

Simple Model for the Complex Dynamics of Dunes

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Abstract. A simple model for the dynamics of dunes is introduced. The model simulates formation processes of typical shapes dunes: transverse dunes, barchans, seif dunes, star dunes, parabolic dunes and elongated parabolic dunes according to several environmental factors. The results have qualitative correspondence to the real systems. Considering the simplicity of the algorithm and the consequent easiness of handling, this model is expected to provide us with wide applicability for the investigation of various dynamics of dunes, especially the interplay between vegetation and dunes.

Keywords: Dunes, Dynamics, Simulation, Vegetation, Morphology

INTRODUCTION

It is well known that the shape of dunes depends upon several environmental factors surrounding them, for example, the amount of available sand in each desert area, wind directional variability, vegetation density covering sand surface, etc. Among all, the interplay between the dynamics of dunes and the growth of plants remains unexplained because of the complex nature of the dynamics of the system and the difficulty of the inspection of the theoretical hypothesis through direct observations. On the other hand, many valuable observational studies have been conducted and have provided us with detailed environmental conditions at individual arid areas in which typical types of vegetated dunes are seen (Hack, 1941; Pye, 1982; Tsoar *et al.*, 1982; Halsey *et al.*, 1990; Thomas *et al.*, 1990; Cook *et al.*, 1993). One of the pioneering studies among them was made by Hack (1941) in which he introduced a phase diagram to show the relation between the wind condition, available sand, the ratio of surface covered by plants and the dominant type of dunes observed in each and (or semiarid) area. Here, taking previous observational studies and computational studies (de Castro, 1995) into consideration, we propose a minimal model which realizes the qualitative dynamics of dunes in mildly vegetated arid areas (Nishimori *et al.*, 2001). Thereafter, the formation processes of transverse dunes, parabolic dunes and elongate parabolic dunes are simulated. Effectively, Hack's phase diagram of vegetated dunes is numerically testified. Below, we make a brief explanation of our model. Firstly, the outline of our previous model (Nishimori *et al.*, 1998) for 'un-vegetated' dunes is introduced which is shown to reproduce various types of

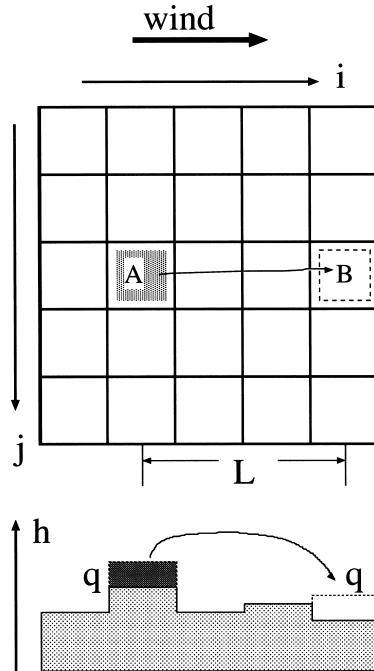


Fig. 1. Top: Top view of 2-dimensional lattice used for the present model. At each cell, field variable $h(i, j)$, height of sand surface, is allocated. Wind is blowing from the right to the left. At a saltation step sand grains at cell A in the top figure will jump into cell B. In this process the surface height at A decreases by q and the height at B increases by q . Bottom: The corresponding profile. Note the size of each cell is sufficiently larger than individual sand grains, thus, each saltation step in this model represents the movement of a large number of sand grains.

dunes according to environmental factors. Secondly, a simple extension of the above model is made to simulate the evolution process of vegetated dunes. Afterwards, we make discussions on the methodology how to investigate complex phenomena using minimal models.

MODEL FOR UNVEGETATED DUNES

The model consists of horizontally extended 2-dimensional lattice (Fig. 1). Each cell of the lattice corresponds to the area of the ground sufficiently larger than individual sand grains. At each cell (i, j) , at each coarse-grained time step n , a continuous field variable $h_n(i, j)$ is allocated to denote the average height of the sand surface within the cell. Therefore, the evolution of $h_n(i, j)$ at one time step does not express the movement of individual sand grains. It rather describes the resulting surface height change after the collective motion of many grains during the unit time period sufficiently shorter than the characteristic time of a dune formation but much larger than the time scale of individual sand grains dynamics.

