Problem 3. Page 1

There are two classes. class 1 vs class 2.

if $\underline{x} \in$ class 1, $\|\underline{x} - \mu_1\|_2 \le \|\underline{x} - \mu_2\|_2$.

$$(\underline{x} - \mu_1)^T (\underline{x} - \mu_1) < (\underline{x} - \mu_2)^T (\underline{x} - \mu_2).$$

$$\underline{x}^T\underline{x} - \mu_1^T\underline{x} - \underline{x}^T\mu_1 + \mu_1^T\mu_1 < \underline{x}^T\underline{x} - \mu_2^T\underline{x} - \underline{x}^T\mu_2 + \mu_2^T\mu_2$$

$$(\mu_2^T - \mu_1^T)\underline{x} + \underline{x}^T(\mu_2 - \mu_1) + \mu_1^T\mu_1 - \mu_2^T\mu_2 < 0.$$

$(\mu_2^T - \mu_1^T)\underline{x}$ and $\underline{x}^T(\mu_2 - \mu_1)$ is scalar. so.

$$2(\mu_2 - \mu_1)^T\underline{x} + \mu_1^T\mu_1 - \mu_2^T\mu_2 < 0.$$

$$g_{12}(\underline{x}) = 2(\mu_1 - \mu_2)^T\underline{x} + \mu_2^T\mu_2 - \mu_1^T\mu_1$$

$\underline{x} \in$ class 1 $\Leftrightarrow$ $g_{12}(\underline{x}) > 0$.

$\underline{x} \in$ class 2 $\Leftrightarrow$ $g_{12}(\underline{x}) < 0$.

$g_{12}(\underline{x})$ is linear classifier.

(b) $\mu_1 = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$ $\mu_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$$g_{12}(\underline{x}) = 2\left[\begin{pmatrix} 0 \\ -2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]^T \underline{x} + [0\ 1]\begin{bmatrix} 0 \\ 1 \end{bmatrix} - [0\ -2]\begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$= 2[0\ -3]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 4 + 1$$

$$= 2(x_1 0 - 3x_2) - 3$$

blem 1

$g_{12}(\underline{x}) = -x_1 - x_2 + 5$

$g_{13}(\underline{x}) = -x_1 + 3$

$g_{23}(\underline{x}) = -x_1 + x_2 - 1$

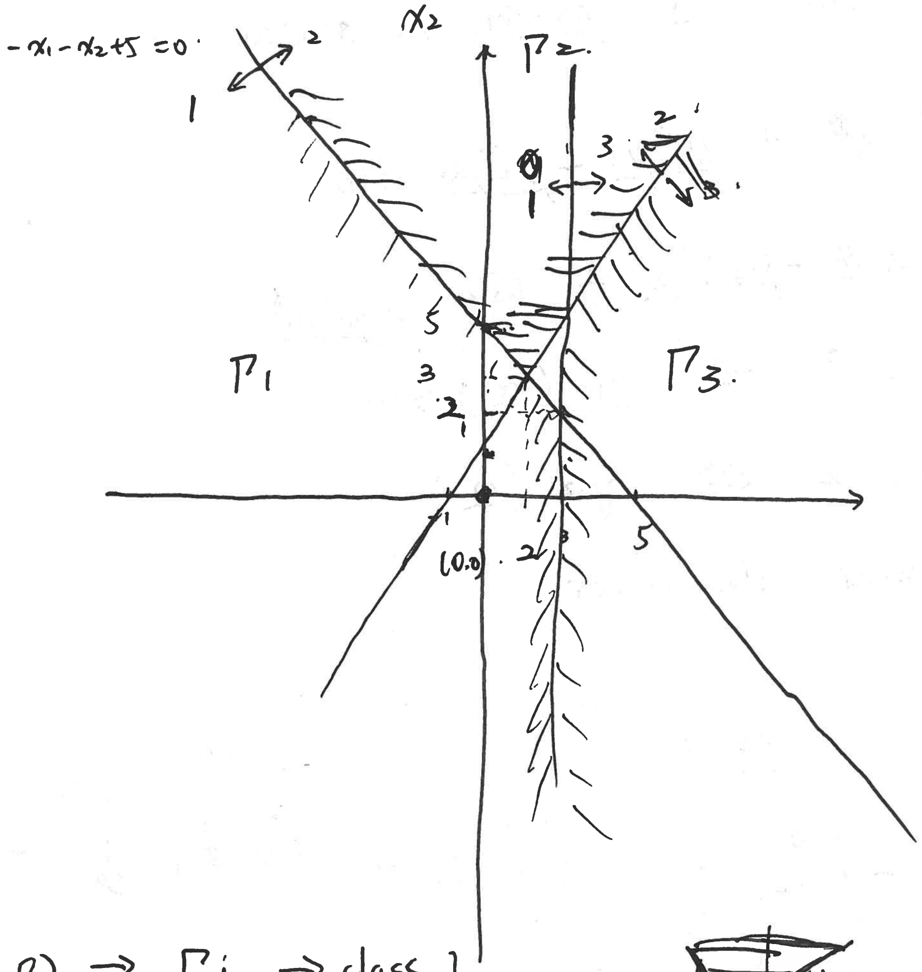
class 2 $\begin{cases} g_{12}(\underline{x}) < 0 \\ g_{23}(\underline{x}) > 0 \end{cases}$

class 3 $\begin{cases} g_{13}(\underline{x}) < 0 \\ g_{23}(\underline{x}) < 0 \end{cases}$

class 1 $\begin{cases} g_{12}(\underline{x}) > 0 \\ g_{13}(\underline{x}) > 0 \end{cases}$

class 1 $\begin{cases} -x_1 - x_2 + 5 > 0 \\ -x_1 + 3 > 0 \end{cases}$

class 3 $\begin{cases} -x_1 + 3 < 0 \\ -x_1 + x_2 - 1 < 0 \end{cases}$



class 2 $\begin{cases} -x_1 - x_2 + 5 < 0 \\ -x_1 + x_2 - 1 > 0 \end{cases}$

$y = x + 1$

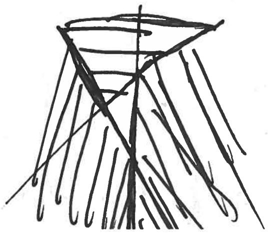
(4, 1)

(1, 5)

(0, 0).

$(0,0) \rightarrow \Gamma_1 \rightarrow$ class 1

$(4,1) \rightarrow \Gamma_3 \rightarrow$ class 3



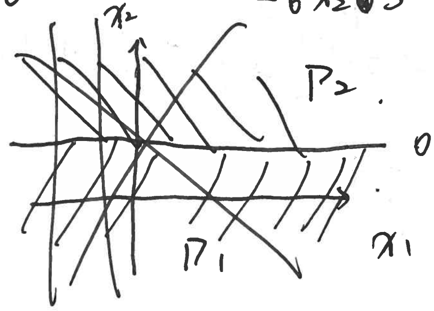
the boundary is shown.

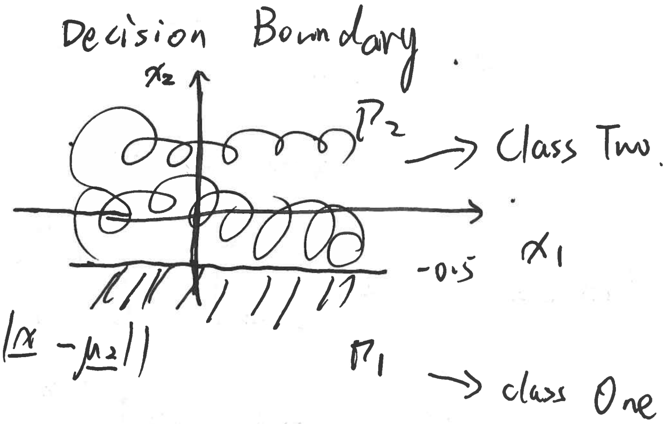
Problem 3 Page 2.

$6x_2 < -3$

$x_2 < -0.5$

$g_{12}(\underline{x}) > 0.$ $\quad -6x_2 - 3 > 0.$





(c). for class 1 $\quad \|\underline{x}-\underline{\mu_1}\|_2 < \|\underline{x}-\underline{\mu_2}\|$

$\|\underline{x}-\underline{\mu_1}\|_2 < \|\underline{x}-\underline{\mu_3}\|.$

$$(\underline{x}-\underline{\mu_1})^T(\underline{x}-\underline{\mu_1}) < (\underline{x}-\underline{\mu_2})^T(\underline{x}-\underline{\mu_2}).$$

$$\underline{x}^T\underline{x} - \underline{\mu_1}^T\underline{x} - \underline{x}^T\underline{\mu_1} + \underline{\mu_1}^T\underline{\mu_1} < \underline{x}^T\underline{x} - \underline{\mu_2}^T\underline{x} - \underline{x}^T\underline{\mu_2} + \underline{\mu_2}^T\underline{\mu_2}.$$

$$2(\underline{\mu_1}^T - \underline{\mu_2}^T)\underline{x} + \underline{\mu_2}^T\underline{\mu_2} - \underline{\mu_1}^T\underline{\mu_1} > 0.$$

$$2(\underline{\mu_1}^T - \underline{\mu_3}^T)\underline{x} + \underline{\mu_3}^T\underline{\mu_3} - \underline{\mu_1}^T\underline{\mu_1} > 0.$$

$$g_{12}(\underline{x}) = 2(\underline{\mu_1}^T - \underline{\mu_2}^T)\underline{x} + \underline{\mu_2}^T\underline{\mu_2} - \underline{\mu_1}^T\underline{\mu_1}$$

$$g_{13}(\underline{x}) = 2(\underline{\mu_1}^T - \underline{\mu_3}^T)\underline{x} + \underline{\mu_3}^T\underline{\mu_3} - \underline{\mu_1}^T\underline{\mu_1}$$

$$g_{23}(\underline{x}) = 2(\underline{\mu_2}^T - \underline{\mu_3}^T)\underline{x} + \underline{\mu_3}^T\underline{\mu_3} - \underline{\mu_2}^T\underline{\mu_2}$$

$\underline{x} \in S_k$ iff $g_{kj}(\underline{x}) > 0$ for all $j \neq k$.

all g_{jk} is linear classifier.

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$$g_{12}(\underline{x}) = 2(\underline{\mu_1} - \underline{\mu_2})^T \underline{x} + \underline{\mu_2}^T\underline{\mu_2} - \underline{\mu_1}^T\underline{\mu_1}$$

$$= 2\left(\begin{bmatrix} 0 \\ -2 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)^T \underline{x} + \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 & -2 \end{bmatrix}\begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -6 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + (1) - 4$$

$$= -6x_2 - 3.$$

$$g_{13}(\underline{x}) = 2\left(\begin{bmatrix} 0 \\ -2 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \end{bmatrix}\right)^T \underline{x} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}^T\begin{bmatrix} 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & -2 \end{bmatrix}\begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 4 - 4.$$

$$= -4x_1 - 4x_2$$

$$g_{23}(\underline{x}) = 2\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \end{bmatrix}\right)^T \underline{x} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}^T\begin{bmatrix} 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -4 \\ 2 \end{bmatrix}^T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 3.$$

$$= -4x_1 + 2x_2 + 3.$$