but it can also be considered from the point of view of Markov chain theory. The transition matrix is

$$\mathbf{P} = \begin{array}{c} \\ \mathrm{W} \\ \mathrm{P} \\ \mathrm{S} \end{array} \begin{array}{c} \begin{array}{ccc} \mathrm{W} & \mathrm{P} & \mathrm{S} \end{array} \\ \begin{pmatrix} .5 & .25 & .25 \\ .5 & .25 & .25 \\ .5 & .25 & .25 \end{pmatrix} \end{array} .$$

□

Example 11.6 In the Dark Ages, Harvard, Dartmouth, and Yale admitted only male students. Assume that, at that time, 80 percent of the sons of Harvard men went to Harvard and the rest went to Yale, 40 percent of the sons of Yale men went to Yale, and the rest split evenly between Harvard and Dartmouth; and of the sons of Dartmouth men, 70 percent went to Dartmouth, 20 percent to Harvard, and 10 percent to Yale. We form a Markov chain with transition matrix

$$\mathbf{P} = \begin{array}{c} \\ \mathrm{H} \\ \mathrm{Y} \\ \mathrm{D} \end{array} \begin{array}{c} \begin{array}{ccc} \mathrm{H} & \mathrm{Y} & \mathrm{D} \end{array} \\ \begin{pmatrix} .8 & .2 & 0 \\ .3 & .4 & .3 \\ .2 & .1 & .7 \end{pmatrix} \end{array} .$$

□

Example 11.7 Modify Example 11.6 by assuming that the son of a Harvard man always went to Harvard. The transition matrix is now

$$\mathbf{P} = \begin{array}{c} \\ \mathrm{H} \\ \mathrm{Y} \\ \mathrm{D} \end{array} \begin{array}{c} \begin{array}{ccc} \mathrm{H} & \mathrm{Y} & \mathrm{D} \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ .3 & .4 & .3 \\ .2 & .1 & .7 \end{pmatrix} \end{array} .$$

□

Example 11.8 (Ehrenfest Model) The following is a special case of a model, called the Ehrenfest model,[3] that has been used to explain diffusion of gases. The general model will be discussed in detail in Section 11.5. We have two urns that, between them, contain four balls. At each step, one of the four balls is chosen at random and moved from the urn that it is in into the other urn. We choose, as states, the number of balls in the first urn. The transition matrix is then

$$\mathbf{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1/4 & 0 & 3/4 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 3/4 & 0 & 1/4 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array} .$$

□

[3] P. and T. Ehrenfest, "Über zwei bekannte Einwände gegen das Boltzmannsche H-Theorem," *Physikalishce Zeitschrift,* vol. 8 (1907), pp. 311-314.