Problem 1. Slide_disp.m returns slide displacement which along with secant.m is used in dwell_width.m to calculate dwell width. But none of my value in secant is converging.

Problem 2.

Starting with the second-order Taylor's series approximation of f(x) around the point x = xn + δ:

$$f(x_n+\delta) = f(x_n) + \delta \frac{df}{dx}\bigg|_{x=x_n} + \frac{\delta^2}{2}\frac{d^2 f}{dx^2}\bigg|_{x=x_n} \qquad = f(x_n) + \delta f'(x_n) + \frac{\delta^2}{2} f''(x_n)$$

To find the root of the equation f(x) = 0 using Halley's method, we use the following iterative formula:

$$\mathrm{x}_{n+1} = x_n - \frac{f(x_n)}{g(x_n)}$$

where g(x) is an approximation of f(x)/f'(x) that improves the convergence of the iteration. We can use the second-order Taylor's series to derive g(x):

\begin{aligned}

$$\mathrm{f(x)} = \mathrm{f(x}_n) + (x-x_n)f'(x_n) + \frac{(x-x_n)^2}{2} f''(x_n) + O((x-x_n)^3)$$

$$\mathrm{f'(x)} = \mathrm{f'(x}_n) + (x-x_n)f''(x_n) + O((x-x_n)^2) \ \frac{f(x)}{f'(x)} = \frac{f(x_n)+(x-x_n)f'(x_n)+\frac{(x-x_n)^2}{2}f''(x_n)+O((x-x_n)^3)}{f'(x_n)+(x-x_n)f''(x_n)+O((x-x_n)^2)} = \frac{f(x_n)}{f'(x_n)} + (x-x_n) - \frac{(x-x_n)^2}{2}\frac{f''(x_n)f'(x_n)-[f'(x_n)]^2}{[f'(x_n)]^2} + O((x-x_n)^3)$$

Substituting this expression for g(x) into Halley's formula, we get:

$$\mathrm{x}_{n+1} = x_n - \frac{f(x_n)}{g(x_n)} = x_n - \frac{f(x_n)}{\frac{f(x_n)}{f'(x_n)} + (x_{n+1}-x_n) - \frac{(x_{n+1}-x_n)^2}{2}\frac{f''(x_n)f'(x_n)-[f'(x_n)]^2}{[f'(x_n)]^2} + O((x_{n+1}-x_n)^3)} = x_n - \frac{f(x_n)f'(x_n)}{f(x_n)f''(x_n)-[f'(x_n)]^2+\frac{f(x_n)}{2}(f''(x_n)f'(x_n)-[f'(x_n)]^2)\Delta_{n+1}+O((\Delta_{n+1})^3)}$$

where

$$\Delta_{n+1} = x_{n+1} - x_n.$$