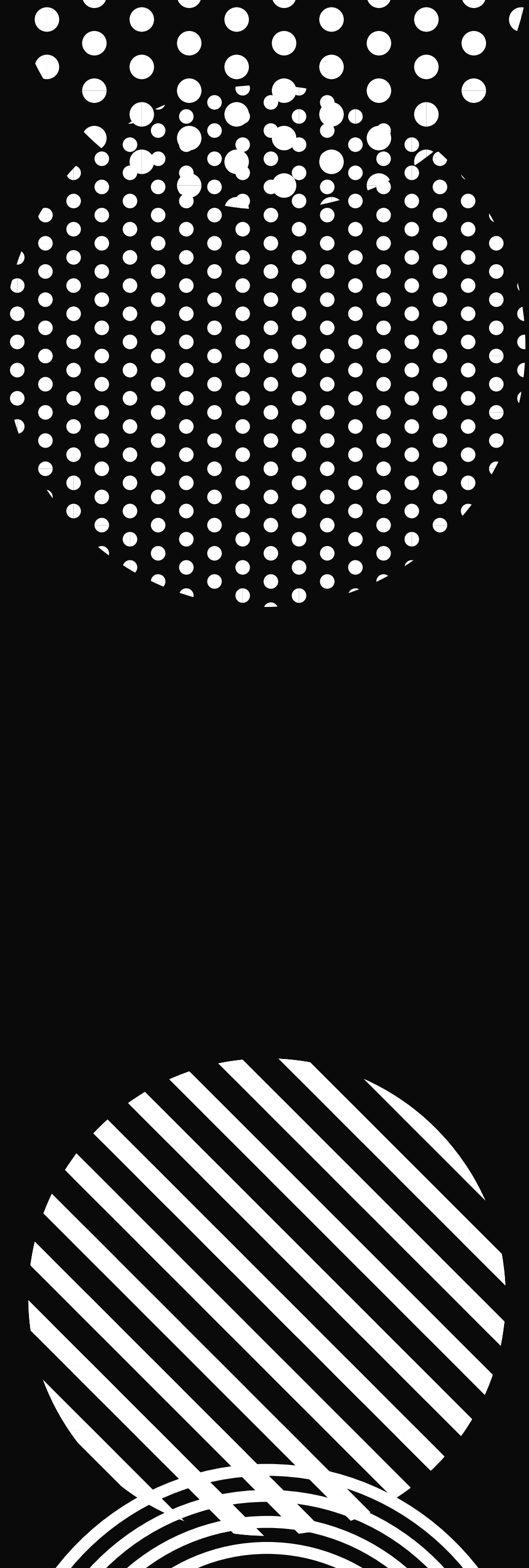
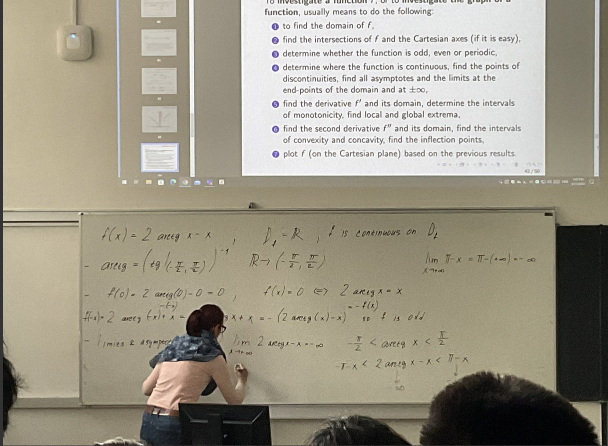




$$-\frac{\pi}{2} < \operatorname{arctg} x < \frac{\pi}{2}$$

$$-\pi - x < 2\operatorname{arctg} x - x < \pi - x$$

$$+\infty \qquad \downarrow -\infty \qquad -\infty$$

$$-\pi - x < 2\operatorname{arctg} x - x < \pi - x$$

$$+\infty \qquad \downarrow +\infty \qquad -\infty$$

$$\lim_{x \to +\infty} 2\operatorname{arctg} x - x = -\infty$$

$$\lim_{x \to -\infty} 2\operatorname{arctg} x - x = +\infty$$

asymptotes

$+\infty$

$$y = kx + q$$

$$k = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{2\operatorname{arctg} x - x}{x}$$

$$= \lim_{x \to +\infty} \frac{2\operatorname{arctg} x}{x} - 1 = \frac{\pi}{+\infty} - 1$$

$$= -1$$

$$q = \lim_{x \to +\infty} f(x) - kx = \lim_{x \to +\infty} 2\arctan x \cancel{-x} \cancel{+x}$$

$$= \lim_{x \to +\infty} 2 \arctan x = \pi$$

$y = -x + \pi$ for $+\infty$

$-\infty$

$$k = \lim_{x \to -\infty} \frac{f(x)}{x} = -1$$

$$q = \lim_{x \to -\infty} f(x) - kx = \lim_{x \to -\infty} 2 \arctan x = -\pi$$

$y = -x - \pi$ for $-\infty$

Monotonicity and extrema

$$f'(x) = \frac{2}{1+x^2} - 1$$

>0

$D_{f'} = \mathbb{R}$

$$f'(x) = 0 \Leftrightarrow \frac{2}{1+x^2} = 1 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1$$

x	$(-\infty, -1)$	$(-1, 1)$	$(1, +\infty)$
$f'(x)$	$-$	$+$	$-$
monotonicity	strictly dec $(-\infty, -1]$	strictly increasing on $[-1, 1]$	strictly dec $[1, +\infty)$

defined on -1 and 1

all included cuz continuous so the value's not gonna jump

-1 = pt for loc. min

$f(-1) = 2\,\mathrm{arctg}(-1) + 1 = -\frac{\pi}{2} + 1 \rightarrow$ loc. min

$+1$ = pt of loc. max

$f(+1) = 2\,\mathrm{arctg}(1) - 1 = \frac{\pi}{2} - 1 \rightarrow$ loc. max

No global extrema

Concavity

$$f''(x) = \left(\frac{2}{1+x^2} - 1\right)'$$

$$= \left(2(1+x^2)^{-1} - 1\right)'$$

$$= \frac{-2}{(1+x^2)^2}(2x) = \frac{-4x}{(1+x^2)^2}$$

$$>0$$

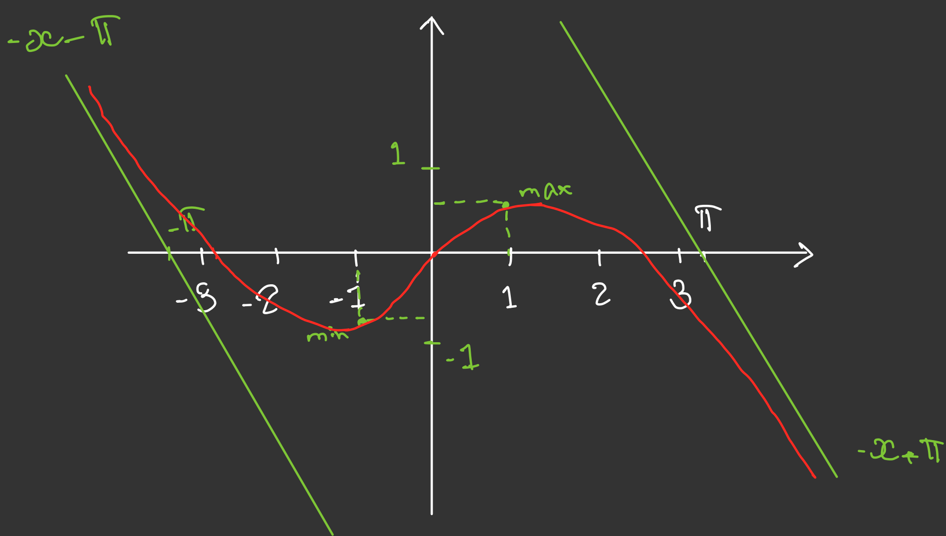
$$D_{f''} = \mathbb{R}$$

$$f''(x) \overset{>}{=} 0 \Leftrightarrow x \overset{<}{=} 0$$

x	$(-\infty, 0)$	$(0, +\infty)$
f''	$+$	$-$
$f(x)$	convex $(-\infty, 0]$	concave $[0, +\infty)$

0 is inflection pt.

$-\frac{\pi}{2} + 1 \approx -0.6$



$f(x) = \ln|x| - 5x$, f is continuous on D_f

$D_f = \mathbb{R} \setminus \{0\}$

$f(x) = 0 \Leftrightarrow \ln|x| = 5x$

can't solve

limits and asymptotes at 0

$$\lim_{x \to 0_+} \ln|x| - 5x = \lim_{x \to 0_+} \ln x - 5x = -\infty - 0$$
$$= -\infty$$
$$= \lim_{x \to 0_-} \ln|x| - 5x$$

$x = 0$ is vertical asymptote

limits

$$\lim_{\substack{x \to +\infty \\ -\infty}} \ln|x| - 5x = \lim_{\substack{x \to +\infty \\ -\infty}} \ln x - 5x$$

$$= \lim_{\substack{x \to +\infty \\ -\infty}} x\left(\frac{\ln x}{x} - 5\right) = \underset{+\infty}{-\infty}$$

$\to 0$

$$\lim_{\substack{x\to+\infty \\ -\infty}} \frac{f(x)}{x} = \lim_{\substack{x\to+\infty \\ -\infty}} \overbrace{\frac{\ln x}{x}}^{\to 0} - 5 = -5 = k$$

$$\lim_{x\to+\infty} f(x) - kx = \lim_{x\to+\infty} \ln|x| \cancel{-5x} + \cancel{5x}$$

$$= +\infty = q$$

$y = kx + q$

no asymptote at $+\infty$
$-\infty$

cuz $q = +\infty$

2 cases

$$f'(x): \quad x>0: \frac{1}{x} - 5$$

$$x<0: -\frac{1}{-x} - 5 = \frac{1}{x} - 5$$

$$x<0: \ln|x|' = \ln(-x)' = \frac{1}{-x}(-1) = \frac{1}{x}$$

$$D_{f'} = \mathbb{R} \setminus \{0\}$$

$$f'(x) = 0 \Leftrightarrow \frac{1}{x} = 5 \Leftrightarrow x = \frac{1}{5}$$

$f'(x) > 0 \Leftrightarrow x \in (0, \frac{1}{5})$

x	$(-\infty, 0)$	$(0, \frac{1}{5})$	$(\frac{1}{5}, +\infty)$
$f'(x)$	$-$	$+$	$-$
$f(x)$ is	strictly dec $(-\infty, 0)$	strictly inc $(0, \frac{1}{5}]$ not defined on 0	strictly dec $[\frac{1}{5}, +\infty)$

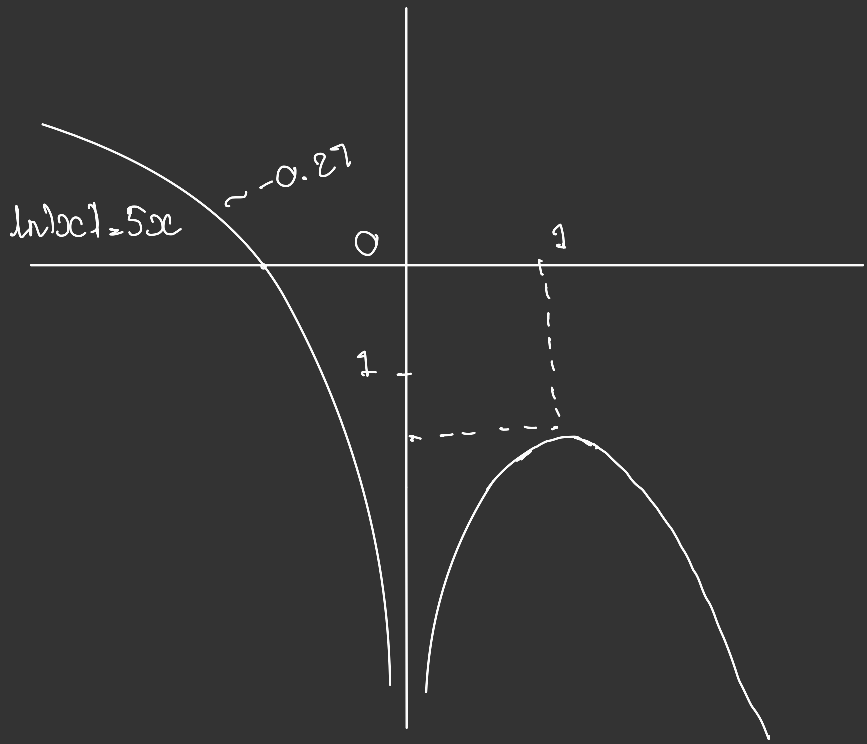
$\frac{1}{5}$ = pt of loc. max

$f(\frac{1}{5}) = -1 \ln 5 - 1$

$f''(x) = -\frac{1}{x^2}$, $D_{f''} = R \setminus \{0\}$

$f''(x) < 0 \quad \forall x \in D_f$

f is concave on D_f



$\ln|x| = 5x$
~ -0.27
0
1
1