Problem 2.

1. Two-point Lagrange interpolation formula for y(x) between x0 and x1:

$$y(x) = y_0 \frac{(x - x_1)}{(x_0 - x_1)} + y_1 \frac{(x - x_0)}{(x_1 - x_0)}$$

2. Taking the derivative of the polynomial and substituting $x = x_0$ gives the forward difference formula:

$$y'(x_0) = \frac{y_0}{h} - \frac{y_1}{h} = \frac{(y_1 - y_0)}{h}$$

3. Three-point Lagrange interpolation formula for $y(x)$ between x_{-1}, x_0, and x_1:

$$y(x) = y_{-1} \frac{(x - x_0)(x - x_1)}{(x - x_{-1})(x_0 - x_{-1})} + y_0 \frac{(x - x_{-1})(x - x_1)}{(x_0 - x_{-1})(x_1 - x_{-1})} + y_1 \frac{(x - x_{-1})(x - x_0)}{(x_1 - x_{-1})(x_0 - x_1)}$$

4. Taking the derivative of the polynomial and substituting $x = x_0$ gives the central difference formula:

$$y'(x_0) = \frac{(y_1 - y_{-1})}{2h}$$

5. Taking the derivative of the polynomial again and substituting $x = x_0$ gives the three-point formula for the second derivative:

$$y''(x_0) = \frac{(y_1 - 2y_0 + y_{-1})}{h^2}$$

6. The forward difference formula is obtained from an interpolation formula that utilizes just two points, but the center difference formula is produced from an interpolation method that uses three points. Unlike the forward difference formula, which only uses data from one point, the central difference formula gathers data from points on both sides of the place of interest. As a result of including more information in the approximation, the central difference formula would be more accurate than the forward difference formula.