# ASTR4750 – Project 2: Stellar Polytropes and Lane-Emden Equations

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**Using polytropic models and Lane-Emden equation, we created a program that computes stellar interiors. Under the assumption of spherical symmetry and neglecting temporal evolution, we calculate mass and radius of polytropic models and plot density, pressure, temperature profiles of the star as a function of radius. We also compute the total luminosity of the sun with the computed physical density and temperature versus radius ratio.**

**Introduction.** Polytropes are self-gravitating gaseous spheres that were, and still are, very useful as crude approximation to more realistic stellar models. [1] They are the models of the famous Lane-Emden equation that describes a dimensionless radius versus density ratio and thus the pressure.

The life of a star is a war between gravity and internal pressure. While gravity pull the material of the star inward, internal pressure, supplied by the state of stellar gas and radiation, resists gravity pushing the stellar envelop outward.

Under the assumption of spherical symmetry and given initial condition, we have equations that describe mass conservation and hydrostatic equilibrium. We will compute Lane-Emden equation for cases n = 0, 1, 2, 3, 4, 5. From there, we will produce temperature, density, and pressure profiles as a function of dimensionless radius. Given specific input parameters of the sun such as radius ($R_\odot$)and mass( $M_\odot$), $\epsilon(\rho, T)$ , the nuclear energy generation rate, $T_6$, temperature in units of $10^6$, X, the hydrogen mass fraction, and using n = 3 polytropic model to represent the sun, we compute physical density and temperature versus radius ratio and thus the total luminosity of the sun.

**Analysis.** The equation of Lane-Emden is expressed as:

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n. \quad (1)$$

Where $\xi = Ar$ and $\rho = \rho_c\theta^n$

$$A = \frac{4\pi G\rho_c^{(1-\frac{1}{n})}}{K(n+1)} \quad (2)$$

For mass conservation and hydrostatic equilibrium, we compute the total stellar mass ($M_*$)and radius ($R_*$), and the central density ($\rho_c$):

$$M_* = 4\pi\rho_c\left[\frac{R^3}{\xi^3}\left(-\xi^2\frac{d\theta}{d\xi}\right)\right]_{\xi=\xi_n} \quad (3)$$

$$R_* = r_n * \xi_1 \text{ (4) with}$$

$$r_n^2 = \frac{(n-1)P_c}{4\pi G\rho_c^2}$$

$$\frac{\rho}{\rho_c} = \left(-\frac{3}{\xi}\frac{d\theta}{d\xi}\right)_{\xi=\xi_n} \quad (4)$$

**Second-order differential equation.** To solve for Lane-Emden equation numerically, we convert second ODE into two first ODE equations. In my program, I define a function that takes parameters y, x, n where I set y to be an array contain $\theta(\xi)$ and $\frac{d\theta}{d\xi}$ . I created a loop that goes from 0 to 5 for n and calculate ODE by importing Python SciPy package *SciPy. integrate.odeint* .

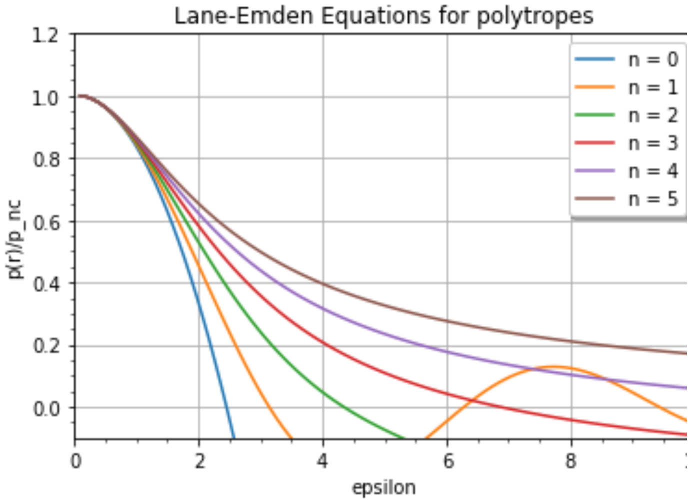


*Figure 1: Polytropic models with x-axis is a dimensionless radius and y-axis is $\theta(\xi)$*

**Results and Analysis**

**Part 1&2.** The first part in modeling the stellar interiors is to calculate solutions of $\theta(\xi)$ for n=0,1,2, 3, 4, and 5 and plot them for density $(\theta^n(\xi))$. In figure 2, I plotted the density versus $\xi$ and compared the analytical solution for n = 0 with my numerical solution. The values of density agree with each other.

