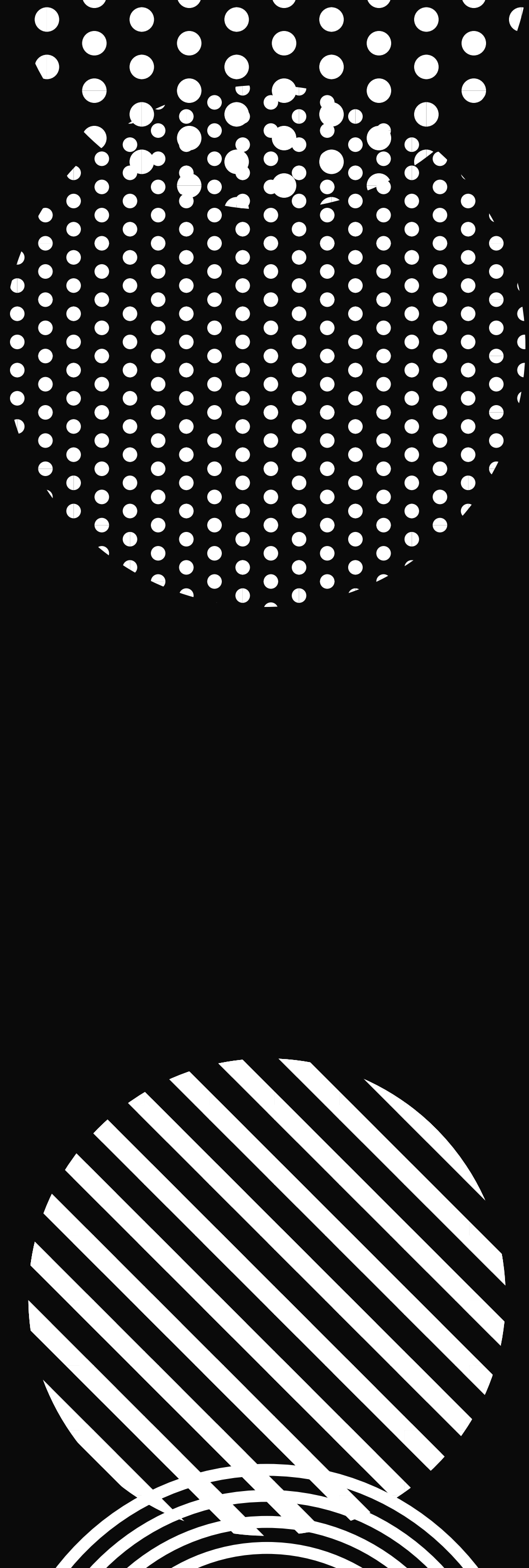


$$\lim_{x\to+\infty} \underset{\substack{\downarrow\\+\infty}}{x}\cdot \underset{\substack{\searrow\\0}}{e^{-x}} = \lim_{x\to+\infty} \frac{x}{e^x} \overset{\frac{+\infty}{+\infty}\text{, can use L'hopital}}{=} \lim_{x\to+\infty} \frac{1}{e^x} = \frac{1}{+\infty} = 0$$

$\frac{0}{0}$, $\frac{\text{anything}}{\pm\infty}$ - can use L'hopital

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{x^3-1} \overset{L'H}{=} \lim_{x\to 1} \frac{\frac{1}{2\sqrt{x}}}{3x^2} = \frac{\frac{1}{2}}{3} = \frac{1}{6}$$

$$\lim_{x\to 0} \frac{e^{\operatorname{arctg} x} - \cos x}{x + \sin x} = \lim_{x\to 0} \frac{e^{\operatorname{arctg} x}\cdot \frac{1}{1+x^2} + \sin x}{1+\cos x}$$

$\operatorname{arctg} x \to 0$, $e^{\operatorname{arctg} x} \to 1$, $\frac{1}{1+x^2} \to 1$, $\sin x \to 0$, $1+\cos x \to 2$

$$= \frac{1}{2}$$

6.1. $f(x) = x\cdot e^{-x}$ find global extrema $[0, 2]$

end-pts. $f(0) = 0$, $f(2) = 2\cdot\frac{1}{e^2}$

$$f'(x) = e^{-x} - x\cdot e^{-x} = e^{-x}(1-x)$$

$$e^{-x}(1-x)=0$$
$>0 \quad \Updownarrow$
$$1-x=0$$
$$x=1$$

$$f(1)=\frac{1}{e}$$

0 is a point of global minimum. $(f(0)=0)$

global maximum $\rightarrow \frac{1}{e}$

point of $=1$

6.3. a). $f(x)=\cos x+\frac{1}{2}\cos 2x$, on $[0, 2\pi]$

$$f(0)=1+\frac{1}{2}=\frac{3}{2}$$

$$f(2\pi)=1+\frac{1}{2}=\frac{3}{2}$$

$$f'(x)=-\sin x-\sin 2x$$

$$-\sin-2\sin x\cdot\cos x=0$$

$$\sin x(1+2\cos x)=0$$

$\sin x = 0$ (or) $1 + 2\cos x = 0$

$x \in \{0, \pi, 2\pi\}$ ∨ $\cos x = -\frac{1}{2}$

$x \in \{\frac{2\pi}{3}, \frac{4\pi}{3}\}$

$x \in \{0, \pi, 2\pi, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

$f(\pi) = -1 + \frac{1}{2} \cdot 1 = -\frac{1}{2}$

$f(\frac{2\pi}{3}) = -\frac{1}{2} + \frac{1}{2}(-\frac{1}{2}) = -\frac{3}{4}$

$f(\frac{4\pi}{3}) = -\frac{1}{2} + \frac{1}{2}\cos . 2 . \frac{4\pi}{3}$

$= -\frac{1}{2} + \frac{1}{2}\cos\frac{8\pi}{3} \rightarrow \frac{2\pi}{3} + 2\pi$

$= -\frac{3}{4}$

Conclusion: $0, 2\pi \rightarrow$ pts of gobal maximum
$$\underset{\frac{3}{2}}{\shortparallel}$$

$\frac{2\pi}{3}, \frac{4\pi}{3} \rightarrow$ pts of global minimum
$$\underset{-\frac{3}{4}}{\shortparallel}$$

$f(x) = x \cdot \sqrt{2-x^2}$ find intervals of monotonicity
local extrema.

Domain : $2 - x^2 \geq 0$

$x \in [-\sqrt{2}, +\sqrt{2}]$

$$f'(x) = \sqrt{2-x^2} + \frac{x}{2\sqrt{2-x^2}} \cdot (-2x)$$

$$= \sqrt{2-x^2} - \frac{\cancel{2}x^2}{\cancel{2}\sqrt{2-x^2}}$$

$$= \frac{2-x^2-x^2}{\sqrt{2-x^2}} = \frac{2(1-x^2)}{\underbrace{\sqrt{2-x^2}}_{>0}}$$

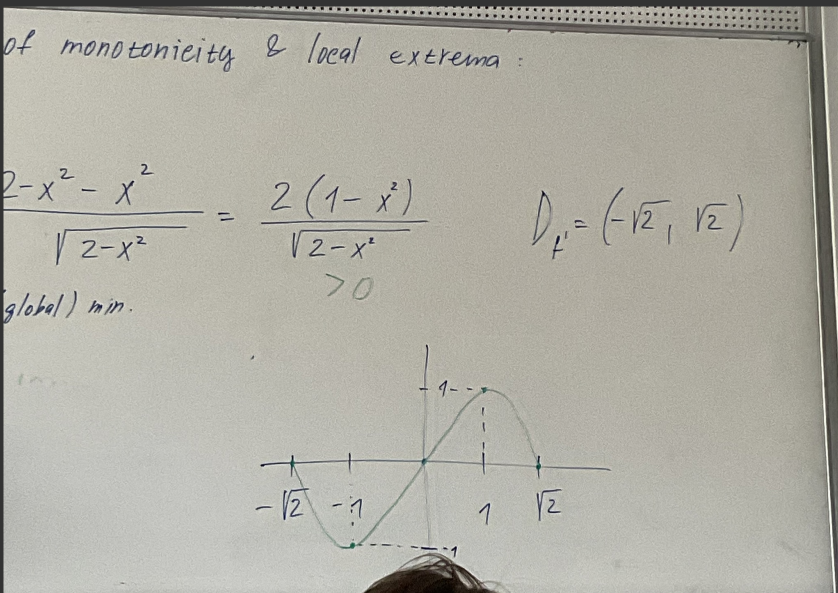
$D_{f'} = (-\sqrt{2}, \sqrt{2})$

$f'(x) = 0 \Leftrightarrow 1 - x^2 = 0 \Leftrightarrow x = \pm 1$

x	$(-\sqrt{2}, -1)$	$(-1, 1)$	$(1, \sqrt{2})$
f'	$-$	$+$	$-$
f	strictly decrease	s. inc.	s. dec

