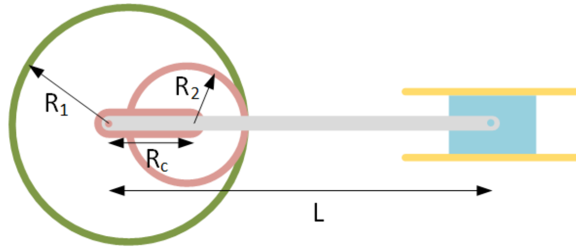
Derivation of kinematics

Problem first used in Spring 2023. This draft 3.18.203, after math bug fixed.

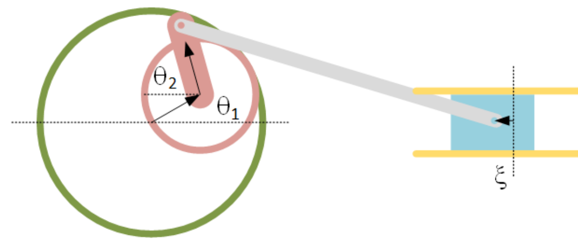


Base position.

We start with θ_1 as the known, independent variable. As the pink wheel travels around the green wheel, the arc length traveled on the green wheel's perimeter is

$$s_{green} = R_1 \theta_1$$

and the arc length traveled on the pink wheel's perimeter is



Rotated.

$$s_{pink} = R_2(\theta_1 + \theta_2)$$

Note on 3.18.2023: I previously made a mistake in the above expression for s_{pink}. The new expression above is the correct one. There is a good discussion about circles rotating inside circles on Stack Exchange here: https://math.stackexchange.com/questions/935739/circle-rotating-within-a-circle-roulette . That discussion shows a picture of exactly the situation modeled here.

Since we assume the pink wheel travels without slipping (i.e. because of gears), the same arc length is traveled on both wheels, so we have

$$s_{green} = s_{pink}$$

And therefore

$$\theta_2 = \pi - \theta_1 \left(\frac{R_1}{R_2} - 1 \right)$$

Note that the π first term just captures the way my coordinate system works: When the wheels are at base position θ_2 is defined to point to the left (its value is $\theta_2 = \pi$). Then as the wheels rotate, θ_2 rotates clockwise, so it decreases with increasing θ_1 .

Next we want to get the (x, y) position of the pin linking the rod extending from the pink wheel to the long grey drive shaft. We assume the origin is the center of the green wheel. We walk out to the center of the pink wheel:

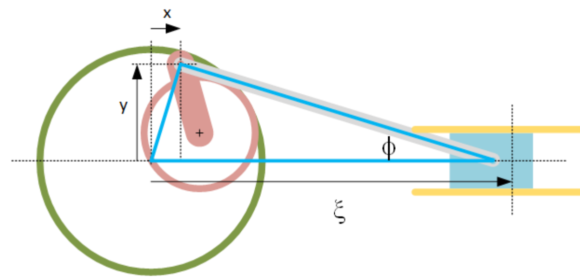
$$x_{pink} = R_2 \cos(\theta_1) \qquad y_{pink} = R_2 \sin(\theta_1)$$

From that point walk out to the pin:

$$x = R_2 \cos(\theta_1) + R_c \cos(\pi - \theta_2) \qquad y = R_2 \sin(\theta_1) + R_c \sin(\pi - \theta_2)$$

Now we want to get the horizontal distance to the slide.

Consult the drawing at right. We have



$$\sin(\phi)=y/L$$

or

$$\phi=\arcsin(y/L)$$

Then

$$\xi=x+L\cos(\phi)$$

This is the value I want – ξ characterizes the horizontal motion of the slide. Although it's straightforward to compute ξ from the input θ_1 and the fixed parameters, going backwards from statements about ξ to the corresponding value of θ_1 is much more difficult because it involves a bunch of trig functions.

Important side note: Since ξ is defined in the coordinate system where the center of the green circle is the origin, I subtract the min value of ξ (when $\theta_1=\pi$) when making plots and calculating the width of the dwell region.