2022/02/04	BIE-DML : Exam Test					**3A**
Family Name, Surname	P 1	P 2	P 3	P 4	P 5	$\sum$

In this part of the exam you can get **at most 36 points**. You have **90 minutes** for this test, no devices (except non-cell phone calculators) are allowed. Do not forget to describe any logical step to explain your thoughts when solving problems. Not only the correct result is evaluated, but especially the correctness and **clarity of the presentation** of the procedure!
A confusing correct solution will be evaluated with minor number of points.

The test consists of **two parts**: theoretical (questions 1, 2) and calculations (questions 3, 4, 5). To succeed in this part of the exam it is necessary to get **at least 5 points from the theoretical part** and **at least 13 points from calculations**.

Question 1. (4 points) Define the **union, intersection** and **difference** of two sets X, Y in universe $\mathcal{U}$. (Don't use Venn's diagram).
Write down the formula describing the principle of **inclusion-exclusion** for three sets A, B, C.

Question 2. (5 points) Define the greatest common divisor of two numbers $a, b \in \mathbb{Z}$; include all cases. Determine $\gcd(92, 39)$ using Euclid's algorithm.

Question 3. (9 points)

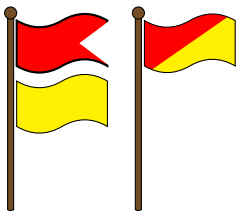
a) **(1 point)** Find a logical equivalent formula to $A \Rightarrow B$ which contains only $\neg$ and $\vee$.

b) **(2 points)** Express $A \Rightarrow B$ equivalently using only $\neg$ and $\wedge$.

c) **(3 points)** Find an equivalent formula to $(A \wedge B) \Rightarrow (C \wedge D)$ which only uses $\neg$ and $\vee$.

d) **(3 points)** Find an equivalent formula to $(A \wedge B) \Rightarrow (C \wedge D)$ which only uses $\neg$ and $\wedge$.

Question 4. (9 points) In yachting races, signals are sent by hanging various flags on masts, while it is important both the order of the masts and the order of the flags on them matter. Some masts may remain empty.



a) **(3 points)** How many different options are there to hang 3 different flags on 2 masts if all flags have to be used?

b) **(3 points)** How many different options are there to hang 3 different flags on 2 masts when all flags **do not have to be** used?

c) **(3 points)** How many different ways are there to hang n different flags on k masts under the same conditions as in **a)**?

You do not have to evaluate your answers, but only properly explained results will be accepted.

Question 5. (9 points) A factory produces the same number of products (not more than 25 000) on the production line every day. The products are afterwards packed in boxes of one of three types throughout the day. When packed in boxes of type A (with a capacity of 32 products) 14 products remain unpacked, when packed in boxes of type B (25 pieces each) 16 products remain unpacked and when using box type C (27 pieces each) 8 products remain. Unwrapped products can no longer be used the next day, so they are discarded.

a) **(7 points)** Determine the number of products the line produces per day before discarding unpacked products.

b) **(2 points)** Determine the minimum number of products by which can be the production line increased in a daily plan so that no products are thrown away, no matter what box is packed.
