# Lab 2a: Building the map for chromaticity conversion


MATH 5110: Applied Linear Algebra and Matrix Analysis
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## 1 Introduction

The goal of this Lab is to build a map which converts points on the standard chromaticity diagram into $(R, B, G)$ triplets.

### 1.1 The standard chromaticity diagram

The horseshoe shape in the figure below contains all the different colors which the human eye can perceive.

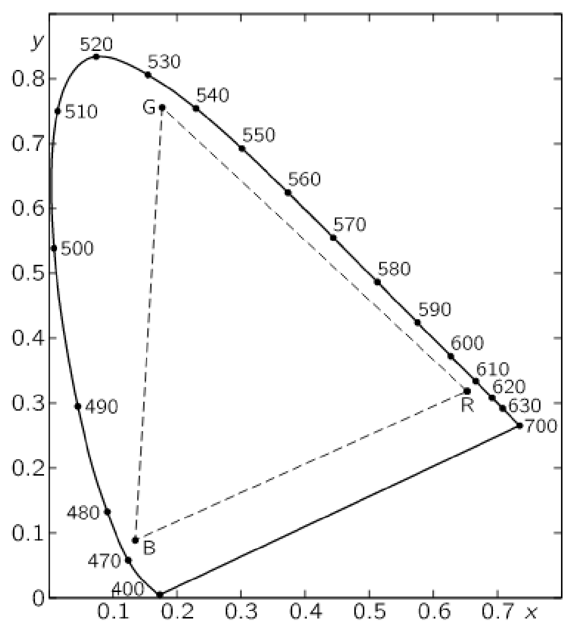


Fig. 35–4. The standard chromaticity diagram.

Each point $(x, y)$ can be viewed as a linear combination of the three primary colors Red, Blue, Green, which appear as the vertices of the triangle inside the figure. For example the color White is the uniform mixture of $R, B, G$ so we have

$$W = \frac{1}{3}\ (R + B + G) \tag{1}$$

The corresponding point $W$ (not marked) is at the centroid of the triangle. The operational meaning is that we get the color White by mixing the primary colors R, B, G with equal weights.

You may notice that we are taking a linear combination of three vectors $R, B, G$ in $\mathbb{R}^2$, which means that the representation in (1) cannot be unique. In fact since $R, B, G$ are three vectors in $\mathbb{R}^2$ they must be linearly dependent, and hence there must be nonzero numbers $u, v, w$ such that

$$uR + vB + wG = 0 \tag{2}$$

Then we could write

$$W = \left(\frac{1}{3} + \alpha u\right) R + \left(\frac{1}{3} + \alpha v\right) B + \left(\frac{1}{3} + \alpha w\right) G \tag{3}$$

for any number $\alpha$. We make the representation unique by requiring that the *sum of coefficients is 1*. That is, suppose $\vec{X}$ is any point in the $(x, y)$-plane, then we can find unique numbers $r, b, g$ such that

$$\vec{X} = rR + bB + gG, \quad \text{and} \quad r + b + g = 1 \tag{4}$$

The curved part of the boundary of the horseshoe corresponds to the 'pure' colors which contain light of one fixed wavelength. Notice that we can get these points by taking linear combinations of $R, B, G$ but at least one of the coefficients must be negative. What does this mean operationally? For example suppose that we find for some color $X$ that

$$X = \frac{1}{2}R + \frac{1}{4}B - \frac{1}{4}G \tag{5}$$

What does it mean to create a mixture with a negative weight for $G$? The answer is that we should rewrite the relation as

$$X + \frac{1}{4}G = \frac{1}{2}R + \frac{1}{4}B \tag{6}$$

Then the operational meaning is that the mixtures on the left and right sides of (6) are the same, that is they will create the same color. This is the situation for all the pure colors on the curved boundary, as well as all the 'purple' colors on the straight line part of the boundary. Any point outside the horseshoe is a color that the human eye cannot see.

### 1.2 The points $R, B, G$

The positions of $R, B, G$ are given as follows:

| | $R$ | $G$ | $B$ |
|---|---|---|---|
| $x$ | 0.67 | 0.21 | 0.14 |
| $y$ | 0.33 | 0.71 | 0.08 |

## 1.3 Mapping from $(x, y)$ to $(R, B, G)$

### 1.3.1 Task 1

Find numbers $(u, v, w)$ so that

$$uR + vB + wG = 0 \tag{7}$$

Compute

$$s = u + v + w \tag{8}$$

### 1.3.2 Task 2

Find the $2 \times 2$ matrix $A$ which sends $\vec{X}$ to the coefficients of its representation as a linear combination of $R$ and $B$. That is, your matrix should have the following property: when we write

$$\vec{X} = \begin{pmatrix} x \\ y \end{pmatrix} = cR + dB \tag{9}$$

then

$$A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix} \tag{10}$$

### 1.3.3 Task 3

Find numbers $r, b, g$ (depending on $c, d$ and $u, v, w, s$) such that

$$\vec{X} = cR + dB = rR + bB + gG, \quad r + b + g = 1 \tag{11}$$

### 1.3.4 Task 4

Apply your result from Task 3 to compute the $(R, B, G)$ representation for the color $\vec{X} = (0.3, 0.5)$.

### 1.3.5 Task 5

For a given color $\vec{X}$ the *complementary color* $\vec{X}'$ is defined as the unique color such that a uniform mixture of $\vec{X}$ and $\vec{X}'$ creates white, that is

$$\frac{1}{2}\vec{X} + \frac{1}{2}\vec{X}' = W = \frac{1}{3}\ (R + G + B) \tag{12}$$

Find the $(R, B, G)$ representation for the complementary color of $\vec{X} = (0.3, 0.5)$.