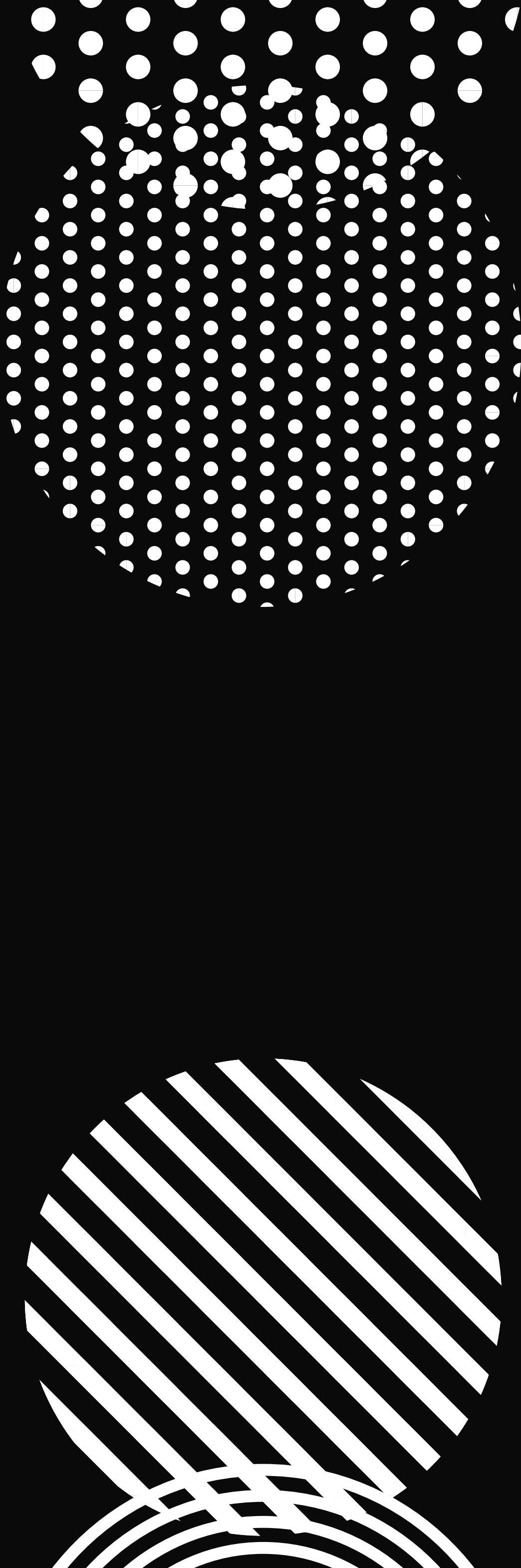


$f = \ln(x)$

$g = \frac{1}{x}$

$f \circ g = \ln \frac{1}{x}$ $\qquad D_{f \circ g} = (0, +\infty)$

$g \circ f = \frac{1}{\ln x}$ $\qquad D_{g \circ f} = (0, 1) \cup (1, +\infty)$

$f(x) = \log_2 \frac{3 + \sin x}{2}$

$f_1 = \sin x$

$f_2 = 3 + x$

$f_3 = \frac{x}{2}$

$f_4 = \log_2 x$

$f(x) = f_4 \circ f_3 \circ f_2 \circ f_1$

$D_f = ?$

$\frac{3 + \sin x}{2} > 0$

$\sin x > -3$ (always true)

$\therefore x \in \mathbb{R} \qquad D_f = (-\infty, +\infty)$

Range of $f = ?$

$$(-\infty, +\infty) \xrightarrow{\sin x} [-1, +1] \xrightarrow{3+x} [2, 4] \xrightarrow{\frac{x}{2}} [1, 2]$$

$$\xrightarrow{\log_2 x} [0, 1]$$

$$f(x) = \frac{1}{\sqrt{9+x^2}}$$

$9 + x^2 > 0$

$x^2 > -9 \qquad D_f = \mathbb{R}$

$$(-\infty; +\infty) \xrightarrow{x^2} [0, +\infty) \xrightarrow{9+x} [9, +\infty)$$

$$\xrightarrow{\sqrt{x}} [3, +\infty)$$

$$\xrightarrow{\frac{1}{x}} (0; \tfrac{1}{3}]$$

2.15.

