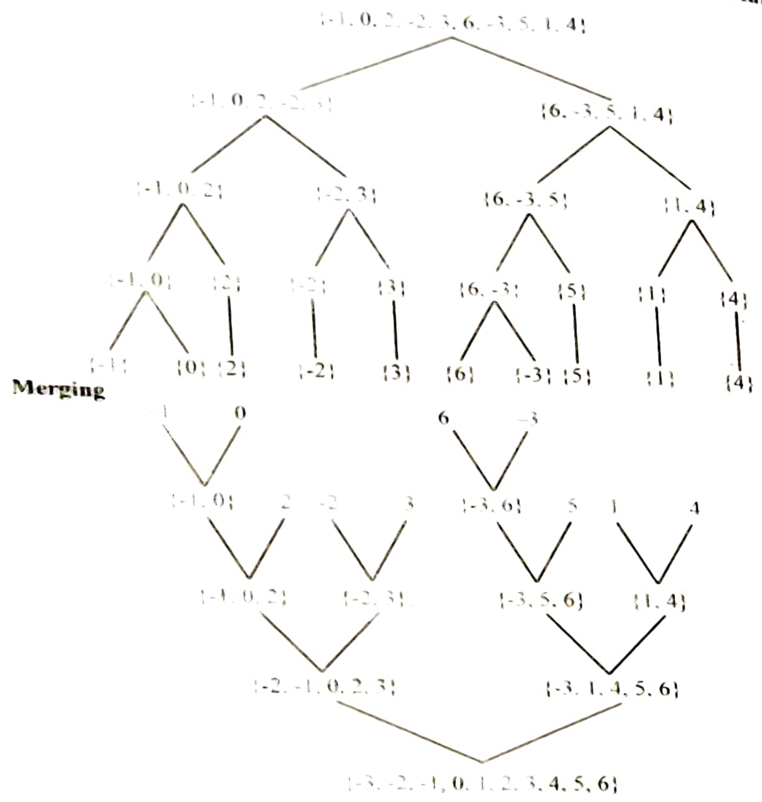


Prefix code will be

R	[illegible]	I	S	G	[illegible]	D	0
1110	1111	000	001	100	101	110	01

c. **Apply the merge sort to the following given list of element.**
{-1, 0, 2, -2, 3, 6, -3, 5, 1, 4}. **(05 Marks)**

Ans. Splitting



Third Semester B.E. Degree Examination, CBCS - Dec 2018 / Jan 2019

Discrete Mathematics Structures

Time: 3 hrs. Max. Marks: 100

Note : *Answer any FIVE full questions, selecting ONE full question from each module.*

Module-1

1. a. **Define proposition, tautology, contradiction. Determine whether the following compound statement is a tautology or not.**
$\{(p \vee q) \rightarrow r\} \leftrightarrow \{\neg r \rightarrow \neg (p \vee q)\}$ **(06 Marks)**

Ans. Proposition :- A sentence true or false but not both is called proposition.

Tautology : A compound proposition which is always True irrespective of the truth value that occurs in the preposition.

Contradiction : Is a compound proposition which is always false irrespective g the truth value that occurs in the proposition.

$\{(p \vee q) \rightarrow r\} \leftrightarrow \{\neg r \rightarrow \neg (p \vee q)\}$

p	q	r	$\neg r$	$p \vee q$	$(p \vee q) \rightarrow r$	$\neg r \rightarrow \neg (p \vee q)$	$s \rightarrow t$
0	0	0	1	0	0	0	1
0	0	1	0	0	1	1	1
0	1	0	1	1	0	1	0
0	1	1	0	1	1	1	1
1	0	0	1	1	0	1	0
1	0	1	0	1	1	1	1
1	1	0	1	1	0	1	0
1	1	1	0	1	1	1	1

$\therefore$ Not tautology

b. **Using the laws of logic, show that $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$** **(07 Marks)**

Ans.

$\Leftrightarrow (p \rightarrow q) \wedge \sim q$ $\quad \because$ Absorption law

$\Leftrightarrow (\sim p \vee q) \wedge \sim q$ $\quad \because p \rightarrow q \Leftrightarrow \sim p \vee q$

$\Leftrightarrow \sim p \wedge (\sim p \vee q)$ $\quad \because$ Commutative law

$\Leftrightarrow (\sim q \wedge \sim p) \vee (\sim q \wedge \sim q)$ $\quad \because$ Distributive law

$\Leftrightarrow (\sim q \wedge \sim p) \vee F_0$ $\quad \because$ Inverse law

$\Leftrightarrow \sim q \wedge \sim p$ $\quad \because$ Identity law

$\Leftrightarrow \sim (q \vee p)$ $\quad \because$ De morgon's law

c. **Establish the validity of the following argument**

$\forall x, p(x) \vee q(x)$
$\exists x, \neg p(x)$
$\forall x, \neg q(x) \vee r(x)$
$\forall x, s(x) \rightarrow \neg r(x)$
$\therefore \exists x, \neg s(x)$

(07 Marks)

Ans. We have $\{\forall x, p(x) \vee q(x)\} \wedge \{\exists x, \neg p(x)\}$

$\Rightarrow \{p(a) \vee q(a)\} \wedge \{\neg p(a)\}$

$\Rightarrow q(a)$ dis junction allgorism

$\{\forall x, p(x) \vee q(x)\} \wedge \{x, \neg q(x)\} \wedge \forall x, \neg q(x) \vee r(x)$

$\Rightarrow q(a) \wedge \{\neg q(a) \vee r(a)\}$

$\Rightarrow r(a)$ dis junction allgorism

$r(a) \wedge \{s(a) \rightarrow \neg r(a)\}$

$\Rightarrow \neg s(a)$ modues theorem

$\Rightarrow \exists x (\neg s(n))$

Hence given argument is valid

OR

2\. a. **Define converge, inverse and contra positive of a conditional. Find converse, inverse and contra positive of $\forall x, (x > 3) \rightarrow (x^2 > 9)$, where universal set is R.** **(06 Marks)**

Ans. Let p, q are two proposition also $p \rightarrow q$ is the conditional proposition then

i) Proposition $q \rightarrow p$ is called convage
ii) Proposition $\sim p \rightarrow \sim q$ is called Inverse
iii) Proposition $\sim q \rightarrow \sim p$ is called contra positive

Converse : $\forall x, x^2 > 9 \rightarrow (x > 3)$
Inverse : $\forall x, x \not> 3 \rightarrow (x^2 \not> 9)$
Contra positive : $\forall x, (x^2 \not> 9) \rightarrow (x \not> 3)$

b. **Test the validity of the following arguments:**

i) If there is a strike by students, the exam will be postponed but the exam was not postponed.
$\therefore$ there was no strike by students.

ii) If Ravi studies, then he will pass in DMS.
If Ravi doesn't play cricket, then he will study.
Ravi failed in DMS.
$\therefore$ Ravi played cricket **(06 Marks)**

Ans. i) The argument is as follows

$p \rightarrow q$
$\sim p$
$\overline{\therefore \sim p}$

We do not have any rule $(p \rightarrow q) \wedge \sim p$.
Hence we shall prepare the truth table for the compound proposition

$[(p \rightarrow q) \wedge \sim p] \rightarrow \sim q$

p	q	p → q	~p	~q	(p → q) ∧ ~p	p → q
T	T	T	F	F	F	T
T	F	F	F	T	F	T
F	T	T	T	F	T	F
F	F	T	T	T	T	T

c. **Define dual of logical statement. Write the dual of the statement $(p \vee T_0) \wedge (q \vee F_0) \vee (r \wedge S \wedge T_0)$.** **(02 Marks)**

Ans. The word 'Dual' in a general perspective is "Consisting two parts"
$(p \vee T_0) \wedge (q \vee F_0) \vee (r \wedge S \wedge T_0)$ is dual

d. **Let $p(x) : x \geq 0$**
$q(x) : x^2 \geq 0$ and $r(x) : x^2 - 3x - 4 = 0$
Then, for the universe completing of all real numbers, find the truth values of:
i) $\exists x\{p(x) \wedge q(x)\}$ ii) $\forall x\{(p(x) \rightarrow q(x)\}$ iii) $\exists x\{p(x) \wedge r(x)\}$ **(06 Marks)**

Ans. i) if $x = 1$ both $p(x) \exists g(x)$ is true
$\therefore \exists x = p(x), q(x)$ is true
ii) $\forall x$ **$(p(x) \rightarrow q(x)$ is true** $\Rightarrow p(x) \rightarrow q(x)$ is true always
$\therefore$ value is 1

Module-2

3\. a. **Prove that for any positive integer n, $\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$ F_n denote the Fibonacci number.** **(06 Marks)**

Ans.

For $n = 1$ $\frac{F_0}{2} = 1 - \frac{F_3}{2}$ Sine $F_0 = 0$ $F_3 = 2$

$P(1)$ is true

Assume $P(K)$ is true

$$\sum_{i=1}^{K} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{K+1}}{e^{2K}}$$

$$\sum_{i=1}^{K+1} \frac{F_{i-1}}{2^i} = \sum_{i=1}^{K} \frac{F_{i-1}}{2^i} + \frac{F_K}{2^{K+1}}$$

$$= \left(1 - \frac{K+2}{2^K}\right) + \frac{F_K}{2^{K+1}}$$

$$= 1 - \frac{1}{2^{K+1}}(2F_{K+2} - F_K)$$

$$= \frac{1}{2^K} + [F_{K+1} + F_{K+2}]$$

P(K+1) is true hence it is true $\forall n \in N$

b. **How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000?** **(07 Marks)**

Ans.

$$\frac{6!}{2!1!1!1!1!}+\frac{6!}{1!2!2!1!}+\frac{6!}{1!2!2!1!}$$

$= 360+180+180$

$= 720$

c. **Determine the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$.** **(07 Marks)**

Ans.

$$(a+2b-3c+2d+5)^{16}=\binom{16}{2,3,2,5,4}a^2(2b)^3(-3c)^2(2d)^5\times 5^4$$

$$=3\times 2^3\times\frac{5^3\times 10!}{(4!)^3}$$

$$=a^2b^3c^2d^5$$

OR

4. a. **Prove by using principle of mathematical induction**

$$\sum_{i=1}^{n} i.2^i = 2+(n-1).2^{n+1}$$

(06 Marks)

Ans.

for $n=1 \sum_{i=1}^{1} i.2^i = 2+(1-1)2^{1+1}$

$\Rightarrow 2=2$ is true

$S(k)$ is true $\sum_{i=1}^{k} i.2^i = 2+(K-1).2^{k+1}$ for $n=k+1$

$$\sum_{i=1}^{k+1} i.2^i = \sum_{i=1}^{k} i.2^i+(K+1).2^{k+1}$$

$= 2+(K-1)2^{k+1}+(K+1)2^{k+1}$

$= 2+(2K)2^{k+1}$

$= 2+K.2^{k+2}$

$S(K+1)$ is true

$S(K)$ is true $\forall n \in N$

b. **A committee of 12 is to be selected from 10 men and 10 women. In how many ways can the selection be carries out if**

i) There are no restrictions

ii) There must be six men and six women

iii) There must be an even number of women. **(07 Marks)**

Ans. **i) There are no restrictions**

$$\binom{20}{12}$$

ii) There must be six men and six women

$$\binom{10}{6}\binom{10}{6}$$

iii) There must be an even number of women.

$$\sum_{i=1}^{5}\binom{10}{12-2i}\binom{10}{2i}$$

c. **Determine the number of integer solutions of $x_1+x_2+x_3+x_4=32$ where $x_i\geq 0$, $1\leq i\leq 4$.** **(07 Marks)**

Ans. All x_1, x_2, x_3, x_4 are non negative integer so the total solution are

$$\binom{4+32-1}{32}=\binom{35}{32}$$

All the x_1, x_2, x_3, x_4 are positive integer

$x_i > 0$

$x_i \geq 1$

$x_i - 1 \geq 1-1$

$x_i - 1 \geq 0$

$y_i \geq 0$

Let's $x_1+x_2+x_3+x_4=32$

$x_1+x_2+x_3+x_4-4=32-4$

$(x_1-1)+(x_2-1)+(x_3-1)+(x_4-1)=28$

$y_1+y_2+y_3+y_4=28$

Now all y_1, y_2, y_3, y_4 are non negative integer

$$\binom{4+28-1}{28}=\binom{31}{28}$$

Module-3

5. a. **If A = {1, 2, 3, 4, 5} and there are 6720 injective functions f: A → B, what is |B|?** **(03 Marks)**

Ans. If |A| = m and |B| = n W.K.T the number of injective function from A → B is P

By data |A| = 5 and $^{n}P_5 = 6720$, n = ?

