

Chapter → 6 Probability distribution

② Binomial distribution:

Probability mass function of random variable X is $P(X=x)$
 $= n C_x P^x q^{n-x}$ OR $C(n,x) P^x q^{n-x}$

where, $x=0,1,2,\dots,n$.

This can be written as $X \sim B(n,p)$ and read as
 X follows the binomial distribution with parameters
 n and p .

Properties:

- i) Binomial distribution is a discrete probability distribution because random variable X takes values $0,1,2,\dots,n$.
- ii) Binomial distribution is also known as biparametric distribution because it has two parameters n and p .
- iii) The mean of binomial distribution is np and variance is npq .
- iv) Mean > Variance OR $np > npq$.
- v) Additive property → Let X and Y be two independent variables such that $X \sim B(n,p)$, $Y \sim B(n,p)$ then $(X+Y) \sim B(n_1+n_2, p)$.

③ Poisson distribution:

It is used if:-

- i) Probability of success is very small i.e., $p \rightarrow 0$
- ii) If the no. of trial is very large i.e., $n \rightarrow \infty$
- iii) If $\lambda = np$

$$\Rightarrow P = \frac{\lambda}{n}$$

Probability mass function of random variable X with parameter λ is

$$P(X=x) = f(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} \quad \text{where, } x=0,1,2,3,\dots,\infty.$$

This can be written as $X \sim P(\lambda)$ and read as: X follows poisson distribution with parameter λ .

Properties

- i) Poisson distribution is a discrete probability distribution because the random variable X takes the values $0, 1, 2, \dots, \infty$.
- ii) Poisson distribution is also known as uniparametric distribution because it has only one parameter λ . If we know the value of λ then we can easily determine the probability.
- iii) The mean and variance of Poisson distribution is λ .
- iv) Let X and Y be two independent random variables such that $X \sim P(\lambda_1)$ and $Y \sim P(\lambda_2)$ then $(X+Y) \sim P(\lambda_1 + \lambda_2)$.

Numericals related to Binomial Distribution (from KEC book exercise).

- Q20. In a binomial distribution with 6 independent trials the probability of 3 and 4 success is found to be 0.2457 and 0.08189 respectively. Find the parameters, mean, variance.

Soln Given, no. of trial (n) = 6,

$$P(3 \text{ success}) = 0.2457 = P(X=3)$$

$$P(4 \text{ success}) = 0.08189 = P(X=4)$$

$$P(X=3) = 6C_3 p^3 (1-p)^3 = 0.2457 \quad \text{--- (i)} \quad (\because P(X=x) = nC_x p^x q^{n-x})$$

$$P(X=4) = 6C_4 p^4 (1-p)^2 = 0.08189 \quad \text{--- (ii)}$$

dividing (i) by (ii),

$$\frac{6C_3 p^3 (1-p)^3}{6C_4 p^4 (1-p)^2} = \frac{0.2457}{0.08189} = \frac{20(1-p)}{15p}$$

$$\text{or, } 3 = \frac{20(1-p)}{15p}$$

$$\Rightarrow 20 - 20p = 45p$$

$$\Rightarrow 65p = 20$$

$$\Rightarrow p = \frac{20}{65}$$

$$\Rightarrow p = \frac{4}{13}$$

Now, mean = np

$$= 6 \times \frac{4}{13}$$

$$= \frac{24}{13}$$

Variance = npq

$$= 6 \times \frac{4}{13} \times \frac{9}{13}$$

$$\approx 1.27$$

$$\left(\begin{array}{l} \because q = 1-p \\ = 1 - \frac{4}{13} \\ = \frac{9}{13} \end{array} \right)$$

Important Note:

- ① Question मा at most जरूर use जरूर & at least जरूर \geq .
- ② $1 - (\text{one minus})$ जरूर बेला sign असके change हुँदे $\geq \leftarrow \rightarrow \leftarrow$
 $\leq \leftarrow \rightarrow \rightarrow$
 for e.g. $P(X \geq 1) \geq 0.99$
 or, $1 - P(X \leq 1) \geq 0.99$
- ③ Question Binomial distribution को कि Poisson को कसारी हुँद्याउने,
 A Parameter question मा average(λ) दिएको ह भने Poisson
 $\lambda = np$ & $P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!}$ formula use जरूर
 होइन parameter n and p दिएका जस Binomial distribution
 $P(X=x) = nC_x p^x q^{n-x}$ OR $C(n, x) p^x q^{n-x}$ formula use जरूर
 १) Number(N) को value Question मा होइए कष्टक probability of
 occurrence होइए कम दिएको ह ज्ञाने Poisson हुँद नस्त Binomial.
 For e.g. $p=0.0005$, $N=1000$
 २) होइए number का single को probability निकाल्नु होइए Poisson
 नस्त Binomial.

Q.No.29. A programmer succeeds twice as often as it fails while developing a specific program. Find the chance that in the next six attempts, there will be at least four successes.

Soln According to the question, the ratio of success and failure is 2:1.

$$\text{So, } P(\text{success}) = \frac{2}{2+1} = \frac{2}{3}$$

$$\Rightarrow p = \frac{2}{3}$$

$$P(\text{failure}) = q = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\begin{aligned} \text{Now, } P(\text{at least 4 success}) &= P(X \geq 4) \\ &= P(X=4) + P(X=5) + P(X=6) \\ &= 6C_4 \left(\frac{2}{3}\right)^4 \cdot \left(\frac{1}{3}\right)^0 + 6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^1 + 6C_6 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^0 \\ &= 15 \times 0.197 \times 0.111 + 6 \times 0.132 \times 0.333 \\ &\quad + 1 \times 0.087 \times 1 \\ &= 0.328 + 0.263 + 0.087 \\ &= 0.678 \end{aligned}$$

Q.No.32. A discrete random variable X has mean equal to 6 and variance equal to 2. If it is assumed that the underlying distribution of X is binomial, what is the probability that $5 \leq X \leq 7$?

Soln Given, Mean (np) = 6

$$\text{Variance } (npq) = 2$$

$$\Rightarrow 6q = 2$$

$$\text{or, } q = \frac{2}{6} = \frac{1}{3}$$

$$\Rightarrow p = 1 - \frac{1}{3} = \frac{2}{3}$$

(We know that $p = 1 - q$)

Now, from $np = 6$

$$\Rightarrow n \times \frac{2}{3} = 6$$

$$\Rightarrow n = \frac{18}{2} = 9$$

Then, $P(5 \leq X \leq 7)$

$$= P(X=6)$$

$$= 9C_6 \left(\frac{2}{3}\right)^6 \cdot \left(\frac{1}{3}\right)^3$$

$$= 84 \times 0.087 \times 0.037$$

$$= 0.27$$

Q. Example 22. It is believed that 80% of Nepalese do not have health insurance. Suppose this is true and let x be the number of health insurance in a random sample of 12 nepalese.

i) What is the probability of no health insurance?

ii) Find the mean and variance of x .

iii) Find $P(x > 2)$.

Solution-

Since x = number of health insurance.

i) Probability of no. health insurance (p) = 80%
 $= 0.8$

$$q = 1 - p = 1 - 0.8 = 0.2$$

No. of Nepalese people (n) = 12

ii) Mean $E(x) = np$
 $= 12 \times 0.8$
 $= 9.6$

& Variance $V(x) = npq$
 $= 12 \times 0.8 \times 0.2$
 $= 1.92$

iii) $P(x > 2) = 1 - P(x \leq 2)$
 $= 1 - \{P(X=0) + P(X=1) + P(X=2)\}$
 $= 1 - \{C(12,0) \cdot (0.8)^0 \cdot (0.2)^{12} + C(12,1) \cdot (0.8)^1 \cdot (0.2)^{11}\}$
 $= 1 - 0$
 $= 1$

Hence the probability of no health insurance is 0.8
Mean is 9.6, variance is 1.92 and $P(x > 2)$ is 1.

Numericals related to Poisson Distribution:

Q.No. 25. (Model Question). A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%.

a) The inspector randomly picks 20 items from a shipment. What is the probability that there will be at least one defective item among these 20?

b) Suppose that the retailer receives 10 shipments in a month and the inspector randomly tests 20 devices per shipment. What is the probability that there will be exactly 3 shipments each containing at least one defective device among the 20 that are selected and tested from the shipment?

Q.No.45 A manufacturer of pen drives knows that 3% of his product is defective. If he sells in boxes of 200 and guarantees that not more than 2 pen drives will be defective, what is the probability that a box will fail to meet the guaranteed quality?

Soln

$$P(\text{defective}) = 3\% = 0.03$$

$$\text{no. of box}(n) = 200$$

$$\text{then } \lambda = np$$

$$= 200 \times 0.03$$

$$= 6.$$

question \bar{H} average \bar{n}
 \bar{n} constant $\lambda = \bar{n} \cdot p$

Now,

$$P(\text{Box will fail to meet the guaranteed quality})$$

$$= P(X > 2)$$

$$= 1 - P(X \leq 2)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \left[\frac{e^{-6} \cdot 6^0}{0!} + \frac{e^{-6} \cdot 6^1}{1!} + \frac{e^{-6} \cdot 6^2}{2!} \right]$$

$$= 1 - e^{-6} (1 + 8 + 18)$$

$$= 1 - e^{-6} \times 25$$

$$= 0.93$$

Again,

$$P(\text{Box will meet guaranteed quality})$$

$$= 1 - P(\text{Box will not meet})$$

$$= 1 - P(X > 2)$$

$$= 1 - 0.93$$

$$= 0.07$$

Q.No.43. If a random variable X follows poisson distribution such that $P(X=1) = P(X=2)$. Find mean & variance.

Soln

$$\text{Given, } P(X=1) = P(X=2)$$

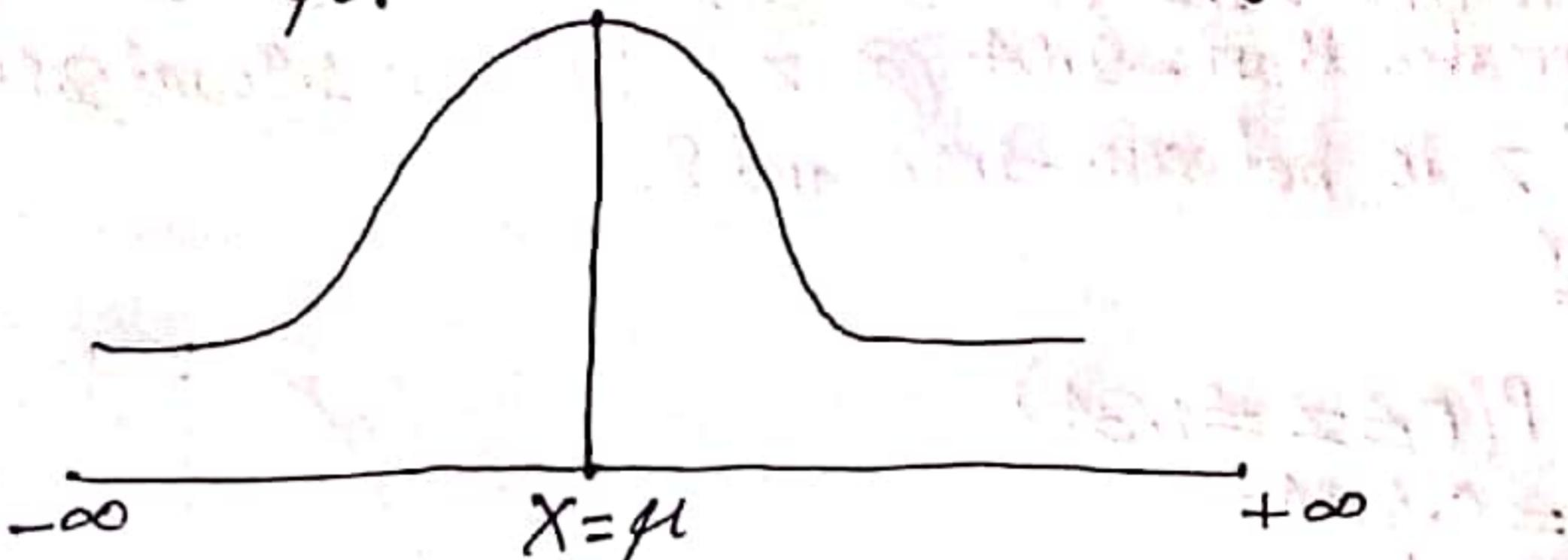
$$\text{or, } \frac{e^{-\lambda} \lambda^1}{1!} = \frac{e^{-\lambda} \lambda^2}{2!}$$

$$\Rightarrow \lambda = 2.$$

$$\text{So, mean} = \text{variance} = \lambda = 2.$$

Properties of normal distribution:

- i) Normal distribution is a continuous probability distribution having parameters μ and σ^2 .
- ii) Normal curve is bell shaped curve and symmetrical about mean μ .



iii) Since the curve is symmetrical, so, mean = median = mode.

iv) The distribution of the curve is maximum when $X = \mu$.

$$\text{i.e., } f(x) = \frac{1}{\sigma\sqrt{2\pi}}$$

v) The coefficient of skewness for normal curve is zero.

$$\text{i.e., } \beta_1 = 0$$

$$\Rightarrow \gamma_1 = 0.$$

vi) The coefficient of kurtosis is $\beta_2 = 3$ or $\gamma_2 = 0$.

vii) The area property of Normal Curve:

$$\text{Let } Z = \frac{X - \mu}{\sigma}$$

$$\text{when } X = \mu + 6$$

$$\text{Then } Z = \frac{\mu + 6 - \mu}{\sigma} = \frac{6}{\sigma} = 1$$

$$\text{when } X = \mu - 6$$

$$\text{Then } Z = \frac{\mu - 6 - \mu}{\sigma} = \frac{-6}{\sigma} = -1$$

Now,

$$P(\mu - 6 \leq X \leq \mu + 6)$$

$$= P(-1 \leq Z \leq 1)$$

$$= P(-1 \leq Z \leq 0) + P(0 \leq Z \leq 1)$$

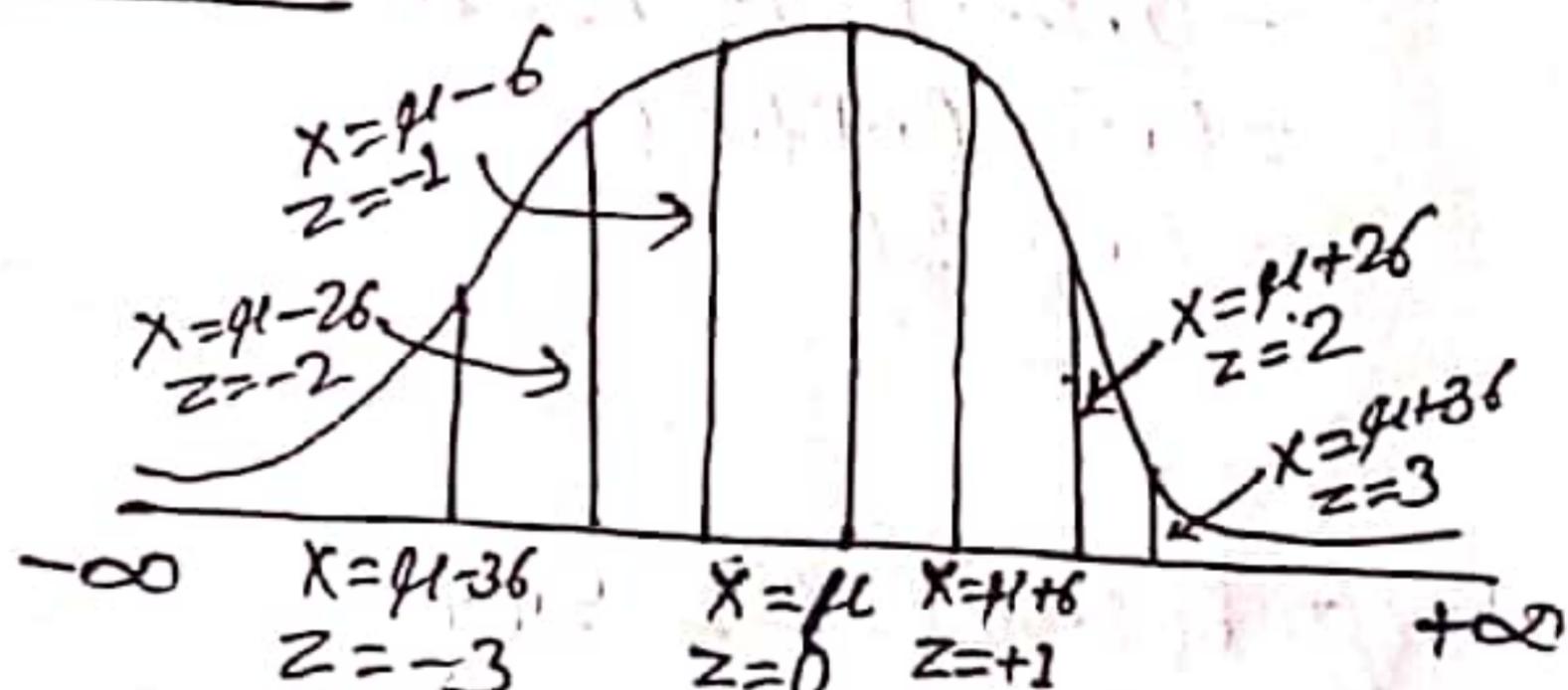
$$= P(0 \leq Z \leq 1) + P(0 \leq Z \leq 1) \quad (\because \text{by symmetry})$$

$$= 2P(0 \leq Z \leq 1)$$

$$= 2 \times 0.3413$$

$$= 0.6826$$

$$= 68.26\%$$

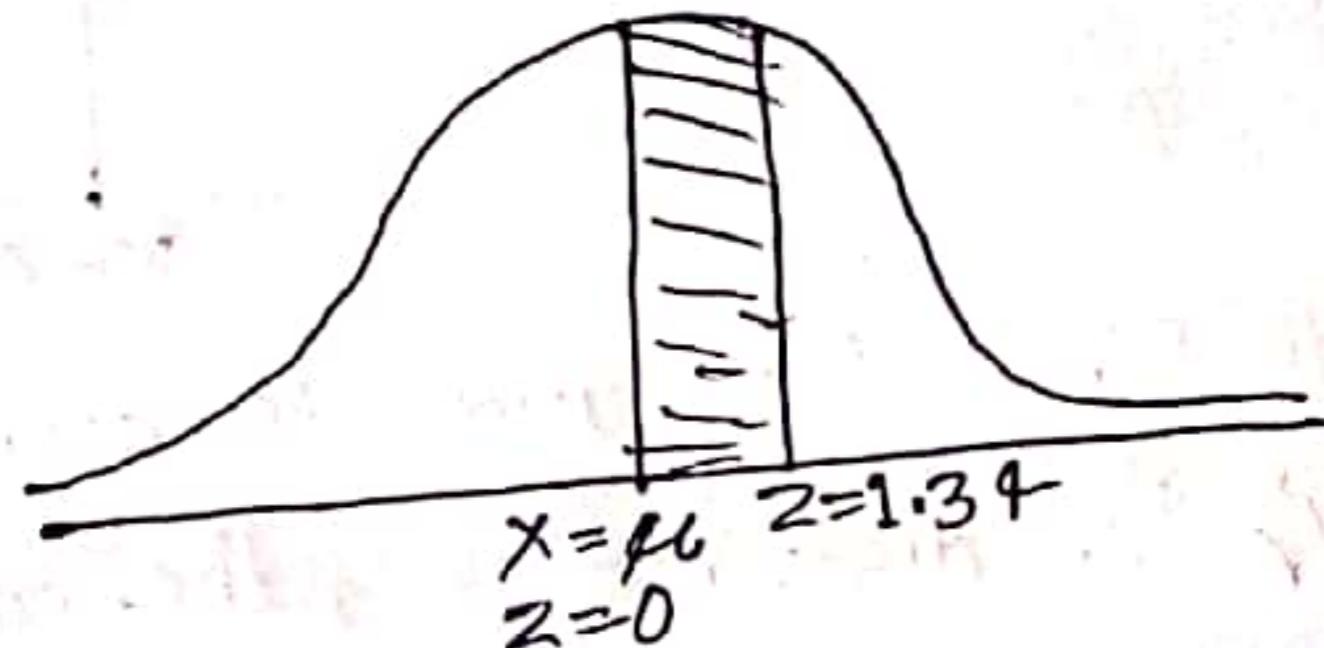


Numericals:

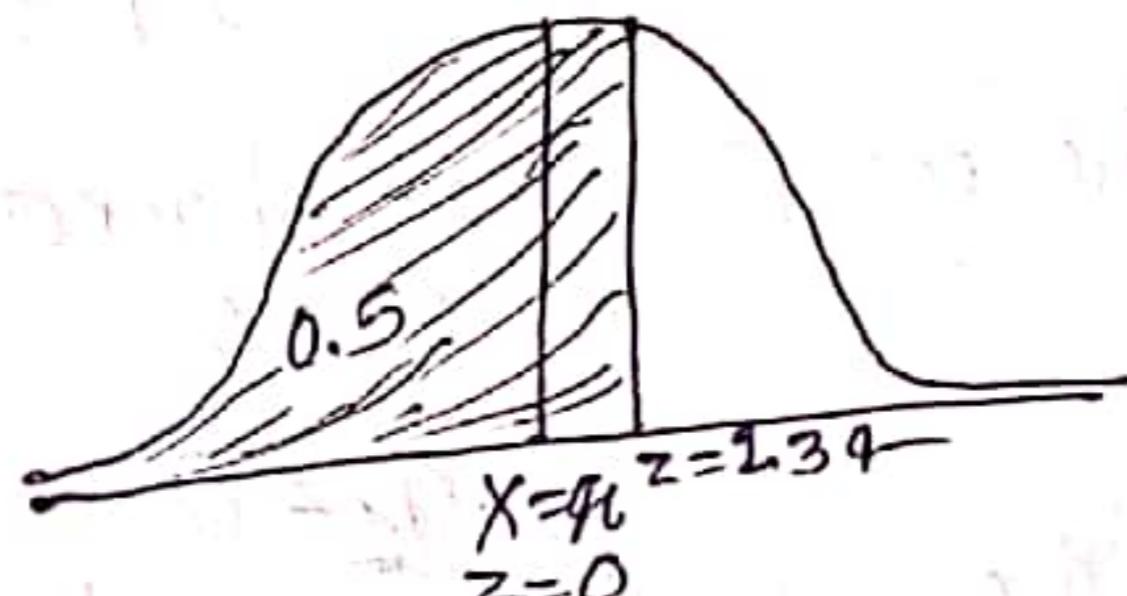
Q.No.53. If Z is the standard normal variable then calculate the following probabilities: (a) Z is between 0 and 1.34
 (b) Z is less than 1.34 (c) Z is more than 1.96 (d) Z is between -1.34 and 0 (e) Z is less than -1.34 (f) Z is greater than -2.04 (g) Z is between 1.96 and 2.84
 (h) Z is between -1.96 and 2.

Soln

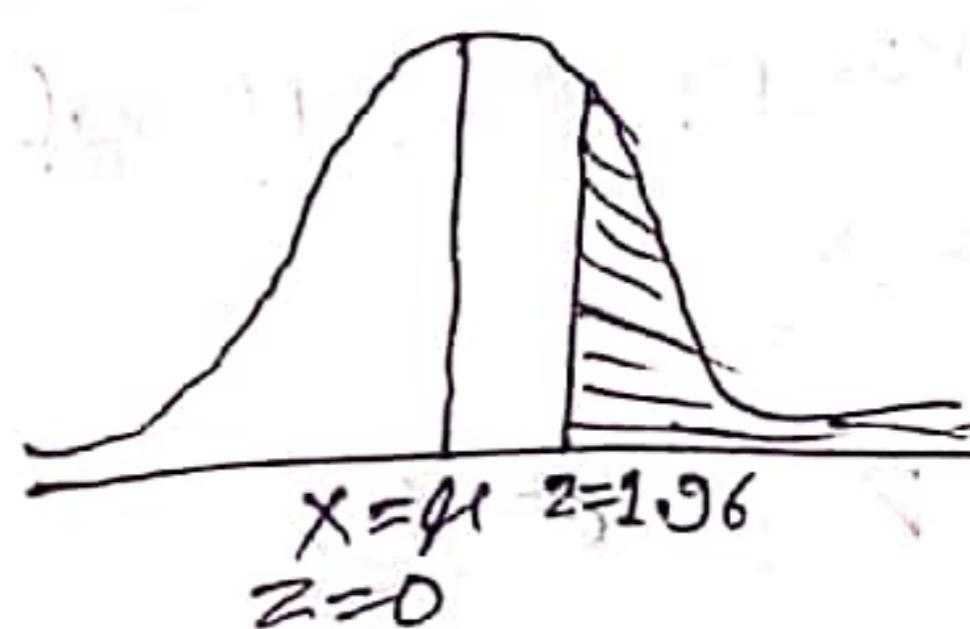
$$\text{(a)} \quad P(0 \leq Z \leq 1.34) \\ = 0.4034 \\ \approx 0.4$$



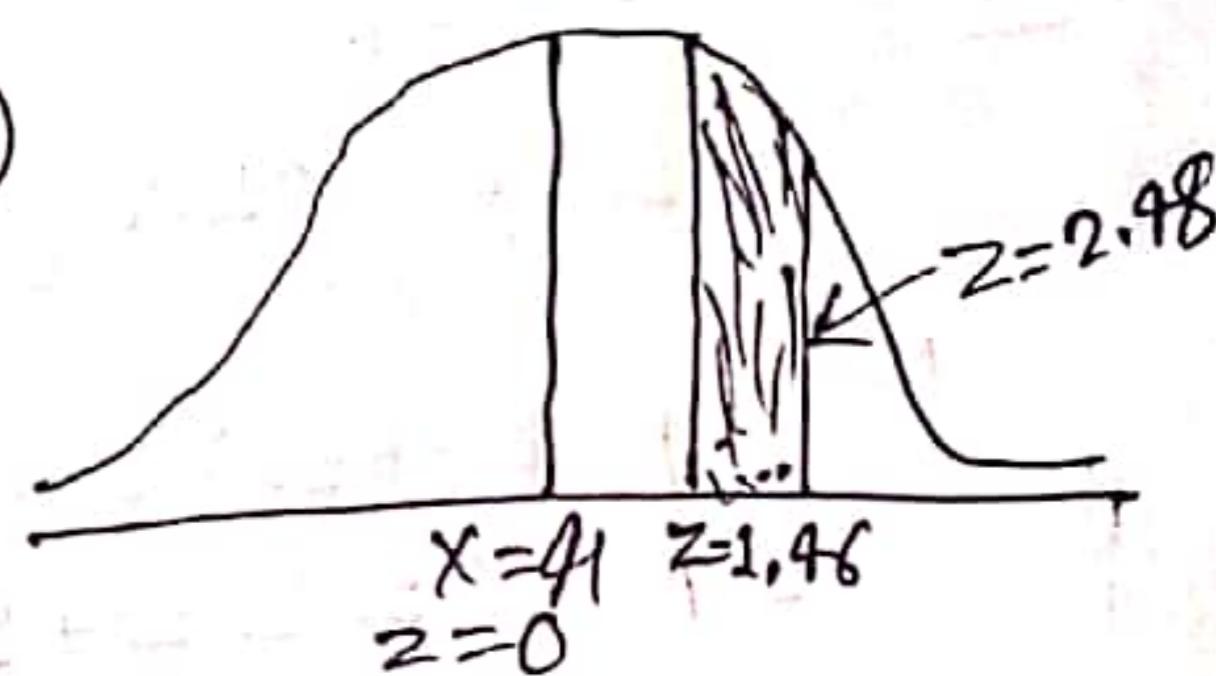
$$\text{(b)} \quad P(Z < 1.34) \\ = 0.5 + P(0 \leq Z \leq 1.34) \\ = 0.5 + 0.4 \\ = 0.9$$