$$f(x) = \frac{\sqrt{x^2-1} \geq 0}{|x| > 0} \qquad D_f = ?,\ \text{SURJECTIVE?}$$
$$R_f = ?$$

$$x^2 - 1 \geq 0$$
$$x \neq 0$$

$$(-\infty, -1) \cup (1, +\infty)$$

Not surjective (no negative values)

$$\frac{\sqrt{x^2-1}}{|x|} = \frac{\sqrt{x^2-1}}{\sqrt{x^2}} = \sqrt{\frac{x^2-1}{x^2}} = \sqrt{1 - \frac{1}{x^2}}$$

$$[1, +\infty) \xrightarrow{x^2} [1, +\infty) \xrightarrow{\frac{1}{x^2}} (0, 1] \xrightarrow{1-x} [0,1) \xrightarrow{\sqrt{x}} [0,1)$$

$$(-\infty, -1] \longrightarrow \longrightarrow \longrightarrow \longrightarrow [0, 1)$$

Range = $[0, 1)$

$a \in \overline{\mathbb{R}}$ CLUSTER POINT $a \in \mathcal{D}_f \cap \mathcal{D}_g$

f, g ... FUNCTIONS

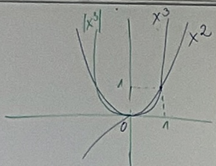
$\cap \mathcal{D}_g \quad \forall x \in \mathcal{U}_a$

$)|$

$f = o(g) \quad x \to a$

$\boxed{\forall C > 0} \quad \exists \mathcal{U}_a \subset \mathcal{D}_f \cap \mathcal{D}_g \quad \forall x \in \mathcal{U}_a \setminus \{a\}$

$$|f(x)| < C \cdot |g(x)|$$



$x \to +\infty$ $\quad a = +\infty$

STRICTLY ...

$x^2 \boxed{=} o(x^3)$

$x \to +\infty$

" $x^2 \in o(x^3)$ "

$\mathcal{U}_a(\varepsilon) = (a-\varepsilon; a+\varepsilon)$ $\quad a-\varepsilon \quad a \quad a+\varepsilon$