According to Bagnold the elementary dynamics of individual sand grains consists of creep and saltation (Bagnold, 1941). Creep is the process where sand grains move along sand surface by sliding or rolling. Saltation is the process in which sand grains make short jumps typically in the order of 10 cm. However, in the real process of dune formation the distinction between creep and saltation is not clear. For example, in the ripple formation model by Anderson *et al.* (Anderson, 1987), they stressed the important role of small jumps called reptation with the intermediate scale between saltation and creep. Apart from the above processes, suspension process plays an important role for sub-aqueous dunes, but is not a decisive factor for the desert dunes because of the large difference of the mass density between the air and sand grains.

Here, we construct a phenomenological model at the space-time scale much larger than these elementary processes. We divide the dynamical process into two:

- i) The *inertial* or *advection* process: This describes the accumulated transport effect by saltation over the unit time scale and the unit space scale of our coarse grained model.
- ii) The *frictional* (or *diffusion*) process: There must be fluctuation effects around the average motion due to erratic wind directions, irregularities of the dune surface, etc. This erratic ‘Brownian’ motion is modified by the slope effect as the chemical potential does in the microscopic Brownian motion. The actual dynamics of our model are as follows:

In the frictional process, we may assume a local conservation law of the amount of sand in the coarse grained space mesh. Therefore, the conservation equation

$$h_{n+1}(i, j) = h_n(i, j) + \left[\sum_{(i', j') \in NN} j_n^{NN}(i', j': i, j) + \sum_{(i', j') \in NNN} j_n^{NNN}(i', j': i, j) \right] \quad (1)$$

holds. Here, NN are nearest cells of (i, j) and NNN are 2nd nearest cells of (i, j) , the quantity $j_n^{NN}(i', j': i, j)$ and $j_n^{NNN}(i', j': i, j)$ are the net flux of sand from (i', j') to (i, j) . This flux is the horizontal component of the flow which is assumed to be proportional to the gravitational force along the slope. Hence, if the local slope is sufficiently gentle, the relations

$$\begin{aligned} j_n^{NN}(i', j': i, j) &= a(h_n(i', j') - h_n(i, j)) + b\delta_{j', j} \quad (i > i') \\ &= a(h_n(i', j') - h_n(i, j)) - b\delta_{j', j} \quad (i' > i) \end{aligned} \quad (2)$$

$$j_n^{NNN}(i', j': i, j) = \frac{a}{\sqrt{2}}(h_n(i', j') - h_n(i, j)) \quad (3)$$

hold, where a and b are positive constants. The symbol δ in the right hand side of

Eq. (2) means Kronecker's delta, and the terms proportional to b in these equations describe the constant drift in the wind direction. However, they are ineffective in this model because of the conservation relation (1).

The advection process is modeled as follows.

$$h_{n+1}(i, j) = h_n(i, j) + \sum_{(i', j')} q_n(i', j') \left(\delta_{i'+L_n(i', j'), i} - \delta_{i', i} \right) \quad (4)$$

Here $L_n(i, j)$ is the average transport length (note that this is not the saltation length) of sand grains which take off from (i, j) , whereas $q_n(i, j)$ is the ‘height transfer’ associated with the grains transfer from (i, j) to $(i + L_n(i, j), j)$ by the inertial process (Fig. 1). The symbol δ in the right hand side of Eq. (4) means Kronecker's delta, and the 2nd and the 3rd terms express the incoming advection flux to (i, j) and the outgoing one from (i, j) , respectively. The transport length $L_n(i, j)$ depends on many factors which are mutually coupled in a complicated manner. For example, the flow field of the wind directly affects the saltation length of individual sand grains, whose accumulation results in the transport length. On the other hand, the flow field sensitively depends on the present profile of the sand surface. At the same time the surface profile varies with time through the transport of sand grains. In the same way, the specification of the amount of the height transfer $q_n(i, j)$ in terms of possible relevant factors, also, is not an easy task.

