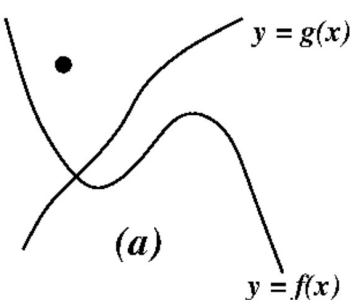
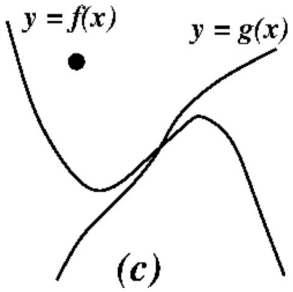
Consider a system with equations as follows, where we assume that $0 < \varepsilon \ll 1$:

$$\begin{aligned} \dot{x} &= f(x) - y \\ \dot{y} &= \varepsilon(g(x) - y)\,. \end{aligned}$$

Consider these possibilities for the nullclines $y = f(x)$ and $y = g(x)$:





(i) What can you say about stability of the steady states in each case?

(ii) Sketch directions of movement in each figure, making sure to show where the vector field is "almost horizontal". Include arrows on all nullclines, too.

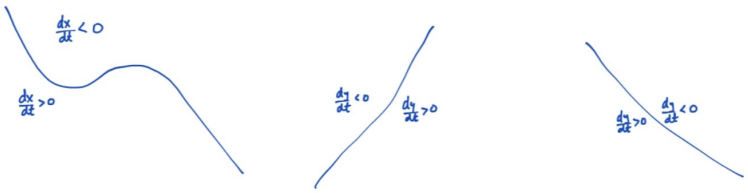
(iii) For each of these, sketch trajectories when starting from the points labeled by the dark circle.

ANS: (i) Since trace is $f'(x_0) - \varepsilon$ and det is $\varepsilon(g'(x_0) - f(x_0))$, we have:

(a) $f'(x_0) < 0$, $g'(x_0) > 0 \Rightarrow$ trace negative, det positive: stable

(c) $f'(x_0) > 0$, $g'(x_0) > f'(x_0) \Rightarrow$ trace positive, det positive: repelling (unstable)

To draw all arrows, all we need are these facts, over and under the respective graphs of f and g (as discussed in notes and class):



(ii)-(iii) Sketched here: