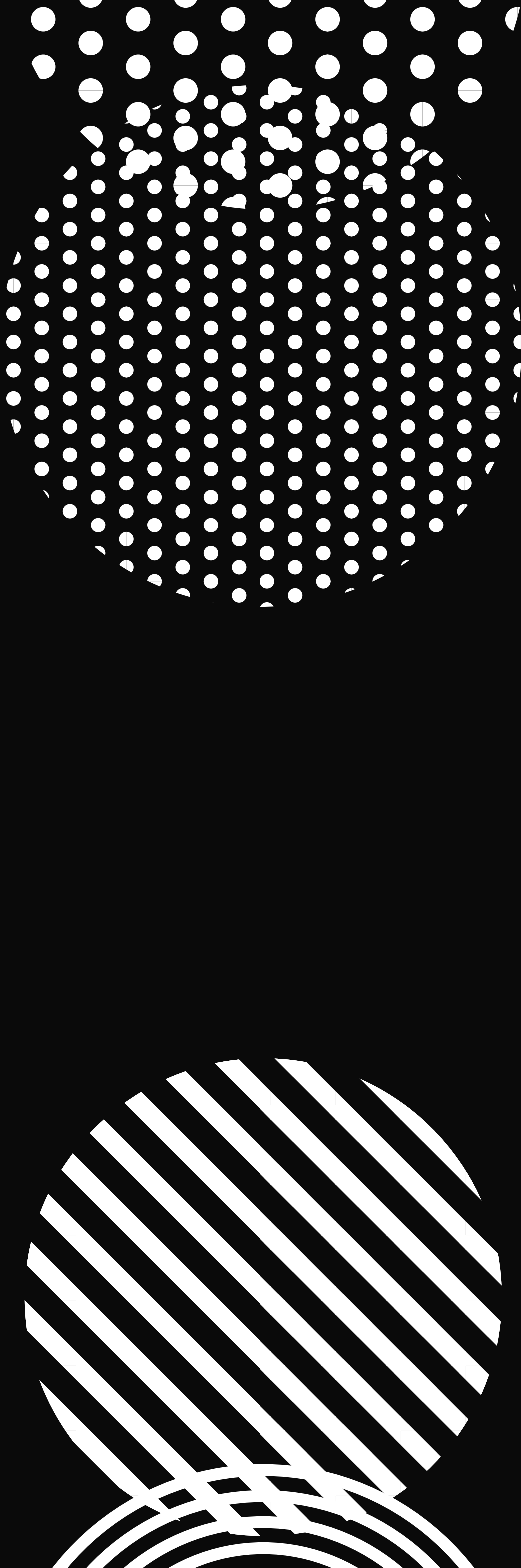


5.19.
$$f(x) = \begin{cases} ax + 1, & x \geq 1 \\ 1 - bx^2 & x < 1 \end{cases}$$

$$\lim_{x \to 1} f(x) = f(1)$$

Continuous?

$$\left.\begin{aligned} &\lim_{x \to 1_+} f(x) = \lim_{x \to 1_+} ax + 1 = a + 1 \\ &\lim_{x \to 1_-} f(x) = 1 - bx^2 = 1 - bx^2 = 1 - b \end{aligned}\right\} \quad a + 1 = 1 - b$$

$$a = -b$$

$$f(x) = \begin{cases} ax + 1 & x \geq 1 \\ 1 + ax^2 & x < 1 \end{cases}$$

Continuous for $a \in \mathbb{R}$.

5.16. $f(x) = \dfrac{\ln(1 + ax^2)}{x}$ $\quad D(f) = \mathbb{R} \setminus \{0\}$

$$\lim_{x \to 0} \frac{\ln(1 + ax^2)}{x}$$

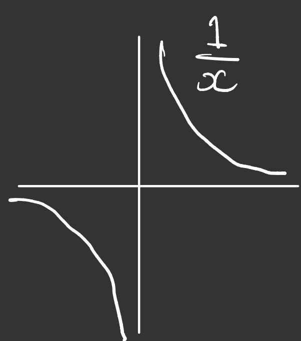
$$\lim_{x \to 0} \frac{\ln(1 + x)}{x} = 1$$

$$= \lim_{x \to 0} \frac{\lim (1+ax^2)}{x} \cdot \frac{ax}{ax}$$

$$= \lim_{x \to 0} \frac{\lim (1+ax^2)}{ax^2} \cdot ax = 1 \cdot ax = 0$$

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

5.17. $f(x) = \begin{cases} ax - 1 & x > 1 \\ b\,\dfrac{e^{x-1} - 1}{(x-1)^2} & x < 1 \end{cases}$