Here, to make a minimal model for the large scale dynamics of dunes, we ignore the details of actual systems and assume a set of rather simple rules for the transport length $L_n(i, j)$ and the height transfer $q_n(i, j)$ as

$$L_n(i, j) = L_0 (\tanh(\nabla_i h_n(i, j)) + 1) \quad (5)$$

$$q_n(i, j) = q_0 (-\tanh(\nabla_i h_n(i, j)) + 1 + \varepsilon) \quad (6)$$

Here, $\nabla_i h_n(i, j)$ means $h_n(i + 1, j) - h_n(i, j)$, i.e., the local slope in i -th (wind) direction, where L_0 , q_0 and ε are positive constants. The above rules reflect the following observations (Rasmussen, 1989):

- (i) In the windward side of a sand hill, the wind velocity is higher than the flat area.
 - (ii) Particularly around the crest of the real sand hill, a sharp peak of the surface wind velocity is observed.
 - (iii) At the lee side of the hill a drastic decrease of the wind velocity is observed.
- The relation between Eqs. (5) and (6) and (i), (ii), (iii) are schematically explained in Fig. 2.

Apart from the above dynamical rules, seasonal change of wind directions and other geological conditions play very important roles in dunes evolution. To investigate the relation between such factors and dunes' morphology, we introduced two control parameters; i) the amount of available sand in a desert area, ii) the

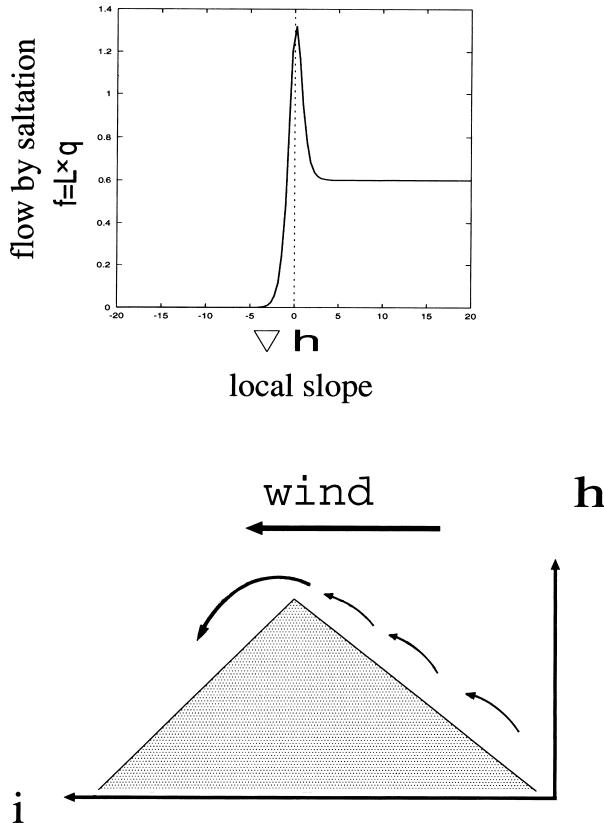


Fig. 2. Saltation flux $f = L \times q$ for the present model as the function of local slope ∇h in wind direction (i -direction). Here, L_0 and q_0 are 1 and ε is 0.3 (see Eqs. (5) and (6)). It reflects the observational fact, namely, large bed load in the windward particularly around the crest (indicated by the arrow), whereas sharply it decreases in the lee side of a dune. This situation is schematically shown in the bottom figure.

directional variability of wind in the desert area. Typical shapes of simulated dunes under various pairs of above control parameters are shown in Fig. 3. The correspondence between the morphology obtained in our simulations and that of real systems (Wasson *et al.*, 1983) seems fairly good as seen in Fig. 4. It means the present simple set of rules is available to study the morphodynamics of various types of dunes.

