2022/02/11	BIE-DML : EXAM TEST					4A
Family Name, Surname	P 1	P 2	P 3	P 4	P 5	$\sum$

In this part of the exam you can get **at most 36 points**. You have **90 minutes** for this test, no devices (except non-cell phone calculators) are allowed. Do not forget to describe any logical step to explain your thoughts when solving problems. Not only the correct result is evaluated, but especially the correctness and **clarity of the presentation** of the procedure!
A confusing correct solution will be evaluated with minor number of points.

The test consists of **two parts**: theoretical (questions 1, 2) and calculations (questions 3, 4, 5). To succeed in this part of the exam it is necessary to get **at least 5 points from the theoretical part** and **at least 13 points from calculations**.

Question 1. (4 points) Define when two **sets** A **and** B **have the same cardinality**. Define also the notions you used in the definition. Give an example of **infinite** sets A and B which have the same cardinality but are not equal. Explain it using the definition above.

Question 2. (5 points) Define the least common multiple of two numbers $a, b \in \mathbb{Z}$; include all cases. Write the formula which relates gcd and lcm.
Determine the value of $lcm(91, 39)$ using the formula above.

Question 3. (8 points) Consider the word ABRAKADABRA.

a) (2 point) How many different words will be obtained by permuting the letters?

b) (4 point) In how many such words there will be no two A's next to each other?

c) (2 point) What is the probability that a word from **a)** (obtained by permuting the letters) has has no two A's next to each other?

Question 4. (9 points) Prove by mathematical induction the following statement for every natural number n: Expression $2^{4n} - 3^{3n}$ is divisible by 11.

a) Specify on which variable is your proof performed.

b) Formulate the inductive step carefully (including the inductive assumption).

c) Check carefully the initial conditions and the range in assumptions in the inductive steps.

Question 5. (10 points) Determine all solutions for x in $\mathbb{Z}$ of the following system of congruences:

$$(70)^{67} \cdot x \equiv 50 \pmod{75}$$
$$x \equiv 1 \pmod{73}$$