$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} ax - 1$$

$$= a - 1$$

approaches 0,

f(x) " -∞

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} b\,\frac{e^{x-1} - 1}{(x-1)^2}$$

$$= \lim_{x \to 1^-} \frac{b}{(x-1)} \cdot \frac{e^{(x-1)} - 1}{(x-1)}$$

$$= \lim_{x \to 1^-} \frac{b}{x-1} \cdot 1$$

$$= b \cdot (-\infty) \begin{cases} \to b > 0 & -\infty \\ \to b = 0 & 0 \\ \to b < 0 & +\infty \end{cases}$$

$a - 1 = 0$, $b = 0$

$$f(x) = \begin{cases} x-1 & x > 1 \\ 0 & x = 1 \\ 0 & x < 1 \end{cases}$$

5. 20.

$$f(x) = \begin{cases} \sqrt{x-2} & , x \geq 2 \\ 1 - b(x-2)^2, & x < 2 \end{cases}$$

$$\left.\begin{aligned} &\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \sqrt{x-2} = 0 \\ &\lim_{x \to 2^-} f(x) = 1 - b(x-2)^2 = 1 \end{aligned}\right\} 0 \neq 1$$

not continuous

$$f(x) = \begin{cases} \dfrac{b\tan(4-x^2)}{(x-2)} & x > 2 \\ 0 & x = 2 \\ a \arcsin(x-2) & x < 2 \end{cases}$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{-b \cdot \sin(4-x^2)}{\underset{(2-x)}{(x-2)}\cos(4-x^2)} \cdot \frac{(2+x)}{(2+x)}$$

$$= \lim_{x\to 2} \frac{\sin(4-x^2)}{(4-x^2)} \cdot \frac{-b(2+x)}{\underbrace{\cos(4-x^2)}_{\cos 0 = 1}}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$= \lim_{x\to 2} 1 \cdot (-b) \cdot 4 = -4b$$

$$-4b = 0, \ b = 0$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} a \cdot \arcsin(x-2)$$

$$= a \cdot 0 = 0$$

continuous $\forall a \in \mathbb{R}, b = 0$

4.34. c)

$$\lim_{x \to e} \frac{\ln(x) - 1}{x - e}$$

$$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$$

$$y = \frac{x}{e} - 1$$

$$\Rightarrow \frac{x}{e} = y + 1$$

$x \to e$, $\frac{e}{e} - 1 = 0$

$$= \lim_{x \to e} \frac{\ln(x) - \ln e}{x - e}$$

$$= \lim_{x \to e} \frac{\ln\left(\frac{x}{e}\right)}{e\left(\frac{x}{e} - 1\right)} = \lim_{y \to 0} \frac{\ln(1+y)}{e \cdot y}$$

$$= \frac{1}{e}$$

4.33. $$\lim_{x \to 0} \frac{\sqrt{1+x^2} - 1}{x} \cdot \frac{\sqrt{1+x^2} + 1}{\sqrt{1+x^2} + 1}$$

$$= \lim_{x \to 0} \frac{1 + x^2 - 1}{x \cdot (\sqrt{1+x^2} + 1)}$$

$$= \lim_{x \to 0} \frac{x}{\sqrt{1+x^2} + 1} = \frac{0}{2} = 0$$

$$\lim_{x\to+\infty} \frac{x^x\left(\frac{x^2}{x^x} + 2 + \frac{\log(x)}{x^x}\right)}{3x^3 + e^{-x^2}}$$

$\frac{x^2}{x^x} \to 0$, $\frac{\log(x)}{x^x} \to 0$, $e^{-x^2} \to 0$

e^{x^2}, let $y = x^2$

$x \to +\infty,\ y \to +\infty$

$\therefore \lim_{y\to+\infty} e^y = +\infty$

$\therefore \lim_{x\to+\infty} e^{x^2} = +\infty$

$\lim_{x\to+\infty} \frac{1}{e^{x^2}} = 0$

In this one x^x goes to infinity faster than $3x^3$.

$\therefore \lim_{x\to+\infty} \frac{x^x}{3x^3} = +\infty$

$\therefore$ the ans. is $+\infty$