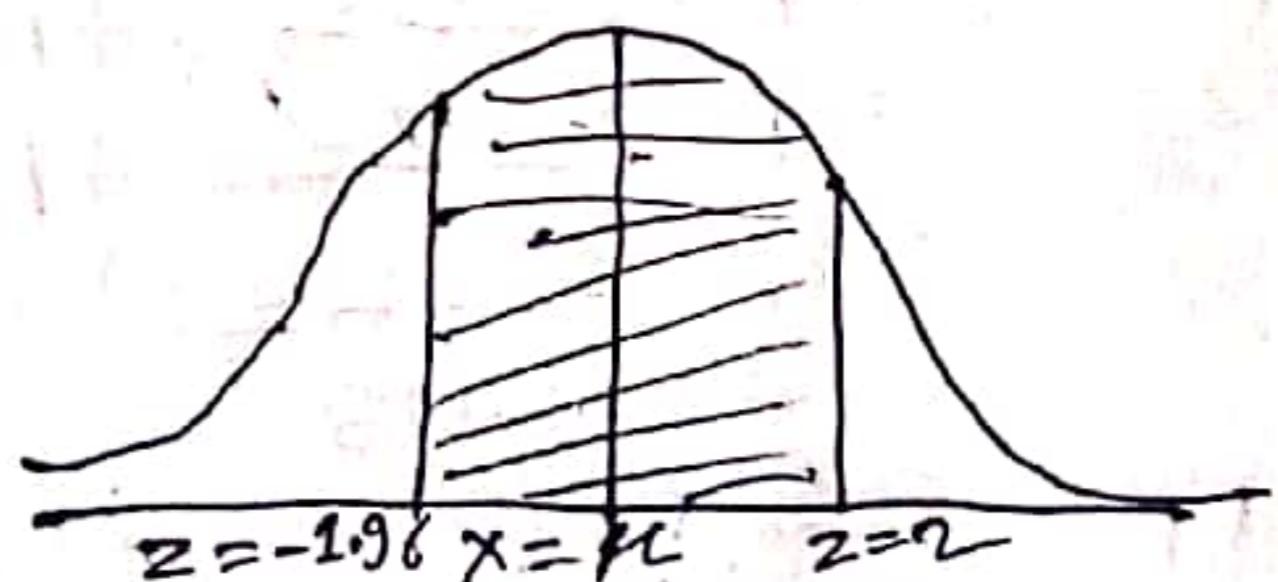
$$\text{(c)} \quad P(Z > 1.96) \\ = 0.5 - P(0 \leq Z \leq 1.96) \\ = 0.5 - 0.4750 \\ = 0.0250$$



$$\text{(d)} \quad P(1.96 \leq Z \leq 2.84) \\ = P(0 \leq Z \leq 2.84) - P(0 \leq Z \leq 1.96) \\ = 0.4977 - 0.4750 \\ = 0.0227$$



$$\text{(e)} \quad P(-1.96 \leq Z \leq 2) \\ = P(-1.96 \leq Z \leq 0) + P(0 \leq Z \leq 2) \\ = P(0 \leq Z \leq 1.96) + P(0 \leq Z \leq 2) \\ = 0.4750 + 0.4772 \\ = 0.9522$$



Q.No.58 The mean yield for one acre plot is 662 kilos with standard deviation 32 kilos. Assuming normal distribution how many one-acre plots in a batch of 1000 plots would you expect to have yield (i) over 700 kilos (ii) below 650 kilos and (iii) What is the lowest yield of the best 100 plots?

Soln

Given, mean yield (μ) = 662

standard deviation = 32

Number of plot (N) = 1000

(i) For the probability (over 700 kilo)

$$= P(X > 700)$$

$$\text{when } X \text{ is 700 then } Z = \frac{700 - 662}{32} = 1.18$$

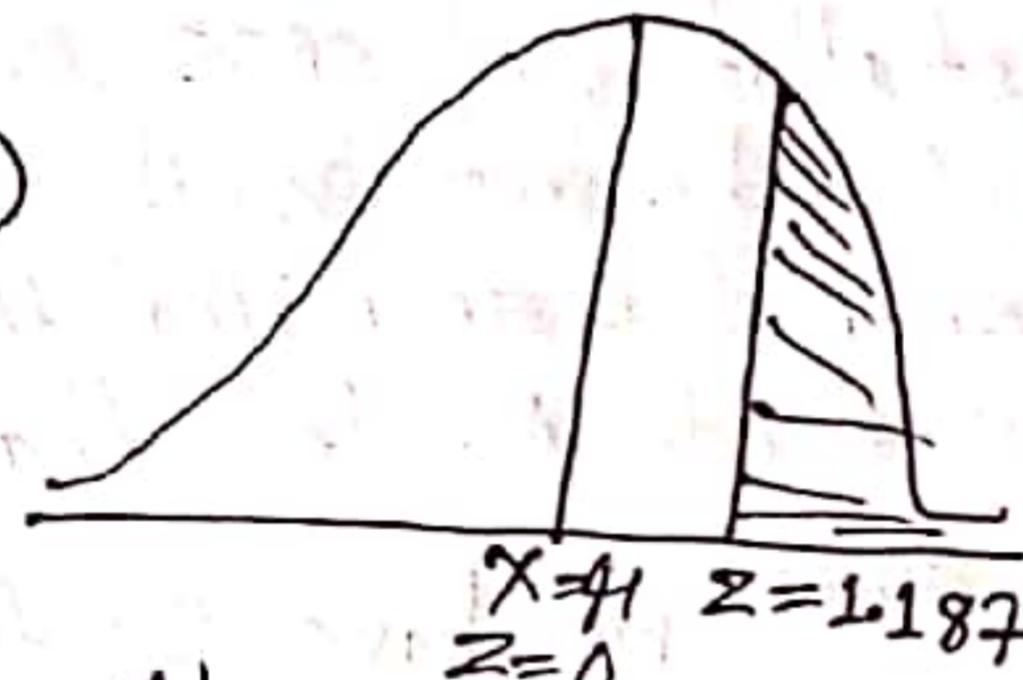
$$\text{Now, } P(X > 700)$$

$$= P(Z > 1.187)$$

$$= 0.5 - P(0 < Z < 1.187)$$

$$= 0.5 - 0.3810$$

$$= 0.1190$$



So, the no. of plots when yield over 700 kilos is

$$N \times P(X > 700)$$

$$= 1000 \times 0.1190$$

(ii) For $P(X < 650)$

When $X = 650$,

$$Z = \frac{650 - 662}{32} = \frac{-12}{32} = -0.375.$$

$$\text{So, } P(X < 650)$$

$$= P(Z < -0.375)$$

$$= 0.5 - P(-0.375 < Z < 0)$$

$$= 0.5 - P(0 < Z < 0.375)$$

$$= 0.5 - 0.144$$

$$= 0.356$$

So, the number of plots which yields less than 650 kilos is

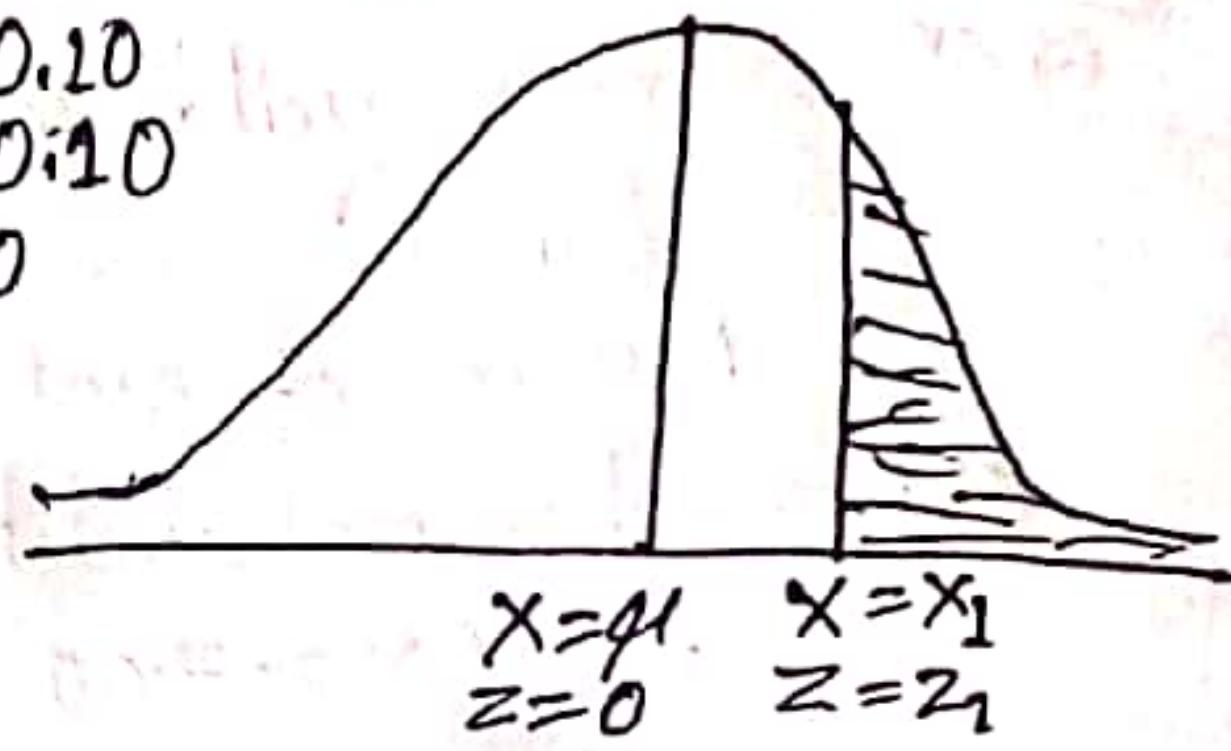
$$N \times P(X < 650) = 1000 \times 0.356$$

$$= 356$$

Q. Let x_1 be the least yield, then $P(X > x_1) = \frac{100}{1000} \times 100$
 $= 10\%$
 $= 0.10$

When $X = x_1$ then, $Z = \frac{x_1 - 662}{32} = z_1$ (say) — (1)

So, $P(X > x_1) = P(Z > z_1) = 0.10$
 $\Rightarrow 0.5 - P(0 < Z < z_1) = 0.10$
 $\Rightarrow P(0 < Z < z_1) = 0.40$
 $\Rightarrow z_1 = 1.29$



Putting value of z_1 in eqn (1)

$$\frac{x_1 - 662}{32} = 1.29$$

$$x_1 = 703.28$$

Q No. 59 Installation of SPSS software package requires downloading 95 files. On the average, it takes 16 secs to download one file with s.d. of 5 sec. What is the probability that the software is installed in less than 25 minutes?

Soln Given, we have 95 files.

$$\text{So, mean } (\mu) = 95 \times 16 \\ = 1520$$

$$\text{And Variance } (\sigma^2) = 95 \times 25 \\ = 2375$$

$$\text{or, } \sigma = \sqrt{2375} = 48.73$$

$$\text{Now, } P(X < 25 \text{ min}) = P(X < 25 \times 60 \text{ sec}) \\ = P(X < 1500)$$

$$\text{When } X = 1500, Z = \frac{1500 - 1520}{48.73} = \frac{-20}{48.73} = -0.41$$

$$\text{So, } P(X < 1500) = P(Z < -0.41) \\ = 0.5 - P(-0.41 < Z < 0) \\ = 0.5 - P(0 < Z < 0.41) \\ = 0.5 - 0.1591 \\ = 0.3419$$

Q.No.65 Incomes of a group of 10,000 computer operators were found to be normally distributed with mean Rs 15,20 and standard deviation Rs 1600. Find (i) highest income of poorest 2000 computer operators (ii) lowest income of richest 1000 computer operators.

Soln

Given, Total no. of computer operators = 10,000.

$$\text{mean}(\mu) = 1520$$

$$\text{standard deviation}(\sigma) = 160.$$

Let x_1 be the highest income of poorest 2000 computer operators.

then, according to the question.

$$P(X \leq x_1) = \frac{2000}{10000} = 0.2$$

Let x_1 be the highest income of poorest 2000 computer operators.

$$\text{Let } X = x_1 \text{ then } z = \frac{x_1 - 1520}{160}$$

$$\text{Now, } P(X \leq x_1) = 0.2 \quad \text{or, } P(z \leq -z_1) = 0.2$$

$$\text{or, } 0.5 - P(-z_1 \leq z \leq 0) = 0.2$$

$$\text{or, } P(0 \leq z \leq z_1) = 0.3 \quad \Rightarrow z_1 = 0.84$$

Putting value of z in eqn (i).

$$\frac{x_1 - 1520}{160} = 0.84$$

$$\text{or, } x_1 = 1385.6$$

(ii). Let x_2 be the lowest income of richest 1000 computer operators.

$$\text{Then } P(X > x_2) = \frac{1000}{10,000} = 0.10.$$

$$\text{When } X = x_2, \text{ then } z = \frac{x_2 - 1520}{160} = z_2 \quad \text{(say)}$$

Now,

$$P(X > x_2) = 0.10$$

$$\text{or, } P(z > z_2) = 0.10$$

$$\text{or, } 0.5 - P(0 \leq z \leq z_2) = 0.10$$

$$\text{or, } P(0 \leq z \leq z_2) = 0.40$$

$$\Rightarrow z_2 = 1.29.$$

Putting value of z_2 in eqn ①.

$$\frac{x_2 - 1520}{160} = 1.28$$

$$\text{or, } x_2 = 1724.8$$

Hence the highest income of poorest 2000 computer operator is 1385.6 and richest computer operators is 1724.8.

Q.No.66. It is known that the life time of a calculator manufactured by Casio Company Ltd. has a normal distribution with mean of 54 months and a standard deviation of 8 months. If the company wants to replace just 22% calculator which starts not working properly, what would be warranty period?

Solⁿ
Given, mean (μ) = 54

standard deviation (σ) = 8

Let x_1 be the warranty period of calculators.

If fail is also asked then used and percentage is 78%.

$$\text{So, } P(X \leq x_1) = 22\% = 0.22$$

$$\text{when } X = x_1 \text{ then } z = \frac{x_1 - 54}{8} = -z_1 \text{ (say) } \text{ f. ①}$$

Now,

$$P(X \leq x_1) = 0.22$$

$$\Rightarrow P(z \leq -z_1) = 0.22$$

$$\Rightarrow 0.5 - P(-z_1 \leq z \leq 0) = 0.22$$

$$\Rightarrow P(0 \leq z \leq z_1) = 0.28$$

$$\Rightarrow z_1 = 0.77$$

from eq ①

$$\frac{x_1 - 54}{8} = 0.77$$

$$\text{or, } x_1 = 54 - 0.77 \times 8$$

$$\text{or, } x_1 = 47.84.$$

Note → Binomial & Poisson Distribution की वाले Question हैं।
पुराने होते भव्या ही question practice ज़रूर important हैं।

Exponential Probability Distribution:

Let X be a continuous random variable assuming non-negative values. It is said to follow exponential distribution with parameter θ if its probability density function

$$f(x) = f(X=x) = \begin{cases} \theta e^{-\theta x}, & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Moments of exponential distribution:

$$g_{k_1}' = E(X^r) = \int_0^\infty x^r f(x) dx.$$

$$\text{i)} g_1' = \frac{1}{\theta}$$

$$\text{ii)} g_2' = \frac{2}{\theta^2}$$

$$\text{iii)} g_3' = \frac{6}{\theta^3}$$

$$\text{iv)} g_4' = \frac{24}{\theta^4}$$

So, In General
$$g_k' = \frac{r!}{\theta^r}$$

$$\begin{aligned} \text{Variance} &= E(X^2) - [E(X)]^2 \\ &= \frac{2}{\theta^2} - \frac{1}{\theta^2} \\ &= \frac{1}{\theta^2} \end{aligned}$$

Q.No.76. Suppose that waiting time (hrs) for bus in a bus station has a negative exponential distribution with parameters $\theta = 5$ hours. What is the probability that a man has to wait at least 15 minutes? Also find expected waiting time for bus.

Soln Given, Parameter (θ) = 5 hours.

$$\begin{aligned} \text{Now, } P(X \geq 15 \text{ min}) &= P(X \geq \frac{15}{60}) = P(X \geq 0.25) \\ &= \int_{0.25}^{\infty} 0. e^{-\theta x} dx \\ &= \int_{0.25}^{\infty} 5 \cdot e^{-5x} dx \\ &= 5 \left[\frac{e^{-5x}}{-5} \right]_{0.25}^{\infty} \\ &= -[e^{-5x}]_{0.25}^{\infty} \\ &= -[e^{-5 \times 0.25}] + [e^{-5 \times 0}] \\ &= e^{-1.25} \\ &= 0.286 \end{aligned}$$

✓ and mean of negative exponential distribution is $\frac{1}{\theta}$
 $= \frac{1}{5}$

Q.No.77 The daily consumption of electricity in a city has an exponential distribution with mean 2000 kilowatt. Find the probability that electricity consumption on a particular day is (i) at least 1500 kilowatt (ii) at most 2500 kilowatt.