That is $n(n-1)(n-2)(n-3)(n-4) = 6720$

If n = 6 LHS = (6)(5)(4)(3)(2) = 720 ≠ RHS

If n = 7 LHS = (7)(6)(5)(4)(3) = 2520 ≠ RHS

If n = 8 LHS = (8)(7)(6)(5)(4) = 6720 = RHS

Hence n = 8

Thus the required |B| = 8

b. Let m, n be positive integers with $1 < n \leq m$ then prove that. **(05 Marks)**
$s(m+1, n) = s(m, n-1) + ns(m, n)$

Ans.

$$s(n+d, n) = n^d\left(s(n, n) + \sum_{j=0}^{d-1} n^j s(n-1) + (d-j)(n-1)\right)$$

$$= 1 + \sum_{n=1}^{d/2 - 1} S\left(l_{n-4} + (d-2j), l_n - 4\right) +$$

$$+ \sum_{j=0}^{d/2 - 1} S\left(n-1 + (d-2j-1), n-1\right)$$

$$\sum_{j=0}^{n} S\left(n-1+(d-2j-1), n-1\right) = 0$$

c. If $f : R \to R$ defined by $f(x) = x^2$, determine whether the function is one-to-one and whether it is onto. If it is not onto, find the range. **(06 Marks)**

Ans. f(1) = f(−1) = 1

∴ f is not one - one

and it is not onto

∴ range f(R) = (0, ∞)

i.e., no negative real number have pre-image

d. Let A = {1, 2, 3, 4, 5} × {1, 2, 3, 4, 5} and define R on A by (x_1, y_1) R (x_2, y_2) if $x_1 + y_1 = x_2 + y_2$, verify that R is an equivalence relation on A. **(06 Marks)**

Ans. For all (x, y) ∈ A

We have

i) $x + y = y + x$

ii) $(x_1, y_1)R(x_2, y_2) \Rightarrow x_1 + y_1 = x_2 + y_2$

$\Rightarrow x_2 + y_2 = x_1 + y_1$

$\therefore (x_2, y_2)R(x_1, y_1)$

iii) $x_1 + y_1 = x_2 + y_2$

$x_2 + y_2 = x_3 + y_3$

$\Rightarrow x_1 + y_1 = x_3 + y_3$

∴ R is equivalent relation

OR

6. a. If $f: R \to R$ defined by $f(x) = x^3$ determine whether f is invertible and if determine f^{-1}. **(05 Marks)**

Ans.

$f(x) = f(y)$

$\Rightarrow x^3 = y^3$

$\to x = y$

$\therefore f(y)$ is one – one

also f is onto

$y = x^3 \Rightarrow x = y^{1/3}$

$\therefore f^{-1} = \{(x, y) \int x = y^{1/3}\}$

b. Define the relation R for two lines l and l_2 by l_1 R l_2 if l_1 is perpendicular to l_2. Determine whether the relation is reflexive, symmetric, antisymmetric or transitive. **(05 Marks)**

Ans.

Since l_1 is not $\perp^{re}$ to L.

∴ R is not reflexive

if $l_1 \perp k_2$ and $l_2 + l_1$

∴ R is symmetric

if $l_1 \perp l_2$ and $l_2 \perp l_1$ but $l_1 \neq l_2$

∴ R is not antisymmetric

if $l_1 \perp l_2$ and $l_2 \perp l_3$ then

l_1 need not to be ⊥ to l_3

∴ R is not transitive

c. Let A = {1, 2, 3, 6, 9, 18} and R on A by xRy if x|y. Draw the Hasse diagram for the poset(A, R). **(05 Marks)**

Ans. x Ry is x divides y by data

1R all the members of A

2R (2, 6, 18)

3R (3, 6, 9, 18)

6R (6, 18)

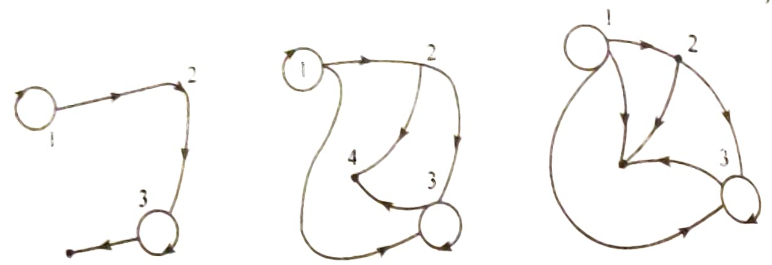
9R (9, 18)

18R (18)

$[1R_2, 1R_3], [2R_6], [3R_6, 3R_9], [6R_{18}, 6R_{18}]$

d. **For A = {1, 2, 3, 4}, let R{(1, 1) (1, 2) (2, 3) (3, 3) (3, 4)} be a relation on A. Draw the directed graph G on A that is associated with R. Do likewise for R^2, R^3.** **(05 Marks)**

Ans.



R R^2 R^3

Module-4

7. a. **Determine the number of positive integers n where $1 \le n \le 100$ and n is not divisible by 2, 3 or 5.** **(06 Marks)**

Ans.

$$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3| = |\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3|$$

$$= |s| - \Sigma|A_i| + \Sigma|A_i \cap A_j| - |A_i \cap A_j \cap A_k|$$

$$= 100 - (50 + 33 + 20) + (16 + 10 + 6)$$

$$= 26$$

b. **How many derangements are there for 1, 2, 3, 4 and 5?** **(07 Marks)**

Ans. The derrangement are 1, 2, 3, 4

$$d_5 = 5!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$$

$$= 5!\left(1 - \frac{1}{1} + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120}\right)$$

$$d_5 = 44$$

c. **Solve the recurrence relation $2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n$, $n \ge 0$, $a_0 = 0$, $a_1 = 1$, $a_2 = 2$.** **(07 Marks)**

Ans. Let

$$a_n = c r^n \text{ for } c, r \neq 0$$

$$x \ge 0 \Rightarrow 2r^3 - r^2 - 2r + 1 = 0$$

$$\Rightarrow (2r - 1)(r - 1)(r + 1) \Rightarrow r - \frac{1}{2}, 1, -1$$

$$\therefore a_n = c_1 (1)^n + c_2(-1)^n + c_3\left(\frac{1}{2}\right)^n$$

$$= c_1 + c_2(-1) + c_3\left(\frac{1}{2}\right)^n$$

$$a_n - \frac{5}{2} + \frac{1}{6}(-1)^n + \left(\frac{-8}{3}\right)\left(\frac{1}{2}\right)^n \quad n \ge 0$$

OR

8. a. **In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns car, dog, pun or byte occurs?** **(06 Marks)**

Ans. Let U be the set of all permutation y the 26 Letters and they are 26! in number that is |U| = 26!