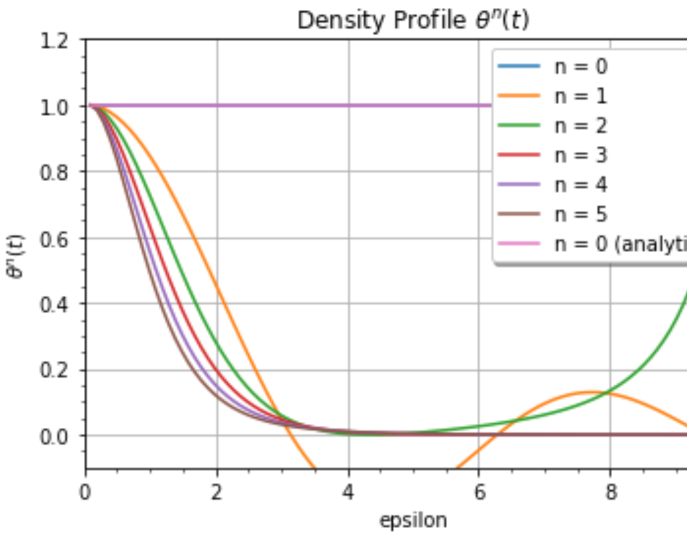


*Figure 2: Density profiles as function of a dimensionless radius.*

**Part 3.** I compute all the dimensionless solutions for temperature and pressure profiles by extracting $\theta(\xi)$ values from *odeint* solutions.

The temperature profile is just a constant multiplied with $\theta^n(\xi)$. The pressure profile is the values of $\theta(\xi)$ to the power of (n+1). Thus, figure 3 represents the results.

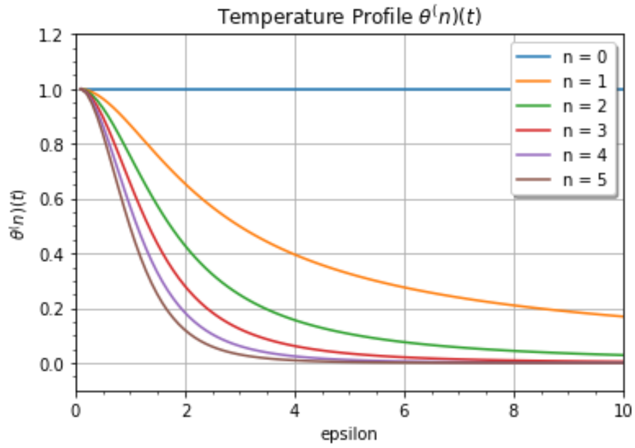


*Figure 3: Temperature Profile $\theta^n(\xi)$ versus $\xi$*

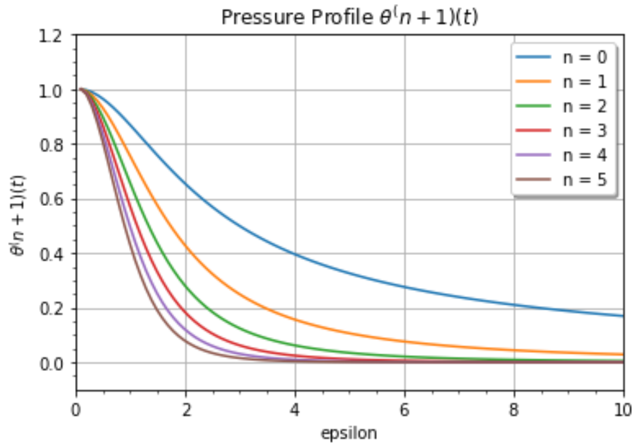


*Figure 4: Pressure profiles $\theta^{n+1}(\xi)$ versus $\xi$*

**Part 4.** Using n = 3 polytropic model to represent the sun. I plot the physical density and temperature versus $r/R_\odot$

