

Cheat Sheet – Bayes Theorem and Classifier

What is Bayes' Theorem?

- Describes the probability of an event, based on prior knowledge of conditions that might be related to the event.

$$P(A|B) = \frac{P(B|A)(\text{likelihood}) \times P(A)(\text{prior})}{P(B)(\text{prior})}$$

- How the probability of an event changes when we have knowledge of another event

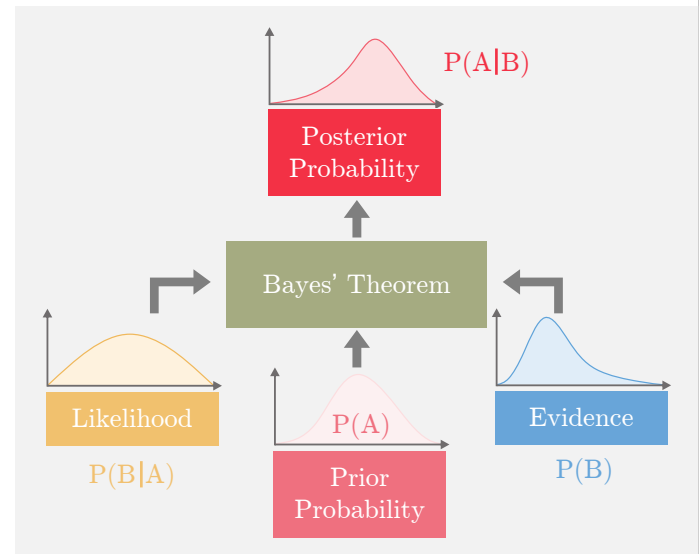
$$P(A) \longrightarrow P(A|B)$$

Usually a better estimate than $P(A)$

Example

- Probability of fire $P(F) = 1\%$
- Probability of smoke $P(S) = 10\%$
- Prob of smoke given there is a fire $P(S|F) = 90\%$
- What is the probability that there is a fire given we see a smoke $P(F|S)$?

$$P(F|S) = \frac{P(S|F) \times P(F)}{P(S)} = \frac{0.9 \times 0.01}{0.1} = 9\%$$



Maximum A Posteriori Probability (MAP) Estimation

The MAP estimate of the random variable y , given that we have observed iid (x_1, x_2, x_3, \dots) , is given by. We try to accommodate our prior knowledge when estimating.

$$\hat{y}_{MAP} = \underset{y}{\operatorname{argmax}} P(y) \prod_i P(x_i|y)$$

y that maximizes the product of **prior** and **likelihood**

Maximum Likelihood Estimation (MLE)

The MAP estimate of the random variable y , given that we have observed iid (x_1, x_2, x_3, \dots) , is given by. We assume we don't have any prior knowledge of the quantity being estimated.

$$\hat{y}_{MLE} = \underset{y}{\operatorname{argmax}} \prod_i P(x_i|y)$$

y that maximizes only the **likelihood**

MLE is a special case of MAP where our prior is uniform (all values are equally likely)

Naïve Bayes' Classifier (Instantiation of MAP as classifier)

Suppose we have two classes, $y=y_1$ and $y=y_2$. Say we have more than one evidence/features (x_1, x_2, x_3, \dots) , using Bayes' theorem

$$P(y|x_1, x_2, x_3, \dots) = \frac{P(x_1, x_2, x_3, \dots | y) \times P(y)}{P(x_1, x_2, x_3, \dots)}$$

Bayes' theorem assumes the features (x_1, x_2, x_3, \dots) are i.i.d. i.e. $P(x_1, x_2, x_3, \dots | y) = \prod_i P(x_i | y)$

$$P(y|x_1, x_2, x_3, \dots) = \prod_i P(x_i | y) \frac{P(y)}{P(x_1, x_2, x_3, \dots)}$$

$$\hat{y} = y_1 \text{ if } \frac{P(y_1|x_1, x_2, x_3, \dots)}{P(y_2|x_1, x_2, x_3, \dots)} > 1 \text{ else } \hat{y} = y_2$$