MTH 5110

1). Suppose that A is a 3×3 matrix with column vectors $\vec{c}_1, \vec{c}_2, \vec{c}_3$, and

$$\mathrm{Det}(A) = \mathrm{Det}\,(\vec{c}_1 \,|\, \vec{c}_2 \,|\, \vec{c}_3) = 2$$

Compute the determinants of the following matrices:

$$(3\vec{c}_1 \,|\, \vec{c}_2 \,|\, -4\vec{c}_3)\,, \quad (\vec{c}_3 \,|\, \vec{c}_2 \,|\, \vec{c}_1)\,, \quad (\vec{c}_1 - 2\vec{c}_2 + \vec{c}_3 \,|\, \vec{c}_2 - \vec{c}_1 \,|\, 5\vec{c}_3)$$

2). Compute the area of the triangle with corners at $(1,1), (3,-2), (2,6)$.

3). The 6×6 matrix A has zeros in all entries except along the diagonal from top right to lower left, where all entries are 1. Compute $|\mathrm{Det}(A)|$.

4). The matrices A and B satisfy the equation $AB = BA^2$. Assuming that B is non-singular find all possible values for $\mathrm{Det}(A)$.

5). Suppose that A is a skew-symmetric 3×3 matrix, meaning that $S^T = -S$. Compute $\mathrm{Det}(S)$.

6). The 4×4 matrix A is diagonalizable and can be written as

$$A = S\,D\,S^{-1}$$

where $D = \mathrm{diag}(2, 3, -1, -1)$. Compute $\mathrm{Det}(A)$.

7). Find the algebraic and geometric multiplicity of each eigenvalue of the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

8). The 2×2 matrix A has eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -2$ with eigenvectors $\vec{v}_1 = (0,1)$ and $\vec{v}_2 = (-1,2)$. Find a diagonal matrix D and an invertible matrix S such that $AS = SD$.