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1. (16 pts) Solve the initial value problem explicitly for y using separation of variables

$$x^2 \frac{dy}{dx} = \frac{4x^2 - x - 2}{(x+1)(y+1)}, \quad y(1) = 1$$

2. (18 pts) A swimming pool whose volume is 10,000 gallons contains water that is 0.01% chlorine. Starting at $t = 0$, city water containing 0.001% chlorine is pumped into the pool at a rate of 5 *gal/min*. The pool water flows out at the same rate. What is the percentage of chlorine in the pool after 1 hour? When will the pool water be 0.002% chlorine?

3. (16 pts) Carbon dating is often used to determine the age of a fossil. For example, a humanoid skull was found in a cave in South Africa along with the remains of a campfire. Archaeologists believe the age of the skull to be the same age as the campfire. It is determined that only 2 % of the original amount of carbon-14 remains in the burnt wood of the campfire. Estimate the age of the skull if the half-life of carbon-14 is about 5568 years.

4. (15 pts) The separation of equation $\frac{dy}{dx} = p(y)q(x)$ requires division by $p(y)$, and this may disguise the fact that the roots of the equation $p(y) = 0$ are actually constant solutions to the differential equation.

(a) To explore this further, separate the equation

$$\frac{dy}{dx} = (x - 3)(y + 1)^{2/3},$$

to derive the solution,

$$y = -1 + (x^2/6 - x + C)^3.$$

(b) Show that $y \equiv -1$ satisfies the original equation $\frac{dy}{dx} = (x - 3)(y + 1)^{2/3}$.

(c) Show that there is no choice of the constant C that will make the solution in part (a) yield the solution $y \equiv -1$. Thus, we lost the solution $y \equiv -1$ when we divide by $(y + 1)^{2/3}$.

5. (18 pts) Beginning at time $t = 0$, fresh water is pumped at the rate of 3 gal/min into a 60-gal tank initially filled with brine. The resulting less-and-less salty mixture overflows at the same rate into a second 60-gal tank that initially contained only pure water, and from there it eventually spills onto the ground. Assuming perfect mixing in both tanks, when will the water in the second tank taste saltiest? And exactly how salty will it then be, compared with the original brine?

6. (17 pts) Use the method discussed under *Bernoulli Equations* to solve the problem

$$\frac{dy}{dx} = \frac{y^2 + 2yx}{x^2}$$

Northeastern University Honor Pledge:

On my honor, I have neither given nor received unauthorized aid doing this Take-Home Test.

Signature: