Problem 1.

a. To prove Q is an orthogonal matrix, $Q^{\mathrm{T}}Q = I$.

$$Q^T Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & c & 0 & s & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -s & 0 & c & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & c & 0 & -s & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & s & 0 & c & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & c^2 + s^2 & 0 & cs - cs & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & cs - cs & 0 & c^2 + s^2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad [\text{Given}, c^2 + s^2 = 1]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = I$$

b. $\begin{bmatrix} c & -s \\ s & c \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} c & s \\ -s & c \end{bmatrix} = \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix}$

$$\begin{aligned} c^2 a_{11} - 2cs a_{12} + s^2 a_{22} &= b_{11} \\ s^2 a_{11} + 2cs a_{12} + c^2 a_{22} &= b_{22} \\ a_{12}(c^2 - s^2) - cs(a_{22} - a_{11}) &= 0 \end{aligned}$$

Putting $t = \frac{s}{\mathrm{c}}$ and $2\tau = \frac{a_{22} - a_{11}}{a_{12}}$

$$\begin{aligned} t^2 + 2\tau t - 1 &= 0 \\ t &= \frac{sign(\tau)}{|\tau| - \sqrt{\tau^2 + 1}} \\ c &= \frac{1}{\sqrt{t^2 + 1}} \\ s &= ct \end{aligned}$$

c. Program which takes as input a matrix A and an off-diagonal position $[i, j]$ and returns the Jacobi matrix Q_{ij} is in folder HW6_Solution/1_jacobi_rotation/jacobi_rotation.m

d. Test program generates random matrices and show that any desired off-diagonal element pair may be zeroed out using matrix Q_{ij} is in folder HW6_Solution/1_jacobi_rotation/jacobi_rotation_test.m

Problem 2.

Program to this problem is in HW6_Solution/2_jacobi_eigen

Problem 3.

Program for this problem is in HW6_Solution/3_pca

a. 3 dimensions of the data hold useful information as given by singular values of the data and 14 dimensions were noise as there were total 17 singular values.
b. The object hidden as a point cloud is a tennis shoe.