The given patterns CAR, DOG, PUN, BYTE are respectively 3, 3, 3, 4 letters words

Let N(i), i = 1 to 4 be the number of permutation consisting of all the pattern individually

$\therefore N(C_1) = N((2) = N((S) = A(1+2_3)! = 2+1, N((4) = 2_3!$

We observe that none of the alphabets in the given 4 patterns are repeated

$N(C_1C_2) = N(C_1C_3) = N(C_2C_3) = (26 - 6 + 2)! = 22!$

$N(C_1C_4) = N(C_2C_4) = N(C_3C_4) = (26 - 7 + 2)! = 21!$

$N(C_1C_2C_4) = (26 - 9 + 3)! = 20!$

$N(C_1C_2C_4) = N(C_1C_3C_4) = N(C_2C_3C_4) = (26 - 10 + 3)! = 19!$

$N(C_1C_2C_3C_4) = (26 - 13 + 4)! = 17!$

b. **Find the root polynomial for 3 x 3 board using the expansion formula.** **(07 Marks)**

Ans. Let B be the following 3x3 board

1	2	3
4	5	6
7	8	9

Board B'

5	6
8	9

Board B''

	2	3
4	5	6
7	8	9

We have $R_B{}^{1}(x) = 1 + 4x + 2x^2$

$R_B{}^{11}(x)$ Here $r_1 = 8$

i) Position of 2 non attacking rooks

(2, 4) (2, 6) (2, 7) (2, 9) ; (3, 4) (3, 5) (3, 7) (3, 8);

(4, 8) (4, 9) (5, 7) (5, 9) ; (6, 7) (6, 8) $\therefore r_2 = 14$

ii) Position of 3 mutually non attacking rooks

(2, 4, 8) (2, 4, 9) (3, 4, 8) (3, 5, 7) $\therefore r_3 = 14$

$R_B{}^{11}(x) = 1 + r_1x = r_2x^2 + r_3x^3 = 1 = 8x + 14x^2 + 14x^3$

Consider expansion formula

$R_p(x) = xR_n(x) + R_n(x)$
$= x(1+4x+2x^2)+(1+8x+14x^2+4x^3)$
$R_p(x) = 1+9x+18x^2+6x^3$

c. **The number of bacteria in a culture is 1000 (approximately) and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day.** **(07 Marks)**

Ans. $a_{n+1} = a_n + 2.5a_n \quad a \geq 0$
$a_n = (3.5)^n a_0$
$a_n = (3.5)^n 1000$
for $n = 12$
$a_0 = 3$
$a_1 = 3+9$
$a_2 = 220$
$a_n = 508$

Module-5

9. a. **Show that the graphs Fig.Q9(a)(i) and (ii) are isomorphic.**

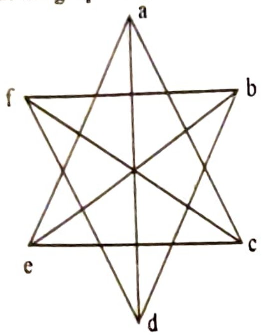


Fig.Q9(a)(i)

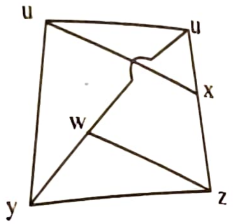


Fig.Q9(a)(ii) **(06 Marks)**

Ans. Both graph have the same numbers of vertices edges and some degree sequences
Associated $a \leftrightarrow u$, $b \leftrightarrow w$, $c \leftrightarrow x$, $d \leftrightarrow y$, $e \leftrightarrow v$, $f \leftrightarrow z$.
This presetive adjacent and cycle
Hence graph are isomorphic

b. **Let G = (V, E) be an undirected graph or multigraph with no isolated vertices. Then prove that G has an Euler circuit if and only if G is connected and every vertex in G has even degree.** **(07 Marks)**

Ans. $P \Rightarrow Q$ we show that if a connected graph G has an Eular circut, Then all $v \in v(G)$ has even degree.
An euler circuit is a closed walk such that even edge is a connected graph G is traversed exactly once. We define $w = v_1, v_2, \ldots\ldots v_k$ such that $v_0 = v_k$ to be that euler circuit in G, y lengths
$|v(G)| > 0$
Case 1 : $0 < i < k$; 1 edge $\{V_{i-1}, V_i\}$ $\{V_i V_{i+1}\}$ deg $(V_i) = 2n$

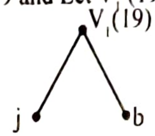
Case 2 : $0 \in 0$, k; beginning of w, we traverse any incident to V_0, that is $\{V_0, V_1\}$.
By defined y Euler circuit.
$Q \Rightarrow P \quad v \in v(G)$ has even degree, then G has euler circuit.

c. **Construct an optimal prefix code for the symbols a, b, c, d, e, f, g, h, i, j that occur with respective frequencies 78, 16, 30, 35, 125, 31, 20, 50, 80, 3.** **(07 Marks)**

Ans. **Step 1 :** Give symbols a, b, c, d, e, f, g, h, i, j are arranged in on decreasing order of their frequencies

j	b	g	c	f	d	h	a	i	e
3	16	20	30	31	35	50	78	80	125

Step 2 : $w(j) + w(b) = 3 + 16 = 19$ and Let V_1 (19) be the vertex



g	c	f	d	h	a	i	e
(20)	(30)	(31)	(35)	(50)	(78)	(80)	(125)

Step 3 : $w(v_1) + w(g) = 19 + 20 = 39$
Let v_2(39) be the vertex
Step 4 : $w(c) + w(f) = 30 + 31 = 61$
Let v_3(61) be the vertex
Step 5 : $w(d) + w(v_2) = 35 + 34 = 74$
Let v_4(74) be the vertex
Step 6 : $w(h) + w(v_3) = 50 + 61 = 111$
Let v_5(111) be the vertex
Step 7 : $w(v_4) + w(a) = 74 + 78 = 152$
Let v_6(152) be the vertex
Step 8 : $w(i) + w(v_5) = 80 + 111 = 191$
Let v_7(191) be the vertex
Step 9 : $w(e) + w(v_6) = 125+152 = 277$
Let v_8(277) be the vertex
Step 10 : $w(v_7) + w(v_8) = 191 + 277 = 468$
Let v be the tree

a	b	c	d	e	f	g	h	i	j
111	110101	0110	1100	10	011	11011	010	00	110100

OR

10. a. **Let G = (V, E) be a connected undirected graph. What is the largest possible value for |V| if |E| = 19 and deg(v) > 4 for all v ∈ V?** ***(06 Marks)***

Ans. By given data
$\Sigma \deg V_i \geq 4(V) = 2|E|$

2|E| = 4
(0 = 4
8 10
|V| = 0

b. **For every tree T = (V, E) if |V| > 2, then prove that T has atleast two pendant vertices.** **(07 Marks)**

Ans. Pendant vertex is a vertex having degree one and it is also called a leaf y the tree

Let T = (V, E) be the tree and V = {$v_1, v_2, \ldots v_n$}

|V| = n ≥ 2

A tree with n vertices has (n – 1) edges

Also Σdeg (v) = 2|E|

deg(V_1) + deg(V_2) + ... + deg(V_{n-1}) + deg(V_n) = 2(n – 1) = 2n-2

If each deg (v_i) = 1 to n ≥ 2 then sum ≥ 2n

the RHS (2_{n-2}) we can easily say that atleast one y the deg (vi) should be less than 2

that is

deg(V_n) = 1 then we have

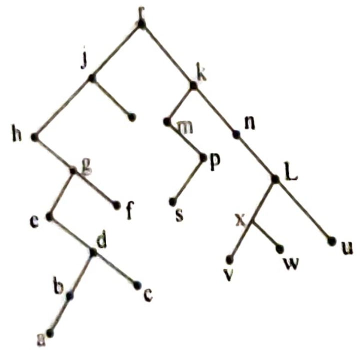
deg(V_1) + deg(V_2) ...... + deg(V_{n-1}) + 1 = 2n-2

deg(V_1) + deg(V_2) ...... + deg(V_{n-1}) = 2n-3

atleast one of the deg(V_i) i = 1 to (n – 1) should be 1

These we conclude that there ex int atlest two pendent vertices (two leaves) in tree having two or more vertices

c. **For the tree shown in Fig.Q 10(c), list the vertices according to a preorder and a postorder traversal.** **(07 Marks)**



