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Applied Stats Assignment 2

1. Answers

- a. $P(\text{Type B or Type O}) = P(\text{Type B}) + P(\text{Type O}) = 0.13 + 0.44 = 0.57$
 $P(\text{Husband not acceptable donor}) = P(\text{Husband Type B}) + P(\text{Husband Type AB})$
 $= 0.13 + 0.06 = 0.19$
- b. $P(\text{Husband Type A and Wife Type B}) = P(\text{Husband Type A}) \times P(\text{Wife Type AB})$
 $= 0.37 \times 0.06 = 0.022$

2. Answers

- a. Student answers “no”
 $P(\text{No}) = P(\text{Didn't go}) \times P(\text{Tails}) = 0.80 \times 0.50 = 0.40$
 20% have gone to an event \Rightarrow 80% have not gone
- b. Student answers “yes”
 $P(\text{Yes}) = 1 - P(\text{No}) = 1 - 0.40 = 0.60$
 Student answers “yes” because head toss
 $P(\text{Didn't go and answered Yes}) = P(\text{Didn't go}) \times P(\text{Yes}) = 0.80 \times 0.60 = 0.48$
- c. Student answers “yes” because head toss (independent)
 $P(\text{Didn't go} / \text{Yes}) = P(\text{Didn't go}) = 0.80$

3. Textbook questions

- a. #11
 - i. X is the number of students out of 20 who watch television for minimum three hours per day
 X is binomial distribution where $n = 20$ and $p = \text{prob of success} = 0.207$
 $E(X) = np = (20)(0.207) = 4.14 \approx 4$
 Probability mass function of X:
 $P(X = x) = \binom{n}{x} p^x q^{n-x} = \binom{20}{x} 0.207^x (1 - 0.207)^{20-x}$ where $x = 0, 1, 2, \dots, 20$
 $P(X = 18) = \binom{20}{18} 0.207^{18} (1 - 0.207)^2 = 5.82 \times 10^{-11}$
 - ii.
 - iii. $P(X = 8) = \binom{20}{8} 0.207^8 (1 - 0.207)^{12} = 0.026$
 In part B it is seen that 18 such students is much less likely to happen. In part C we see that getting eight such students is still less likely but more likely than part B.
- b. #12
 - i. Given, X is Poisson where mean = 4.5

$$P(X = x) = \frac{4.5^x e^{-4.5}}{x!}$$

$$\Rightarrow P(X = 1) = \frac{4.5^1 e^{-4.5}}{1!} = 0.4999048$$

$$\begin{aligned}
 \text{ii. } P(x \leq 2) &= P(X = 0) + P(X = 1) + P(X = 2) \\
 &\Rightarrow \frac{4.5^0 e^{-4.5}}{0!} + \frac{4.5^1 e^{-4.5}}{1!} + \frac{4.5^2 e^{-4.5}}{2!} = 0.1735781 \\
 \text{iii. } P(X \geq 4) &= 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3) \\
 &\Rightarrow \frac{4.5^0 e^{-4.5}}{0!} + \frac{4.5^1 e^{-4.5}}{1!} + \frac{4.5^2 e^{-4.5}}{2!} + \frac{4.5^3 e^{-4.5}}{3!} = 0.657704 \\
 \text{iv. Mean} &= 4.5 \\
 \text{Standard Dev} &= \sqrt{4.5} = 2.12132
 \end{aligned}$$

c. #17

$$\text{i. } \mu = 172.2 \text{ and } \sigma = 29.8$$

$$\begin{aligned}
 Z &= \frac{X - \mu}{\sigma} = \frac{130 - 172.2}{29.8} = -1.4161 \\
 |Z| &= 1.4161
 \end{aligned}$$

$$\begin{aligned}
 \text{ii. To find } P(X < 130): &\text{ From table Area Under Standard Normal Curve} \\
 &\text{Corresponding to } Z = 1.4161 \text{ is area} = 0.4222 \\
 P(X < 130) &= 0.5 - 0.4222 = 0.0778
 \end{aligned}$$

$$\text{iii. } Z = \frac{210 - 172.2}{29.8} = 1.2685$$

$$\begin{aligned}
 &\text{To find } P(X > 210): \text{ From table Area Under Standard Normal Curve} \\
 &\text{Corresponding to } Z = 1.2685 \text{ is area} = 0.3980 \\
 P(X > 210) &= 0.5 - 0.3980 = 0.1020
 \end{aligned}$$

$$\begin{aligned}
 P(\text{exactly 2}) &= \binom{5}{2} 0.0778^3 0.9222^2 + \binom{5}{2} 0.1020^3 0.898^2 \\
 &= 8.5092 + 8.0747 = 16.5839
 \end{aligned}$$

iv.

$$\text{v. Probability of one male between 130 and 210 lbs} = 0.0778 + 0.1020 = 0.1798$$

$$\text{Probability all five are within range} = (0.1798) \times 5 = 0.899$$

$$\text{Probability at least one is outside of range} = 1 - 0.899 = 0.101$$

4. Answers

$$\text{a. } P(980 < z < 1032.6) = P(z < 1032.6) - P(z > 980)$$

R Code:

$$\begin{aligned}
 &= [pchisq(q = 1032.6, df = 1000, lower.tail = TRUE)] - [pchisq(q = 980, df = 1000, lower.tail = TRUE)] \\
 &= 0.7691504 - 0.3316733 = 0.4374772
 \end{aligned}$$

b. Normal Approx

R Code:

$$\begin{aligned}
 &= [pnorm(q = 1032.6, mean = 1000, sd = \sqrt{2 \times 1000}, lower.tail = TRUE)] - \\
 &[pnorm(q = 980, mean = 1000, sd = \sqrt{2 \times 1000}, lower.tail = TRUE)] \\
 &= 0.7669864 - 0.3273604 = 0.4396259
 \end{aligned}$$

5. x – binomial distribution

y – poisson distribution

- a. $\text{mean}(w) = \text{mean}(x) + \text{mean}(y) = np + 1 = (20)(0.06) + 1 = 2.2$
 $\text{var}(w) = \text{var}(x) + \text{var}(y) = np(1 - p) + 2 = (20)(0.06)(0.94) + 1 = 2.128$
- b. $\text{prob}(w = 1) = P(x = 0, y = 1) + P(x=1, y = 0)$

R Code:

```
prob_w1 = [dbinom(x = 0, sine = 20, prob = 0.06) * dpois(x = 1, lambda = 1)] +  
[dbinom(x = 1, sine = 20, prob = 0.06) * dpois(x = 0, lambda = 1)]  
= 0.1067241 + 0.1362436 = 0.2429677
```

- c. Normal Approximation

R Code:

```
std_x = sqrt(3.128)  
Prob_Normal_w1 = [pnorm(q = 1.5, mean = 3.2, sd = std_x)] - [pnorm(q = 0.5,  
mean = 3.2, sd = std_x)]  
= 0.3156644 - 0.121935 = 0.1937294
```