

## MATH 7241: Problem Set #2

Due date: Friday September 30

**Reading:** relevant background material for these problems can be found in the class notes, and in Ross (Chapters 2,3,5) and in Grinstead and Snell (Chapters 1,2 3, 6).

**Exercise 1** [Ross 1-29] Suppose that  $\mathbb{P}(E) = 0.6$ . What can you say about  $\mathbb{P}(E|F)$  when

- (a)  $E$  and  $F$  are mutually exclusive?
- (b)  $E \subset F$ ?
- (c)  $F \subset E$ ?

**Exercise 2** [Ross 1-42] There are three coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up Heads 75 percent of the time. One of the three coins is selected at random and flipped, and it comes up Heads. What is the probability that it was the two-headed coin?

**Exercise 3** In a variation of the classic Monty Hall game show, the host sets up five doors and hides prizes behind two of the doors. The contestant first guesses a door, and then the host opens one of the other four doors to show that it does not conceal a prize. The contestant is offered the opportunity to switch her guess to a different door. Should she switch or stay with her original choice? [Hint: see notes on the Monty Hall question, and try to imitate the solution provided there].

**Exercise 4** A fair die is rolled repeatedly. Let  $X_n$  be the result of the  $n^{\text{th}}$  roll. So  $X_n$  takes values  $\{1, \dots, 6\}$ , each with probability  $1/6$ , and the random variables  $X_1, X_2, \dots$  are all independent. Let

$$N = \min\{n : X_n = X_{n-1} = 4, n \geq 2\}$$

That is,  $N$  is the first time you roll two consecutive 4's. Find  $E[N]$ .

[Hint: condition on the outcomes of the first two rolls, and imitate the argument we used in class for the 'rat in a maze' problem to derive a recursive equation for  $\mathbb{E}[N]$ .]

**Exercise 5** Suppose  $X$  is an exponential random variable. One of the following three formulas is correct:

- (a)  $\mathbb{E}[X^2 \mid X > 1] = \mathbb{E}[(X + 1)^2]$
- (b)  $\mathbb{E}[X^2 \mid X > 1] = \mathbb{E}[X^2] + 1$
- (c)  $\mathbb{E}[X^2 \mid X > 1] = (\mathbb{E}[X] + 1)^2$

Without doing computations, use the memoryless property of the exponential distribution to explain which answer is correct.