Ans. **Pre order :** r, j, h, g, e, d, b, a, c, f, i, k, m, p, s, n, q, t, v, w, u

Post order : a, b, c, d, e, f, g, h, i, j, s, p, m, v, w, t, u, q, n, k, r

Third Semester B.E. Degree Examination, CBCS - June / July 2019

Discrete Mathematics Structures

Time: 3 hrs Max. Marks: 100

Note : *Answer any FIVE full questions, selecting ONE full question from each module.*

Module-1

1. a. **Define tautology. Verify that [p → (q → r)] → [(p → q) → (p → r)] is a tautology.** **(06 Marks)**

Ans. Tautology definition : Refer Q.No. 1.a. of Dec 2018 / Jan 2019

p	q	r	s p → (q → r)	t (p → q) → (p → r)	s → t
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	1	1	1	1

b. **If statement q has truth value 1, determine all truth value- assignments for the primitive statements p, r, s for which the truth value of the statement: (q → [(¬p ∨ r) ∧ ¬s]) ∧ [¬s → (¬r ∧ q)] is 1.** **(04 Marks)**

Ans. We have the following statement value

(q → [(¬p ∨ r) ∧ ¬s]) ∧ [¬s → (¬r ∧ q)]

Assume this statement ([~p ∨ r) ∧ ~s]) ∧ [~s → (~r ∧ q)] is t. If implication of statement q → t is true and q is true then it is only possible when t is true.

$\underbrace{[(\neg p \vee r) \wedge \neg s]}_{u} \wedge \underbrace{[(\neg s) \rightarrow (\neg r \vee q)]}_{v}$ is true

the ~~u and v~~ are both true

if [(¬p ∨ r) ∧ (¬s]) is true then both (¬p ∨ r) and ~s are true.

Sine ~s is true and should be false (0).

For (¬p ∨ r) to be true these are following three case.

(~p ∨ r)	~p	r	p
1	1	0	0
1	0	1	1
1	1	1	0

If (~s) → (~r ∧ q)] is true and ~s is also true then both ~r and q should be true. Now 15 is true then r should be false (0). From the above cases for (~p ∨ r) to be true is possible only when ~p is true (1) so p will be false p, r, and s all three false (0)

c. **Establish the following logical equivalence :** **(10 Marks)**

i) $p \vee q \vee (\neg p \wedge \neg q) \wedge r) \Leftrightarrow p \vee q \vee r$

ii) $[(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r) \Leftrightarrow p \wedge q$

Ans. i) $p \vee q \vee (\neg p \wedge \neg q) \wedge r)$

	Reasons
$\Rightarrow (p \vee q) \vee [\neg(p \vee q) \wedge r]$	$s \rightarrow t \Leftrightarrow \neg s \vee t$ Demorgan's law
$\Rightarrow [(p \vee q) \vee \neg(p \vee q)] \wedge (p \vee q \vee r)$	Distributed law
$\Rightarrow T_0 \wedge (p \vee q \vee r)$	
$\Rightarrow p \vee q \vee r$	

ii) $[(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r) \Leftrightarrow p \wedge q$

	Reasons
$\Rightarrow \neg(\neg p \vee \neg q) \vee (p \wedge q \wedge r)$	$s \rightarrow t \Leftrightarrow \neg s \vee t$
$\Rightarrow (\neg\neg p \wedge \neg\neg q) \vee (p \wedge q \wedge r)$	Demorgon's law
$\Rightarrow (p \wedge q) \vee (p \wedge q \wedge r)$	Law of double negation
$\Rightarrow p \wedge q$	Absorption law

OR

2\. a. **Establish the validity of following arguments :**

i) $(\neg p \vee \neg q) \rightarrow (r \wedge s)$
$r \rightarrow t$
$\neg t$
$\therefore p$

ii) $u \rightarrow r$
$(r \wedge s) \rightarrow (p \vee t)$
$q \rightarrow (u \wedge s)$
$\neg t$
q
$\therefore p$

(08 Marks)

Ans. i) Steps

Steps	Reasons
1) $r \rightarrow t$	Premise
2) $\neg t$	Premise
3) $\neg r$	(1) & (2) modus follows
4) $\neg r \vee \neg s$	(3) disjective ampli
5) $\neg(r \wedge s)$	(4) demorgon's law
6) $(\neg p \vee \neg q) \rightarrow (r \wedge s)$	Premise
7) $\neg(\neg p \vee \neg q)$	(6) & (5) modus follows
8) $p \wedge q$	(7) Demorgon's double negation
9) $\therefore p$	(8) Conjunctive simplification

ii) Steps	Reasons
1) q	Premise
2) $q \rightarrow (u \wedge s)$	Premise
3) $u \wedge s$	(1) & (2) modus follows
4) u	(3) conjuctive simplification
5) $u \rightarrow r$	Premise
6) r	(4) & (5) Determinent
7) s	(3) conjuctive simplification
8) $r \wedge s$	(6) & (7) conjunction
9) $(r \wedge s) \rightarrow (p \vee t)$	Premise
10) $p \vee t$	(8) & (9) Determinent
11) $\neg t$	Premise
12) $\therefore p$	(10) & (11) disjuctive syllogirm

b. **Let p(x), q(x) and r(x) be the following open statements :**
$p(x) : x^2 - 7x + 10 = 0 \quad q(x) : x^2 - 2x - 3 = 0 r(x) < 0$.
Determine truth or falsity of following statements, where universe is all integers. If a statement is false, provide a counter example.

i) $\forall x[p(x) \rightarrow \neg r(x)]$ ii) $\forall x[q(x) \rightarrow r(x)]$
iii) $\exists x[q(x) \rightarrow r(x)]$ iv) $\exists x\ [p(x) \rightarrow r(x)]$. **(08 Marks)**

Ans. i) $\forall x[p(x) \rightarrow \neg r(x)]$; True
ii) $\forall x[q(x) \rightarrow r(x)]$; False $x=3$
iii) $\exists x[q(x) \rightarrow r(x)]$; True
iv) $\exists x\ [p(x) \rightarrow r(x)]$; True

c. **Prove that for all integers 'k' and 'l', if 'k' and 'l' are both even, then $k + l$ is even and kl is even by direct proof** **(04 Marks)**

Ans. k, l are both even then $k = 2c$ and $l = 2d$ where c, d are integer. Then, sum $k + l = 2c + 2d = 2(c+d)$ with $(c+d)$ an integer. $\therefore$ $k + l$ is even $k = 2c$, $l = 2d$. For integers c, d $kl = (2c)(2d)\ k = 2(2cd)$
where 2cd is integer. $\therefore$ kl is even

Module-2

3\. a. **Define wall ordering principle and prove the following by mathematical induction :** **(12 Mark)**

i) $1^2 + 3^2 + 5^2 + ------ + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

ii) $1 \times 3 + 2 \times 4 + 3 \times 5 + ------ + n(n+2) = \frac{n(n+1)(2n+1)}{6}$

Ans. (i)

$$1^2+3^2+\ldots+(2n-1)^2=\frac{1}{3}n(2n-1)(2n+1)$$

$$s(1)\ 1^2=\frac{1}{3}\times 1\times 3$$

s(n) is true from n = k where $k \geq 1$ Then

$$1^2+3^2+5^2+\ldots+(2k-1)^2=\frac{1}{3}k(2k-1)(2k+1)$$

Adding $(2k+1)^2$ to both sides

$$1^2+3^2+5^2+\ldots+(2k-1)+(2k+1)^2$$

$$=\frac{1}{3}k(2k-1)(2k+1)+(2k+1)^2=\frac{1}{3}(2k+1)(2k^2+5k+1)$$

$$=\frac{1}{2}(2k+1)(k+1)(2k+3)$$

$\therefore s(k+1)$ is true

(ii)

$$1.3+2.4+3.5+\ldots+n(n+2)=\frac{n(n+1)(2n+7)}{6}$$

$$s(1)!\ 1(1+2)=1(1+1)(2+7)/6=3$$

Adding $n(n+3)$ both sides

$$s(x+3)=\frac{1}{6}(c(k+1)(2k+7))$$

b. Find the coefficients of:

i. x^9y^3 in the expansion of $(2x-3y)^{12}$

ii. $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$. **(08 Marks)**

Ans.

i) x^9y^3 in the expansion $(2x-3y)^{12}$

$$\binom{12}{3}2^9(-3)^3x^9y^3$$

ii) $$\frac{3\times 2^5\times 5^8\times 16!}{(4!)^2}a^2b^3c^2d^5$$

OR

4. a. A women has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in following situations,

i. There is no restriction on the choice

ii. Two particular persons will not attend separately

iii. Two particular persons will not attend together. **(06 Marks)**

Ans. (i) There is no restriction on the choice

$\Rightarrow c(11,5)=462$ ways $\rightarrow$ no choice

(ii) Two particular persons will not attend separately

$\Rightarrow c(9,3)+c(9,5)=84+126=210$ ways

$\therefore$ not attend separately

(iii) Two particular persons will not attend together

$\Rightarrow (9,4)+c(9,4)+c(9,5)=126+126+126=378$

$\therefore$ 378 ways attend together

b. How many arrangements are there for all letters in word SOCIOLOGICAL? In how many of these arrangements all vowels are adjacent. **(06 Marks)**

Ans. Number of arrangements of all letters is

SOCIOLOGICAL

$$=\frac{12!}{1!3!2!2!2!1!}$$

All vowels are adjacent

$$=\frac{7!}{1!2!2!1!1!}$$

c. For the Fibonacci sequence $F_0, F_1, F_2 \ldots$ prove that $F_n=\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n-\left(\frac{1-\sqrt{5}}{2}\right)^n\right]$ **(08 Marks)**

Ans. For n = 0 and n = 1

$$F_0=\frac{1}{\sqrt{5}}(1-1)=0 \qquad F_1=\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)-\left(\frac{1-\sqrt{5}}{2}\right)\right]$$

Result is true for n = 0 and n = 1 for n = 0,1,2,.....k

where $k \geq 1$. Then $F_{K+1}=F_K+F_{K-1}$

$$=\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^K-\left(\frac{1-\sqrt{5}}{2}\right)^K\right]+\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{K-1}-\left(\frac{1-\sqrt{5}}{2}\right)^{K-1}\right]$$

$$=\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{K-1}\left\{\frac{1+\sqrt{5}}{2}+1\right\}-\left(\frac{1+\sqrt{5}}{2}\right)^{K-1}\left\{\frac{1+\sqrt{5}}{2}+1\right\}\right]$$

$$=\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{K+1}-\left(\frac{1-\sqrt{5}}{2}\right)^{K+1}\right]$$

Module-3

5. a. Let A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5, 6}.

i. How many functions are there from A to B?

ii. How many of these are one to one?

iii. How many are onto?

iv. How man}| functions are there from B to A?

v. How many of these are onto?

vi. How many are one to one? **(06 Marks)**

$A \to B = n^m$ $B \to A = m^n$ $= \frac{n!}{(n-m)!}$

Ans. i) $|A| = m = 4 \; |B| = n = 6$

Number of functions A to B = 6^4

ii) One to one from A to B $\dfrac{n!}{(n-m)!} = 360$

iii) No onto functions from A to B

iv) Functions from B to A = 4^6 = 4096

v) Onto from B to A = 1560

vi) No one to one function from B to A

b. A computer operator is given a magnetic tape that contains 500,000 words of four or fewer lowercase letters. Can it be that the 500,000 words are all distinct? **(06 Marks)**

Ans. Each place is an n letter string can be filled is 26 ways

∴ Total number of possible strings made up of 4 or fewer letter is

$$26^4 + 26^3 + 26^2 + 26 = 4{,}75{,}254$$

If there are 5 lakh strings then atleast one string is repeated

c. Let f, g, h : R → R where $f(x) = x^2$, $g(x) = x + 5$ and $h(x) = \sqrt{x^2 + 2}$. Show that (hog) of = ho(gof). **(08 Marks)**

Ans. $f(x) = x^2$, $g(x) = x + 5$ and $h(x) = \sqrt{x^2 + 2}$

$$((hog)of)(x) = (hog)(f(x))$$
$$= hog(x^2) = h(g(x^2))$$
$$= h(x^2 + 5)$$
$$= \sqrt{(x^2+5)^2 + 2}$$
$$= \sqrt{x^4 + 10x^2 + 27}$$

$$(h(gof))(x) = h((gof)(x))$$
$$= h(g(f(x)))$$
$$= h(g(x^2))$$
$$= h(x^2 + 5)$$
$$= \sqrt{(x^2+5)^2 + 2}$$

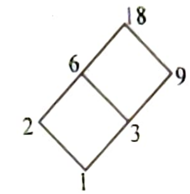
$$\sqrt{x^4 + 10x^2 + 27}$$

$\therefore (hog)of = ho(gof)$

OR

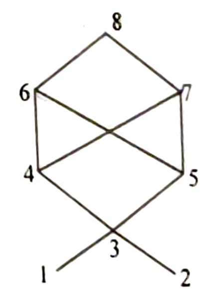
6. a. Let A = {1, 2, 3, 6, 9, 18} and define R on A by xRy if "x divides y", Draw the Hasse diagram for the poset (A, R). Also write the matrix of relation. **(08 Marks)**

Ans.