MODEL FOR VEGETATED DUNES

Here we make a simple extension of the above model to plant trees on the sand surface. For this purpose, in addition to $h_n(i, j)$, a new field variable $c_n(i, j)$, the local density of vegetation is allocated at each site. These two field variables

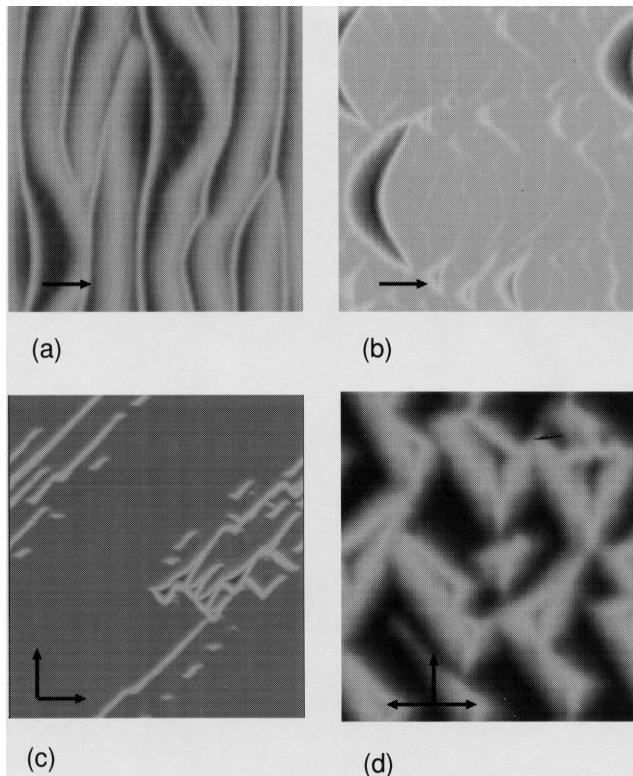


Fig. 3. Various shapes of simulated dunes; (a) transverse dunes, (b) barchans, (c) seif dunes, and (d) star dunes. Arrows indicate wind directions which can change depending on seasons.

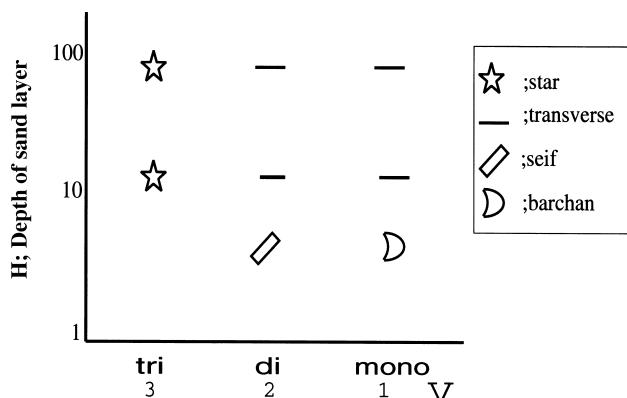


Fig. 4. Types of dunes formed at several points in the control parameters space. The control parameter V is the wind directional variability and H is the average depth of sand layer.

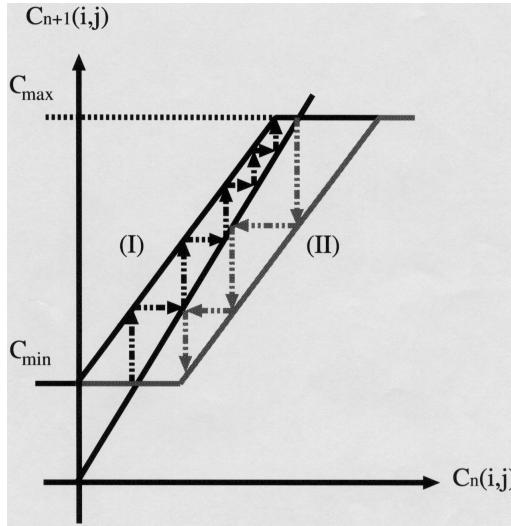


Fig. 5. Schematic explanations of the present model; (I) The Map $c_n(i, j) \rightarrow c_{n+1}(i, j)$ to describe the discretized time evolution of plants density. Without the temporal change $b_n(i, j)$ of surface height, plant density monotonically increases up to the saturation value c_{\max} . (II) With more than a certain speed of surface rise or deflation, plants at the surface are, more or less, damaged because they are buried or cut away, then, $c_n(i, j)$ will decrease with time with the lower limit c_{\min} .

are set to interact each other through a suppression factor $a_{\beta,n}(i, j)$. The specific rules are as follows:

(i) For the evolution of $c_n(i, j)$, the local density of plants, we adopt a set of simple rules. Firstly in an extreme case such that the profile of the sand surface is kept unchanged, plants will grow up to the saturation density without being cut away if the drastic deflation of the ground does not occurs nor are buried by the rapid accumulation of sands, that is, $c_n(i, j)$, will increase until the maximum value c_{\max} . Unlike such a cases, if the temporal change of surface height is very fast, the growth rate of plants is suppressed or some of them may wither and die, then $c_n(i, j)$, decreases down to the minimum density c_{\min} . To reflect the above situation, we use a discrete set of dynamics which is a sectional linear map as shown in Fig. 5. Specifically, the dynamics are expressed as

$$\left. \begin{array}{l} c_{n+1}(i, j) = A(c_n(i, j) - b_n(i, j)) + c_{\min} \quad (b \leq c \leq (c_{\max} - c_{\min}) / A + b) \\ c_{n+1}(i, j) = c_{\max} \quad ((c_{\max} - c_{\min}) / A + b < c) \\ c_{n+1}(i, j) = c_{\min} \quad (c < b) \end{array} \right\} \quad (7)$$

here $b \equiv b_n(i, j) \equiv |h_n(i, j) - h_{n-1}(i, j)|$, $c \equiv c_n(i, j)$ and A is a constant to determine the growth rate of plants.

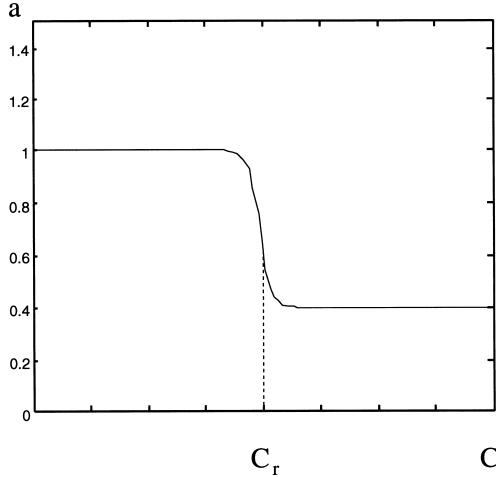


Fig. 6. Suppression factor a of sand flux as a function of local density of vegetation c . Above a critical density c_{cr} of vegetation, the flux of sand drastically decreases as described in Eq. (8).

(ii) For the evolution of $h_n(i, j)$. The crucial effect caused by permitting the growth of plants is such that the sand transport sharply decreases when the cover ratio of sand surface by plants exceeds a critical value. To realize the situation in a simple expression, the suppression factor $a_{\beta,n}(i, j)$ is introduced,

$$a_{\beta,n}(i, j) = \frac{1}{2} \left(\tanh \beta (c_{\text{cr}} - c_n(i, j)) + 1 \right) \quad (8)$$

Here, c_{cr} is the critical vegetation density over which the movement of sand sharply decreases, and β determines the maximum efficiency of suppression (Fig. 6). The value of β depends on whether it is for the saltation flux or for the creep flux. With the suppression factor, the height transfer $q_n(i, j)$ in Eq. (4) and, subsequently, the saltation flux $j_n^{\text{sal}}(i, j) \equiv L_n(i, j) \times q_n(i, j)$ are forced to decrease as

$$q'_n(i, j) = a_{\beta,n}(i, j)^2 q_n(i, j) \quad (9)$$

$$j'_n^{\text{sal}}(i, j) = a_{\beta,n}(i, j)^2 j_n^{\text{sal}}(i, j) \quad (10)$$

where $q'_n(i, j)/j'_n^{\text{sal}}(i, j)$ and $q_n(i, j)/j_n^{\text{sal}}(i, j)$ are, respectively, the height transfer/local saltation flux with vegetation and without vegetation. In the same way, by the vegetation, the diffusional flux (1) is suppressed as

$$j'^{NN}_n(i', j'; i, j) = a_{\beta,n}(i, j)^2 j^{NN}_n(i', j'; i, j) \quad (11)$$

$$j'_n{}^{NNN}(i', j': i, j) = a_{\beta, n}(i, j)^2 j_n{}^{NNN}(i', j': i, j) \quad (12)$$

where the left hand sides are the diffusional flux with vegetation. The time evolution of sand surface is, like the previous model for unvegetated dunes, governed by Eqs. (1) and (2) though with the modified sand flux by the vegetation mentioned above. This suppressed sand dynamics affects the growth of vegetation through the variable b introduced in Eq. (7). In such way, the sand dynamics and the vegetation growth are competing each other in this model to reflect real systems.

