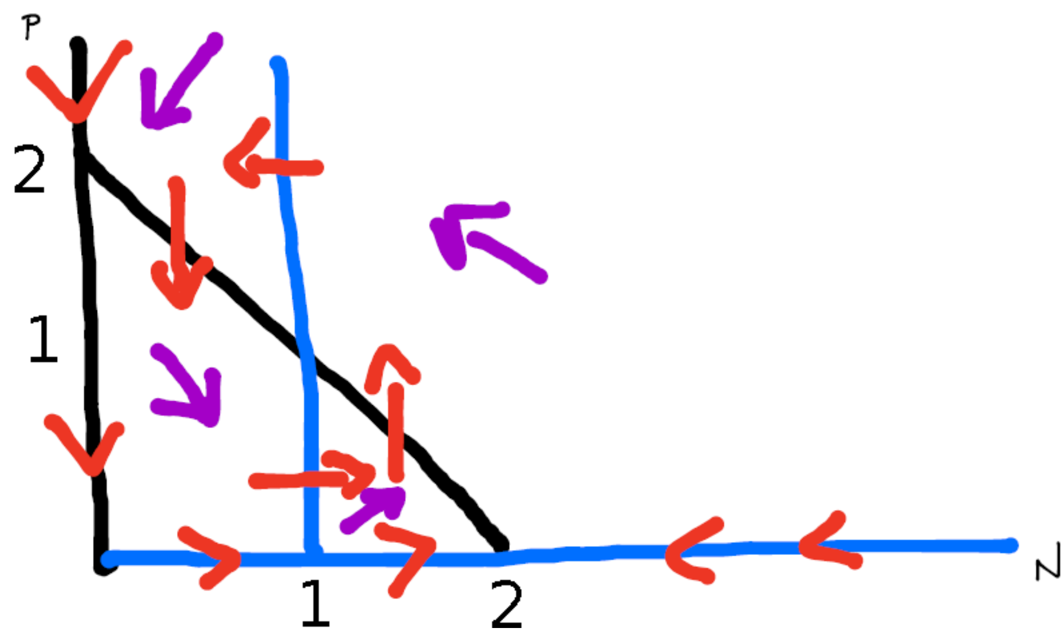
Solution of ODE2 Problem 2a

(2a)

$$\begin{aligned}\frac{dN}{dt} &= \;= 2N - N^2 - PN = N(2 - N - P)\\ \frac{dP}{dt} &= -P + PN = P(-1 + N)\end{aligned}$$



N-nullcline (black, vertical arrows): $\{N = 0\} \bigcup \{P = 2 - N\}$

P-nullcline (blue, horizontal arrows): $\{P = 0\} \bigcup \{N = 1\}$

(steady states are $(0, 0), (2, 0), (1, 1)$)

on N-nullcline (black), we need to look at the sign of dP/dt to decide if arrows point up or down:

$dP/dt > 0$ when $N > 1$, i.e. to right of vertical blue line (and < 0 to left)

on P-nullcline (blue), we need to look at the sign of dN/dt to decide if arrows point up or down:

$dN/dt > 0$ when $P < 2 - N$, i.e. under black diagonal line (and < 0 over)

Solution of ODE2 Problems 2d and 2e

(2d) For the special case $\alpha = 2$ and $\beta = \delta = \varepsilon = \gamma = 1$, determine the stability and sketch the phase plane near the equilibrum $(2, 0)$. (If real eigenvalues, find eigenvalues and eigenvectors for the linearization at that point. If complex, determine if clockwise or counterclockwise spiral.)

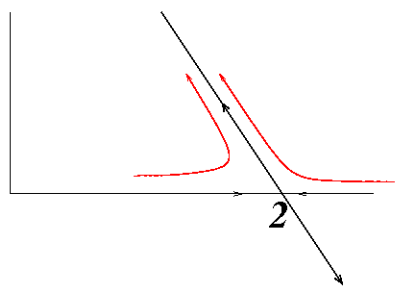
Jacobian matrix is

$$\begin{pmatrix} -2 & -2 \\ 0 & 1 \end{pmatrix}$$

so eigenvalues are $-2, 1$ and we have a saddle

Solving $Jv = \lambda v$, we can pick eigenvectors

$$v = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ for } \lambda = -2\,, \qquad v = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \text{ for } \lambda = 1\,,$$



(2e) Same for $(1, 1)$.

