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INTRODUCTION TO PROBABILITY

Probability is the chance of happening of an event OR probability is the happening of an event or not happening of an event. The word probability is also used in our daily life for e.g. (i) The chance of raining in this morning
(ii) The chance of passing in exam etc.

The probability is a number that lies between 0 and 1. If the probability is equal to 1 then such a probability is called probability of sureity or certainity & if probability is equal to zero then such a probability is called probability of unsureity or uncertainity.

Terminology/related terms:

i) Experiment or Trial → Experiment or trial is such a activity which generates the data.
For example:- If we toss a coin then coin tossing is an experiment.

ii) Sample space → Sample space is the set of all possible outcomes.
For example:- If we toss two coins at a time then sample space $S = \{HH, TH, HT, TT\}$.

iii) Exhaustive or total no. of cases → Exhaustive number of cases is the number of total possible outcomes. For example:- If we toss a coin then sample space $S = \{H, T\}$ and exhaustive no. of cases $= 2$.

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IV). **Mutually exclusive event** → Two events are said to be mutually exclusive if the happening of an event that excludes the happening of other event. For example:- If we toss the coin head and tail are mutually exclusive.

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V). **Equally likely event** → Events are said to be equally likely if the probability of happening of ~~all~~ each events is same.
For example:- If we throw a coin then the probability of head $= \frac{1}{2}$ and probability of tail $= \frac{1}{2}$ so, head and tail are equally likely events.

VI). **Favourable no. of cases** → Favourable number is the number which is favourable to happening of an event. For example:- If we throw a dice then sample space $S = \{1, 2, 3, 4, 5, 6\}$, then favourable no. of cases for getting even number is 3.

VII). **Independent and Dependent events** → Two events are said to be independent if the probability of happening of one event that does not effect the happening of other events. For example:-

Independent → $\boxed{\text{52 cards}}$ $\xrightarrow{\text{1st draw}}$ Probability $\frac{1}{52}$ $\xrightarrow[\text{replacement}]{\text{with}}$ $\boxed{\text{52 cards}}$ $\xrightarrow[\text{draw}]{\text{2nd}}$ Probability $\frac{1}{52}$

Dependent → $\boxed{\text{52 cards}}$ $\xrightarrow{\text{1st draw}}$ Probability $\frac{1}{52}$ $\xrightarrow[\text{replacement}]{\text{without}}$ $\boxed{\text{51 cards}}$ $\xrightarrow[\text{draw}]{\text{2nd}}$ $P = \frac{1}{51}$.

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Approach of probability:-

1) **Classical definition of probability** → Let 'n' be the exhaustive, equally likely and mutually exclusive outcomes and 'm' be the favourable number of cases for the happening an event A then,

Probability of happening of event A = $\frac{\text{favourable number}}{\text{Exhaustive number}}$

$= \frac{m}{n}$ ... (i)

Then, $n-m$ be the favourable number of not happening of event A. Now, the probability of not happening of event (A) = $P(\bar{A})$

$= \frac{n-m}{n}$ ... (ii)

From eqn (i) and (ii)

$$P(A) + P(\bar{A}) = \frac{m}{n} + \frac{n-m}{n}$$

$$= \frac{m}{n} + 1 - \frac{m}{n}$$

$$= 1.$$

Thus, probability of happening of an event + Probability of not happening of an event = 1.

Thus condition does not hold under following condition:

(i) If events are not equally likely.

(ii) If n is not finite.

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Laws of Probability:

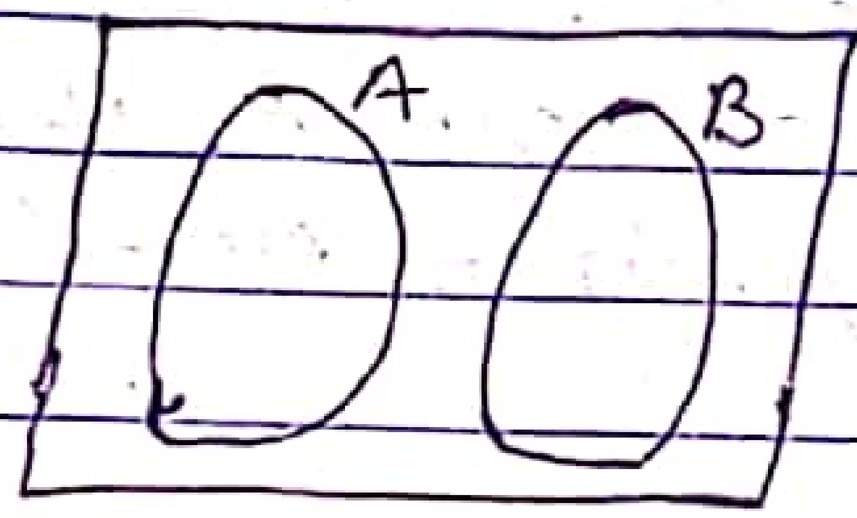
Additive law of probability

Let A and B are two events. If A and B are mutually exclusive events then probability of either A or B is equal to the sum of their individual probabilities that means probability of either A or B

$$\Rightarrow P(A \cup B) = P(A) + P(B).$$

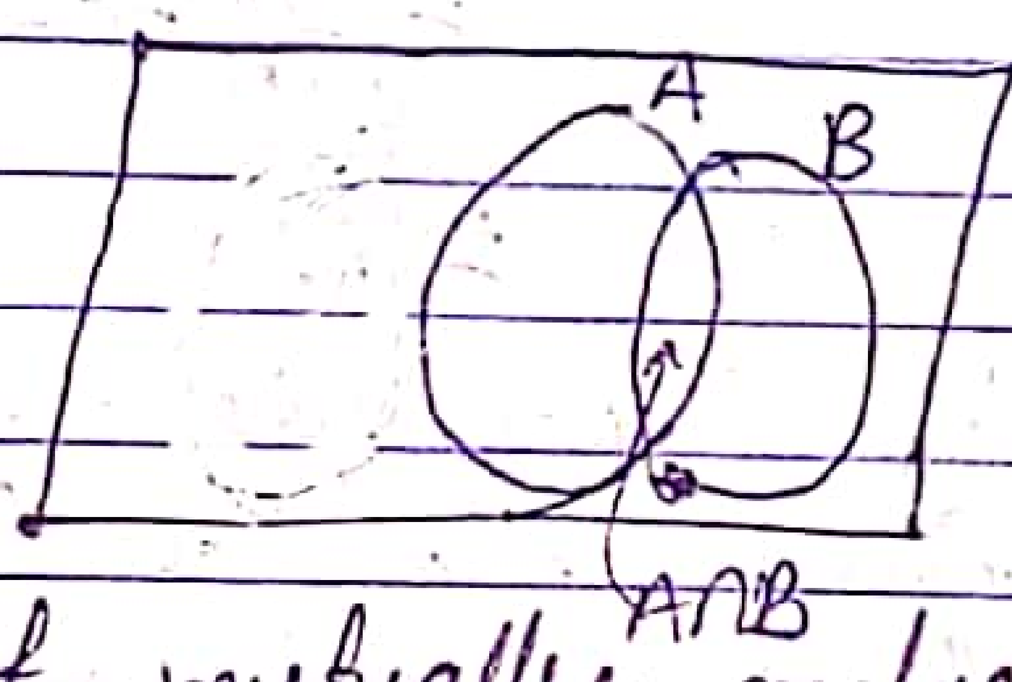
Similarly

$$P(A \cup B \cup C) = P(A) + P(B) + P(C).$$



Mutually exclusive events.

If A and B are not mutually exclusive events then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.



Not-mutually exclusive.

Similarly

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C).$$

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$(A \cup B)$ OR $(A \cap B)$ and

$P(B/A) \rightarrow$ Not B divides A. i.e. B due to already happening of A.

Date.

$P(B/A) = \frac{P(A \cap B)}{P(A)}$

ii) Multiplicative law of probability:-

If A and B are two independent events then probability of getting or happening both events is equal to the product of their individual probabilities

$P(A \cap B)$
$= P(A).P(B)$.

Similarly $P(A \cap B \cap C) = P(A).P(B).P(C)$.

If A and B are dependent events then the probability A and B ~~are~~ is $P(A \cap B) = P(A).P(B/A)$.
$= P(B).P(A/B)$.

where, $P(A) \rightarrow$ Marginal probability of A.
$P(B) \rightarrow$ Marginal probability of B.
$P(A \cap B) \rightarrow$ Joint probability of A and B.
$P(A/B) \rightarrow$ Probability of A given that B has already happen or conditional probability of A given B.

Similarly If A, B and C are dependent events then,

$P(A \cap B \cap C)$
$= P(A \cap B).P(C/A \cap B)$.
$= P(A).P(B/A).P(C/A \cap B)$

⊛ Conditional Probability:-

The concept of conditional probability is derived from the idea of dependent events so, the probability of events A given that B has already

Club →
Spade
Diamond →
Heart →

$$P(B/D) = \frac{P(B\cap D)}{P(D)} = \frac{P(B\cap D)}{P(D)} = \frac{P(B).P(D/B)}{P(D)} = \frac{P(B).P(D/B)}{P(A\cap B)+P(B\cap D)+P(C\cap D)}$$

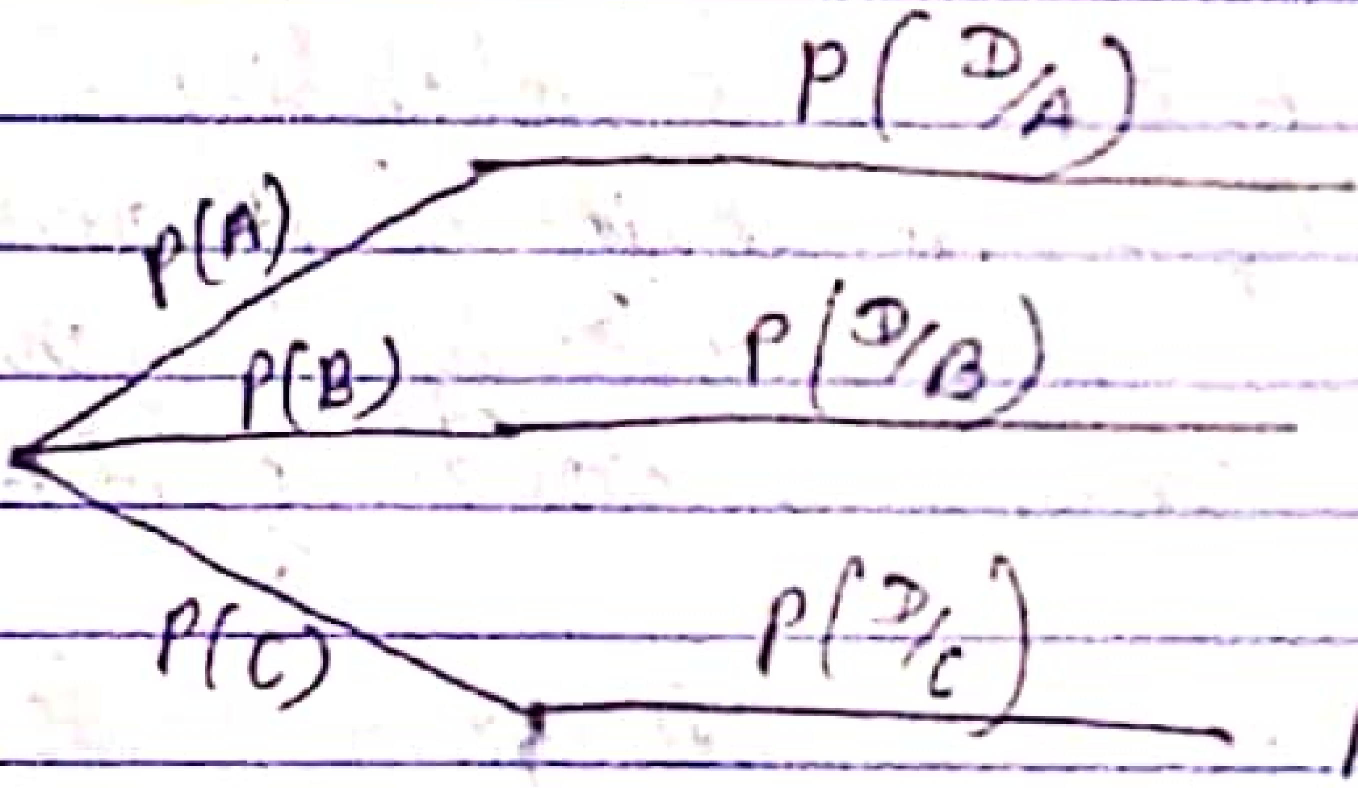
happened is the conditional probability of A given B and probability of A given B is:

$$P(A/B) = \frac{P(A\cap B)}{P(B)} \text{, provided that } P(B)>0.$$

Similarly $P(B/A)$ = conditional probability of B given that A has already happened or conditional probability of B given A.

$$= \frac{P(A\cap B)}{P(A)} \text{, provided that } P(A)>0.$$

Baye's theorem of Probability: Imp ⇐ problems related to these



$P(A).P(D/A) = P(A\cap D)$

$P(B).P(D/B) = P(B\cap D)$

$P(C).P(D/C) = P(B\cap D)$

$$P(A/D) = \frac{P(A\cap D)}{P(D)}$$

$$= \frac{P(A).P(D/A)}{P(A).P(D/A)+P(B).P(D/B)}$$

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Numerical Questions:

Q1. If two dice are thrown, what is the probability that the sum is greater than 9 neither 8 nor 10.

Soln

If we throw two dice then sample space is.

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66

Here, exhaustive number = 36.

Now, favourable no. ~~of~~ for the sum greater than 9 = 6

So, P(sum is greater than 9) $= \frac{6}{36} = \frac{1}{6}$ //

Again,

Favourable no. of cases for the sum neither 8 nor 10 = 28

Now,

P(sum neither 8 nor 10) $= \frac{28}{36} = \frac{7}{9}$ //

2. A secretary types 5 letters and 5 envelopes and puts the 5 letters into the envelopes at random. What is the probability that the letters are not put into correct envelop?

Soln

Here, exhaustive no. of letters put in correct envelop is $^5P_5 = 5!$ ways $= 120$.

5 वटाले 5 वटा संग
Combination [illegible] i.e. permutation.

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Now,

probability (letter are put into correct envelop) $= \frac{1}{5!}$

$= \frac{1}{120}$.

Now, P(letters are not put into correct envelop) $= 1 - \frac{1}{120}$

$= \frac{119}{120}$

3. What is the probability that a leap year selected at random contain 53 saturdays?

Sol^n.

We know, Leap year = 366 days

$= \underbrace{52}_{i.e,\ 364\ days}$ weeks + 2 days

These two days maybe,

i) Sunday + Monday
ii) Monday + Tuesday
iii) Tuesday + Wednesday
iv) Wednesday + Thrusday
v) Thrusday + Friday
vi) Friday + Saturday
vii) Saturday + Sunday

Now, the P(leap year contains 53 saturday) $= \frac{2}{7}$

4. A problem in statistics is given to three students A, B and C whose chances of solving are $1/5$, $2/5$ and $3/5$ respectively. What is the probability that the problem will be solved?

Sol^n.

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Now,

$$P(\text{Problem will not be solved}) = P(\bar{A} \cap \bar{B} \cap \bar{C})$$
$$= P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$$
$$= \left(1-\frac{1}{5}\right) \cdot \left(1-\frac{2}{5}\right) \cdot \left(1-\frac{3}{5}\right)$$
$$= \frac{4}{5} \times \frac{3}{5} \times \frac{2}{5}$$
$$= \frac{24}{125}$$

$\therefore P(\text{Problem will be solved}) = 1 - P(\text{will not be solved})$
$$= 1 - \frac{24}{125}$$
$$= \frac{101}{125}$$

5. The odds against solving a problem by B is 4 to 3. The odds in favor of solving the same problem by A is 7 to 6. What is the probability that problem will be solved if both of them try independently?

Soln

Given, odds against solving problem by B is 4 to 3 or 4:3

Now, $(\text{B solve the problem}) = \frac{3}{4+3} = \frac{3}{7}$

or, $P(B) = \frac{3}{7}$

Odds favour of solving the problem is 7 to 6 or, 7:6

So, $P(A) = \frac{7}{7+6} = \frac{7}{13}$

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at least one type का answer ... कुनै हरु हो जस्तो प्रति समस्या ... AUB को प्रयोग गर्ने

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Now, P (Problem will be solved if both of them try) = $P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{3}{7} + \frac{7}{13} - \frac{3}{13}$

$= \frac{39+49-21}{91}$

$= \frac{67}{91}$

6. From a pack of 52 cards 2 cards are drawn at random. Find the probability that (i) one is king & other is queen. (ii) both are face cards (iii) One is spade and other is club.