To compare the simulation outputs with Hack's (1941) diagram, two kind of quantities are chosen as the control parameters. One is the amount of sand at the source. Namely, the average height, $\langle h_{\text{source}} \rangle$, of the sand surface at the source site, $\{(i, j) | i = 1, 2 \leq j \leq N\}$. Specifically, uniformly random numbers between $2\langle h_{\text{source}} \rangle$ and 0 are allocated to these sites at each time step. The other control parameter is the wind strength which should be a monotonically increasing function of saltation flux. Specifically, the variable q_0 in Eq. (6) is adopted as the index of wind strength. Note that the vegetation density, which is one of the axes in the diagram by Hack (1941), is not adopted as a control parameter because it is rather a resultantly attained quantity after the above two control parameters are fixed.

Except for the source area, simulations are initiated from flat sand surface, while the initial vegetation density $c_0(i, j)$ at each site is set randomly around the average value $\langle c_0(i, j) \rangle$ which is between c_{\max} and c_{\min} . Note the boundary condition in j direction, which is perpendicular to the wind direction, is set periodic to decrease the boundary effect, and the leeward boundary in i direction is set as the free boundary. Below the initial level 0 of sand surface, erosion of sand surface is inhibited to realize the existence of the hard ground or the ground water table. Using these rules, spontaneous formation processes of typical types of dunes are observed.

RESULTS

Typical snapshots of simulated dune field are shown in Fig. 7. In all figures, sand is supplied from the most windward 2 rows, and steady wind is blowing from the left to the right as indicated by an arrow. Parameters to control the amount of sand at the source $\langle h_{\text{source}} \rangle$ and the index of wind strength $f_0 \equiv L_0 \times q_0$ are described under each figure. The left part in each figure shows the spatial distribution of vegetation density $c_n(i, j)$. The darker tone indicates the more densely vegetated place, whereas white parts indicates the areas with bare sand surfaces. The right part in each figure shows the surface height distribution $h_n(i, j)$, where the darker position means the higher surface.

The specific conditions for the appearance of individual steady states are as follows. First of all, when the wind force is too weak, regardless of the amount of sand supply at the source, clearly shaped dunes will not appear in the system (Fig. 7(a)). This is also the case where the amount of sand supply is too small.

