Lab 2a.

The positions of R, B, G are given as follows:

	R	G	B
x	0.67	0.21	0.14
y	0.33	0.71	0.08

1.3.1 Task 1. Find numbers (u, v, w) so that

$$uR + vB + wG = 0$$

Compute

$$s = u + v + w$$

Matlab code:

```
R = [0.67; 0.33];
B = [0.14; 0.08];
G = [0.21; 0.71];

RBG = [R, B, G];

disp("Lab 2a. 1.3.1 Task 1")

S = sym(RBG)
rref(S)
```

Command Window:

```
Lab 2a. 1.3.1 Task 1

S =

[67/100, 7/50, 21/100]
[33/100, 2/25, 71/100]


ans =

[1, 0, -413/37]
[0, 1, 2032/37]
```

Solution:

$$(u, v, w) = w * \left(-\frac{413}{37}, \frac{2032}{37}, 1\right)$$

$$s = w * \frac{1656}{37}$$

$$s = 44.7567\, w$$

1.3.2 Task 2. Find the 2 × 2 matrix A which sends $\vec{X}$ to the coefficients of its representation as a linear combination of R and B. That is, your matrix should have the following property: when we write

$$\vec{X} = \begin{pmatrix} x \\ y \end{pmatrix} = cR + dB$$

then

$$A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix}$$

Solution: We have,

$O \times (0.67, 0.33)^T = (1,0)^T$, $O \times (0.14, 0.08)^T = (0,1)^T$

Using the same method as Lab1b, by constructing M, D1, D2, we compute O as following:

Matlab Code:

```
disp("Lab 2a. 1.3.2 Task 2")

M = [R,B]
D1 = [1;0];
D2 = [0;1];

N = rref([M D1 D2]);
O = N(:,[3:4])
```

Command Window:

```
Lab 2a. 1.3.2 Task 2

M =

    0.6700    0.1400
    0.3300    0.0800


O =

   10.8108  -18.9189
  -44.5946   90.5405
```

1.3.3 Task 3. Find numbers r, b, g (depending on c, d and u, v, w, s) such that

$$\vec{X} = cR + dB = rR + bB + gG, \qquad r + b + g = 1$$

Solution:

$$\begin{aligned} \vec{X} &= cR + dB \\ &= cR + dB + uR + vB + wG \\ &= (c+u)R + (d+v)B + wG \\ &= rR + bB + gG \end{aligned}$$

Require $r + b + g = 1$, so $c + d + s = 1$

Hence $s = 42.7567\, w = 1 - c - d$

$\therefore w = (1 - c - d)/42.7567$

Hence,

$r = c + u = c + 11.1622 * w$

$b = d + v = d - 54.9189 * w$
$g = w$

1.3.4 Task 4. Apply your result from Task 3 to compute the (R, B, G) representation for the color $\vec{X} = (0.3, 0.5)$.

Matlab Code:

```
disp("Lab 2b. 1.3.4 Task 4")

X = [0.3; 0.5];

P = [RBG; [1, 1, 1]];

rbg = P\([X; 1]);

disp(rbg);
```

Command Window:

```
Lab 2b. 1.3.4 Task 4
    0.2257
    0.1972
    0.5771
```

Solution:

First, $(c, d)^T = A * (0.3, 0.5)^T = (-6.2162, 31.8919)^T$

$$\therefore w = -\frac{(1 - c - d)}{42.7567} = 0.5771$$

Hence,

$$r = c + u = c + 11.1622 * w = 0.2257$$

$$b = d + v = d - 54.9189 * w = 0.1972$$
$$g = 0.5771$$

1.3.5 Task 5. For a given color $\vec{X}$ the $complementary\ color\ \overrightarrow{X'}$ is defined as the unique color such that a uniform mixture of $\vec{X}$ and $\vec{X}$ creates white, that is

$$\frac{1}{2}\vec{X} + \frac{1}{2}\vec{X}' = W = \frac{1}{3}(R + G + B)$$

Find the (R, B, G) representation for the complementary color of $\vec{X} = (0.3, 0.5)$.

Matlab Code:

```
disp("Lab 2b. 1.3.5 Task 5")

X_prime = 2 * ((R + B + G) / 3 - X / 2);

disp(P\([X_prime; 1]));
```

Command Window:

```
Lab 2b. 1.3.5 Task 5
    0.4410
    0.4694
    0.0895
```

Solution:

White is $W = (R + B + G)/3 = (0.34, 0.373)$

So, if $\vec{X} = (0.3, 0.5)$ then, $\vec{X}' = 2 * W - \vec{X} = (0.38, 0.247)$

So, after redoing Task 4 with $x = 0.38,\ y = 0.247$

$(c, d)^T = A * (0.38, 0.247)^T = (-0.5649, 5.4176)^T$

$$\therefore w = -\frac{(1 - c - d)}{42.7567} = 0.0901$$

Hence,

$r = c + u = c + 11.1622 * w = 0.4409$

$b = d + v = d - 54.9189 * w = 0.4690$
$g = 0.0901$

Lab 2b.

