CSCI 2610 Homework 2

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September 20th, 2024

Section 1.4

Question 10

(a) $\exists x(C(x) \land D(x) \land F(x))$

(b) $\forall x(C(x) \lor D(x) \lor F(x))$

(c) $\exists x(C(x) \land F(x) \land \neg D(x))$

(d) $\forall x(\neg C(x) \lor \neg D(x) \lor \neg F(x))$

(e) $\exists x C(x) \land \exists x D(x) \land \exists x F(x)$

Question 28

Let:

- x is "everything"

- $T(x)$ is "x is a tool"

- $P(x)$ is "x is in the correct place"

- $C(x)$ is "x is in excellent condition"

The logical expressions are as follows:

(a) $\exists x(\neg P(x))$

(b) $\forall x(T(x) \to (P(x) \land C(x)))$

(c) $\forall x(P(x) \land C(x))$

(d) $\forall x(\neg P(x) \lor \neg C(x))$

(e) $\exists x(T(x) \land \neg P(x) \land C(x))$

Question 36

(a) $\exists x(x \leq -2 \lor x \geq 3)$

(b) $\exists x(x < 0 \lor x \geq 5)$

(c) $\forall x(x < -4 \lor x > 1)$

(d) $\forall x(x \leq -5 \lor x \geq -1)$

Section 1.5

Question 8

(a) $\exists x \exists y \ Q(x, y)$

(b) $\forall x \forall y \ \neg Q(x, y)$

(c) $\exists x \ (Q(x, \text{Jeopardy!}) \land Q(x, \text{Wheel of Fortune}))$

(d) $\forall y \ \exists x \ Q(x, y)$

(e) $\exists x_1 \exists x_2 \ (Q(x_1, \text{Jeopardy!}) \land Q(x_2, \text{Jeopardy!}) \land (x_1 \neq x_2))$

Question 28

(a) True

(b) False

(c) True

(d) False

(e) True

(f) False

(g) True

(h) False

(i) True

(j) True

Question 30

(a) $\forall y \forall x \neg P(x, y)$

(b) $\exists x \forall y \neg P(x, y)$

(c) $\forall y (\neg Q(y) \vee \exists x R(x, y))$

(d) $\forall y (\forall x \neg R(x, y) \wedge \exists x \neg S(x, y))$

(e) $\forall y (\exists x \forall z \neg T(x, y, z) \wedge \forall x \exists z \neg U(x, y, z))$

Section 1.6

Question 4

(a) *Simplification*

(b) *Disjunctive Syllogism*

(c) *Modus Ponens*

(d) *Addition*

(e) *Hypothetical Syllogism*

Question 10.a

1. $w \to (s \vee p)$		*Premise*
2. $\neg w$		*Premise*
3. $\neg s$		*Modus Tollens (1 & 2)*
4. $h \to s$		*Premise*
5. $\neg h$		*Modus Tollens (4 & 3)*

Question 10.b

1. $W(x) \to (S(x) \vee P(x))$		*Premise*
2. $W(M) \vee W(F)$		*Premise*
3. $\neg S(T)$		*Premise*
4. $\neg P(F)$		*Premise*
5. $W(F) \to (S(F) \vee P(F))$		*From Premise 1 for F*
6. $S(F)$		*Modus Tollens (4 & 5)*

7. $W(M)$ *Disjunctive Syllogism (2 & 4)*

8. $W(M) \rightarrow (S(M) \vee P(M))$ *From Premise 1 for M*

9. $W(M) \wedge (P(M) \vee S(M))$ *Conclusion*

Question 10.c

1. $\forall x(I(x) \rightarrow L(x))$ *Premise*

2. $I(d)$ *Premise*

3. $\neg L(s)$ *Premise*

4. $E(s, d)$ *Premise*

5. $I(s) \rightarrow L(s)$ *Universal Instantiation (1)*

6. $L(d)$ *Modus Ponens (2 & 5)*

7. $\neg I(s)$ *Modus Tollens (3 & 6)*

Question 16.a

1. $\forall x(E(x) \rightarrow D(x))$ *Premise*

2. $\neg D(Mia)$ *Premise*

3. $\neg E(Mia)$ *Modus Tollens (1 & 2)*

Conclusion: Correct. Mia is not enrolled in the university.

Question 16.b

1. $C(x) \rightarrow F(x)$ *Premise*

2. $\neg C(Isaac's\,car)$ *Premise*

3. *Logical Fallacy*

Conclusion: Incorrect. The fact that Isaac's car is not a convertible does not imply it is not fun to drive.

Question 16.c

1. $Q \rightarrow A(x)$ *Premise*

2. $Q \rightarrow L(Eight\,Men\,Out)$ *Premise*

3. *Logical Fallacy*

Conclusion: Incorrect. Liking the movie does not necessarily mean it is an action movie.

Question 16.d

1. $\forall x(L(x) \rightarrow T(x))$ *Premise*

2. $L(Hamilton)$ *Premise*

3. $T(Hamilton)$ *Universal Instantiation (1 & 2)*

Conclusion: Correct. Hamilton sets at least a dozen traps.

Question 28

1. $\forall x(P(x) \vee Q(x))$ *Premise 1*

2. $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ *Premise 2*

3. $(P(T) \vee Q(T))$ *UI (1)*

4. $((\neg P(T) \wedge Q(T)) \rightarrow R(T))$ *UI (2)*

5. $\neg R(T) \rightarrow (P(T) \vee \neg Q(T))$ *Contrapositive (4)*

6. $\neg R(x) \rightarrow P(x)$ *Disjunction (3 & 5)*

7. $\forall x(\neg R(x) \rightarrow P(x))$ *UG (6)*