A study of Recombination Operators for the Cyclic Bandwidth Problem

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Background of the problem

Definition

Cyclic Bandwidth problem (CBP) is a labeling problem, first introduced by Leung [5] in 1984 for the design of a ring interconnection network. It is aimed to find an arrangement on a cycle for a couple of computers to ensure every message to be sent to its destination in less than $k + 1$ steps.

Figure: A simple illustration of the CBP.

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Problem statement

Definition

Let $G(V, E)$ be a finite undirected graph of order n and C_n a cycle graph with vertex set $|V'| = n$ and edge set E'. Given a one-to-one mapping φ (an injection or a labeling or an embedding) from V of G to V' of C_n, the cyclic bandwidth (the cost) for G with respect to φ is defined as:

$$B_C(G, \varphi) = \max_{uv \in E} \{|\varphi(u) - \varphi(v)|_n\},$$

where $|x|_n = min\{|x|, n - |x|\}$ $(1 < |x| < n - 1)$ is called the *cyclic distance*, and $\varphi(u)$ denotes the the label associated to vertex u. The objective is to find an embedding φ^*, such $B_C(G, \varphi^*)$ is minimum, i.e.,

$$\varphi^* = \arg\min_{\varphi \in \xi} \{B_C(G, \varphi)\}$$

where ξ is the set of all possible embeddings. The embedding φ^* is named by optimal solution. The decision problem of CBP is proven to be $\mathcal{NP}$-complete [6]. This motivates us to use heuristic algorithm to solve it.

Problem Statement

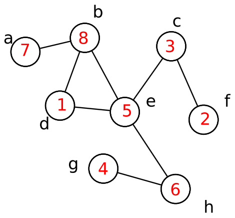

Figure: Guest graph G

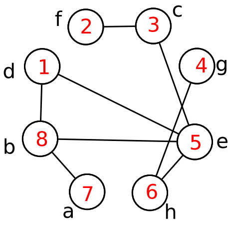

Figure: Host graph H

V	a	b	c	d	e	f	g	h
φ	7	8	3	1	5	2	4	6

φ	1	2	3	4	5	6	7	8
V	d	f	c	g	e	h	a	b

E	(a,b)	(b,d)	(b,e)	(c,e)	(c,f)	(d,e)	(e,h)	(g,h)
Abs dis: $\lvert x \rvert$	$\lvert \varphi(a) - \varphi(b) \rvert = 1$	7	3	2	1	4	1	2
Cyclic dis: $\lvert x \rvert_n$	$min(\lvert x \rvert, n - \lvert x \rvert) = 1$	1	3	2	1	4	1	2

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State of the art

Relevant related work

- Theoretical study: Much work are done to determine the exact cyclic bandwidth for some special graphs or find the bounds of general graphs.
- Practical work:
  - 2012, Romero-Monsivais et al.: A new branch and bound for CBP [10].
  - 2015, Rodriguez-Tello et al.: Tabu search for CBP [9].
  - 2019, Ren et al: An Iterated Three-Phase Search Approach for Solving the CBP [8].
- Memetic Algorithm is one of the most famous and effective meta-heuristic frameworks, which has been largely applied and achieved good performances in $\mathcal{NP}$-hard problems.
- For permutation problems, memetic algorithms have also achieved good performances for the Traveling Salesman Problem (TSP), the Antibandwidth Problem and the Cyclic Antibanwidth Problem.
- There is no memetic or genetic algorithm published to solve CBP.

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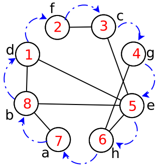
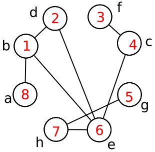
Search space

The main feature of the embedding

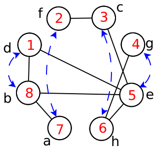
- The symmetry characteristics for the solution: There exists $\frac{(n-1)!}{2}$ embeddings other than $n!$ in the search space ξ.



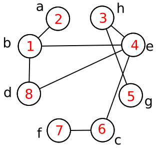
(a) Original solution to make rotation



(b) Rotation in clockwise



(c) Original solution to turn around



(d) Turn around

Representation and fitness function

The main feature of the embedding

- Two representations of embedding are used:
  - φ, the *i*-th value of $\varphi[i]$ means the vertex whose label is *i*; This representation is used in the local search procedure. ($\varphi = (7, 8, 3, 1, 5, 2, 4, 6)$ in the example below)
  - γ, the *i*-th value of $\gamma[i]$ indicates the label which assigned to *i*-th vertex. This is adopted in recombination operators as well as the distance calculation. ($\gamma = (d, f, c, g, e, h, a, b)$ in the the example below)
- Each solution is evaluated using the objective function of the CBP.

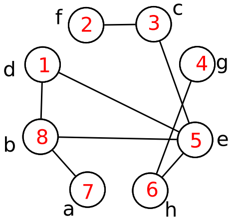


Figure: Illustration of example

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General procedure for the proposed memetic algorithm (MA)

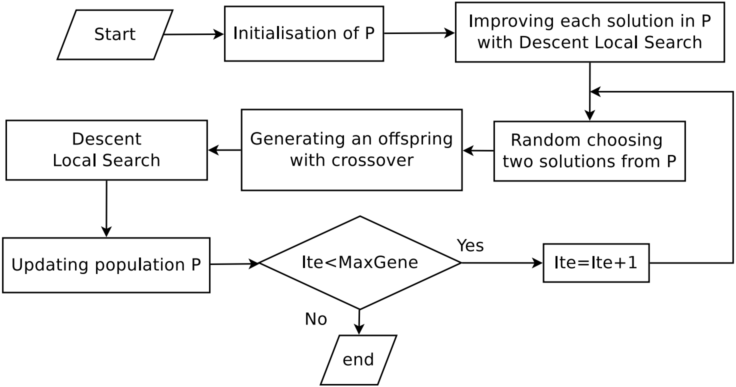

Figure: A flowchart of the proposed MA.