$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 6 & 9 & 18 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 6 \\ 9 \\ 18 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

b. Consider Poset whose Hasse diagram is given below. Consider B = {3, 4, 5}. Find upper and lower bounds of B, least upper bound and greatest lower bound of B. **(04 Marks)**



(Ref Fig.Q6(b)).

Ans. Upper bound = 6, 7, 8 LUB = 7

Lower bound = 6, 7, 8 LUB = 7

c. **Let $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$ and define R on A by $(x_1, y_1) R (x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$.**

i. **Verify that R is an equivalence relation on A**

ii. **Determine equivalence classes $[(1, 3)]$, $[(2, 4)]$ and $[(1, 1)]$**

iii. **Determine partition of A induced by R.** **(08 Marks)**

Ans. i) R is reflexive symmetric, transitive it is an equivalent relation

ii) $[(1,3)] = \{(1,3), (2,2)(3,1)\}$

$[(1,1)] = \{(1,1)\}$

$[(2,4)] = \{(1,5), (2,4)(3,3)(4,2)(5,1)\}$

iii)

$$A = \{(1,1)\} \cup \{(1,2)(2,1)\} \cup \{(1,3)(2,2)(3,1)\} \cup \{(1,4)(2,3)(3,2)(4,1)\}$$
$$\cup \{(1,5)(2,4)(3,3)(4,2)(5,1)\} \cup \{(2,5)(3,4)(4,3)(5,2)\} \cup \{(3,5)(4,4)(5,3)\}$$
$$\cup \{(4,5)(5,4)\} \cup \{(5,5)\}$$

Module-4

7. a. **In how many ways can the 26 letters of English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs?** **(08 Marks)**

Ans. S denote set of all permutations of 26 letters $|S| = 26!$

A be set is which CAR appears $|A_1| = 24!$

(26-

$$|A_2| = |A_3| = 2A! |A_4| = 23!$$
$$|A_1 \cap A_2| = |A_1 \cap A_3| = |A_2 \cap A_3| = (26 - 6 + 2)! = 22!$$
$$|A_1 \cap A_4| = |A_2 \cap A_4| = |A_3 \cap A_4| = (26 - 7 + 2)! = 21!$$
$$|A_1 \cap A_2 \cap A_3| = (26 - 9 + 3)! = 20!$$
$$|A_1 \cap A_2 \cap A_4| = |A_1 \cap A_3 \cap A_4| = |A_2 \cap A_3 \cap A_4| = (26 - 10 + 3)! = 19!$$
$$|A_1 \cap A_2 \cap A_3 \cap A_4| = (26 - 13 + 4)! = 17!$$
$$|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4}| = |S| = \Sigma|A_i| + \Sigma|A_i \cap A_j| + \Sigma|A_i \cap A_j \cap A_k| + |A_1 \cap A_2 \cap A_3 \cap A_4|$$
$$= 26! - (3 \times 24! + 23!) + (3 \times 22! + 3 \times 21!) + (20! + 3 \times 19!) + 17!$$

b. **There are eight letters to eight different people ta be placed in eight different addressed envelops. Find the number of ways of doing this so that atleast one letter gets to right person.** **(04 Marks)**

Ans. Number of ways of placing and letters is and ven is 8!

$\therefore$ Number of ways of placing 8 letters is evelops such that atleast one letter is in the right evelop is $8! - d_8 = (8!) - [(8!) \times e^{-1}] = 25.486$

c. **Four persons P_1, P_2, P_3, P_4 who arrive late for a dinner party find that only one chair at each of five table T_1, T_2, T_3, T_4 and T_5 is vacant. P_1 will not sit at T_1 or T_2, P_2 will not sit at T_2, P_3 will not sit at T_3 or T_4 and P_4 will not sit at T_4 or T_5. Find the number of ways they can occupy the vacant chairs.** **(08 Marks)**

Ans.

	T_1	T_2	T_3	T_4	T_5
P_1					
P_2					
P_3					
P_4					

$$r(c,x) = 1 + 7x + 16x^2 + 13x^3 + 3x^4$$
$$r_1 = 7,\ r_2 = 16,\ r_3 = 13,\ r_4 = 3$$
$$S_0 = 5! = 120$$
$$S_1 = (5-1)! \times r_1 = 168$$
$$S_2 = (5-2)! \times r_2 = 96$$
$$S_3 = (5-3)! \times r_3 = 26$$
$$S_4 = (5-4)! \times r_4 = 3$$

Number of ways is which four persons can occupy the vacant chairs

$S_0 - S_1 + S_2 - S_3 + S_4 = 120 - 168 + 96 - 26 + 3 = 25$

OR

8. a. **Find the recurrence relation and the initial condition for tire sequence 0, 2, 6, 12, 20, 30, 42, ... Hence find the general term of the sequence.** **(10 Marks)**

Ans. Given sequence be(ar) we note that

$a_0 = 0,\ a_1 = 2,\ a_1 - a_0 = 2$

$a_2 = 6,\ a_2 - a_1 = 4,\ a_3 = 12,\ a_3 - a_2 = 6$

$a_4 = 20,\ a_4 - a_3 = 8,\ a_5 = 30,\ a_5 - a_4 = 10$ and so on

$a_n - a_{n-1} = 2n$ or $a_n = a_{n-1} + 2n$ for $n \geq 1$

Recurrence relation working back words

$a_n - a_{n-1} = 2n$	$a_3 - a_2 = 2 \times 3$
$a_{n-1} - a_{n-2} = 2(n-1)$	$a_2 - a_1 = 2 \times 2$
$a_{n-2} - a_{n-3} = 2(n-2)$	$a_1 - a_0 = 2 \times 1$

Adding all these we get

$$a_n - a_0 = 2\{n + (n-1) + (n-2) + \ldots\ldots + 3 + 2 + 1\}$$
$$= \frac{2.n(n+1)}{2} = n(n+1)$$

$\therefore$ General term of given sequence is

$$a_n = n(n+1) - a_0 = n(n+1) - 0 = n^2 + n$$

b. **If $a_0 = 0, a_1 = 1, a_2 = 4$ and $a_3 = 37$ satisfy the recurrence relation $a_{n+2} + ba_{n+1} + ca_n = 0$ for $n > 0$, determine the constants b and c and then solve the relation for an.** **(10 Marks)**