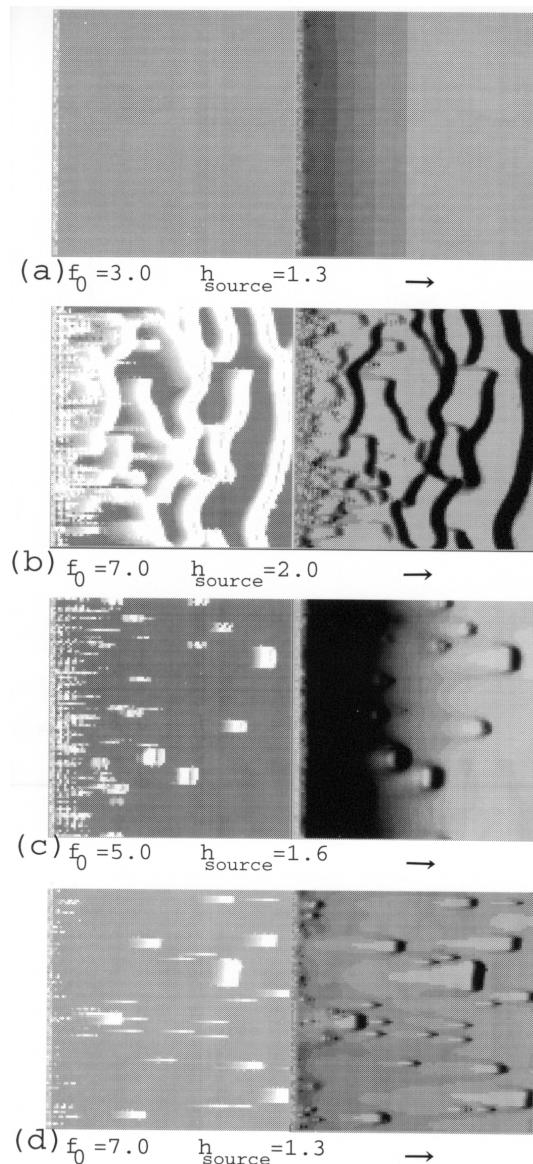


Fig. 7. Snapshots of simulated dunes under various pairs of control parameters, wind force and sand supply. (a) When the wind force is too weak no distinguishable dune is formed. It is also the case for too small amount of sand supply at the source. (b) Under stronger wind with sufficient amount of sand supply transverse dunes prevail while small parabolic dunes are seen just lee of the sand source. The latter will soon grow up to the former. (c) Parabolic dunes, the arms of which extend in the windward direction, are formed under mildly blowing wind with intermediate amount of sand at the source. (d) If the amount of sand is comparatively small within this regime, thin and long parabolic dunes, namely, elongate parabolic dunes will grow. They look like rather linear dunes if without their noses.

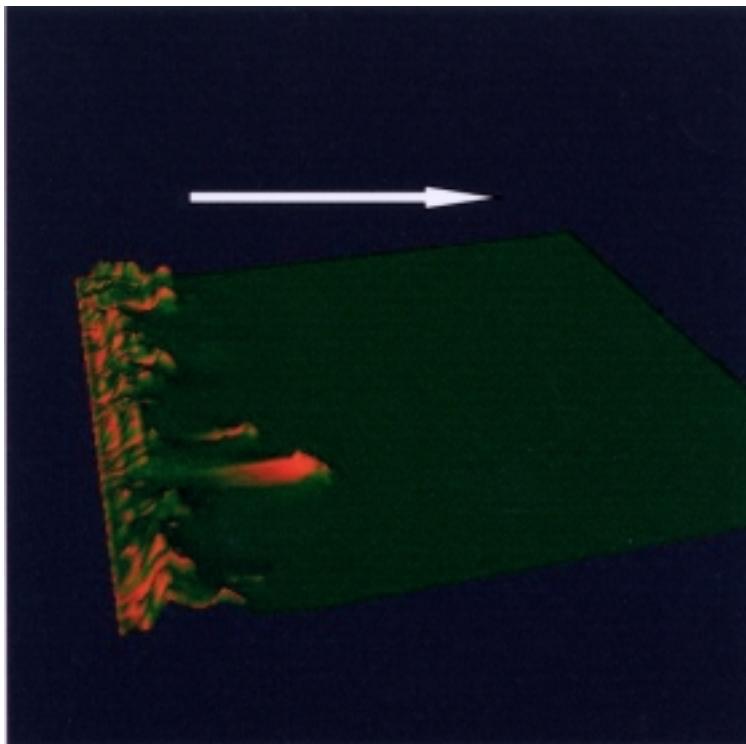


Fig. 8. Under mildly blowing wind (from the left to the right) with proper amount of sand at the source (the left-most area), parabolic dunes will be generated to penetrate into the vegetated inland area. Green color means vegetated area whereas khaki color means bare sand surface.

However, with a stronger wind force and a greater amount of sand supply, two types of clearly shaped active dunes are formed:

- i) When a sufficiently large amount of sand is supplied under strong wind, transverse dunes, barchan dunes or both will dominate in the system. In more detail, small parabolic dunes formed just the lee of the sand source soon develop into barchan dunes, which connect to each other and grow larger as they move, to form transverse dunes the crests of which extend roughly perpendicular to the direction of wind (Figs. 7(b) and 8).
- ii) If the amount of sand supply or the wind force is slightly less than the above, parabolic dunes will prevail in the system (Figs. 7(c) and (d)). They have arms extending to the windward direction. At the centre of each parabolic dune, a hollow develops in which surface erosion proceeds until the unerodible surface appears. The sand at the hollow will move to constitute the crest of the dune. There we can see the tendency that the length of the arms varies depending on the wind force, namely, the arms extend further under a stronger wind. In the case with rather small amounts of sand, elongate parabolic dunes (Pye, 1982) with the

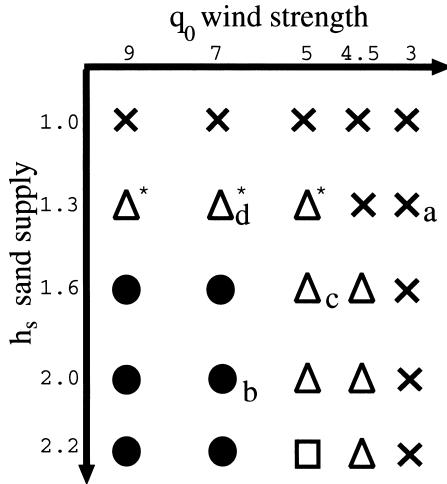


Fig. 9. Phase diagram to show the dominant types of dunes under various pairs of control parameters.

Here the directions of axes are set for the easiness to compare with Hack's (1941) phase diagram. The alphabets in the diagram indicate the conditions corresponding to respective snapshots in Fig. 7. Symbols Δ s and ●s indicate, respectively, parabolic dunes, and, barchan (or transverse) dunes, whereas x mean the conditions for no dune formation. The Δ s accompanied by * mean the conditions for the development of rather thin parabolic dunes with long arms. At the condition with the symbol \square many irregular mounds are formed which are not clearly categorized as particular type of dunes.

shapes like hair-pins will appear (Fig. 7(d)). The arms of such elongate dunes, if without their noses, may appear to be pairs of linear dunes.

The above results are summarized in a phase diagram as shown in Fig. 9. This diagram is not directly compared with that of Hack (1941) because of the fewer axes in our diagram. Also, in the diagram, an ambiguous area is left in which condition irregular mounds of sand are formed, which are not easily categorized into other dominant types of dunes. There is, however, good qualitative correspondence to the previous observational studies including that of Hack (1941), especially, regarding the systematic change of the dunes' morphology from the transverse (or barchan) dunes to the parabolic or the elongate parabolic dunes according to the amount of available sand (or to the wind force). Moreover no dune formation is realized under very light winds or under very small amounts of sand supply.

DISCUSSION

The above results indicate that our simple model of dunes contains intrinsic dynamics by which the morphodynarnics of dunes are decisively affected, and that through this model time evolution of dune systems under various situations are expected to be effectively simulated. Also, the simplicity of the model would

enables us to make analytical investigations of the system. On the other hand, it should be noted that, in modeling the ‘minimal’ dynamics of the system, several aspects of them were intendedly cut away which may cause some degrees of discrepancies between the reality and simulated dunes.

To fill the gap between the reality and the minimal model, recently, several models have been proposed some parts of which dynamics are deduced from detailed fluid model. Especially a simple analytical expression of turbulent boundary layer near dunes surface (Jackson *et al.*, 1975) has been incorporated into numerical models to catch some quantitative features of real barchans (Sauermann *et al.*, 2001; Kroy *et al.*, 2002; Andreotti *et al.*, 2002). Yet, calculations under more complex and interesting boundary conditions, e.g., unsteady wind in strength and directions, naturally or artificially vegetated sand surface, etc. are still far beyond to be fulfilled on the basis of the detailed models. Also the interaction between isolated dunes and their collective dynamics in large time and spatial scales are of great interest but hard to be solved if one try to start from the details of the underlying systems. It is supposed the difficulty to treat such complex situations is staying not only in the amount of the accompanying calculations but rather in the entangled relationship between what are engaged in the whole dynamics the degree of which is not clearly estimated. In other words, we hardly know what equations we should calculate before starting and accomplishing the calculations.

Considering the above facts and the present results of our model, heuristic approaches of exploring minimal rules are expected to play a certain role for the study of dunes. Especially, the investigation of universal natures which do not depend on the details of the underlying systems, seems the most effectively performed by this kind of approach. The methodology for the minimal model approach is yet to be developed (Werner, 1995; Momiji *et al.*, 2000; Kroy and Herrmann, 2002) for the efficient understanding of complex dynamics of dunes.

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