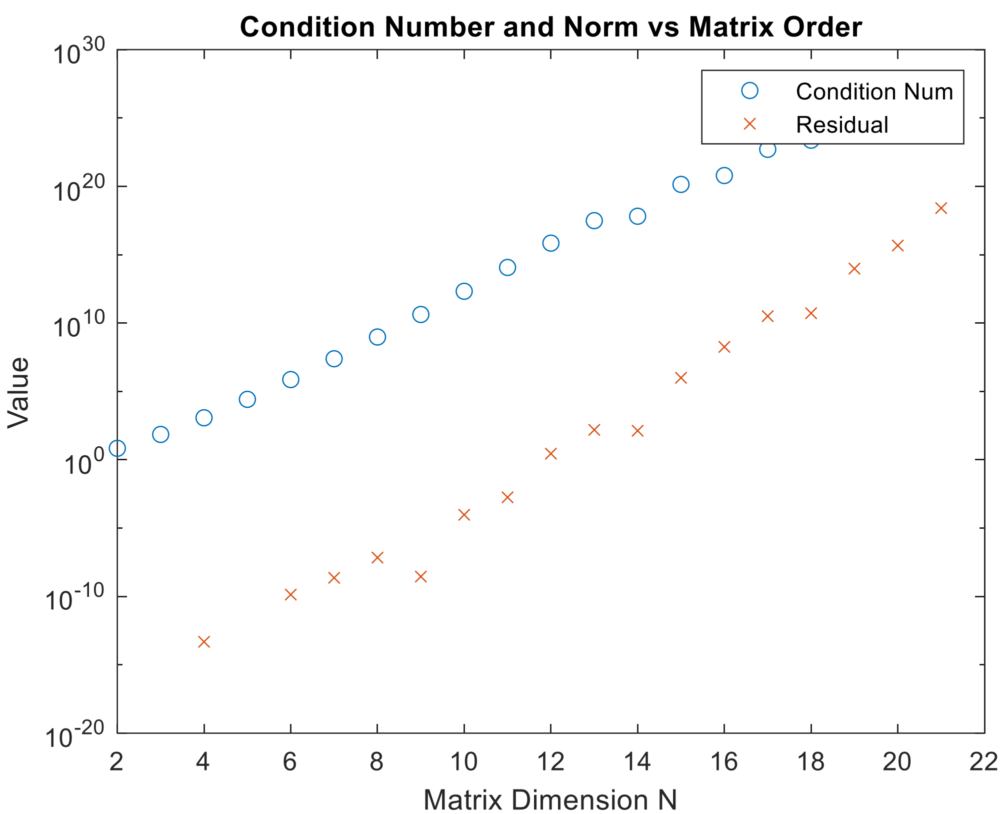
Problem 1.

Order of magnitude separate the value of the residual and the value of the condition number is 10^{10}. This is because Vandermonde matrices are ill-conditioned, meaning that small perturbations in the input data can result in large errors in the solution.

To mitigate this problem, we can try to improve the conditioning of the Vandermonde matrix by rescaling the input data by multiplying with a tolerance value.

Following graph have been achieved through the code rel_condnum_error.m



Problem 2.

1. $A = \begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix}$

Eigen values of A are λ which can be computed as:

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1-\lambda & a \\ a & 1-\lambda \end{bmatrix} = 0$$

$$(1 - \lambda)(1 - \lambda) - a^2 = 0$$

$$\lambda_1 = (1 + a)$$

$$\lambda_2 = (1 - a)$$

2. $\sigma_i = |\lambda_i|$
$$\sigma_1 = |\lambda_1|$$
$$\sigma_1 = |1 + a|$$
When $a = 1$, A is singular.
3. Program for this question is plot_Quadratic_Form.m
4. If the matrix A is singular, it means that it is not invertible, and there is no unique solution to the equation A x = b. In this case, the quadratic form u^T A u becomes undefined, because it requires the matrix to be invertible.
If a quadratic form is calculated using a singular matrix, the result may be unstable or meaningless. This is because small changes in the input vector u can result in large changes in the output, due to the ill-conditioning of the singular matrix.

Problem 3.

1. This question has been solved by taking the angle as 90 – theta.

c = cos (90 - theta)

s = sin (90 – theta)

At a_x: $r_{ax} - f_1 = 0$

At a_y: $r_{ay} = 0$

At b_x: $r_{bx} - cf_2 - f_3 = 0$

At b_y: $r_{by} - sf_2 = 0$

At c_x: $f_1 + c\, f_2 - f_4 - cf_5 = 0$

At c_y: $sf_2 + sf_5 = 0$

At d_x: $f_4 - f_7 = 0$

At d_y: $f_6 = 0$

At e_x: $f_3 + cf_5 - cf_8 - f_9 = 0$

At e_y: $-sf_5 - sf_8 - f_6 = 0$

At f_x: $f_7 + cf_8 - f_{10} - cf_{11} = 0$

At f_y: $sf_8 + sf_{11} = 0$

At g_x: $f_{10} = 0$

At g_y: $f_{12} = 0$

At h_x: $f_9 + cf_{11} = 0$

At h_y: $-sf_{11} - f_{12} = -w$

2. $f_e = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -w \end{bmatrix}$

3. $\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \\ f_{10} \\ f_{11} \\ f_{12} \end{bmatrix} = \begin{bmatrix} -20.0000 \\ 11.1803 \\ 15.0000 \\ -10.0000 \\ -11.1803 \\ 0 \\ -10.0000 \\ 11.1803 \\ 5.0000 \\ 0 \\ -11.1803 \\ 0 \end{bmatrix}$ Crane will not collapse with 10 ton weight.

4. Since $|f_1| < 25$, crane will collapse due to 15 tons weight.
5. Since $|f_1| < -25$, crane will collapse due to 20 tons weight.