$\mathcal{U}_3(1) = (2;4)$ $\quad 2 \quad 3 \quad 4$

$|x-3| < 1$

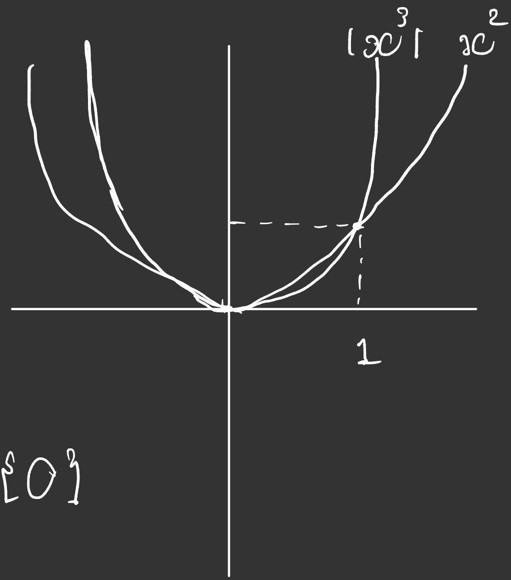
$|x| < 1$

$(-1;1)$

$\mathcal{U}_{+\infty}(5) = (5; +\infty)$

$2x^3 = O(3x^2)$

$x \to 0$



$\forall c > 0, \exists U_0 \subset D_{2x^3}$

$\cap D_{3x^2}$

$\forall x \in U_0 \setminus \{0\}$

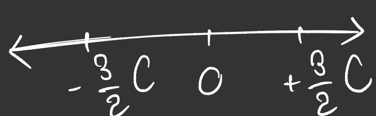
$|2x^3| < c \cdot |3x^2|$

$2 \cdot x^2 \cdot |x| < c \cdot 3x^2 \; / x^2$

$2|x| < c \cdot 3$

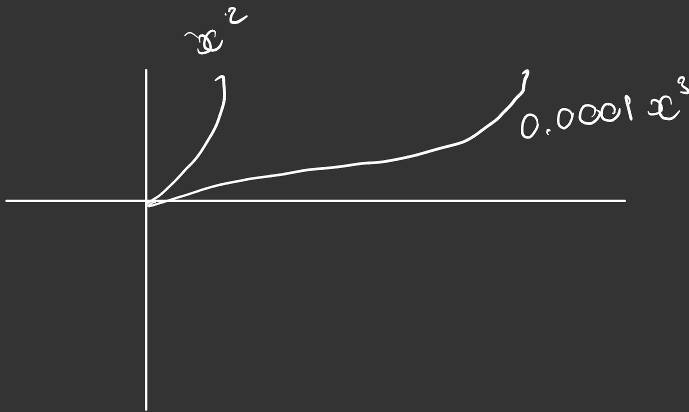
$U_0(\frac{3}{2}c)$

$|x| < \frac{3}{2}c$



$$x^2 = O(0.0001\,x^3)$$

$$x \to +\infty$$



$$\exists\, C > 0,\ \exists\, U_{+\infty} \subset D_{x^2} \cap D_{0.0001x^3}$$

$$\forall x \in U_{+\infty}$$

$$|x^2| \leqslant C \cdot |0.0001\,x^3|$$

$$x \to +\infty$$

$$x^2 \leqslant C \cdot 0.0001\,x^3$$

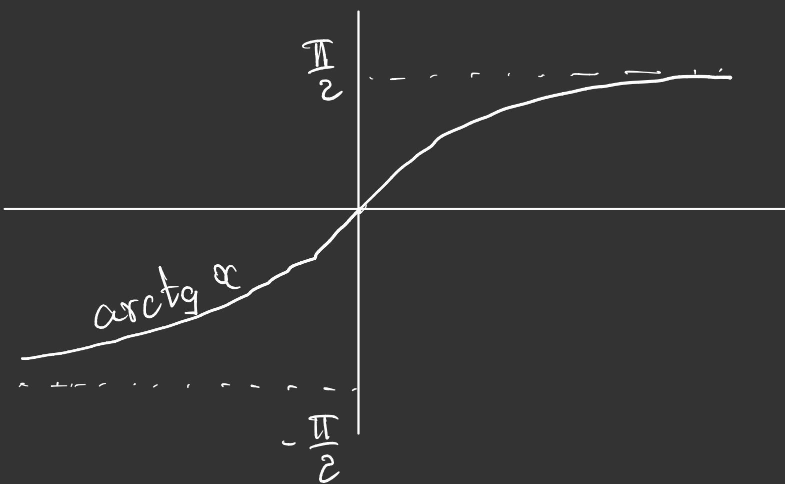
E.g. $C = 10{,}000$ $\quad x^2 \leqslant 1 \cdot x^3 \,/\, x^2$

$$1 \leq x$$

$$U_{+\infty}(1)$$

$$\operatorname{arctg} x = O(1)$$

$$x \to a \qquad a \in \mathbb{R}$$



$$\exists C > 0 \quad \exists U_a \subset D_{\operatorname{arctg} x} \cap D_1 , \ \forall x \in U_a$$

$$|\operatorname{arctg} x| \leq C \cdot |1|$$

eg. $C = \frac{\pi}{2}$

$|\text{arctg}\, x| \leqslant \frac{\pi}{2}$ $\mathcal{U}_a(1)$

Why not

$2x\sqrt{x} + 3x^2 = o(\sqrt{x})$

$x \to 0$

$\forall C > 0, \exists\, \mathcal{U}_0 \subset D_f \cap D_g \quad \forall x \in \mathcal{U}_0 \setminus \{0\}$