1.4.1 Task 1. Compute the vectors $\vec{P}, \vec{Q}$ for the orbit. Note that $\vec{P}$ should be parallel to $\vec{x}(0)$, and the length of $\vec{P}$ should be 1. Then given $\vec{W}$ and $\vec{P}$ you can find $\vec{Q}$ using $\vec{Q} = \vec{W} \times \vec{P}$

Matlab Code:

```
disp("Lab 2b. 1.4.1 Task 1")

W = 1 / 3 * [-1; -2; 2];

x0_E = 117.67 * [4; 1; 3];

P = x0_E / norm(x0_E);

Q = cross(W, P);

disp("P = "); disp(P);
disp("Q = "); disp(Q);
disp("W = "); disp(W);
```

Command Window:

```
Lab 2b. 1.4.1 Task 1
P =
    0.7845
    0.1961
    0.5883

Q =
   -0.5230
    0.7191
    0.4576

W =
   -0.3333
   -0.6667
    0.6667
```

1.4.2 Task 2. Find the 3 × 3 matrix $[Id]_{EU}$ which changes bases from U to E and use it to find the inverse $[Id]_{UE}$.

Matlab Code:

```
disp("Lab 2b. 1.4.2 Task 2")

Id_EU = [P, Q, W];

disp("Id_EU = (P Q W) ="); disp(Id_EU);

Id_UE = Id_EU\eye(3);

disp("Id_UE = (Id_EU)^(-1) = (Id_EU)^T = "); disp(Id_UE);
```

Command Window:

```
Lab 2b. 1.4.2 Task 2
Id_EU = (P Q W) =
    0.7845   -0.5230   -0.3333
    0.1961    0.7191   -0.6667
    0.5883    0.4576    0.6667

Id_UE = (Id_EU)^(-1) = (Id_EU)^T =
    0.7845    0.1961    0.5883
   -0.5230    0.7191    0.4576
   -0.3333   -0.6667    0.6667
```

1.4.3 Task 3. The position of the satellite at time t is $\hat{R}_3(\vec{x}\,(0))$, which in the standard basis is equal to

$$[\vec{x}(t)]_E \;=\; [\hat{R}_3]_{EE}\,[\vec{x}\,(0)]_E$$

where $[\hat{R}_3]_{EE}$ is the matrix which implements the rotation about the vector $\overrightarrow{W}$ by angle ωt. By changing to the U-basis and using the matrix (12), find the position of the satellite (in the standard basis) at times t = 0.5, 1, 1.5 hours.

Matlab Code:

```
disp("Lab 2b. 1.4.3 Task 3")

disp("R3_EE = id_EU * R3_UU * id_UE")

disp("For t = 0.5: theta = omega * t = 3.91*0.5")

R3_UU_0_5 = [[cos(3.91 * 0.5), -sin(3.91 * 0.5), 0];
    [sin(3.91 * 0.5), cos(3.91 * 0.5), 0];
    [0, 0, 1]]

R3_EE_0_5 = Id_EU * R3_UU_0_5 * Id_UE

x_0_5 = R3_EE_0_5 * x0_E;

disp("x(t=0.5) = R3_EE * x(0)")
disp("x(0.5) = ")
disp(x_0_5);
```

Command Window:

```
Lab 2b. 1.4.3 Task 3
R3_EE = id_EU * R3_UU * id_UE
For t = 0.5: theta = omega * t = 3.91*0.5

R3_UU_0_5 =

   -0.3748   -0.9271         0
    0.9271   -0.3748         0
         0         0    1.0000


R3_EE_0_5 =

   -0.2221   -0.3125   -0.9236
    0.9236    0.2362   -0.3020
    0.3125   -0.9201    0.2362

x(t=0.5) = R3_EE * x(0)
x(0.5) =
 -467.3315
  355.8971
  122.2313
```

Matlab Code:

```
disp("For t = 1: theta = omega * t = 3.91*1")
R3_UU_1 = [[cos(3.91 * 1), -sin(3.91 * 1), 0];
    [sin(3.91 * 1), cos(3.91 * 1), 0];
    [0, 0, 1]]

R3_EE_1 = Id_EU * R3_UU_1 * Id_UE

x_1 = R3_EE_1 * x0_E;

disp("x(t=1.0) = R3_EE * x(0)")
disp("x(1) = ")
disp(x_1);
```

Command Window:

```
For t = 1: theta = omega * t = 3.91*1

R3_UU_1 =

   -0.7190    0.6950         0
   -0.6950   -0.7190         0
         0         0    1.0000


R3_EE_1 =

   -0.5280    0.8453    0.0813
   -0.0813    0.0450   -0.9957
   -0.8453   -0.5323    0.0450

x(t=1.0) = R3_EE * x(0)
x(1) =
 -120.3487
 -384.4654
 -444.6397
```

Matlab Code:

```
disp("For t = 1.5: theta = omega * t = 3.91*1.5")

R3_UU_1_5 = [[cos(3.91 * 1.5), -sin(3.91 * 1.5), 0];
    [sin(3.91 * 1.5), cos(3.91 * 1.5), 0];
    [0, 0, 1]]

R3_EE_1_5 = Id_EU * R3_UU_1_5 * Id_UE

x_1_5 = R3_EE_1_5 * x0_E;

disp("x(t=1.5) = R3_EE * x(0)")
disp("x(1.5) = ")
disp(x_1_5);
```

Command Window:

```
For t = 1.5: theta = omega * t = 3.91*1.5

R3_UU_1_5 =

    0.9138    0.4061         0
   -0.4061    0.9138         0
         0         0    1.0000


R3_EE_1_5 =

    0.9234    0.2899    0.2516
   -0.2516    0.9521   -0.1737
   -0.2899    0.0971    0.9521

x(t=1.5) = R3_EE * x(0)
x(1.5) =
  557.5499
  -67.6857
  211.0893
```