

# MATH 7241 Fall 2022: Problem Set #5

Due date: Friday October 28

**Reading:** relevant background material for these problems can be found on Canvas ‘Notes 4: Finite Markov Chains’. Also Grinstead and Snell Chapter 11.

**Grinstead and Snell:** see pages 442-443 and pages 444, 467-468 on Canvas. The text is available online (free!) at

<http://www.dartmouth.edu/~chance/>

Click on the link “A GNU book”.

**Exercise 1** Grinstead & Snell, p. 442: #3.  
[Note: ergodic means irreducible].

**Exercise 2** Grinstead & Snell, p. 443: #5.  
[Note: fixed probability vector means stationary distribution].

**Exercise 3** ‘Finite Markov Chains – Problems’ file: Exercise 5

**Exercise 4** GS, p. 444: #17.

**Exercise 5** For a Markov chain, suppose that state  $i$  is transient, and that state  $i$  is accessible from state  $j$  (meaning that there is some integer  $m$  such that  $p_{ji}(m) > 0$ ). Show that  $p_{ij}(n) \rightarrow 0$  as  $n \rightarrow \infty$ . [Hint: find an inequality relating  $\sum_n p_{ii}(n)$  and  $\sum_n p_{ij}(n)$ ]

**Exercise 6** GS, p. 467: #7  
[Note: ergodic means irreducible, and fixed vector means stationary distribution. For part (c), guess that the chain is reversible and solve the reversibility equations. For part (d), set up a recursion formula by conditioning on the first step].