Jacobian matrix is

$$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$$

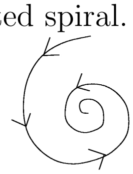
and trace $= -1 < 0$, det $= 1 > 0$, so stable

since trace^2 - 4 det $= -3 < 0$. we have a stable spiral

since

$$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

we know it is a counterclockwise-oriented spiral.

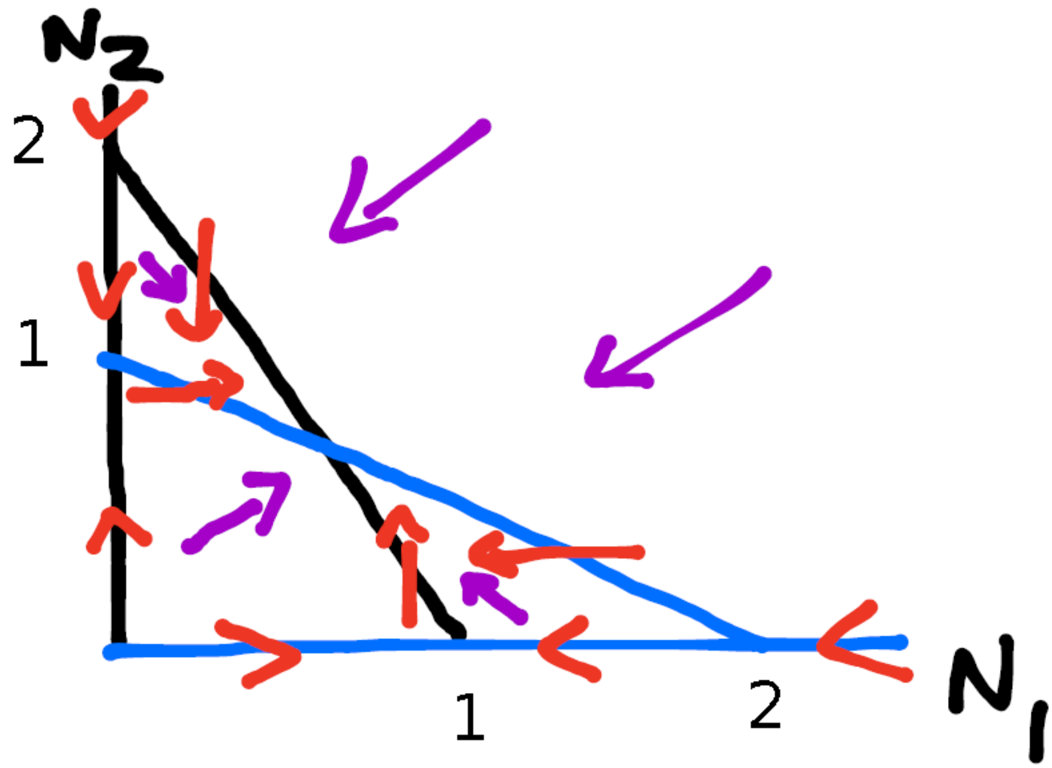


– the picture is consistent with information from nullclines

Solution of ODE2 Problem 4d

(4d)

$$\frac{dN_1}{dt} = N_1(1 - N_1 - (1/2)N_2)$$
$$\frac{dN_2}{dt} = \alpha N_2(1 - N_2 - (1/2)N_1)$$



N_1-nullcline (black, vertical arrows): $\{N_1 = 0\} \bigcup \{N_2 = 2 - 2N_1\}$

N_2-nullcline (blue, horizontal arrows): $\{N_2 = 0\} \bigcup \{N_2 = 1 - N_1/2\}$

on N_1-nullcline (black), we need to look at the sign of dN_2/dt to decide if arrows point up or down:

$dN_2/dt > 0$ when $N_2 < 1 - N_1/2$, i.e. under blue line (and < 0 over)