$-1 \to$ pt of local / global minimum, -1 is loc, glob min

$1 \to$ " " , 1 "



f is st. dec on $[(-\sqrt{2}, -1)]$ and $[(1, \sqrt{2})]$

" st. inc. on $[(-1, 1)]$

$-\sqrt{2}$ is pt of local max, 0 loc. max

$\sqrt{2}$ loc min, 0 loc. min

Convex, Concave

$f(x) = \ln\sqrt{x^2+1}$ find intervals of convexity

$= \frac{1}{2}\ln(x^2+1)$ and concavity

$D_f = \mathbb{R}$

$f'(x) = \frac{1}{2}\cdot\frac{1}{x^2+1}\cdot 2x$

$= \frac{x}{x^2+1}$

$D_{f'} = \mathbb{R}$

$f''(x) = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$ >0

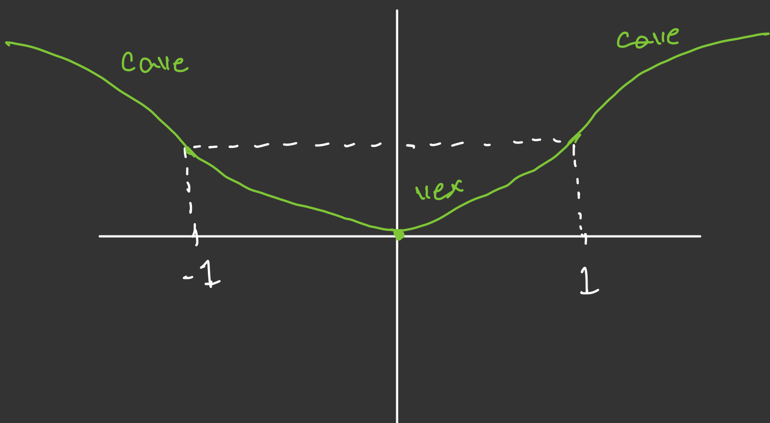
$D_{f''} = \mathbb{R}$

$f''(x) = 0 \Leftrightarrow 1-x^2 = 0 \Leftrightarrow x = \pm 1$

$>0 \Leftrightarrow x \in (-1,1)$

x	$(-\infty, -1)$	$(-1, 1)$	$(1, +\infty)$
f''	$-$	$+$	$-$
f	concave	convex	concave

$-1, 1$ are inflection points



$f(x) = \ln(e^{-x} + e^{x} - 2)$ find asymptotes.

$D_f = \mathbb{R} \setminus \{0\}$

$$e^{x} + e^{-x} - 2 \geq 0$$

$$e^{2x} - 2e^{x} + 1 \geq 0$$

$$(e^{x} - 1)^2 \geq 0$$

$$e^{x} \geq 1$$

$\pm\infty$: $y = kx + q$

$$k = \lim_{x \to +\infty} \frac{f(x)}{x}$$

$$= \lim_{x \to +\infty} \frac{\ln(e^{-x} + e^{x} - 2)}{x} \qquad x \overset{?}{\geq} 0$$

$$\overset{L'H}{=} \lim_{x \to +\infty} \frac{-e^{-x} + e^{x}}{1(e^{-x} + e^{x} - 2)}$$

$|e^{x} - 1| > 0$

$e^{x} - 1 > 0 \qquad 1 - e^{x} > 0$

$e^{x} > 1 \qquad e^{x} < 1$

$x > 0 \qquad x < 0$

$$= \lim \frac{\cancel{e^{x}}(1 - \underbrace{e^{-2x}}_{\to 0})}{\cancel{e^{x}}(1 + \underbrace{e^{-2x}}_{\to 0} - \underbrace{2e^{-x}}_{\to 0})}$$

$$= 1$$

$$q = \lim_{x \to +\infty} \quad f(x) - k \cdot x = \ln(e^{-x} + e^{x} - 2) - x$$

$$= \lim_{+\infty} \underbrace{\ln e^{x}}_{x} + \ln(\underbrace{e^{-2x}}_{0} + 1 - \underbrace{2e^{-x}}_{0}) - x$$

$$= \ln(1) = 0$$

$\boxed{y = x}$ $+\infty$

$x=0$: $\lim\limits_{x \to 0} f(x) = \pm\infty$

$$\lim_{x \to 0} (\overbrace{\underset{\downarrow \atop 1}{e^{-x}} + \underset{\downarrow \atop 1}{e^{x}} - 2}^{0}) = -\infty$$

$$\lim_{x \to 0} \ln(x) = -\infty$$

$x = 0$ is asymptote at $x=0$

asymptote at $-\infty$: $k = \lim\limits_{x \to -\infty} \dfrac{\ln(e^{-x} + e^{x} - 2)}{x}$

$$= \lim_{x \to -\infty} \frac{-e^{-x} + e^{x}}{e^{-x} + e^{x} - 2}$$

$$= \lim_{x \to -\infty} \frac{\cancel{e^{-x}}(-1 + \underbrace{e^{2x}}_{\to 0})}{\cancel{e^{-x}}(1 + \underbrace{e^{2x}}_{\to 0} - \underbrace{2e^{x}}_{\to 0})}$$

$= -1$

$q = 0$