$|2x\sqrt{x} + 3x^2| < C \cdot |\sqrt{x}|$

$|A + B| \leqslant |A| + |B|$

$|2x\sqrt{x} + 3x^2| \leqslant |2x\sqrt{x}| + |3x \cdot x|$

$< \sqrt{x}$ for $|x| < 1$

$\leqslant 2\sqrt{x}\,|x| + 3|x| \cdot \sqrt{x}$

$= 5\sqrt{x}\,|x|$

$5\sqrt{x}\,|x| < C \cdot \sqrt{x} \quad / \sqrt{x}$

$5|x| < C$

$|x| < \frac{C}{5}$

$\mathcal{U}_0\left(\frac{c}{5}\right)$

$\sqrt{x^2+4x-3} \geq 0$

$|x^2-4| > 0$

SUMMER 2021/2022 BIE-MA1 : ASSESMENT TEST-SAMPLE					TEST 1S
Family name, Given name	1	2	3	4	**Sum**

Problem 1. (10 points) Let f be the function defined by

$$f(x) = 2 - \frac{\sqrt{-x^2+4x-3}}{|x^2-4|}$$

≥ 0

> 0

a) Find the natural domain of f.

b) Is f surjective onto $\mathbb{R}$?

c) Does

$$\lim_{x \to 2} f(x)$$

exist? If yes, find the vale of the limit. If not, explain why the limit does not exist.

2 - positive no.

$f(x) \notin (2, +\infty)$

2 - +ve number

$2 - x$

$x \in [0, +\infty)$

$[0, +\infty) \xrightarrow{2-x}$

$(-\infty, 2]$

Problem 2. (10 points) Consider the sequence defined by

$$a_n = \frac{1}{3n-5}$$

a) Is the sequence (eventually) monotonic? If yes, specify the type of monotonicity (increasing, strictly increasing, decreasing,...).

b) Is the sequence bounded from below, bounded from above, or bounded (from both sides)? Find the appropriate bounds.

c) Show that $\frac{1}{n^2} = o(a_n)$ for $n \to +\infty$.

$2 - (+\infty)$

$2 + (-\infty)$

Problem 3. (10 points) Compute the following limits of sequences (or if the limit does not exist, explain why):

a)

$$\lim_{n \to +\infty} \frac{\ln(n^3+2n+1)}{\ln(2n^5+n)}.$$

b)

$$\lim_{n \to +\infty} \frac{(-1)^n n^2 + \sqrt[4]{n} + \cos(n^{10}) + 3^{-n}}{n! + n^2}.$$

Problem 4. (10 points) For which values of the parameter $a, b \in \mathbb{R}$ is the function

$$f(x) = \begin{cases} \frac{b\tan(4-x^2)}{(x-2)}, & x > 2 \\ 0, & x = 2 \\ a\arcsin(x-2) & x < 2 \end{cases}$$

continuous at $x = 2$?

***

$$f(x) = \frac{2 - \sqrt{-x^2 + 4x - 3}}{|x^2 - 4|}$$

$$2.0000C002 \qquad \lim_{x \to 2^+} \frac{2 - \cancel{x}\sqrt{-1 + \frac{4}{x} - \frac{3}{\cancel{x^2}}}}{x^2\left(1 - \frac{4}{x^2}\right)}$$

$$1 - \frac{4}{x^2} \quad 1$$

$$+$$

$$1 - 1 = 0$$

$$2 - \sqrt{-1 + 2 - \frac{3}{4}}$$