Solution of ODE6 Problem 1(a,b,c,d,e)

```
% S = [A,E,C1,P,C2]'
% R(S) = [k1*A*E , km1*C1 , k2*C, k3*A*C1 ,km3*C2 , k4*C2]'

G = [-1  1  0 -1  1  0
     -1  1  1  0  0  0
      1 -1 -1 -1  1  1
      0  0  1  0  0  1
      0  0  0  1 -1 -1]

rank(G)

% answer is 3

c1 = [1 0 1 1 2]

c1*G
% A + C1 + P + 2*C2

c2 = [0,1,1,0,1]
% E + C1 + C2

c2*G
```

The equations are:

$$\dot{a} = -k_1 ae - k_3 ac_1 + k_{-1}c_1 + k_{-3}c_2$$

$$\dot{e} = -k_1 ae + (k_{-1} + k_2)c_1$$

$$\dot{c_1} = k_1 ae - k_3 ac_1 - (k_2 + k_{-1})c_1 + (k_4 + k_{-3})c_2$$

$$\dot{c_2} = k_3 ac_1 - (k_{-3} + k_4)c_2$$

$$\dot{p} = k_2 c_1 + k_4 c_2$$

Solution of ODE6 Problem 2

$$S = \begin{pmatrix} m \\ e \\ c \\ p \end{pmatrix} \quad R(S) = \begin{pmatrix} m^3e^2 \\ c \\ c \end{pmatrix} \quad \Gamma = \begin{pmatrix} -3 & 3 & 0 \\ -2 & 2 & 2 \\ 1 & -1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$\mathrm{rank}(\Gamma) = 2$

dimension of left nullspace = 4 -2 = 2

basis: for example, $\{(1,0,3,3),(0,1,2,0)\}$ (of course, there are infinitely many possible answers!)

Solution of ODE6 Problem 3

Let $ES_0 = C_0$, $ES_1 = C_1$, $FS_1 = D_1$, and $FS_2 = D_2$. Here are the vector of species S, stoichiometry matrix Γ and vector of reaction rates $R(S)$:

$$S = \begin{pmatrix} E \\ S_0 \\ C_0 \\ S_1 \\ C_1 \\ S_2 \\ F \\ D_2 \\ D_1 \end{pmatrix}, \quad \Gamma = \begin{pmatrix} -1 & 1 & 1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 \end{pmatrix},$$

$$R(S) = \begin{pmatrix} ES_0 \\ C_0 \\ C_0 \\ ES_1 \\ C_1 \\ C_1 \\ FS_2 \\ D_2 \\ D_2 \\ FS_1 \\ D_1 \\ D_1 \end{pmatrix}.$$

From here, we can write the equations for the system:

$$\frac{dS}{dt} = \Gamma R(S).$$

For example:

$$\frac{dE}{dt} = -ES_0 + C_0 + C_0 - ES_1 + C_1 + C_1$$

and so on.

It is easy to see (by inspection of the equations and some intuition) that these are all constant:

$$F + D_1 + D_2\,, \quad E + C_0 + C_1\,, \quad S_0 + S_1 + S_2 + C_0 + C_1 + D_1 + D_2$$

and therefore

$(0, 0, 0, 0, 0, 0, 1, 1, 1)\Gamma = 0$,

$(1, 0, 1, 0, 1, 0, 0, 0, 0)\Gamma = 0$, and

$(0, 1, 1, 1, 1, 1, 0, 1, 1)\Gamma = 0$.

The rank of Γ is 7. Since Γ has 9 rows, its left null space has dimension 3 and therefore

$$\{(0, 0, 0, 0, 0, 0, 1, 1, 1),\ (1, 0, 1, 0, 1, 0, 0, 0, 0),\ (0, 1, 1, 1, 1, 1, 0, 1, 1)\}$$

is a basis.

Solution of ODE6 Problem 8

We consider this chemical network (using $k_1 a$ for the first forward reaction, and k_{-1} for the backward reaction):

$$X \underset{k_{-1}}{\overset{k_1 a}{\rightleftharpoons}} 2X$$
$$X \xrightarrow{k_2 b} C$$

Then:

$$S = \begin{pmatrix} x \\ c \end{pmatrix} \quad R(S) = \begin{pmatrix} k_1 a x \\ k_{-1} x^2 \\ k_2 b x \end{pmatrix} \quad \Gamma = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} dx/dt &= k_1 a x + k_{-1} x^2 - k_2 b x \\ dc/dt &= k_2 b x \end{aligned}$$