

Math 5110 Applied Linear Algebra

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Eigenvalues and Eigenspaces

Question 1. An $n \times n$ matrix A is called nilpotent if there exists an integer k such that $A^k = 0$. Find all possible eigenvalues of A .

Question 2. Let $A \in \mathbb{R}^{2 \times 2}$ defined by

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

where $a, b, c \in \mathbb{R}$. (Notice that A is symmetric, that is, $A^T = A$.)

(1) Prove that A has only real eigenvalues.

(2) Under what conditions on a, b, c does A have a multiple eigenvalue?

Question 3. Let $A \in \mathbb{F}^{n \times n}$ be an invertible matrix. Show that every eigenvector of A is also an eigenvector of A^{-1} . What is the relationship between the eigenvalues of A and A^{-1} ?

Question 4. Suppose that A is a square matrix with real entries and real eigenvalues. Prove that every eigenvalue of A has an associated real eigenvector.