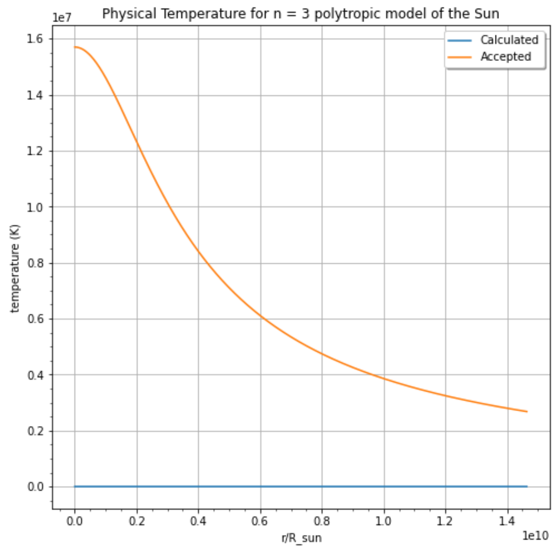


*Figure 5: Temperature Profile of polytropic model of the sun versus temperature that uses accepted value of central temperature $(15.7 * 10^6 K)$.*

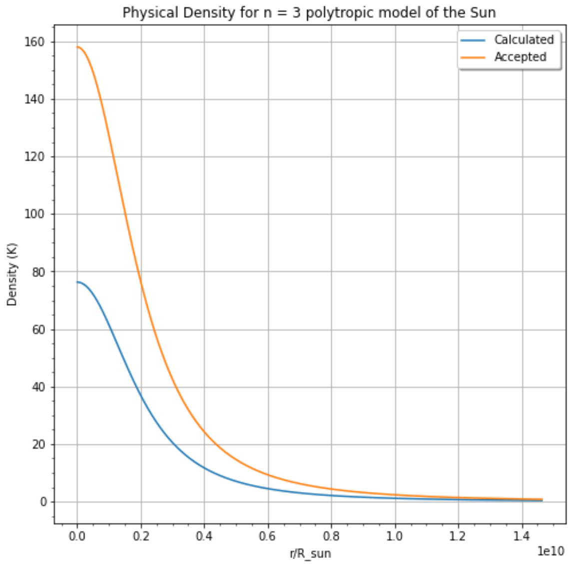


*Figure 6: Physical Density of the Sun compared with the density that uses the accepted value of central density (158 g/$cm^3$)*

My calculated values of physical density do not agree with the expected values of density.

**Part 5.** I computed total luminosity of the Sun using the density, temperature profiles in part 4. Given the equation of the nuclear energy reaction rate to be:

$$\varepsilon(\rho, T) = 2.46 * 10^6 \rho^2 X^2 T_6^{-\frac{2}{3}} e^{-33.81 T_6^{-\frac{1}{3}}} \ erg\ s^{-1} cm^{-3}$$

However, since my temperature in part 4 is problematic, the values of $\varepsilon(\rho, T)$ turns out to be 0. Thus, I cannot compute the total luminosity.

**Part 6.** Total computing time. To calculate the total computing time, I executed the command at the beginning of the code:

```
! pip install ipython-autotime
%load_ext autotimer
```

Thus, the total computing time is 6.69s.

**Conclusion.**

In this project, I am able to use Lane-Emden equation and get solutions for polytropic models. Using n = 3 polytropic model to illustrate the Sun, I implemented python code that calculate the density, pressure, and temperature profiles of the Sun. One improvement I need to make in the future would be fixing my output for physical temperature and density so that they agree with the accepted values.