2022/01/28	BIE-DML : Exam Test					**2A**
Family Name, Surname	P 1	P 2	P 3	P 4	P 5	$\sum$

In this part of the exam you can get **at most 36 points**. You have **90 minutes** for this test, no devices (except non-cell phone calculators) are allowed. Do not forget to describe any logical step to explain your thoughts when solving problems. Not only the correct result is evaluated, but especially the correctness and **clarity of the presentation** of the procedure!
A confusing correct solution will be evaluated with minor number of points.

The test consists of **two parts**: theoretical (questions 1, 2) and calculations (questions 3, 4, 5). To succeed in this part of the exam it is necessary to get **at least 5 points from the theoretical part** and **at least 13 points from calculations**.

Question 1. (5 points) Formulate the Euler's theorem (power reduction in modular arithmetic) and write the formula for Euler's function φ if its argument is $n = p_1^{a_1} \cdot p_2^{a_2} \cdots p_k^{a_k}$, where $p_1, \ldots, p_k$ are distinct primes and $a_1, \ldots, a_k \in \mathbb{N}$.
Determine the value of $\varphi(96)$.

Question 2. (4 points) Assume that we have a language of predicate logic with constants 0 and 1, unary function symbol f, unary predicate symbol p and binary predicate symbol r.
Consider the formula below and list **all** terms and **all** atomic formulas which are contained there:

$$(r(f(1), 0) \vee \neg p(1)) \Leftrightarrow \big(p(0) \wedge (\forall x)(\exists y)\ r(f(x), f(y))\big).$$

Question 3. (8 points) There are seven identical wooden cubes in the children's corner. Three children split these wooden cubes and play with the cubes from their own pile. In how many different ways could the children split the cubes between them if we distinguish individual children and **do not allow** a situation where some child (children) has no cube?

a) (2 points) Determine the number of ways if children divided all the cubes.

b) (3 points) Determine the number of ways if children do not necessarily use all the cubes.

b) (3 points) Determine the number of ways for **b)** for c children and b cubes, $b \geq 7, c \geq 3, b \geq c$. Your reasons should be understandable. Simply writing a formula like the ones above is not enough.

Question 4. (9 points) Calculate the following expression for several small n's (like $n = 1$, $n = 2$, $n = 3$,...). Guess the sum up formula and prove your guess by mathematical induction for any $n \in \mathbb{N}^+$:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n \cdot (n+1)}$$

i) formulate the implication of an inductive case (including inductive assumption)

ii) write which principle of mathematical induction you use.

Question 5. (10 points) Determine all integer solutions for x of the given system of congruences:

$$\begin{aligned} x &\equiv 8 \pmod{99} \\ 12345^{6789} \cdot x &\equiv 1 \pmod{113} \end{aligned}$$