Ans. $a_0 = 0,\ a_1 = 1,\ a_2 = 4,\ a_3 = 37$

$a_{n+2} + ba_{n+1} + ca_n = 0$ for $n \geq 0$

For $n = 0$ and $n = 1$

$a_2 + ba_1 + ca_0 = 0$ and $a_3 + ba_2 + ca_1 = 0$

Substituting values of a_0, a_1, a_2 and a_3 we get

$4 + b + 0 = 0$ and $37 + 4b + c = 0$

These yield $b = -4$ and $c = -21$

Recurrence solution reads $a_{n+2} - 4a_{n+1} - 21a_n = 0$

or $a_n - 4a_{n-1} - 21a_{n-2} = 0$ for $n \geq 2$

Characteristic equation is $k^2 - 4k - 21 = 0$ whose roots are $k_1 = 7$ and $k_2 = -3$

General solution for a_n is $a_n = A \times 7^n + B \times (-3)^n$

Using given conditions $a_0 = 0$ and $a_1 = 1$ we get

$0 = A + B$ and $1 = 7A - 3B$

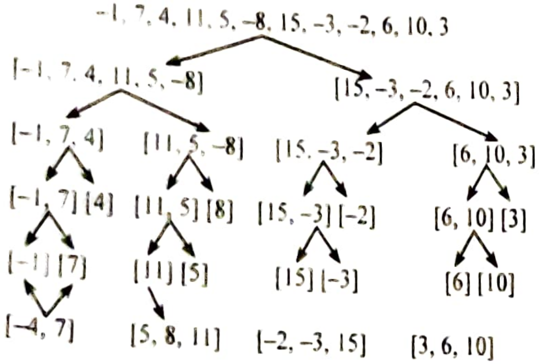
Which yields $A = -B = \dfrac{1}{10}$ puting is $a_n = A \times 7^n + B(-3)^n$

We get $a_n = \dfrac{1}{10}\{7^n - (-3)^n\}$

Module-5

9. a. Merge sort the list -1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3. **(06 Marks)**

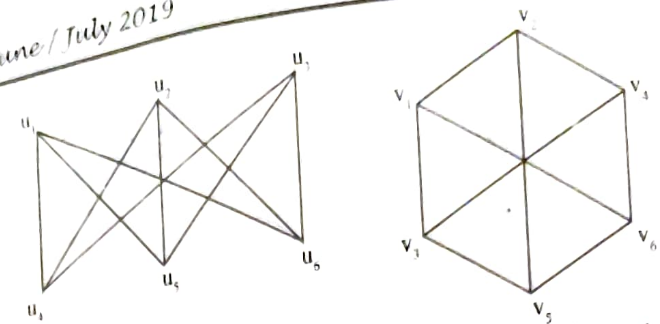
Ans.



[-1, -2, -3, -8, 3, 4, 5, 6, 7, 0, 11, 15]

]

b. Determine whether the following graphs are isomorphic or not **(06 Marks)**



Ans. Graphs are isomorphic list degree of vertices is both graph, count edges and vertices is graph, show correspondence between edges

Sub graph : Let $Q = (V, E)$ and $H = (V_1, E_1)$ be tow graph, H is said to be a subgraph of G if $V_1 \leq V$ and $E_1 \leq E$

c. Define the following with an example to each.

i) Simple graph

ii) Complete graph

iii) Regular graph

iv) Spanning sub graph

v) Induced subgraph

vi) Complete Bipartite graph

vii) Tree

viii) Complement of graph. **(08 Marks)**

Ans. **i) Simple graph :** Refer Q.No. 9.a. of June / July 2017

ii) Complete graph : Refer Q.No. 9.c. of June / July 2018

iii) Regular graph : Refer Q.No. 9.a. of June / July 2017

iv) Spanning sub graph : Refer Q.No. 9.a. of Model Question paper - 1

v) Induced subgraph : Refer Q.No. 9.a. of June / July 2017

vi) Complete Bipartite graph : Refer Q.No. 9.a. of Dec 2016 / Jan 2017

vii) Tree : A tree is a connected a cyclic graph there is a unique path between every pair of vertices in G.

viii) Complement of graph : let G(V.E) be a simple graph and let K consists of all 2-element subset of V then H = (V, K/E) is the complement of G.

OR

10. a. Define trail, circuit, path, cycle. In the graph shown below determine :

i. a walk from b to d that is not a trail

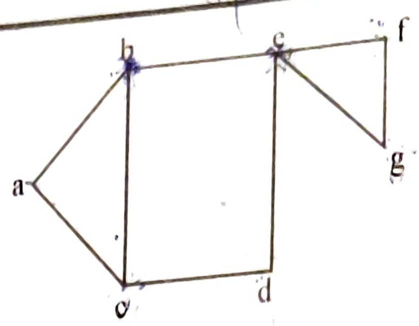
ii. b-d trail that is not a path

iii. a path from b to d

iv. a closed walk from b to b that is not a circuit

v. a circuit from b to b that is not cycle

vi. a cycle form b to b. **(10 Marks)**



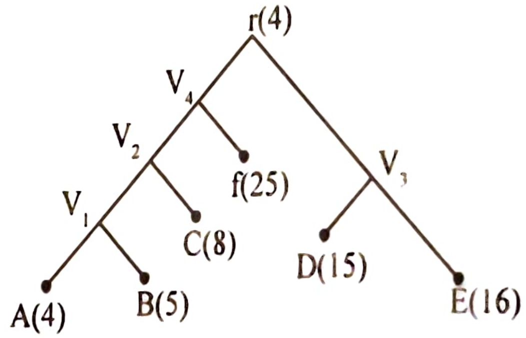
befgebcd

Ans. Definition of trial circuit path cycle   efg

1) b – c – f – g – e – b
2) b – c – a – b – e – d
3) b – c – d
4) b – e – f – g – e – b
5) b – e – d – e – f – g – e – d
6) b – a – c – d

b. Define optimal tree and construct an optimal tree for a given set of weights {4, 15, 25, 5, 8, 16}. Hence find the weight of optimal tree. **(06 Marks)**

Ans.



$$W(T)=\sum_{i=1}^{n} w_i l(w_i)$$

$$=(4\times4)+(5\times4)+(8\times3)+(15\times2)+(16\times2)+(25\times2)$$

$$=172$$

c. Prove that in a graph. The sum of degrees of all vertices is an even number and is equal to twice the number of edges in the graph. **(04 Marks)**

Ans. Consider a graph with n vertices if K vertices are of odd degree. Remabring n-k vertices are of even degree.

Denote vertices with odd degree by $V_1, V_2, \dots V_k$ vertices with even degree by $V_{K+1}, V_{K+2} \dots V_n$

∴ Sum of degrees of vertices's is

$$\sum_{i=1}^{n} \deg(vi)=\sum_{i=1}^{n} \deg(vi)+\sum_{i=K+1}^{n} \deg(vi)$$

∴ By hand sharing property

$$\sum_{rev} \deg(V)=2|E|$$

Show with an example.