

### **Solution of SDE1 problem 1**

(a)  $P_0 = 100$ ,  $P_1 = 300$ ,  $P_2 = 900$ ,  $P_3 = 2700$ ,  $P_4 = 8100$ ,  $P_5 = 24300$

(b) (i)  $P^+ = 3P$

(ii)  $\Delta P = 2P$

(c) There is not enough information to determine  $f$  and  $d$  individually, but, since  $f - d + 1 = \lambda = 3$ , we can say that the difference between  $f$  and  $d$  is 2:  $f - d = 2$ .

### **Solution of SDE1 problem 2**

(a)  $P^+ = 2P$ , time unit is 1/2 hour, and  $P_0 = 1$ .

(b) We need to compute to  $t = 10$  (10 half-hours). Clearly the solution is  $P = 2^t P_0$ , so there are 1024 cells after 5 hours.

### **Solution of SDE1 problem 5**

(a)  $k > 1$

(b)  $r > 0$

(c)  $0 \leq k < 1$

(d)  $-1 \leq r < 0$

See next page for SDE2 problems

## Solution of SDE2 problem 2

(a):

(i)  $\Delta P = 2P(1 - P/10)$

(ii)  $\Delta P = 2P - .2P^2$

(iii)  $\Delta P = .2P(10 - P)$

(iv)  $P^+ = 3P - .2P^2t$

(b):

(i)  $\Delta P = 1.5P(1 - P/(7.5))$

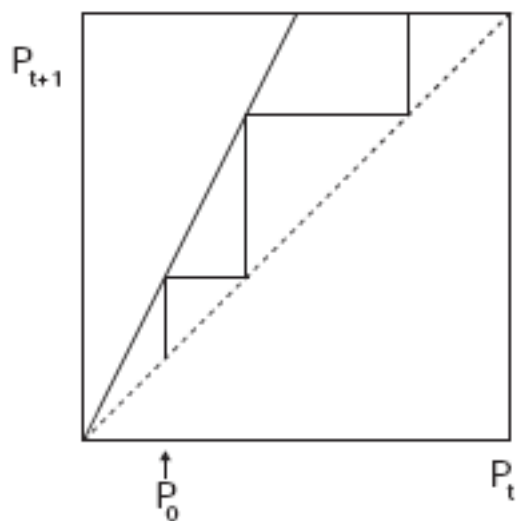
(ii)  $\Delta P = 1.5P - .2P^2$

(iii)  $\Delta P = .2P(7.5 - P)$

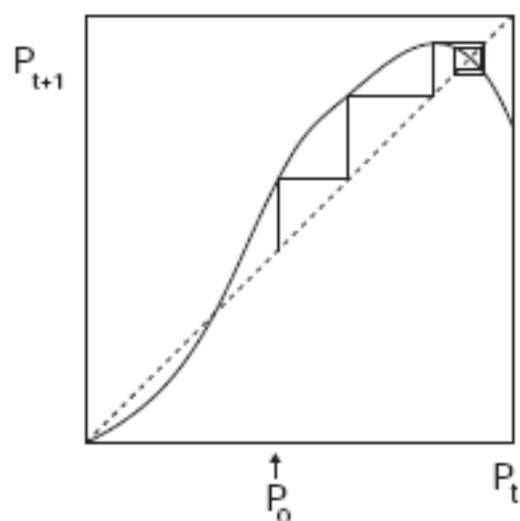
(iv)  $P^+ = 2.5P - .2P^2$

## Solution of SDE2 problem 5

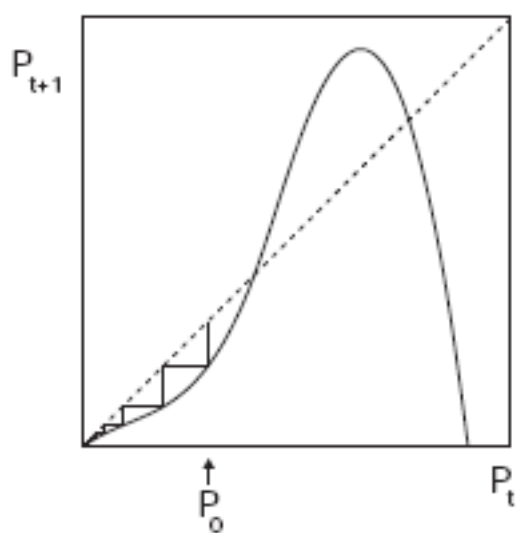
a.



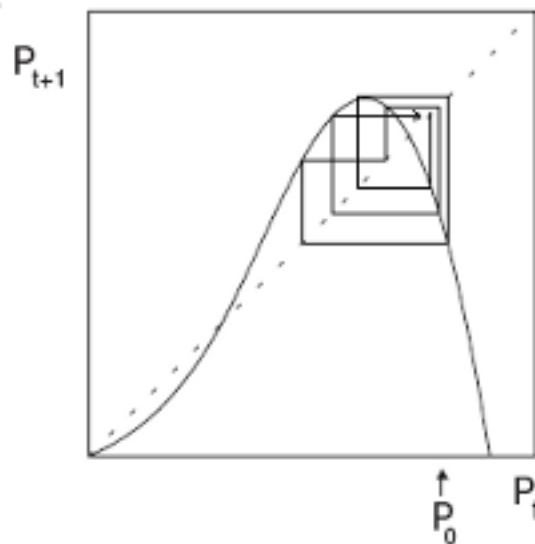
b.



c.



d.



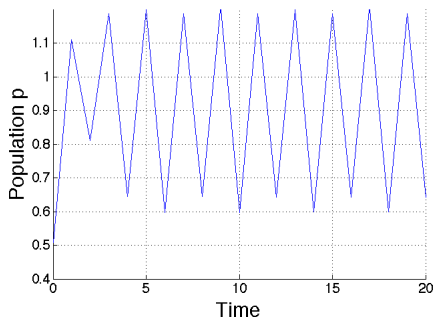
### Solution of SDE3 problem 1

- (a)  $P^* = 0, 15$
- (b)  $P^* = 0, 44$
- (c)  $P^* = 0, 20$
- (d)  $P^* = 0, \alpha/\beta$
- (e)  $P^* = 0, (\varepsilon - 1)/\delta$

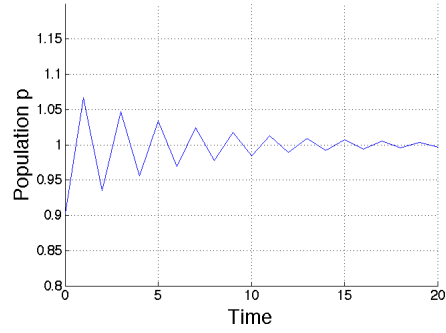
### Solution of SDE3 problem 2

- (a) At  $P^* = 0$ , the linearization is  $F' \approx 1.3$ . Since  $|1.3| > 1$ ,  $P^* = 0$  is unstable.  
At  $P^* = 15$ , the linearization is  $F' \approx .7$ . Since  $|.7| < 1$ ,  $P^* = 15$  is stable.
- (b)  $P^* = 0$  is unstable since  $|3.2| > 1$ ;  $P^* = 44$  is unstable since  $|-1.2| > 1$ .
- (c)  $P^* = 0$  is unstable since  $|1.2| > 1$ ;  $P^* = 20$  is stable since  $|.8| < 1$ .
- (d)  $P^* = 0$  is stable if  $|1 + \alpha| < 1$  (i.e.,  $-2 < \alpha < 0$ ) and unstable if  $|1 + \alpha| > 1$  (i.e.,  $\alpha < -2$  or  $\alpha > 0$ );  
 $P^* = \alpha/\beta$  is stable if  $|1 - \alpha| < 1$  (i.e.,  $0 < \alpha < 2$ ) and unstable if  $|1 - \alpha| > 1$  (i.e.,  $\alpha < 0$  or  $\alpha > 2$ ).
- (e)  $P^* = 0$  is stable if  $|\varepsilon| < 1$  and unstable if  $|\varepsilon| > 1$ ;  $P^* = (\varepsilon - 1)/\delta$  is stable if  $|2 - \varepsilon| < 1$  (i.e.,  $1 < \varepsilon < 3$ ) and unstable if  $|2 - \varepsilon| > 1$  (i.e.,  $\varepsilon < 1$  or  $\varepsilon > 3$ ).

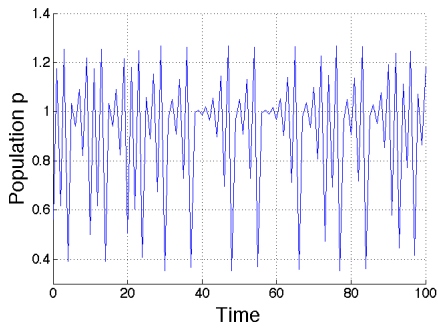
# Solution of SDE4 problem 1



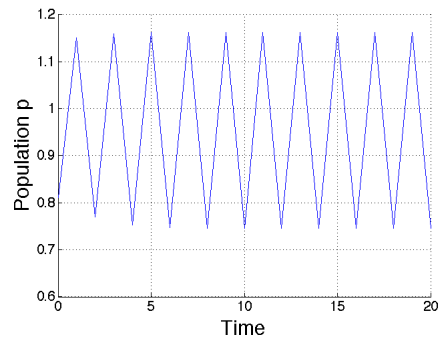
2.45



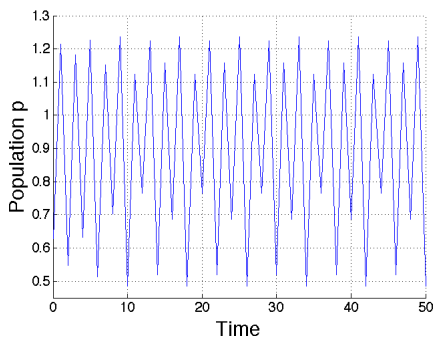
1.85



2.7

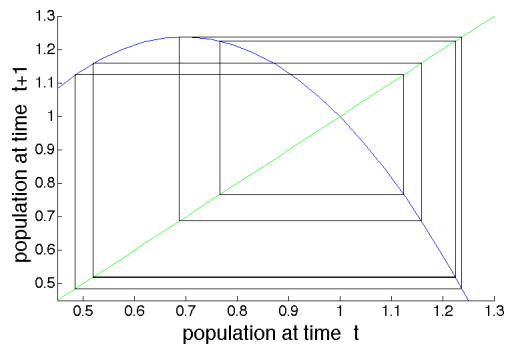


2.2

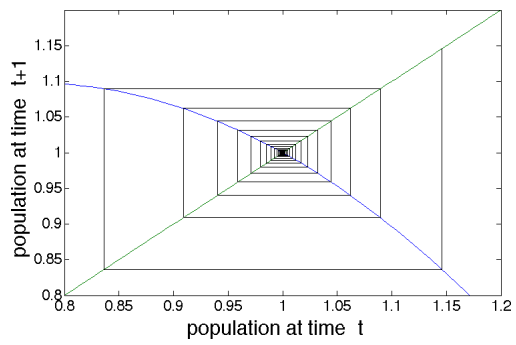


2.56

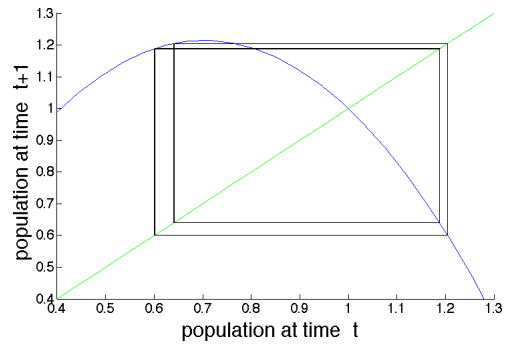
## Solution of SDE4 problem 2



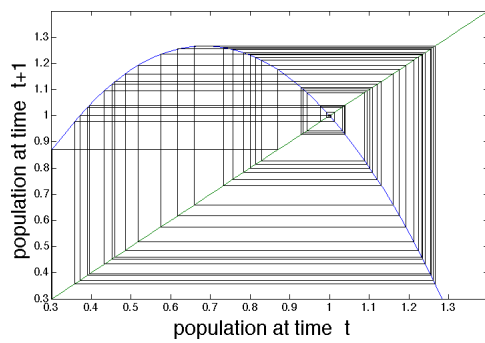
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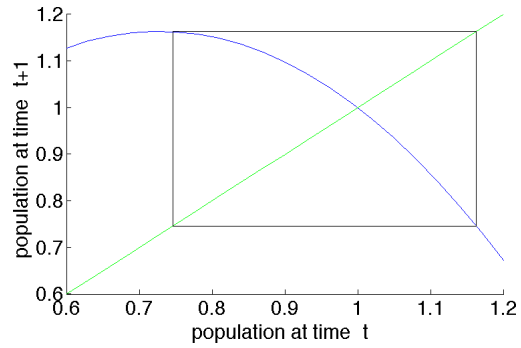
1.85



2.45



2.7



2.2