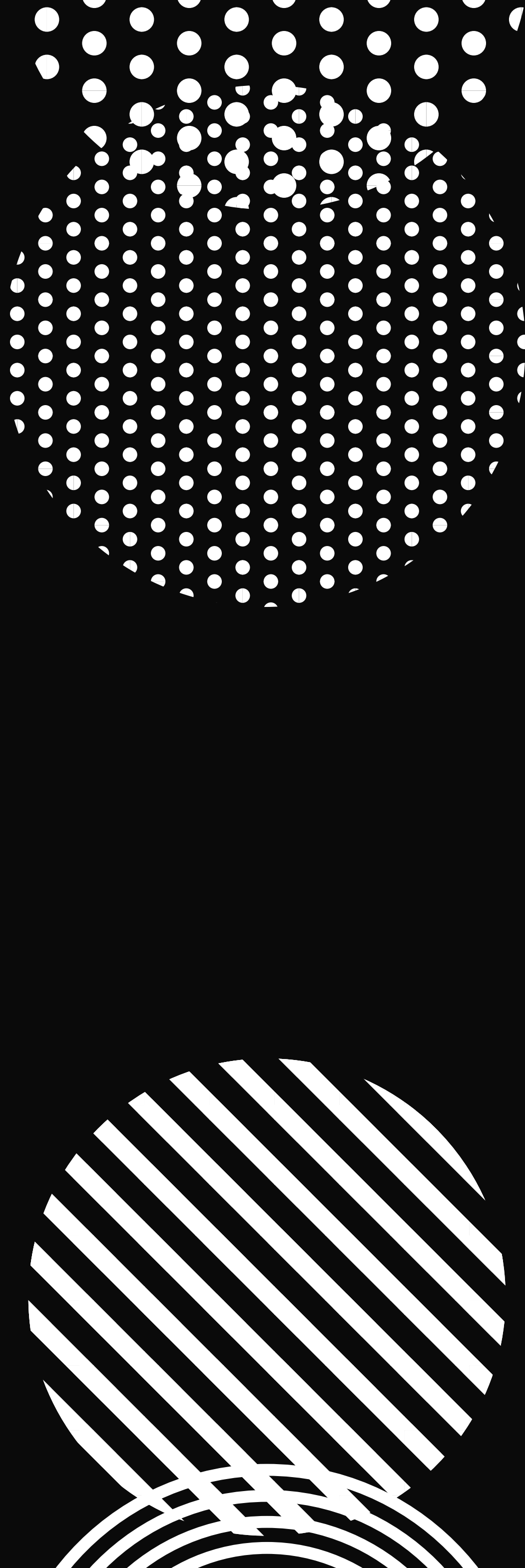


$$\lim_{x \to \infty} \cos x$$

$$x_n := 2n\pi \longrightarrow +\infty$$

$$\lim_{n \to \infty} \cos x_n = 1$$

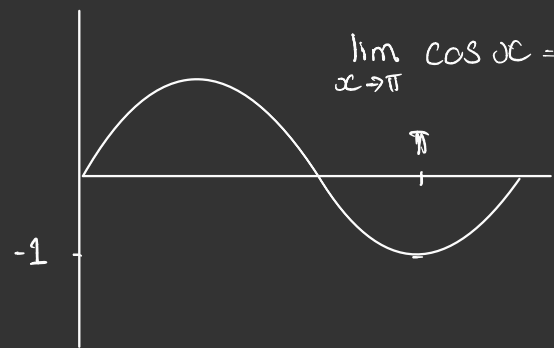
$$y_n : (2n+1)\pi \longrightarrow +\infty$$

$$\lim_{n \to \infty} \cos y_n = -1$$

Heines Thm

$\Rightarrow \lim_{x \to +\infty} \cos x$ does not exist.

$$\lim_{x \to \pi} \cos x = -1$$



$$\lim_{x \to 0} \cos \frac{1}{x}$$

$$f(x) = \cos \frac{1}{x}$$

$$x_n \to 0$$

$$x_n := 2n\pi$$

$$\frac{1}{x_n} = 2n\pi \Rightarrow x_n = \frac{1}{2n\pi} \longrightarrow 0$$

$$\lim_{n \to \infty} \cos \frac{1}{x_n} = 1$$

$y_n \to 0$

$$\frac{1}{y_n} = (2n+1)\pi$$

$$y_n = \frac{1}{(2n+1)\cdot\pi} \longrightarrow 0.$$

$$\cos\frac{1}{y_n} = \cos(2n+1)\pi = -1$$

$$\lim_{n\to\infty} \cos\frac{1}{y_n} = -1$$

By Heines Thm, the $\lim\limits_{x\to 0} \cos\frac{1}{x}$ does not exist.