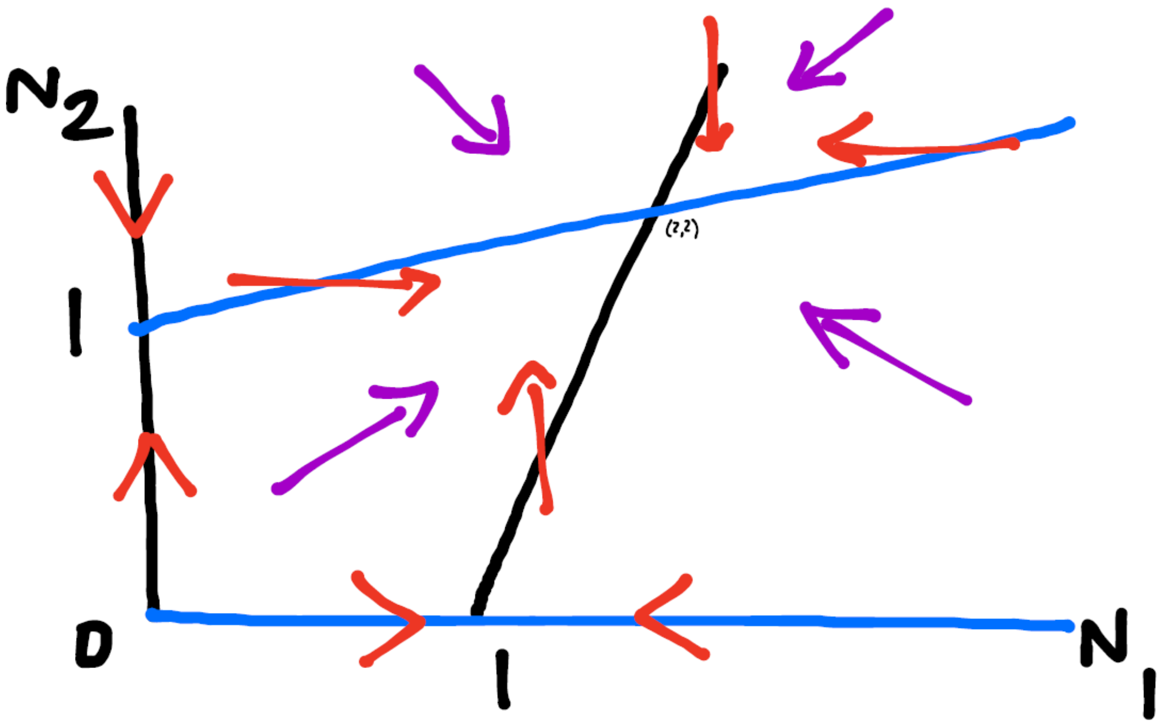
on N_2-nullcline (blue), we need to look at the sign of dN_1/dt to decide if arrows point up or down:

$dN_1/dt > 0$ when $N_2 < 2 - 2N_1$, i.e. under the black line (and < 0 over)

Solution of ODE2 Problem 6d

(6d)

$$\frac{dN_1}{dt} = N_1\left(1 - \frac{N_1}{1+(1/2)N_2}\right)$$
$$\frac{dN_2}{dt} = N_2\left(1 - \frac{N_2}{1+(1/2)N_1}\right)$$



N_1-nullcline (black, vertical arrows): $\{N_1 = 0\} \bigcup \{N_2 = 2N_1 - 2\}$

N_2-nullcline (blue, horizontal arrows): $\{N_2 = 0\} \bigcup \{N_2 = N_1/2 + 1\}$

on N_1-nullcline (black), we need to look at the sign of dN_2/dt to decide if arrows point up or down:

$dN_2/dt > 0$ when $N_2 < N_1/2 + 1$, i.e. under blue line (and < 0 over)

