

$$f(x) = \sqrt{2 \sin(x) + 1}$$

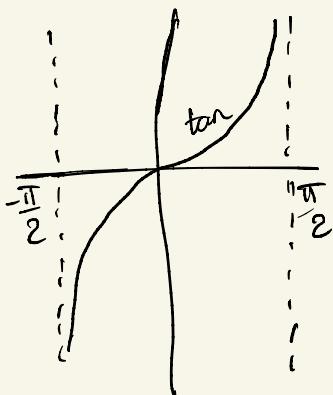
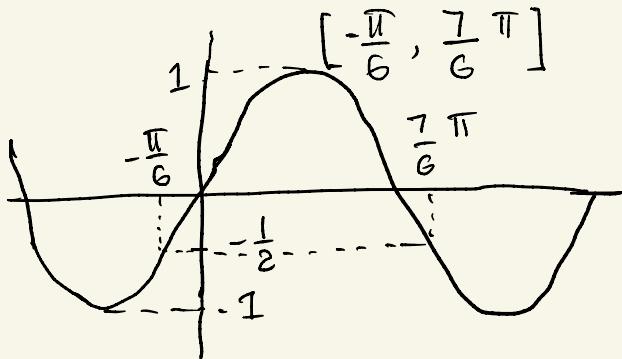
$$\text{Dom } f = ?$$

$\sin - \cos - 2\pi$  period

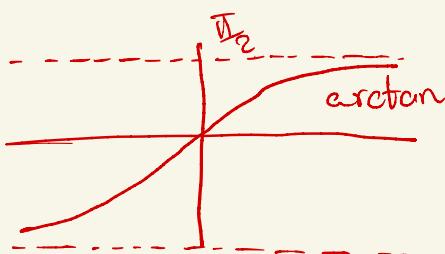
$\tan - \cot - \pi$  period

$$2 \sin x + 1 \geq 0$$

$$\sin x \geq -\frac{1}{2}$$



$f^{-1}$  arctan



$$\text{domain} = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = \text{range}$$

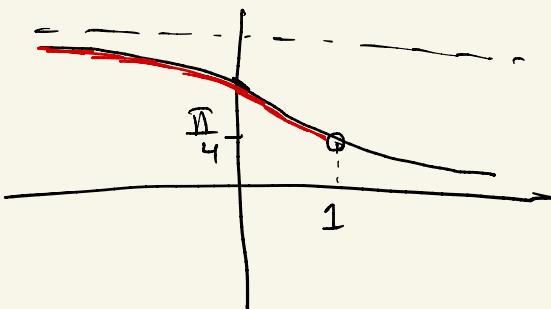
$$\text{range} = (-\infty, \infty) = \text{domain}$$

$$\arctan x = \frac{3}{2} \pi$$

No sol<sup>n</sup>: - just values approaching  $\frac{3}{2} \pi$ -

$$\operatorname{arccot} x > \frac{\pi}{4}$$

$$x \in (-\infty, 1)$$



$$\cot x = \frac{\cos x}{\sin x}$$

$$\cot \frac{\pi}{4} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

$$\operatorname{arccot} x > \frac{\pi}{4}$$

$$x < 1$$

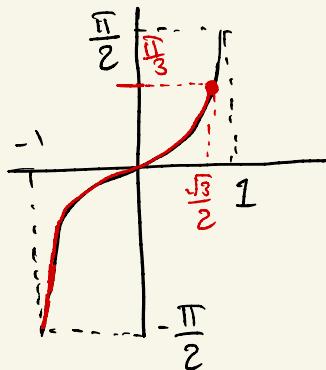
cot -decreasing function

$$A < B$$

$$-A > -B$$

$$\sin^{-1} \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$\arcsin x$

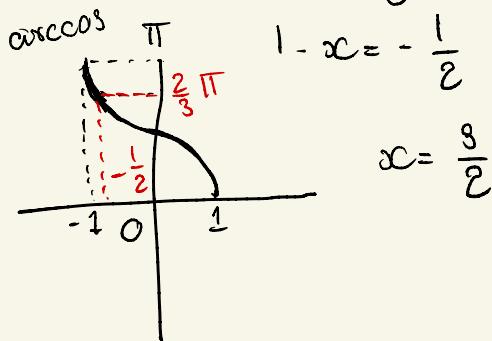


$$\arcsin x \leq \frac{\pi}{3}$$

$$x \in [-1, \frac{\sqrt{3}}{2}]$$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\arccos(1-x) = \frac{2}{3}\pi$$



sign fun

$$\operatorname{sgn}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

Quadratic fun:

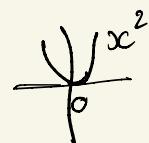
$$f(x) = ax^2 + bx + c$$

$a < 0$ , downward concave

$a > 0$ , upward "