$$\lim_{n\to\infty} \frac{\sqrt[3]{n} - 3^n}{\cos(n^8) + 2^{-n} + n!} = \lim \frac{3^n\left(\frac{\sqrt[3]{n}}{3^n} - 1\right)}{n!\left(\frac{\cos n^8}{n!} + \frac{1}{2^n\cdot n!} + 1\right)}$$

$\frac{3^n}{n!} \to 0$, $\frac{\sqrt[3]{n}}{3^n} \to 0$, $\frac{1}{2^n\cdot n!} \to 0$

$$\frac{-1}{n!} \leqslant \frac{\cos n^8}{n!} \leqslant \frac{1}{n!} \to 0$$

$0 \leftarrow \frac{-1}{n!}$, $\frac{\cos n^8}{n!} \to$

By squeeze Thm, 0

$$0\cdot\frac{0-1}{0+0+1} = 0$$

$$\lim_{n\to\infty} \frac{n^3 e^n + 3}{2^n - (n+1)^3 e^n} = \lim_{n\to\infty} \frac{\cancel{n^3}\,\cancel{e^n}\left(1 + \frac{3}{n^3 e^n}\right)}{(n+1)^3 \cancel{e^n}\left(\frac{2^n}{(n+1)^3 e^n} - 1\right)}$$

$$(n+1)^3 \to \cancel{n^3}\left(1+\frac{1}{n}\right)^3$$

$$= \lim_{n\to\infty} \frac{1 + \frac{3}{n^3 e^n}}{\left(1+\frac{1}{n}\right)^3\left(\frac{2^n}{(n+1)^3 e^n} - 1\right)}$$

$$\frac{3}{n^3 e^n} \to 0 \qquad \left(1+\frac{1}{n}\right)^3 \to 1 \qquad \frac{1}{n} \to 0$$

$$\lim_{n\to\infty} \left(\frac{2}{e}\right)^n \cdot \frac{1}{(n+1)^3} = 0$$

$$\therefore \frac{1}{-1} = -1$$

$$\lim_{n\to\infty} \frac{3^n}{(1+3)(1+3^2)(1+3^3)\dots(1+3^n)}$$

$$= \lim_{n\to\infty} \frac{\cancel{3^n}}{(1+3)(1+3^2)\dots(\cancel{3^n}(\frac{1}{3^n}+1))}$$

$$a > b$$

$$\frac{1}{a} < \frac{1}{b}$$

$$0 \leq \frac{1}{(1+3)(1+3^2)\dots(\frac{1}{3^n}+1)} \leq \frac{1}{3 \cdot 3^{n-1}} = \frac{1}{3^n}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$0 \qquad\qquad 0 \qquad\qquad 0$$

$$2 \cdot \frac{\sin\left(\frac{2}{x}\right)}{\frac{1}{x}}$$

$$x = \frac{1}{\frac{1}{x}}$$

$$x \cdot \sin\left(\frac{2}{x}\right)$$

$$\sin\left(\frac{2}{x}\right) \cdot \frac{1}{\frac{1}{x}}$$

$$\lim_{x \to +\infty} x \cdot \sin\left(\frac{2}{x}\right)$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$= \lim_{x \to +\infty} \frac{\sin \frac{2}{x}}{\frac{1}{x}} = \lim_{x \to +\infty} 2 \cdot \frac{\sin\left(\frac{2}{x}\right)}{\left(\frac{2}{x}\right)} = 2 \cdot 1 = 2$$

Lim of $f \circ g$; $f(x) = \frac{\sin x}{x}$, $g(x) = \frac{2}{x}$, $D_{f \circ g} = \mathbb{R} \setminus \{0\}$

1. $\lim_{+\infty} \frac{2}{x} = 0$

2. $\lim_{x \to 0} \frac{\sin x}{x} = 1$

3. $\forall x \in D_g : \frac{2}{x} \neq 0$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$$

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \to \pm\infty} \left(1+\frac{1}{x}\right)^x = e \overset{\text{Heine Thm}}{\Rightarrow} \lim_{n \to \infty} \left(1+\frac{1}{n}\right)^n = e$$

$$\lim_{x \to 0} \frac{5\ln(x+1)}{\sin 3x} = \lim_{x \to 0} \frac{5 \cdot \ln(x+1)}{\sin 3x} \cdot \frac{3x}{3x}$$

$$= \lim_{x \to 0} 5 \cdot \underbrace{\frac{\ln(x+1)}{x}}_{1} \cdot \frac{3x}{\sin 3x} \cdot \frac{1}{3}$$

$$\frac{3x}{\sin 3x} = \frac{1}{\frac{\sin 3x}{3x}}$$

$$f \circ g: \quad f(x) = \frac{\sin x}{x}, \quad g(x) = 3x$$

1. $\lim_{x \to 0} 3x = 0$

2. $\lim_{x \to 0} \frac{\sin x}{x} = 1$

3. $\forall x \in D_g \setminus \{0\}: \ 3x \neq 0$

$\therefore$ ans is $5 \cdot 1 \cdot \frac{1}{3} = \frac{5}{3}$

$$\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^{n}$$

$$= \lim_{n\to\infty}\left[\left(1+\frac{1}{2n}\right)^{2n}\right]^{\frac{1}{2}} = e^{\frac{1}{2}}$$

limit of subsequence

$$\lim_{n\to\infty} a_n = \alpha$$

every subseq of a_n has the same limit, α.

