Third Semester B.E. Degree Examination, Aug./Sept.2020
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find $L\{e^{-2t} t \cos 2t\}$. **(06 Marks)**

b. Express the function in terms of unit step function and hence find Laplace transform of :

$$f(t) = \begin{cases} 1 & 0 \le t \le 1 \\ t & 1 < t \le 2 \\ t^2 & t > 2 \end{cases}.$$ **(07 Marks)**

c. Solve the equation $y''(t) + 3y'(t) + 2y(t) = 0$ under the condition $y(0) = 1$, $y'(0) = 0$. **(07 Marks)**

OR

2 a. Find :

i) $L^{-1}\left\{\dfrac{s+3}{s^2 - 4s + 13}\right\}$ ii) $L^{-1}\left\{\log \dfrac{(s^2+1)}{s(s+1)}\right\}$. **(06 Marks)**

b. Find $L^{-1}\left\{\dfrac{s^2}{(s^2+a^2)^2}\right\}$ using convolution theorem. **(07 Marks)**

c. A periodic function of period 2a is defined by

$$f(t) = \begin{cases} E & 0 \le t \le a \\ -E & a < t \le 2a \end{cases}$$

Where E is a constant and show that trim $L\{f(t)\} = \dfrac{E}{S} \tan h\left(\dfrac{as}{2}\right)$. **(07 Marks)**

Module-2

3 a. Express $f(x) = x^2$ as a Fourier series in the interval $-\pi < x < \pi$. Hence deduce that $\dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} - - - - = \dfrac{\pi^2}{12}$. **(07 Marks)**

b. Obtain the Fourier seires expression of $f(x) = \begin{cases} \pi x & 0 < x < 1 \\ \pi(2-x) & 1 < x < 2 \end{cases}$. **(07 Marks)**

c. Obtain the half range cosine series for the function $f(x) = (x-1)^2$ $0 \le x \le 1$. **(06 Marks)**

OR

4 a. Obtain the Fourier series of $f(x)=\left(\frac{\pi-x}{2}\right)$ $0 < x < 2\pi$. Hence deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}-- = \frac{\pi}{4}$. **(07 Marks)**

b. Obtain the half range cosine series of $f(x) = x\sin x$ $0 \le x \le \pi$. **(07 Marks)**

c. Express f(x) as a Fourier series upto first harmonic.

x	0	1	2	3	4	5
f(x)	4	8	15	7	6	2

(06 Marks)

Module-3

5 a. Find the Fourier cosine transform of
$$f(x)=\begin{cases} x & \text{for } 0<x<1 \\ (2-x) & \text{for } 1<x<2 \\ 0 & \text{for } x>2 \end{cases}.$$ **(07 Marks)**

b. Find the Fourier transform by $f(x)=e^{-|x|}$. **(07 Marks)**

c. Obtain the inverse Z – transform by $u(z)=\frac{z}{(z-2)(z-3)}$. **(06 Marks)**

OR

6 a. Find the Fourier transform by
$$f(x)=\begin{cases} 1-|x| & |x|<1 \\ 0 & |x|>1 \end{cases}$$
and show that $\int_0^\infty \frac{\sin^2 t}{t^2}dt=\frac{\pi}{2}$. **(07 Marks)**

b. Find the z-transform of : i) $\cos n\theta$ ii) $\sin n\theta$. **(06 Marks)**

c. Solve using Z –transform $u_{n+2} - 4u_n = 0$ given that $u_0 = 0$ and $u_1 = 2$. **(07 Marks)**

Module-4

7 a. Using Taylor's series method solve $y(x) = x + y$, $y(0) = 1$ then find y at x = 0.1, 0.2 consider upto 4th degree. **(07 Marks)**

b. Solve $y'(x)=1+\frac{y}{z}$, $y(1)=2$ then find $y(1.2)$ with $n = 0.2$ using modified Euler's method. **(06 Marks)**

c. Solve $y'(x) = x - y^2$ and the data is $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$ then find $y(0.8)$ by applying Milne's method and applying corrector formula twice. **(07 Marks)**

OR

8 a. Solve $y'(x) = 3x + \frac{y}{2}, y(0) = 1$ then find y(0.2) with n = 0.2 using modified Euler's method. **(06 Marks)**

b. Solve $y(x) = 3e^x + 2y$, y(0) = 0 then find y(0.1) with h = 0.1 using Runge-Kutta method of fourth order. **(07 Marks)**

c. Solve $y'(x) = 2e^x - y$ and data is

x	0	0.1	0.2	0.3
y	2	2.010	2.040	2.090

Then find y(0.4) by using Adam's Bash forth method. **(07 Marks)**

Module-5

9 a. By applying Milne's predictor and corrector method to compute y(0.4) give the differential equation $\frac{d^2y}{dx^2} = 1 - \frac{dy}{dx}$ and the following table by initial value. **(07 Marks)**

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.3990
y'	1	1.2103	1.4427	1.6990

b. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(06 Marks)**

c. Find the extremal of the functional $\int_{x_1}^{x_2} (y' + x^2 y'^2)dx$. **(07 Marks)**

OR

10 a. By Runge Kutta method solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x = 0.2 correct to four decimal places. Using initial condition y(0) = 1, y'(0) = 0. **(07 Marks)**

b. Prove that the shortest distance between two points in a plane is a straight line. **(06 Marks)**

c. Find the curve on which the functional $\int_0^1 [y'^2 + 12xy]dx$ with y(0) = 0, y(1) = 1. **(07 Marks)**