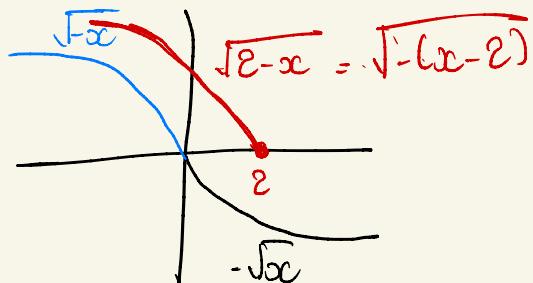
$$\text{vertex} = -\frac{b}{2a}$$

$$(x-a)^2 + b$$



$a \rightarrow$  right shifts on x-axis

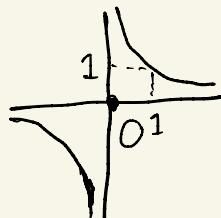
$b \rightarrow$  up shifts on y-axis



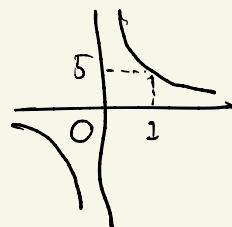
$f(x) = x^2$   
injective from  $[0, +\infty)$

## Linear Rational Functions

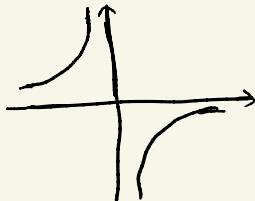
$$\frac{1}{x}$$



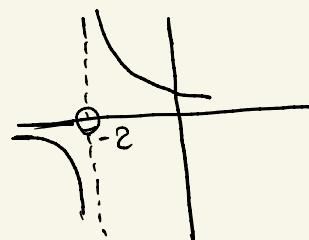
$$\frac{5}{x}$$



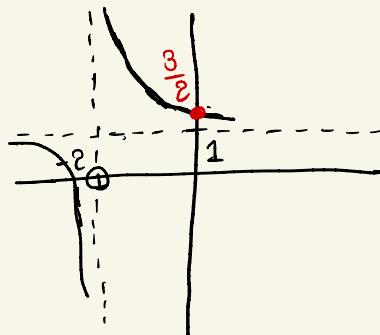
$$-\frac{1}{x}$$



$$\frac{1}{x+2}$$

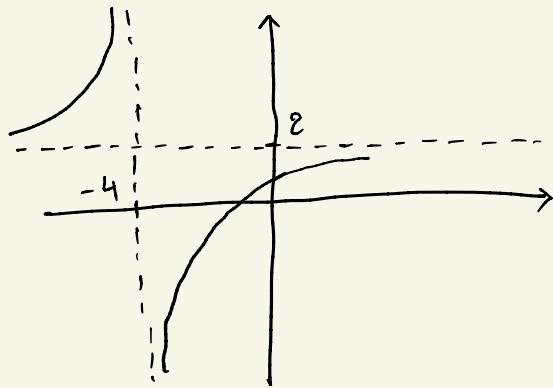


$$\frac{1}{x+2} + 1$$



$$\begin{aligned}\frac{x+3}{x+2} &= \frac{x+2}{x+2} + \frac{1}{x+2} \\ &= \frac{1}{x+2} + 1\end{aligned}$$

$$\begin{aligned}\frac{2x+3}{x+4} &= \frac{2(x+4)}{x+4} - \frac{5}{x+4} \\ &= 2 - \frac{5}{x+4}\end{aligned}$$



# Exponential Functions

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$f(x) = a^x$$

$$a \in (0, +\infty)$$

$$1^{+\infty} \neq 1$$

$$1^\infty = 1$$

$$2^\infty = \infty$$

if  $a \in (0, 1)$ , approaches 0,  
for  $a^\infty$ .

$$\left(\frac{1}{3}\right)^x - \sqrt{3} > 0$$

$$3^{-x} - 3^{\frac{1}{2}} > 0$$

$$3^{-x} > 3^{\frac{1}{2}}$$

$$\log_3 3^{-x} > \log_3 3^{\frac{1}{2}}$$

$$-x > \frac{1}{2}$$

$$x < -\frac{1}{2}$$

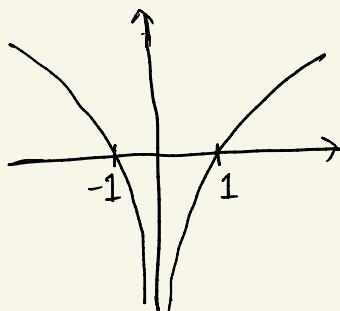
$$\left(\frac{1}{3}\right)^x > \left(\frac{1}{3}\right)^{-\frac{1}{2}}$$

$$\log_{\frac{1}{3}} \left(\frac{1}{3}\right)^x > \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-\frac{1}{2}}$$

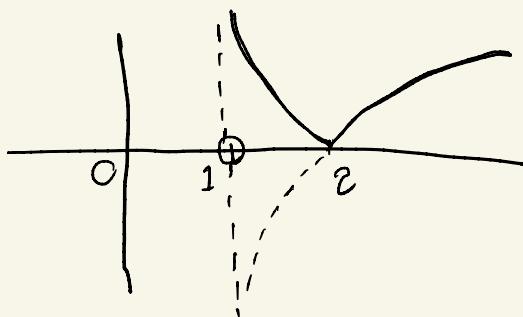
$$x < -\frac{1}{2}$$

$\log_{\frac{1}{3}} x$  is a decreasing function.

$\log x^{\varepsilon}$ , domain =  $\mathbb{R} \setminus \{0\}$



$\log_{10}|(x-1)|$ , domain =  $(1, +\infty)$



Neighbourhood of  $+\infty$  =  $(d^{\alpha}, +\infty)$