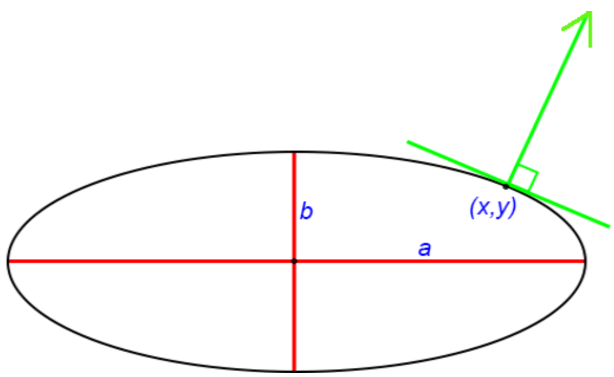
When computing a normal vector, remember that the normal vector is generally different at each point on the boundary. In the very special case that a piece of the boundary is a straight line, then the formula for the normal vector is everywhere the same. But if the piece is not a straight line, then the vector will depend on which specific point we are looking at.

For example, consider the ellipse

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = c^2$$

and take a point (x, y) on this ellipse. What is the normal vector at this point? We need to find a vector that is perpendicular to a tangent vector, that is, a vector whose dot (or "inner") product with a tangent vector is zero (green arrow in picture).



If we think of the ellipse as parametrized by a parameter s, so $x = x(s), y = y(s)$ then taking derivatives using the chain rule, and dividing by 2, we get

$$\frac{x}{a^2}\frac{dx}{ds} + \frac{y}{b^2}\frac{dy}{ds} = 0$$

and, because

$$\left(\frac{dx}{ds}, \frac{dy}{ds}\right)$$

is tangent to the curve, it follows that the vector

$$\eta = \left(\frac{x}{a^2}, \frac{y}{b^2}\right)$$

is a normal vector at the point (x, y). For example, if $a = 2, b = 1, c = 3$ and $x = y = 1$ then a normal is $(1/4, 1)$ (or $(1, 4)$, or any other multiple).