Consider the following reaction network:

$$0 \to X \to Y \to 0$$

with the respective kinetic constants 10, 1, 2, i.e. the propensities are:

$$\rho_1 = 10$$
,  $\rho_2 = x$ ,  $\rho_3 = 2y$ ,

(a) Find the steady state means of X and Y and the steady state covariance matrix (that is, the variances of X and Y and the covariance of (X,Y), at steady state). Please use the fluctuation-dissipation approach discussed in class.

Hint: there are two species, and we have these matrices:

$$\Gamma = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}, \quad A = \begin{pmatrix} -1 & 0 \\ 1 & -2 \end{pmatrix}, \quad B = \Gamma \begin{pmatrix} 10 & 0 & 0 \\ 0 & \mu_x & 0 \\ 0 & 0 & 2\mu_y \end{pmatrix} \Gamma^T$$

where  $\mu_x$  and  $\mu_y$  should be the means of X and Y. You should find that the covariance is zero!

(b) Run the following script:

gillespie\_XY.m

from

https://drive.google.com/drive/folders/1oIRZ3qAb48BVvuZymdquawLzKqW8Sz42?usp=sharing I suggest that you run these script a few times, as each time you will get a different answer.

- (i) Print out a plot (from one of the runs).
- (ii) Compare the results from the simulation(s) to the theoretical calculation.
- (iii) Subtle question: why is "mean(X(:,1))" the wrong way to compute the mean? (Looking at how I computed the mean, in the code, will help you answer this question. You will learn a lot from answering this, actually.)