Soln

Given, mean ($\frac{1}{\theta}$) = 2000

$$\Rightarrow \theta = \frac{1}{2000} = 0.0005.$$

$$\text{Now, } P(X \geq 1500) = \int_{1500}^{\infty} 0. e^{-\theta x} dx.$$

$$= \int_{1500}^{\infty} 0.0005 \cdot e^{-0.0005x} dx$$

$$= 0.0005 \left[\frac{e^{-0.0005x}}{-0.0005} \right]_{1500}^{\infty}$$

$$= - \left[e^{-0.0005x} \right]_{1500}^{\infty}$$

Note $e^{\infty} = 1$

$$= 0 + e^{-0.0005 \times 1500}$$

$$= e^{-0.75}$$

$$= 0.47$$

(b). Now, $P(\text{at most } 2500)$

$$= P(X \leq 2500)$$

$$= \int_0^{2500} 0. e^{-0.2x} dx.$$

$$= \int_0^{2500} 0.0005 \cdot e^{-0.0005x} dx$$

$$= 0.0005 \left[\frac{e^{-0.0005x}}{-0.0005} \right]_0^{2500}$$

$$= -e^{-0.0005 \times 2500} + e^0$$

$$= -0.286 + 1$$

$$= 0.714,$$

Normal approximation to Binomial.

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{X_{\text{adj}} - np}{\sqrt{npq}}$$

X_{adj} means X adjusted.
i.e. $X \pm 0.5$
 $\underline{\text{ex}} P(X \geq 20)$
 $= P(X_{\text{adj}} \geq 19.5)$ taking \leftarrow
OR
 $P(X_{\text{adj}} \geq 20.5)$

- Q For overseas flight an airline has three different choices on its dessert menu, ice cream, apple pie and chocolate cake. Based on past experience the airline feels that each dessert is equally chosen, of the random sample of 90 passengers is selected what is the probability that at least 20 will choose ice cream for dessert.
- ② At least 20 will choose ice cream for dessert.

(b) Exactly 20 will choose ~~ice~~ icecream for dessert.

(c) Less than 20 will choose icecream for dessert.

Soln
Given,

$$\text{No. of passenger}(n) = 90.$$

$$\text{Probability of choosing any dessert } (P) = \frac{1}{3} = 0.33$$

$$\Rightarrow q = 1 - P \\ = 1 - 0.33 \\ = 0.67$$

(d) $P(\text{at least } 20 \text{ will choose icecream as a dessert})$

$$= P(X \geq 20)$$

$$= P(X_{\text{adj}} \geq 19.5)$$

$$\text{When } X_{\text{adj}} = 19.5 \text{ then, } Z = \frac{X_{\text{adj}} - np}{\sqrt{npq}}$$

$$= \frac{19.5 - 90 \times 0.33}{\sqrt{90 \times 0.33 \times 0.67}}$$

Now,

$$P(X \geq 20)$$

$$= P(Z \geq -2.28)$$

$$= 0.5 + P(-2.28 < Z \leq 0)$$

$$= 0.5 + P(0 \leq Z \leq 2.28)$$

$$= 0.5 + 0.4887$$

$$= 0.9587$$

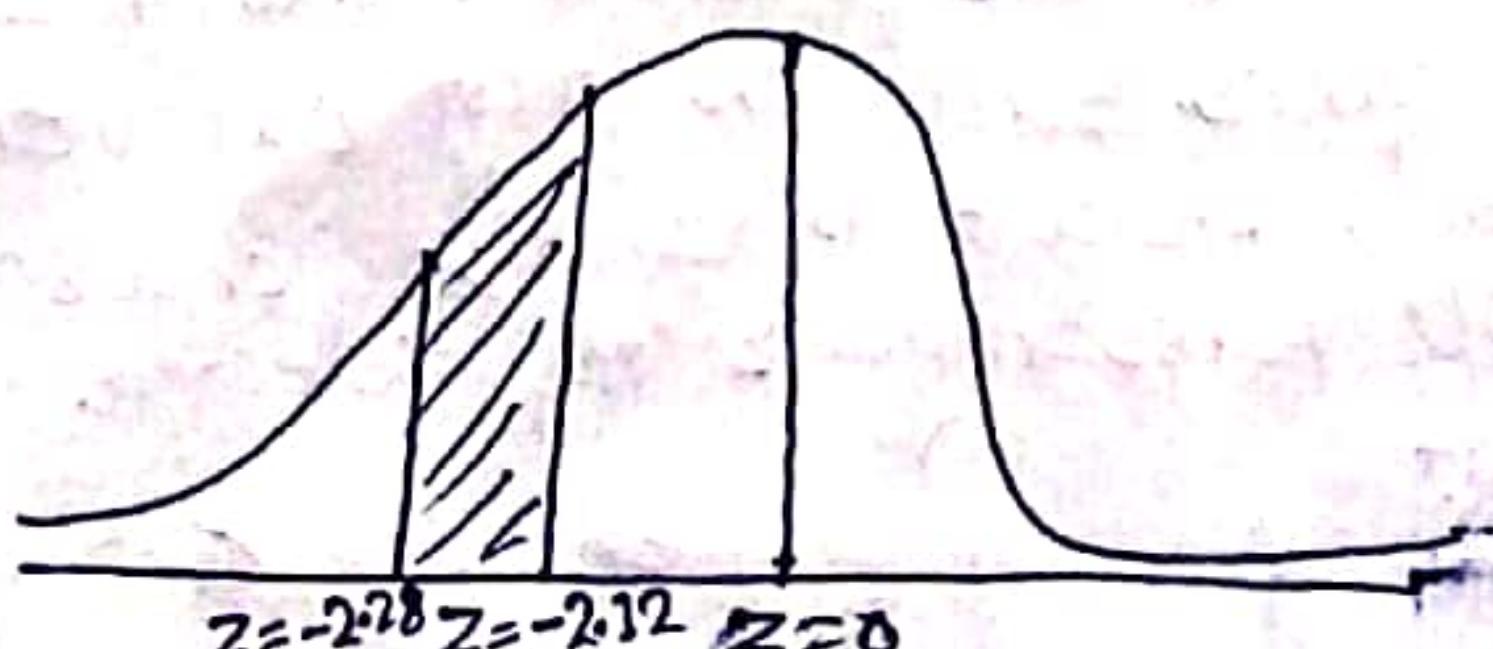
(e) $P(\text{exactly } 20 \text{ will choose icecream}).$

$$= P(X = 20)$$

$$= P(19.5 < X_{\text{adj}} < 20.5)$$

$$\text{When } X_{\text{adj}} = 19.5, \text{ then } Z = \frac{20.5 - 30}{\sqrt{90 \times 0.33 \times 0.67}}$$

$$= -2.12$$



$$\begin{aligned}
 \text{So, } P(19.5 < X_{\text{adj}} < 20.5) \\
 &= P(-2.28 < z < -2.12) \\
 &= P(-2.28 < z < 0) - P(-2.12 < z < 0) \\
 &=
 \end{aligned}$$

Q. P (less than 20)

$$\begin{aligned}
 &= P(X < 20) \\
 &= P(X_{\text{adj}} < 20.5)
 \end{aligned}$$

When $X_{\text{adj}} = 20.5$ then $z = -2.12$

Now,

$$\begin{aligned}
 &P(X_{\text{adj}} < 20.5) \\
 &= P(z < -2.12) \\
 &= 0.5 - P(-2.12 < z < 0) \\
 &= 0.5 - P(0 < z < 2.12) \\
 &= 0.5 - 0.4830 \\
 &= 0.0170,
 \end{aligned}$$

Q.No.78 A printer can print 15 pages per minute on average. Using normal distribution as approximation of Poisson distribution find probability that printer can print more than 25 pages per minute.

Soln

Given, parameter (λ) = mean = 15.

Now, P(more than 25 pages)

$$\begin{aligned}
 &= P(X > 25) \\
 &= P(X_{\text{adj}} > 24.5)
 \end{aligned}$$

$$\begin{aligned}
 \text{When } X_{\text{adj}} > 24.5, z &= \frac{X_{\text{adj}} - \lambda}{\sqrt{\lambda}} \\
 &= \frac{24.5 - 15}{\sqrt{15}} \\
 &= 2.32
 \end{aligned}$$

Now, $P(X_{\text{adj}} > 24.5)$

$$= P(z > 2.32)$$

$$= 0.5 - P(0 < z < 2.32)$$

$$= 0.5 - 0.4898$$

$$= 0.0102,$$

Q.No.79 The life time of Lenovo cell phone has gamma distribution with parameter 2. Find the probability that cell phone has life (i) more than 2 years (ii) Between 3 years to 5 years.

Soln

Given, Parameter (α) = 2

Now, $P(\text{Cell phone life more than 2 years})$.

$$= P(X > 2)$$

$$= \int_2^{\infty} \frac{e^{-x} \cdot x^{\alpha-1}}{\Gamma(\alpha)} \cdot dx$$

$$= \int_2^{\infty} \frac{e^{-x} \cdot x}{\Gamma(2)} dx$$

$$= \int_2^{\infty} x \cdot e^{-x} dx$$

$$= \left[x \int e^{-x} dx - \int \left[\frac{d}{dx} \int e^{-x} dx \right] dx \right]_2^{\infty}$$

$$= \left[x \left[\frac{e^{-x}}{-1} \right] - \int \left(\frac{e^{-x}}{-1} \right) dx \right]_2^{\infty}$$

$$= \left[-x \cdot e^{-x} - e^{-x} \right]_2^{\infty}$$

$$= 0 - e^{-\infty} + 2e^{-2} + e^{-2}$$

$$= 3e^{-2}$$

$$= 0.406$$

Now, $P(3 < X < 5)$

$$= \int_3^5 \frac{e^{-x} \cdot x^{\alpha-1}}{\Gamma(\alpha)} \cdot dx$$

$$= \int_3^5 e^{-x} \cdot x \cdot dx$$

$$= \left[-x \cdot e^{-x} - e^{-x} \right]_3^5$$

$$= -5e^{-5} - e^{-5} + 3e^{-3} + e^{-3}$$

$$= -6e^{-5} + 4e^{-3}$$

$$= 0.158,$$

$$\because f(x) = \frac{e^{-x} \cdot x^{\alpha-1}}{\Gamma(\alpha)} \quad 0 < x < \infty$$

$$\text{mean is } \lambda \\ \text{and } \sigma \text{ is } \sqrt{\lambda} \\ \frac{x-\mu}{\sigma}$$

$$0 < x < \infty$$

$$\lambda$$

$$\sigma$$

$$\mu$$

$$\sigma$$

$$\lambda$$

Joint probability density function:

Let x and y be the continuous random variable ($-\infty \leq x \leq \infty, -\infty \leq y \leq +\infty$) then the function $f(x,y)$ is said to be a joint probability density function of random variable x and y if it satisfy the following condition:

① If $f(x,y) \geq 0$.

$$\text{ii)} \quad \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) dx dy = 1, \quad -\infty \leq x \leq \infty \\ -\infty \leq y \leq \infty$$

Marginal Probability density function:

$$f(x) = \int_{-\infty}^{\infty} f(x,y) \cdot dy - \textcircled{1} \quad \text{where, } -\infty \leq y \leq \infty$$

$$f(y) = \int_{-\infty}^{\infty} f(x,y) dx \quad \text{--- (7)}$$

where, $-\infty \leq x \leq \infty$

Let $f(x,y)$ be the joint probability density function of continuous random variables X and Y . Then, marginal probability density function of X is obtained as in (9) given below, $-\infty \leq x \leq \infty$.

$f(x)$ is probability density function if it satisfies following conditions:

- 1) If $f(x) \geq 0$.

and the

且若 $\int_{-\infty}^{\infty} f(x) dx = 1$.

Similarly $\int_{-\infty}^{+\infty} f(x) dx = 1.$

$f(y)$ is probability density function (pdf) if it satisfies following conditions.

$$\nexists y \in S \text{ such that } f(y) < 0.$$

$$\text{Pf } \int_{-\infty}^{\infty} f(y) \cdot dy = 1.$$

Conditional probability density function:

Let $f(x, y)$ be the joint probability density function of continuous random variable X and Y and $f(x)$ and $f(y)$ be the normal probability density function of x and y respectively. Then conditional probability density function of X given Y is.

$$f(x|y) = \frac{f(x,y)}{f(y)}$$

Similarly $f(y|x) = \frac{f(x,y)}{f(x)}$.

conditions
if $f(x,y) \geq 0$
if $\int_{-\infty}^{\infty} f(x) dx = 1$.

Note: If X and Y are independent random variable then joint probability is $f(x,y) = f(x) \cdot f(y)$.

Numerical Questions

1. If two random variables have the joint probability density function

$$f(x,y) = \begin{cases} 6 \frac{(x+y^2)}{5} & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability that (i) $0.2 < x < 0.5$ and $0.4 < y < 0.6$ (ii) $x > 0.4$ and $y < 0.5$.

[Ans: 0.43, 0.28]

2. Suppose that x & y have joint density function

$$f(x,y) = \begin{cases} (x+y), & \text{if } 0 < x, y < 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find: (i) Marginal density function of x & y (ii) Covariance between x & y .

[Ans: $(x+1)/2, (y+1)/2, -0.006$]

3. If two random variables X_1 and X_2 have the joint probability density function

$$f(x_1, x_2) = \begin{cases} \frac{2}{3} (x_1 + 2x_2) & \text{for } 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the conditional density of X_1 given X_2 and X_2 given X_1

$$\left(\text{Ans: } \frac{2(x_1 + 2x_2)}{1 + 4x_2}, \frac{x_1 + 2x_2}{x_1 + 1} \right)$$

4. Let (X, Y) be two dimensional random variable with joint pmf $P(x,y) = \frac{x-y+3}{48}$; $x=0,1,2,3$; $y=0,1,2,3$. Find: (i) marginal pmf of X and Y (ii) conditional distribution of X for given $Y=1$ (iii) conditional distribution of Y for given $X=2$.

$$\left(\text{Ans: } \frac{2x+3}{24}, \frac{9y-2}{24}, \frac{x+2}{14}, \frac{5-y}{14} \right)$$

5. Given the following bivariate probability distribution of X & Y .

	X	-1	0	1
Y				
0		1/15	2/15	1/15
1		3/15	2/15	1/15
2		2/15	1/15	2/15

Find: (i) $P(X=1, Y \leq 1)$ (ii) $P(Y \leq 1)$ (iii) $P(X=-1)$

$$[\text{Ans: } 2/15, 2/3, 2/5]$$

6. A company has 3 hardware engineering consultants and 4 software engineering consultants. From these 2 engineering consultants are selected at random. Let X denote the number of hardware engineering consultant and Y denotes the number of software engineering consultant. Find the (i) Joint probability distribution of X & Y . (ii) Marginal distribution of X and that of Y (iii) Conditional distribution of X for given Y .

7. A two-dimensional discrete random variable (X, Y) has the joint probability distribution as given below:

	X	0	1	2	3
Y					
-1		0.03	0.02	0.05	0
0		0.01	0.25	0.45	0.01
1		0	0.03	0.05	0.10

Determine: (i) marginal distribution of X & Y (ii) Conditional distribution of X for given Y and that of Y for given X (iii) $P(X=0, Y \leq 2)$ (iv) $P(X \geq 0, Y < 3)$ (v) $P(X \leq 0 / Y=1)$

8. Suppose two-dimensional random variable (X, Y) has pdf given by

$$f(x,y) = \begin{cases} x^2 + kxy; & 0 \leq x \leq 1; 0 \leq y \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

Determine: (i) constant k (ii) marginal pdf of X and that of Y (iii) conditional pdf of X given Y and that of Y given X (iv) Are X and Y independent.

$$\text{Ans: } \left(\frac{1}{3}, \frac{6x^2 + 2x}{3}, \frac{2+y}{6}, \frac{2(3x^2 + xy)}{2+y}, \frac{3x^2 + xy}{6x^2 + 2x}, \text{dependent} \right)$$

9. The joint pdf of two dimensional random variable (X, Y) is $f(x, y) = \begin{cases} 2x; & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$

$$[\text{Ans: } 2x, 1, \text{independent}]$$

10. Let two dimensional random variable (X, Y) have joint pdf $f(x,y) = \begin{cases} k(6-x-y); & 0 \leq x \leq 2, 2 \leq y \leq 4 \\ 0 & \text{elsewhere} \end{cases}$

Determine: (i) Constant k (ii) $P(X \leq 1 \cap Y < 3)$ (iii) $P(Y \leq 3)$ (iv) $P(X \leq 1 / Y \leq 3)$

Numerical Questions:

Q. No. 1 Solⁿ

Given, Joint probability density function $f(x,y) = \frac{6(x+y^2)}{5}$
 $0 < x < 1, 0 < y < 1$
 = 0 otherwise.

Now,

$$P(0.2 < x < 0.5 \text{ and } 0 < y < 0.6)$$

$$= \int_{0.2}^{0.5} \int_{0.4}^{0.6} f(x,y) dx dy$$

$$= \int_{0.2}^{0.5} \left[\int_{0.4}^{0.6} \frac{6(x+y^2)}{5} dy \right] dx$$

$$= \int_{0.2}^{0.5} \frac{6}{5} \left[xy + \frac{y^3}{3} \right]_{0.4}^{0.6} dx$$

$$= \frac{6}{5} \int_{0.2}^{0.5} \left[0.6x + \frac{0.216}{3} - 0.4x - \frac{0.064}{3} \right] dx$$

$$= \frac{6}{5} \int_{0.2}^{0.5} \left(\frac{0.2x + 0.047x}{3} \right) dx$$

$$= \frac{6}{5} \cdot \left[\frac{0.2x^2 + 0.047x^2}{6} \right]_{0.2}^{0.5}$$

$$= \frac{6}{5} \left[\frac{0.2(0.5)^2}{2} + \frac{0.047(0.5)^2}{2} \right]_{0.2}^{0.5}$$

$$= \frac{6}{5} \left[\frac{0.2 \times 0.5^2}{2} + 0.05 \times 0.5 - \frac{0.2 \times 0.1^2}{2} - 0.05 \times 0.2 \right] \\ = 0.093,$$

(9) $P(x > 0.4 \text{ and } y < 0.5)$.

$$= \int_{0.4}^1 \int_0^{0.5} f(x,y) dx dy$$

$$= \int_{0.4}^1 \int_0^{0.5} \frac{6}{5} (x+y^2) dx dy.$$

$$= \frac{6}{5} \int_{0.4}^1 \left[\int_0^{0.5} (x+y^2) dy \right] dx,$$

$$= \frac{6}{5} \int_{0.4}^1 \left[xy + \frac{y^3}{3} \right]_0^{0.5} dx.$$

$$= \frac{6}{5} \int_{0.4}^1 \left(0.5x + \frac{0.125}{3} \right) dx$$

$$= \frac{6}{5} \left[\frac{0.5x^2}{2} + 0.0416x \right]_{0.4}^1$$

$$= \frac{6}{5} \left[\frac{0.5 \times 1}{2} + 0.0416 - \frac{0.5 \times (0.4)^2}{2} - 0.0416 \times 0.4 \right]$$

$$= 0.28,$$

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Q.No.3 Soln

Given, $f(x_1, x_2) = \frac{2}{3}(x_1 + 2x_2)$ for $0 < x_1 < 1, 0 < x_2 < 1$.
 = 0 otherwise.

We know,

Conditional probability density function of x_1 and x_2 is

$$f(x_1/x_2) = \frac{f(x_1, x_2)}{f(x_2)}$$

$$\text{and } f(x_2/x_1) = \frac{f(x_1, x_2)}{f(x_1)}$$

Now,

Marginal probability density function of x_1 is $f(x_1)$.

$$\text{and } f(x_1) = \int_0^1 f(x_1, x_2) dx_2.$$

$$= \frac{2}{3} \int_0^1 (x_1 + 2x_2) dx_2.$$

$$= \frac{2}{3} \left[x_1 x_2 + 2x_2^2 \right]_0^1$$

$$= \frac{2}{3} [1 + \frac{1}{2}]$$

Since value
of x_2 is 1
 $\frac{1}{2}$

$$f(x_2) = \int_0^1 f(x_1, x_2) dx_1.$$

$$= \frac{2}{3} \int_0^1 (x_1 + 2x_2) dx_1,$$

$$= \frac{2}{3} \left[\frac{x_1^2}{2} + 2 \cdot x_2 \cdot x_1 \right]_0^1$$

$$= \frac{2}{3} \left[\frac{1}{2} + 2x_2 \right]$$

$$\text{Now, } f\left(\frac{x_1+x_2}{x_1}\right) = \frac{f(x_1, x_2)}{f(x_1)} = \frac{2/3(x_1+2x_2)}{2/3(1/2+2x_2)} \\ = \frac{2(x_1+2x_2)}{1+4x_2}$$

$$\text{Similarly, } f\left(\frac{x_2+x_1}{x_2}\right) = \frac{f(x_2, x_1)}{f(x_2)} = \frac{2/3(x_1+2x_2)}{2/3(1+x_1)} \\ = \frac{x_1+2x_2}{1+x_1}$$

Q.No.8. Soln

given, $f(x,y) = x^2 + kxy ; 0 \leq x \leq 1 ; 0 \leq y \leq 2.$
 $= 0 \text{ elsewhere.}$

(P) For finding the value of 'k'.

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) dx dy = 1.$$

$$\text{or, } \int_0^1 \int_0^2 f(x^2+kxy) dx dy = 1,$$

$$= \int_0^1 \int_0^2 (x^2+kxy) dx dy$$

$$\text{or, } \int_0^1 \left[\int_0^2 (x^2+kxy) dy \right] dx = 1$$

$$\text{or, } \int_0^1 \left[\frac{x^2y + kxy^2}{2} \right]_0^2 dx = 1$$

$$\text{or, } \int_0^1 (2x^2 + 2kx) dx = 1.$$

$$\text{or, } \left[\frac{2x^3}{3} + kx^2 \right]_0^1 = 1$$

$$\text{or, } \frac{2}{3} + k = 1$$

$$\text{or, } k = 1 - \frac{2}{3}$$

$$\text{or, } k = \frac{1}{3}.$$

(ii) Here,

$$f(x) = \int_0^2 f(x, y) dy$$

$$= \int_0^2 (x^2 + \frac{2}{3}xy) dy$$

$$= \left[x^2y + \frac{x^2y^2}{6} \right]_0^2$$

$$= \frac{2x^2 + 4x}{6}$$

$$= \frac{6x^2 + 2x}{3}$$

(iii) Here, $f(y) = \int_0^1 f(x, y) dx$

$$= \int_0^1 (x^2 + \frac{2}{3}xy) dx$$

$$= \left[\frac{x^3}{3} + \frac{x^2y}{6} \right]_0^1$$

$$= \frac{1}{3} + \frac{y}{6}$$

$$= \frac{2+y}{6}$$



(ii) & (iii) आपके लिए.

Q.No. 10 Solⁿ

Given,

$$f(x,y) = \begin{cases} 2x + k(6-x-y), & 0 \leq x \leq 2, 2 \leq y \leq 4 \\ 0, & \text{elsewhere.} \end{cases}$$

(i) Solⁿ for finding the value of 'k'.

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) \cdot dx \cdot dy = 1.$$

$$\text{or, } \int_0^2 \int_2^4 k(6-x-y) \cdot dx \cdot dy = 1.$$

$$\text{or, } k \int_0^2 \left[\int_2^4 (6-x-y) dy \right] dx = 1.$$

$$\text{or, } k \int_0^2 \left[6y - xy - y^2/2 \right]_2^4 dx = 1.$$

$$\text{or, } k \int_0^2 (24 - 4x - 8 - 12 + 2x + 2) dx = 1.$$

$$\text{or, } k \int_0^2 (6 - 2x) dx = 1.$$

$$\text{or, } k \left[6x - x^2 \right]_0^2 = 1$$

$$\text{or, } k [12 - 4] = 1$$

$$\text{or, } k = \frac{1}{8}$$

$$\text{(ii). Now, } f(x,y) = \frac{1}{8} (6-x-y).$$

So, the probability $(x \leq 1 \wedge y \leq 3)$

$$= \int_0^1 \int_2^3 f(x,y) \cdot dx \cdot dy.$$

$$\begin{aligned}
 &= \int_0^1 \left[\int_0^3 \frac{1}{8} (6-x-y) dy \right] dx \\
 &= \frac{1}{8} \int_0^1 \left[6y - xy - \frac{y^2}{2} \right]_0^3 dx \\
 &= \frac{1}{8} \cdot \int_0^1 (18 - 3x - \frac{9}{2} - 12 + 2x + 2) dx \\
 &= \frac{1}{8} \int_0^1 (3.5 - x) dx \\
 &= \frac{1}{8} \left[3.5x - \frac{x^2}{2} \right]_0^1 \\
 &= \frac{1}{8} \left[3.5 - \frac{1}{2} \right] \\
 &= \frac{3}{8}
 \end{aligned}$$

Q17. $P(Y \leq 3)$. Since ≤ 3 otherwise 0

$$= \int_2^3 f(y) dy$$

where, $f(y) = \int_0^2 f(x,y) dx$

$$\begin{aligned}
 &= \int_0^2 \frac{1}{8} (6-x-y) dx \\
 &= \frac{1}{8} \left[6x - \frac{x^2}{2} - xy \right]_0^2 \\
 &= \frac{1}{8} \cdot (10 - 2y) \\
 &= \left(\frac{5-y}{4} \right)
 \end{aligned}$$

Now, $P(Y \leq 3)$

$$= \int_2^3 \left(\frac{5-y}{4} \right) dy$$

$$= \frac{1}{4} \left[5y - \frac{y^2}{2} \right]_2^3$$

$$= \frac{1}{4} [25 - \frac{9}{2} - 10 + 2]$$

$$= \frac{1}{4} [2.5]$$

$$= \frac{2.5}{4}$$

or, $\frac{2.5 \times 2}{4 \times 2} = \frac{5}{8}$

⑨. $P(X \leq 1 / Y \leq 3)$

$$= \frac{P(X \leq 1, Y \leq 3)}{P(Y \leq 3)}$$

$$= \frac{3}{8}$$

$$\frac{5}{8}$$

$$= \frac{3}{5}$$