Problem 1.

Given,

$$u_p = A_p(\sin(\theta_n) + B_p \cos(\theta_n)$$

$$\text{where } \theta_n = \frac{p\pi n}{N+1}\ , p = eigenvalue\ number,$$
$$n = index\ into\ the\ eigenvector, N = matrix\ size$$

To find the corresponding eigenvalues

$D^{-1}N\, u_p = \lambda_p u_p$------ 1

Substituting u_p in equation 1, we get

$$D^{-1}N\left(A_p \sin\theta_n + B_p \cos\theta_n\right) = \lambda_p\left(A_p \sin\theta_n + B_p \cos\theta_n\right)$$

Expanding equation by given $D^{-1}N$

$$-\frac{1}{2}B_p \cos(\theta_{n-1}) - \frac{1}{2}A_p \sin(\theta_{n-1}) - \frac{1}{2}B_p \cos(\theta_{n+1}) - \frac{1}{2}A_p \sin(\theta_{n+1}) = \lambda_p\left(A_p \sin\theta_n + B_p \cos\theta_n\right)$$

$\therefore \cos(\theta_{n-1}) + \cos(\theta_{n+1}) + 2\lambda_p \cos\theta_n = 0$ -----------2

Identities:

$$\cos(\theta_{n-1}) = \cos\theta_n \cos\left(\frac{n\pi}{N+1}\right) + \sin\theta_n \sin\left(\frac{n\pi}{N+1}\right)$$
$$\cos(\theta_{n+1}) = \cos\theta_n \cos\left(\frac{n\pi}{N+1}\right) - \sin\theta_n \sin\left(\frac{n\pi}{N+1}\right)$$

Using identities on equation 2, we get

$$\cos\theta_n \cos\left(\frac{n\pi}{N+1}\right) + \sin\theta_n \sin\left(\frac{n\pi}{N+1}\right) + \cos\theta_n \cos\left(\frac{n\pi}{N+1}\right) - \sin\theta_n \sin\left(\frac{n\pi}{N+1}\right) + 2\lambda_p \cos\theta_n = 0$$

$2\lambda_p \cos\theta_n = -\,2\cos\theta_n \cos\left(\frac{n\pi}{N+1}\right)$ ----------------3

Dividing by $2\cos\theta_n$ on both sides of equation 3, we get

$$\lambda_p = -\cos\left(\frac{n\pi}{N+1}\right)$$

Problem 2.

For Copper bar, with $\alpha = 1.11e - 4\ m^2/sec$

When N = 100, The maximum temperature is 155.71 C.

When N = 250, The maximum temperature is 159.02 C.

For AISI 1010 carbon steel with $\alpha = 1.88e - 5\ m^2/sec$
When N = 100, The maximum temperature is 919.36 C.
When N = 250, The maximum temperature is 938.90 C.