Problem 3.

$g(\underline{x})$? $\quad g_1(\underline{x}) > g_2(\underline{x})$ if $\underline{x} \in S_1$.

(a)

$\underline{x} \in S_1 \Rightarrow \quad \|\underline{x} - \mu_1\|_2 < \|\underline{x} - \mu_2\|_2$.

$$(\underline{x} - \underline{\mu_1})^T(\underline{x} - \underline{\mu_1}) < (\underline{x} - \underline{\mu_2})^T(\underline{x} - \underline{\mu_2})$$

$$\underline{x}^T\underline{x} - \underline{\mu_1}^T\underline{x} - \underline{x}^T\underline{\mu_1} + \underline{\mu_1}^T\underline{\mu_1} < \underline{x}^T\underline{x} - \underline{\mu_2}^T\underline{x} - \underline{x}^T\underline{\mu_2} + \underline{\mu_2}^T\underline{\mu_2}$$

$$2\underline{\mu_1}^T\underline{x} - \mu_1^T\mu_1 > 2\underline{\mu_2}^T\underline{x} - \underline{\mu_2}^T\underline{\mu_2}.$$

so. $g_1(\underline{x}) = 2\underline{\mu_1}^T\underline{x} - \underline{\mu_1}^T\underline{\mu_1} \qquad g_2(\underline{x}) = 2\underline{\mu_2}^T\underline{x} - \underline{\mu_2}^T\underline{\mu_2}$

if $g_1(\underline{x}) > g_2(\underline{x})$ $\quad \underline{x} \in S_1$ $\quad$ else $g_1(\underline{x}) < g_2(\underline{x})$. $\underline{x} \in S_2$

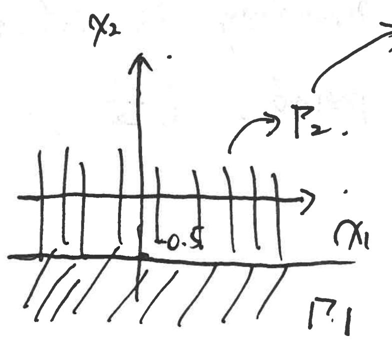
(b)

$g_1(\underline{x}) = g_2(\underline{x})$ is decision boundary.

$$g_1(\underline{x}) = 2\begin{bmatrix} 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 4 = -4x_2 - 4$$

$$g_2(\underline{x}) = 2\begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 1 = 2x_2 - 1.$$

$g_1(\underline{x}) = g_2(\underline{x}) \Rightarrow \quad -4x_2 - 4 = 2x_2 - 1 \qquad 6x_2 = -3 \quad x_2 = -0.5$



class Two decision boundary.

decision boundary

class One decision boundary

(c).

$g_1(\underline{x}) = 2\underline{\mu_1}^T\underline{x} - \underline{\mu_1}^T\underline{\mu_1}$ $\qquad$ if $\underline{x} \in S_1$ $\begin{cases} g_1(\underline{x}) > g_2(\underline{x}). \\ g_1(\underline{x}) > g_3(\underline{x}). \end{cases}$

$g_2(\underline{x}) = 2\underline{\mu_2}^T\underline{x} - \underline{\mu_2}^T\underline{\mu_2}$

$g_3(\underline{x}) = 2\underline{\mu_3}^T\underline{x} - \underline{\mu_3}^T\underline{\mu_3}.$ $\qquad$ for S_2, S_3 is the same.

the classifier is $w^T x + b$. $\quad w = 2\mu_i \quad b = \mu_i^T\mu_i$

so linear classifier.