Soln

Given, no. of cards = 52

Exhaustive no. of selecting 2 card $= 52C_2$

$= \frac{52!}{50!\,2!}$

$= \frac{52 \times 51 \times 50!}{50! \times 2!}$

$= 1326$

i) Favourable no. of case of selecting 1 king and 1 queen

$= 4C_1 \times 4C_1$

$= 16$

Now P(1 king and 1 queen) $= \frac{16}{1326}$

$= \frac{8}{663}$

ii) Favourable no. of case of selecting both face cards

$= 12C_2$

$= 66$

Now, P(both face card) $= \frac{60}{1326}$

$= \frac{11}{221}$

iii) Favourable no. of cases for selecting one spade and other club is $13C_1 \times 13C_1 = 169$

So P(1 spade & 1 club) $= \frac{169}{1326} = \frac{13}{102}$.

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8. If $P(A)=0.4$, $P(B)=0.3$ and $P(A\cup B)=0.58$. Are A & B independent?

Soln

$P(A)=0.4$
$P(B)=0.3$
$P(A\cup B)=0.58$ are A and B independent?

Since $P(A\cup B)=0.58 \neq P(A)+P(B)=0.4+0.3=0.7$,
So, A and B are not mutually exclusive.
then $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
or, $0.58=0.4+0.3-P(A\cap B)$
or, $P(A\cap B)=0.12$.

and $P(A).P(B)=0.4\times 0.3$
$=0.12$.

equal भएमा dependent

So, $P(A\cap B)=P(A).P(B)=0.12$.
Hence, A and B are independent event.

11. The probability that a 50 years old man will alive at 60 is 0.83 and the probability that a 45 yrs old woman will be alive at 55 is 0.87, What is the probability that a man who is 50 and women who is 45 (i) both will alive 10 yers (ii) at least one will alive 10 years.

Soln

$P(\text{Man will alive 10 year})=0.83$
Let $P(A)=0.83$
then, $P(\bar{A})=1-0.83=0.17$

$P(\text{women will alive 10 year})=0.87$
Let $P(B)=0.87$
Now, $P(\bar{B})=1-0.87=0.13$

Now,

$P(\text{both will alive}) = P(A \cap B)$
$= P(A) \cdot P(B)$
$= 0.83 \times 0.87$
$= 0.722$

(ii) $P(\text{at least one will alive}) = P(A \cup B)$
$= P(A) + P(B) - P(A \cap B)$
$= 0.83 + 0.87 - 0.722$
$= 0.977$

13. Two cards are drawn at random one after another from a pack of 52 cards: What is the probability of getting
(i) First black card and second red card.
(ii) First ace and second ace, if first draw is not replaced before second draw.

Soln.
Given, total number of cards = 52
$P(1^{st} \text{ red and } 2^{nd} \text{ Black}) = ?$

Now,
(i) $P(R_1 \cap B_2)$
$= P(R_1) \cdot P(B_2 / R_1)$ (∵ this is the case of dependent i.e. without replacement)
$= \frac{26}{52} \times \frac{26}{51}$
$= \frac{13}{51}$

(ii) $P(1^{st} \text{ ace and } 2^{nd} \text{ ace})$
$P(A_1 \cap A_2) = P(A_1) \cdot P(A_2 / A_1)$
$= \frac{4}{52} \times \frac{3}{51}$
$= \frac{1}{221}$

$\frac{4}{3} \times \frac{5}{4}$ $\frac{3}{5}$

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15. The student body of a college is composed of 60% girls and 40% boys. 40% of girls and 60% of ~~bys~~ boys shows interest in mobile apps. What is the probability that a student who shows interest in mobile apps given that she is a girl?

Soln

The given information can be shown in following table.

	Interest in mobile apps (I)	Not interest in mobile app (N)	Total
Boys	24	16	40
Girls	24	36	60
Total	48	52	100

Now,

$$P(\text{interest in mobile apps}/\text{girl}) = P(I/G)$$

$$= \frac{P(I \cap G)}{P(G)}$$

$$= \frac{24/100}{60/100}$$

$$= \frac{24}{60}$$

$$= 0.4$$

18. A six faced dice is so biased that it is twice likely to show an even number as an odd numbers thrown is even?

Soln.