$$a_n = \left(1+\frac{1}{n}\right)^{n}$$

is a sub seq

$$a_{2n} = \left(1+\frac{1}{2n}\right)^{2n}$$

$$\lim_{n\to\infty} a_n = e$$

$$\lim_{n\to\infty} a_{2n} = e$$

go back up

$$\lim_{n\to\infty} \frac{\ln(n^2+4n) \to \infty}{\ln(n^4+5) \to \infty}$$

$$\lim_{x\to\infty} \ln\left(1+\frac{4}{n}\right) = \ln \lim_{n\to\infty}\left(1+\frac{4}{n}\right) = \ln 1 = 0$$

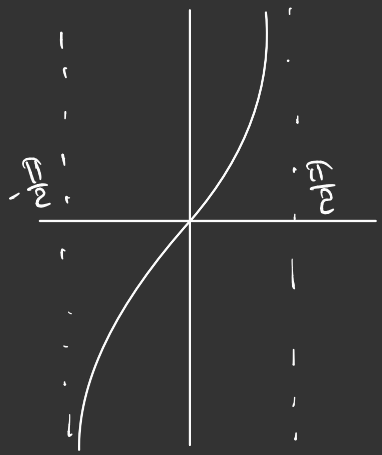
$$\frac{\ln n^2\left(1+\frac{4}{n}\right)}{\ln n^4\left(1+\frac{5}{n^4}\right)} = \frac{\ln n^2 + \ln\left(1+\frac{4}{n}\right)}{\ln n^4 + \ln\left(1+\frac{5}{n^4}\right)}$$

$$= \frac{2\cancel{\ln n}}{4\cancel{\ln n}} \frac{\left(1+\frac{\ln\left(1+\frac{4}{n}\right)}{2\ln n}\right)}{\left(1+\frac{\ln\left(1+\frac{5}{n^4}\right)}{4\ln n}\right)}$$

$$\lim \frac{2}{4} \frac{\left(1+\frac{\ln\left(1+\frac{4}{n}\right)}{\ln n}\right) \to 0}{\left(1+\frac{\ln\left(1+\frac{5}{n^4}\right)}{\ln n}\right) \to 0} = \frac{1}{2}$$

$$\lim_{x \to 0} \operatorname{arctg}\left(\frac{1}{x}\right)$$

$$\operatorname{arctg}(x) = tg(x)^{-1} \Big|_{\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}$$



$$\operatorname{arctg}(x): R \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\lim_{x \to \frac{\pi}{2}} tg(x) = +\infty$$

$$\lim_{x \to +\infty} \operatorname{arctg}(x) = \frac{\pi}{2}$$

$$f(x) = \operatorname{arctg}(x)$$

$$g(x) = \frac{1}{x}$$

1. $\lim_{x \to 0^+} \frac{1}{x} = +\infty$

2. $\lim_{x \to +\infty} \operatorname{arctg} x = \frac{\pi}{2}$

1. $\lim_{x \to 0^-} \frac{1}{x} = -\infty$

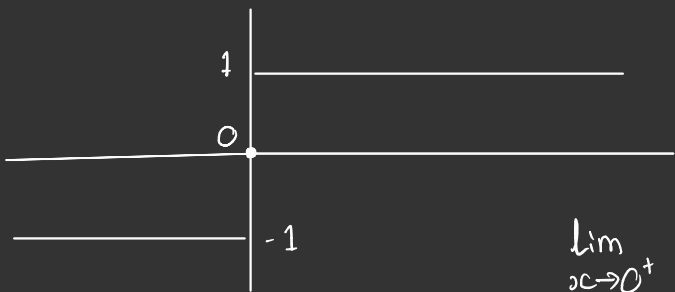
2. $\lim_{x \to -\infty} \operatorname{arctg} x = -\frac{\pi}{2}$

The limits of $x \to +\infty$ and $x \to -\infty$ are not equal, the limit does not exist.

If $\lim_{x \to x_0} f(x) = f(x_0)$ we say f is continuous at $x = x_0$.

$$f(x) = \operatorname{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

not continuous at $x = 0$.



$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 1 = 1$$

$$\neq$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} -1 = -1$$