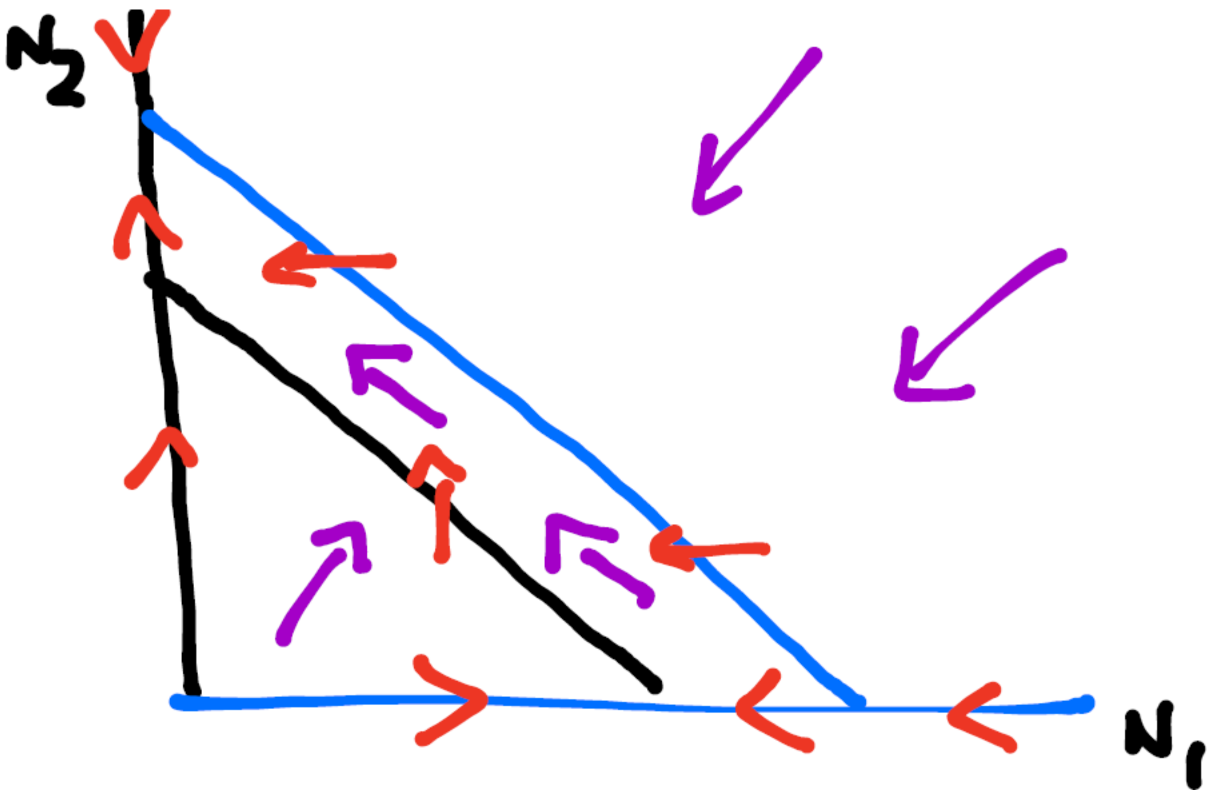
on N_2-nullcline (blue), we need to look at the sign of dN_1/dt to decide if arrows point up or down:

$dN_1/dt > 0$ when $N_2 > 2N_1 - 2$, i.e. over the black line (and < 0 under)

Solution of ODE2 Problem 8e

(8e)

$$\begin{aligned}\frac{dN_1}{dt} &= N_1(2-(N_1+N_2)-1)\\ \frac{dN_2}{dt} &= N_2(2-(N_1+N_2)-1/2)\end{aligned}$$



N_1-nullcline (black, vertical arrows): $\{N_1 = 0\} \bigcup \{N_1 + N_2 = 1\}$

N_2-nullcline (blue, horizontal arrows): $\{N_2 = 0\} \bigcup \{N_1 + N_2 = 3/2\}$

on N_1-nullcline (black), we need to look at the sign of dN_2/dt to decide if arrows point up or down:

$dN_2/dt > 0$ when $N_2 < 3/2 - N_1$, i.e. under blue line (and < 0 over)

on N_2-nullcline (blue), we need to look at the sign of dN_1/dt to decide if arrows point up or down:

$dN_1/dt > 0$ when $N_2 < 1 - N_1$, i.e. under the black line (and < 0 over)