According to the question,

$P(\text{odd numbers}) = 2/6 = \frac{1}{3}$ = probability of odd number.

$P(\text{even number}) = 4/6 = 2/3$ = probability of even number

G	G	G
G	G	R
G	R	G
G	R	R
R	G	G
R	G	R
R	R	G
R	R	R

odd को sum even हुन्छ

Now,

$P(\text{Sum of two number is even numbered}) = P(EE \text{ or } OO)$

$= P(E \cap E) + P(O \cap O)$

$= P(E).P(E) + P(O).P(O).$

$= \frac{2}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{3}$

$= \frac{5}{9}$ //

Moreover (if asked)

$P(\text{sum of two number is odd}) = P(EO \text{ or } OE)$

$= P(E \cap O) + P(O \cap E)$

$= P(E).P(O) + P(O).P(E)$

$= \frac{2}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3}$

$= \frac{4}{9}.$ //

$(1 - P(\text{even}))$ गरे नि हुन्छ

20. There are three traffic light on your way home. As you arrive at each light assume that it is either red (R) or green (G) and that it is green with probability 0.7, construct the sample space by listing all possible eight sample events. Assign the probability to each sample event. Are the events equally likely? What is the probability that you stop no more than one time.

Soln

Since we are passing through 3 traffic lights each traffic light having either Red (R) or Green (G) signal, then sample space (S) is given as,

S = { GGG, GGR, GRG, GRR, RGG, ~~RRG, RRR~~, RGR, RRG, RRR }

$P(G) = 0.70$

$P(R) = 1 - 0.70$

$= 0.30$

Now;

$P(\text{Stop no more than 1 time}) = P(GRG \text{ or } RGG \text{ or } GGR \text{ or } GGG)$

$= P(GRG) + P(\cancel{G}RGG) + P(GGR) + P(GGG)$

$= PG.PR.PG + PR.PG.PG + PG.PG.PR + PG.PG.PG$

$= 0.7 \times 0.3 \times 0.7 + 0.3 \times 0.7 \times 0.7 + 0.7 \times 0.7 \times 0.3 + 0.7 \times 0.7 \times 0.7$

$= 0.78$

22) In the past several years, credit card companies have made an aggressive effort to solicit new accounts from college students. Suppose that a sample of 200 students at your college indicated the following information as to whether the student possessed a bank credit ~~ent~~ card &/or a travel and entertainment credit card.

Bank credit card.	Travel & Entertainment credit card.	
	Yes	No
Yes	60	60
No.	15	65.

a) What is the probability that the student has a bank credit card?

b) What is the probability that the student has a bank credit card and a travel and entertainment card?

c) What is the probability that the student has a bank credit card or a travel and entertainment card?

d) ~~Know~~ Known that the student has a bank credit card, what is the probability that he or she has a travel and entertainment card?

intersection ($\cap$) आए → not-mutually exclusive $P(A\cap B) = P(A).P(B)$
or, union ($\cup$) आए → mutually exclusive. $A\cup B = P(A) + P(B) - P(A\cap B)$

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Soln

For the calculation of probability.

Bank credit card	Travel and entertainment card		Total
	Yes (T)	No (NT)	
Yes (Y)	60	60	120
No (N)	15	65	80
Total	75	125	200.

Now,

i) P (student has bank credit card) $= \frac{120}{200} = 0.6$

Additional P(student has travel and Entertainment card)
$= \frac{75}{200} = 0.375$

ii) P(student having book credit card and Travel and entertainment card) $= P(T\cap Y)$
$= \frac{60}{200}$
$= 0.3$

iii) P(student having bank credit card or travel and entertainment card) $= P(Y \text{ or } T)$
$= P(Y) + P(T) - P(Y\cap T)$
$= \frac{120}{200} + \frac{75}{200} - \frac{60}{200}$
$= 0.6 + 0.375 - 0.3$
$= 0.675$

iv) P(he or she has travel and entertainment card having bank credit card) $= P(T/Y)$
$= \frac{P(T\cap Y)}{P(Y)} = \frac{60/200}{120/200} = 0.5$

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23. A company has made available to it's employees (without charge) extensive health club facilities that be used before work, during lunch hour, after work, and on weekends. Records for the last year indicate that of 250 employees, 110 used the facilities at sometime of 170 males employed by company, 65 used facilities.

a) Set up the (2x2) table to evaluate the probabilities.

b) What is the probability that an employee chosen at random is a female and has used the health club facilities?

c) What is the probability that an employee chosen at random in male?

d) What is the probability that an employee chosen at random is a male or has not used the health club facilities?

Soln

Given information can be shown in the following 2x2 table which is used for calculating probability.

(a)

Gender	Health Club facilities		Total
	Yes (Y)	No (N)	
Male (M)	65	105	170
Female (F)	45	35	80
Total	110	140	250

Now,

(b) P(Female and use the Health club facilities) = $P(F \cap Y)$

$= 45/250$

$= 0.18$

(c) P(Random is male) $= \frac{170}{250} = 0.68$

(d) P(Male and has not used the Health club facilities)

$= P(M \text{ or } N)$

$= P(M) + P(N) - P(M \cap N)$

$= \frac{170}{250} + \frac{140}{250} - \frac{105}{250}$

$= 0.82$

24\. We are given a box containing 5000 IC chips, of which 1000 are manufactured by company X and the rest by company Y. Ten percent of the chips made by company X and five percent of the chips made by company Y are defective. If a randomly chosen chip is found to be defective, find the probability that it came from company X.

Soln

Let A represent the IC manufactured by company X and B represent the IC manufactured by company Y.

Now,

$P(\text{IC manufactured by } X) = \frac{1000}{5000} = 0.2$

or, $P(A) = 0.2$

So, the $P(\text{IC manufactured by } Y) = 1 - P(A)$

$= 1 - 0.2$

$= 0.8$

or, $P(B) = 0.8$

Let D be the defective chips.

then, $P(\text{Defective given by company } X) = P(D/A) = 10\%$

$= 0.10$

& $P(\text{Defective given by company } Y) = P(D/B) = 5\%$

$= 0.05$

Now, the $P(\text{Company } X/\text{defective}) = P(A/D)$ ← read as A given D

$= \frac{P(A) \cdot P(D/A)}{P(A) \cdot P(D/A) + P(B) \cdot P(D/B)}$

$= \frac{0.2 \times 0.10}{0.2 \times 0.10 + 0.8 \times 0.05}$

$= 0.33$

25. In a certain assembly plant, three machines A_1, A_2 and A_3 make 30%, 45% and 25% respectively of the product. It is known from past experience that 2%, 3% and 2% of the products made by each machine A_1, A_2 & A_3 respectively are defective. If a product was chosen randomly and found to be defective, what is the probability that it was made by machine A_3.

Soln

$P(A_1) = 30\% = 0.3$

$P(A_2) = 45\% = 0.45$

$P(A_3) = 25\% = 0.25$

$P(D/A_1) = 2\% = 0.02$

$P(D/A_2) = 3\% = 0.03$

$P(D/A_3) = 2\% = 0.02$

Now,

$$P(A_3/D) = \frac{P(A_3).P(D/A_3)}{P(A_1).P(D/A_1) + P(A_2).P(D/A_2) + P(A_3).P(D/A_3)}$$

$=$

$=$

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28. Urn A contains 2 white, 1 black and 3 red balls. Urn B contains 3 white, 2 black and 4 red balls and urn C contains 4 white, 3 black and 2 red balls. One urn is chosen at random and two balls are drawn. They happen to be red and black. What is the probability that both balls came from urn B?

Sol^n

Given,

A: 2W, 1B, 3R — 6 Ball

B: 3W, 2B, 4R — 9 Ball

C: 4W, 3B, 2R — 9 Ball

Since two ball are drawn ~~from~~ that will be red and ~~white~~ black and let E represents red and black.

