Solution of ODE6 Problem 7

(a)

$$S = \begin{pmatrix} d \\ m \\ p \\ r \\ c \end{pmatrix} \quad R(S) = \begin{pmatrix} \alpha d \\ \beta m \\ \theta m \\ \delta p \\ k_1 dr \\ k_{-1} c \\ \varepsilon c \end{pmatrix} \quad \Gamma = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{pmatrix}$$

$$\begin{aligned} \dot{d} &= -k_1 dr + k_{-1} c \\ \dot{m} &= \alpha d - \beta m + \varepsilon c \\ \dot{p} &= \theta m - \delta p \\ \dot{r} &= -k_1 dr + k_{-1} c \\ \dot{c} &= k_1 dr - k_{-1} c \end{aligned}$$

(b) If $\varepsilon < \alpha$, there is less expression when the transcription factor is bound to the promoter, compared to when the promotre is free. So the TF R is a repressor.

(c) Similarly, if $\varepsilon > \alpha$, the TF R is an activator, because there is higher expression when it is bound to the promoter.

(d) If we make k_{-1} smaller, then there will be more C, and hence the effect of having $\varepsilon > \alpha$ will be magnified.

(e) We must solve

$$\begin{aligned} -3dr + c &= 0 \quad [1] \\ d - m + \varepsilon c &= 0 \quad [2] \\ m - p &= 0 \quad [3] \\ d + c &= 0 \quad [4] \\ r + c &= 0 \quad [5]. \end{aligned}$$

From [1,4,5] we get $-3(2-c)(2-c) + c = 0$ so $c = 4/3$ or $c = 3$.

But $c = 3$ is not valid, since $d = 2 - c$ must be non-negative.

Thus, $c = 4/3$ and so $d = 2/3$.

Substituting back in [2], we get $m = \frac{4\varepsilon+2}{3}$.

Substituting back in [3], we get $p = m = \frac{4\varepsilon+2}{3}$.

Solution of ODE6 Problem 10

(a) At steady-state, $\alpha = \beta m$, so $m = \alpha/\beta$.

And $\theta m = \delta p$ implies $p = (\theta/\delta)m$, so $p = \frac{\alpha\theta}{\beta\delta}$.

(b) Plug-in and check!

(c) As $e^{-\beta t} \to 0$ and $e^{-\delta t} \to 0$ as $t \to \infty$, the solution converges to:

$$\frac{\alpha\theta}{\delta(\beta-\delta)} - \frac{\alpha\theta}{\beta(\beta-\delta)} = \frac{\alpha\theta}{\beta\delta}$$

which (or course) is the steady value of the protein p.

Solution of ODE7 Problem 2

$$000000000000 \rightarrow AAAABBBBCCCC \rightarrow BBBBBBBBCCCC$$

because in the second step, cells that start near A move toward the "B" equilibrium, while those near B stay near B, and those near C stay near C.

Solution of ODE7 Problem 5

(a) We solve the equations $k_1 s - k_2 pq = 0$ and $k_3 s - k_4 q$ and get $p = \frac{k_1 k_4}{k_2 k_3}$, $q = \frac{k_3}{k_4} s$

(b) The graphs are as follows, for $s = 0.5, 3, 20$ respectively: