MTH 7241: Fall 2022

Practice Problems for Quiz 1

1). Four balls are chosen at random from a box which contains two Red balls and some number of Green balls. The probability that both Red balls are chosen is twice the probability that neither Red ball is chosen. How many Green balls are in the box?

2). A maze for rats is constructed with two doors; door 1 immediately leads to the exit, door 2 leads back to the maze after 1 minute. Assume that a rat is equally likely to choose either door at all times, and that if several rats are in the maze then they choose independently.
a). A rat is put in the maze. Find the expected time until it escapes.
b). Two rats are put in the maze. Find the expected time until the first escape occurs, and find the expected time until both escape.
c). Suppose n rats are put in the maze. Find the expected time until the first escape occurs. [Hint: you may want to condition on the first choices made by all the rats].

3). A biased coin has probability p of coming up Heads. The coin is tossed repeatedly. Let N_2 be the number of tosses until the first occurrence of the sequence (Heads, Tails). Use the conditional expectation method to compute $E[N_2]$. [Hint: follow the methodology used in class, when we computed the expected number of tosses until the first occurrence of Heads. You will find it useful to first separately compute $E[N_2|H_1]$ where H_1 is the event that the first toss comes up Heads].

4). Mary's bowl of spaghetti contains n strands. She selects two ends at random and joins them together. She does this until there are no ends left. What is the expected number of spaghetti hoops in the bowl?

5). A patient walks in who has a fever and chills. The doctor wonders, "What is the chance that this patient has tuberculosis given the symptoms I am seeing?" Let A be the event that the patient has TB, let B be the event that the patient has fever and chills. Assume that TB is present in 0.01% of the population, whereas 3% of the population exhibits fever and chills. Assume that $P(B|A) = 0.5$. What is the answer to the doctor's question?

6). Let X_i, $i = 1, \ldots, 10$ be independent random variables, each being uniformly distributed over $[0, 1]$. Use the Central Limit Theorem to estimate the probability that $X_1 + \cdots + X_{10}$ exceeds 7.

7). The random variable X has 10 possible values, with the pdf

Outcome	1	2	3	$\cdots$	10
Probability	p_1	p_2	p_3	$\cdots$	p_{10}

X is measured 1000 times and the frequency of each of its possible values is recorded. The goodness of fit test is used to determine if the observed data supports this pdf.

a) State the number of degrees of freedom, state the null hypothesis, and state the distribution of the test statistic under the null hypothesis. Use the tables to find the critical value of the test statistic at the 5% significance level.

b) The test statistic for the data is found to be 18.34. Decide if the observed data supports the null hypothesis at the 5% significance level.

c) Decide if the observed data supports the null hypothesis at the 1% significance level.