Now the probability of (selecting urn A) $= \frac{1}{3}$

$\Rightarrow P(A) = \frac{1}{3}$

Similarly $P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{3}$

Probability (Red and Black / urn A) $= P(E/A)$

$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}$$

$$= \frac{3}{15}$$

$$= 0.2$$

Probability (Red and Black / urn B) $= P(E/B)$

$$= \frac{{}^2C_1 \times {}^4C_1}{{}^9C_2}$$

$$= \frac{8}{36}$$

$$= 0.222$$

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and $P(E/C) = \frac{3C_1 \times 2C_1}{9C_2} = \frac{6}{36} = 0.167$

Now, the Probability (Urn B / Black and Red)

$= P(B/E)$

$= \frac{P(B).P(E/B)}{P(A).P(E/A) + P(B).P(E/B) + P(C).P(E/C)}$

29. The chances of X, Y, Z becoming manager of a Worldlink Pvt. Ltd. are 4:2:3. The probabilities that bonus scheme to the staff will be introduced if X, Y, Z become managers are 0.3, 0.5 and 0.8 respectively. If the bonus scheme has been introduced, what is the probability that X is appointed as the manager?

Soln.

Given, $P(X \text{ becomes manager}) = \frac{4}{4+2+3} = \frac{4}{9}$

$\Rightarrow P(X) = \frac{4}{9}$

$P(Y \text{ becoming manager}) = \frac{2}{9}$

$\Rightarrow P(Y) = \frac{2}{9}$

$P(Z \text{ becoming manager}) = \frac{3}{9}$

$\Rightarrow P(Z) = \frac{3}{9}$

Let B be the events represent the bonus scheme then according to the question.

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$P(\text{Bonus scheme}/X) = P(B/X)$
$= 0.3$

$P(\text{Bonus scheme}/Y) = P(B/Y)$
$= 0.5$

$P(\text{Bonus scheme}/Z) = P(B/Z)$
$= 0.8$

Now,

$P(X \text{ is appointed as manager}/\text{Bonus scheme})$

$= P(X/B)$

$$= \frac{P(X).P(B/X)}{P(X).P(B/X)+P(Y).P(B/Y)+P(Z).P(B/Z)}$$

$$= \frac{\frac{4}{9}\times 0.3}{\frac{4}{9}\times 0.3+\frac{2}{9}\times 0.5+\frac{3}{9}\times 0.8}$$

$= 0.26$

32. Three urns are given each containing red and white chips as indicated.

Urn 1: 6 red and 4 white.
Urn 2: 2 red and 6 white.
Urn 3: 1 red and 8 white.

(i) An urn is choosen at random and a chip is drawn from the urn, the chip is red. Find the probability that the choosen urn was urn 1.

(ii) An urn is choosen at random and two chips are drawn without replacement from the urn. If both chips are red, find the probability that urn 1 was choosen. Under the condition what is the probability that urn 5 was choosen.

Soln

A_1	A_2	A_3
6R	2R	1R
4W	6W	8W
10	8	9

Here,

$P(\text{selecting urn } A_1) = 1/3$

$P(A_1) = 1/3$

$P(A_2) = 1/3$

$P(A_3) = 1/3$

Since the choosen chip is red. or, $P(R/A_1)$

So, probability $(Red/A_1) = 6/10$.

Similarly,

$P(R/A_2) = 2/8$

$P(R/A_3) = 1/9$.

(i) $P(\text{Choosen urn is } 1/\text{Red chip}) = P(A_1/R)$

$$= \frac{P(A_1)\,P(R/A_1)}{P(A_1).P(R/A_1) + P(A_2).P(R/A_2) + P(A_3).P(R/A_3)}$$

$$= \frac{\frac{1}{3} \times \frac{6}{10}}{\frac{1}{3}\times\frac{6}{10} + \frac{1}{3}\times\frac{2}{8} + \frac{1}{3}\times\frac{1}{9}}$$

$= 0.624$

red without replacement

(ii) $P(Rw/A_1) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$

$\because P(A_1) = 1/3 = P(A_2) = P(A_3)$

$P(Rw/A_2) = \frac{2}{8} \times \frac{1}{7} = \frac{1}{28}$

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$$P\left(R_W/A_3\right) = \frac{1}{9} \times 0 = 0.$$

Now,

$$P(A_1/R_w) = \frac{P(A_1).P\left(R_w/A_1\right)}{P(A_1).P\left(R_w/A_1\right) + P(A_2).P\left(R_w/A_2\right) + P(A_3).P\left(R_w/A_3\right)}.$$

$$= \frac{\frac{1}{3} \times \frac{1}{3}}{\frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{28}}$$

$$= 0.903$$

Under the condition the probability that urn 3 was chosen is 0 since $P\left(R_w/A_3\right)$ is zero.