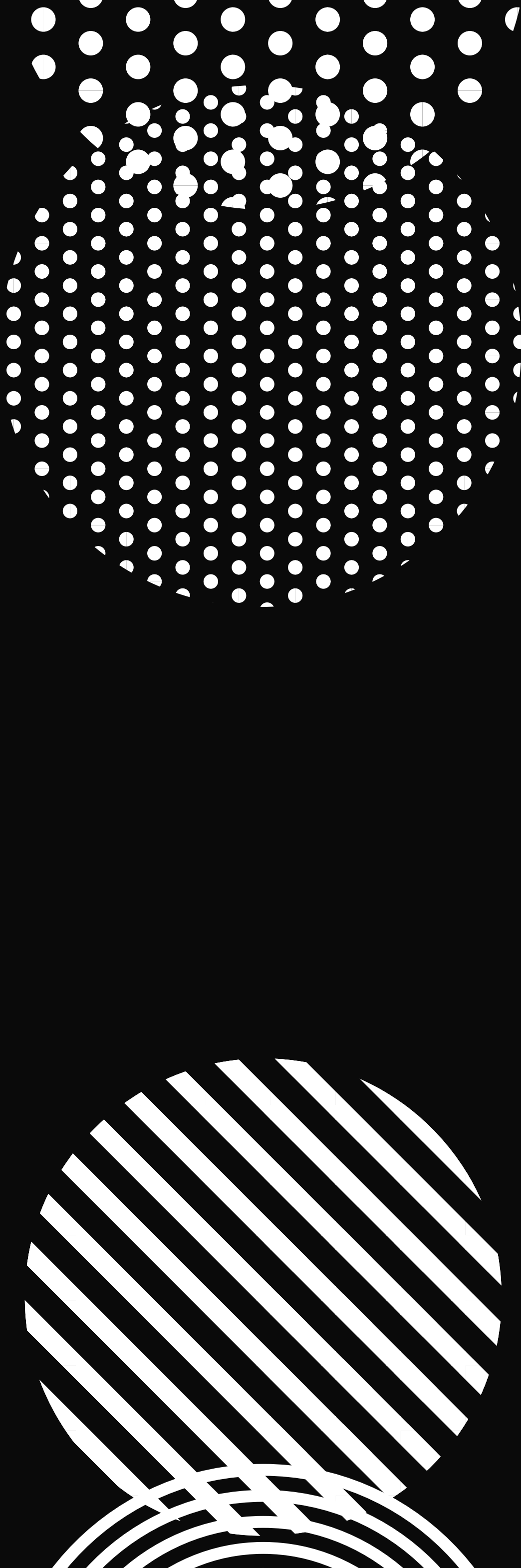
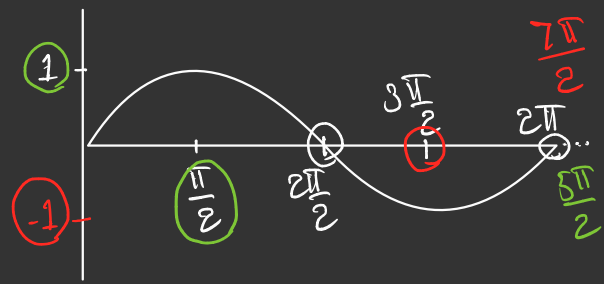


Limits

$$\lim_{n \to \infty} \sin\left(\frac{n \cdot \pi}{2}\right)$$

$$a_n = \sin\left(n \cdot \frac{\pi}{2}\right)$$

$1, 0, -1, 0, 1, 0, -1, \dots$



$n = 1, 5, 9, \dots, n = 4k+1$

$a_n = 1$

$n = 2, 4, 6, 8, \dots, n = 2k$

$a_n = 0$

$n = 3, 7, 11, \dots, n = 4k-1$

$a_n = -1$

$(k \in \mathbb{N})$

$$a_{n_k} = \sin\left((4k+1) \cdot \frac{\pi}{2}\right) = \sin\left(2k\pi + \frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1$$

$$\lim_{k \to \infty} a_{n_k} = 1$$

$$a_{n_k} = \sin\left(2k \cdot \frac{\pi}{2}\right) = \sin(k\pi) = 0, \quad \lim_{k\to\infty} a_{l_k} = 0$$

a_{n_k}, a_{l_k} are subseq^{n}: of a_n.

$\lim a_{n_k} \neq \lim a_{l_k} \Rightarrow \lim a_n$ does not exist

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$\frac{1}{n} \to 0$

$2^{\frac{1}{n}} \to \infty$

$2^0 \to 1$

$$\lim_{n\to\infty} \sqrt[n]{2^n + n^n}$$

$$\sqrt[n]{n^n} \leqslant \sqrt[n]{2^n + n^n} \leqslant \sqrt[n]{2n^n}$$

$$n \leqslant \sqrt[n]{2^n + n^2} \leqslant \sqrt[n]{2} \cdot n$$

$n \to +\infty$, $\sqrt[n]{2} \to 1$, $n \to +\infty$

$\sqrt[n]{2^n + n^2} \to$ By squeeze thm., $+\infty$

$$\lim_{n\to\infty} \frac{(-1)^{n^2+n}\cdot n}{2+(-1)^n}$$

$$(-1)^{n^2+n} \cdot n = n$$

$$\frac{n}{3} \leq \frac{(-1)^{n^2+n}\cdot n}{2+(-1)^n} \leq n$$

$\frac{n}{3} \downarrow +\infty$

$2+(-1)^n$: 3, 1

$n \downarrow +\infty$

squeeze thm

$+\infty$

$$\lim_{n\to\infty} \sqrt[n]{7n^3+n^2+5} = 1$$

$(-1)^n + 2$: 3, 1

$$\lim_{n\to\infty} \frac{(-1)^n+2}{(n^2+1)} = 0$$

$(n^2+1) \searrow \infty$

$$\lim_{n\to\infty} \sqrt[n]{n} = 1$$

$$\sqrt[n]{n^3} \leq (7n^3 + n^2 + 5)^{\frac{1}{n}} \leq (13n^3)^{\frac{1}{n}}$$

$n^2 \leq n^3$, $5 \leq 5n^3$

$$\sqrt[n]{n^3} = (\sqrt[n]{n})^3 \to 1$$

$\downarrow$ 1

$$(13n^3)^{\frac{1}{n}} = \sqrt[n]{13}\cdot(\sqrt[n]{n})^3$$

$\sqrt[n]{13} \downarrow 1$, $\sqrt[n]{n} \downarrow 1$

$$\frac{1}{n^2+1} \leq \frac{(-1)^n+2}{n^2+1} \leq \frac{3}{n^2+1}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$0 \qquad\qquad 0 \qquad\qquad 0$$

$$\lim_{x\to+\infty} \frac{x^2+2x-3}{x^3-3x+2} = \lim_{x\to+\infty} \frac{x^2+2x-3}{x^3-3x+2} = \frac{\cancel{x^2}\left(1+\overset{0}{\frac{2}{x}}-\overset{0}{\frac{3}{x^2}}\right)}{x^{\cancel{3}}\left(1-\frac{3}{x^2}+\frac{2}{x^3}\right)}$$

$$= \lim_{x\to+\infty} \frac{1}{x} = 0$$

$$\lim_{x\to 1} \frac{x^2+2x-3}{x^3-3x+2} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+3)}{\cancel{(x-1)}(x^2+x-2)}$$

$$= \lim_{x\to 1} \frac{(x+3)}{(x^2+x-2)}$$

$$= \lim_{x\to 1} \frac{x+3}{(x-1)(x+2)}$$

$$= \lim_{x\to 1} \frac{1}{x-1}, \quad \lim_{x\to 1^+} \frac{1}{x-1} = +\infty$$

$$\neq$$

$$\lim_{x\to 1^-} \frac{1}{x-1} = -\infty$$

$\Rightarrow \lim \frac{1}{x-1} \cdot \frac{x+3}{x+2}$ does not exist.

$$\lim_{x\to+\infty} \left(x - \sqrt{x^2+3x}\right) = \lim_{x\to+\infty} \frac{\left(x-\sqrt{x^2+3x}\right)\times\left(x+\sqrt{x^2+3x}\right)}{\left(x+\sqrt{x^2+3x}\right)}$$

$-\infty - \infty$

if $x \to -\infty$

$x - \sqrt{x^2\left(1+\frac{3}{x}\right)}$

$$= \lim_{x\to-\infty} \underset{\downarrow\,-\infty}{x} - \underset{\downarrow\,\infty}{|x|}\underset{\downarrow\,1}{\sqrt{1+\frac{3}{x}}} \quad \frac{3}{x}\to 0$$

$= -\infty$

$$= \lim_{x\to+\infty} \frac{\cancel{x^2} - \cancel{x^2} - 3x}{\underset{\downarrow\,\infty}{x} + \underset{\downarrow\,\infty}{\sqrt{x^2+3x}}} \quad 3x \to \infty$$

$$= \lim_{x\to+\infty} \frac{-3\cancel{x}}{\cancel{x}\left(1+\underset{\downarrow\,1}{\sqrt{1+\frac{3}{x}}}\right)} = -\frac{3}{2}$$

$$\lim_{x\to+\infty}\frac{x+\sin x}{x+\operatorname{arctg} x}=\lim_{x\to+\infty}\frac{\cancel{x}\left(1+\frac{\sin x}{x}\right)}{\cancel{x}\left(1+\frac{\operatorname{arctg} x}{x}\right)}$$

$$=\lim_{x\to+\infty}\frac{1+\frac{\sin x}{x}}{1+\frac{\operatorname{arctg} x}{x}}=1$$

$$\frac{\sin x}{x}\to 0 \qquad \frac{\operatorname{arctg} x}{x}\to 0$$

$$\lim_{+\infty}\frac{\sin x}{x}=0 \qquad -\frac{1}{x}\leqslant\frac{\sin x}{x}\leqslant\frac{1}{x}$$

$$\downarrow\qquad\downarrow\qquad\downarrow$$

$$0\qquad 0\qquad 0$$

$$\lim_{+\infty}\frac{\operatorname{arctg} x}{x}=0$$

$$-\frac{\pi}{2x}\leqslant\frac{\operatorname{arctg} x}{x}\leqslant\frac{\pi}{2}\cdot\frac{1}{x}$$

$$\downarrow\qquad\downarrow\qquad\downarrow$$

$$0\qquad 0\qquad 0$$

$$\lim_{x \to 1} \frac{(1-x)^2}{\sqrt{x^2-2x+1}} = \lim_{x \to 1} \frac{(1-x)^2}{(\sqrt{(x-1)^2})}$$

$$= \lim_{x \to 1} \frac{(1-x)(1+x)}{|(x-1)|}$$

$$\lim_{x \to +\infty} \frac{\cos x}{x} = 0$$

$$-\frac{1}{x} \leq \frac{\cos x}{x} \leq \frac{1}{x}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$0 \quad 0 \quad 0$$

$$\lim_{x \to 1^+} \frac{(x-1)^2}{x-1} = 0$$

$$\lim_{x \to 1^-} \frac{(x-1)^2}{1-x} = 0$$