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Local search procedure

Descent local search

- To highlight the function of recombination operators, a simple descent local search structure is adopted.
- Only the embedding with a better objective value $B_C(G, \varphi') < B_C(G, \varphi)$ is taken and the local search stops when there is no improving solutions in the neighborhoods.
- Some preliminary definitions:
  - $B_C(u, \varphi) = \max_{v \in A(u)}\{|\varphi(u) - \varphi(v)|_n\}$ where $A(u)$ denotes the set of vertices adjacent to u of cardinality $deg(u)$
  - The critical vertex set $C(\varphi)$ is defined:$C(\varphi) = \{u \in V : B_C(u, \varphi) = B_C(G, \varphi)\}$
- The neighborhood is defined as follows: $N(\varphi) = \{\varphi' = \varphi \oplus swap(u, v) : u \in C(\varphi), v \in V\}$, where $swap(u, v)$ denotes swapping the labels of vertex u and v.

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Recombination procedure

Recombination procedure

- It is aimed to generate new diversified and potentially improving solutions. Only one offspring is generated for each generation.
- The population is updated using a simple strategy: inserting the improved solution φ after the local search and removing the solution having the “worst” objective function value.
- 5 recombination operators are used: Order Crossover (OX), Order-based Crossover (OX2), Cycle Crossover (CX), Partially Mapped Crossover (PMX) and Distance Preserved Crossover (DPX).

Recombination procedure

Distance Preserved Crossover

- Distance Preserved Crossover (DPX) [3]:
  Given two parents embeddings:
  φ_F=(1 3 2 4 5 6 7 8), φ_M=(2 4 6 8 7 5 3 1).
  Identifying all the patterns:
  P(φ_F)={{1,3},{3,2},{2,4},{4,5},{5,6},{6,7},{7,8},{8,1}}
  P(φ_M)={{2,4},{4,6},{6,8},{8,7},{7,5},{5,3},{3,1},{1,2}}
  Finding the repeated patterns between solutions (noting that {7,8} and {8,7} are the same pattern):
  R(φ_F, φ_M)={{1,3}, {2,4} {7,8}}
  Getting the distance between φ_F and φ_M:
  D(φ_F, φ_M)=$|V|$-$|R(\varphi_F, \varphi_M)|$=8-3=5
  Reconstructing the offspring conserving the common patterns and avoiding all the rest patterns in φ_F and φ_M:
  φ_C=(1 3 6 2 4 7 8 5)
  P(φ_C)={{1,3},{3,6},{6,2}{2,4},{4,7},{7,8},{8,5},{5,1}}
  D(φ_F, φ_C)=5
  D(φ_M, φ_C)=5

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Instances and settings

Instances and settings

- Experimental results of the previous MA using 5 different recombination operators are presented in the following section.
- Twenty representative benchmark graphs are selected from a test-suite of 113 instances (`https://www.tamps.cinvestav.mx/~ertello/cbmp.php`) from 59 to 2048 vertices.
- Considering the stochastic nature of the algorithm, each instance is independently solved 50 times under the environment of Linux using Intel Xeon E5-2695 2.1 GHz CPU and 2GB RAM.
- The stopping condition of each execution is set to 20000 generations ($MaxGene = 20000$) and the population size $|P|$ is set to 20.

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Computational results

	CX		DPX		OX		OX2		PMX	
Graph	Best	Avg	Best	Avg	Best	Avg	Best	Avg	Best	Avg
nos6	327	331.28	327	329.74	266	287.98	**216**	**227.84**	327	331.98
path1000	461	475.42	462	474.02	254	301.04	**226**	**247.54**	468	482.68
nos4	44	46.12	43	45.24	32	39.32	**28**	**34.48**	42	45.78
tree10x2	39	42.72	35	40.72	**28**	32.50	**28**	**29.26**	36	41.56
cycle1000	457	476.66	466	473.38	252	296.98	**226**	**246.94**	459	480.86
mesh2D8x25	88	93.04	89	91.82	59	75.18	**57**	**62.94**	87	93.28
caterpillar29	203	211.48	203	208.70	138	162.98	**100**	**127.32**	198	210.14
mesh3D6	102	103.88	101	102.96	86	93.08	**73**	**78.26**	102	104.28
hypercube11	1022	1022.76	1022	1022.14	1019	1021.26	**952**	**1010.48**	1022	1022.54
cycle475	200	215.16	206	213.36	105	128.36	**99**	**110.76**	192	217.30
mesh2D28x30	409	413.40	410	412.06	336	371.76	**270**	**287.46**	406	414.06
mesh3D11	660	662.04	660	661.28	625	650.30	**507**	**522.82**	660	662.40
can__715	354	355.80	355	355.14	347	353.92	**293**	**316.70**	354	355.74
impcol_b	28	28.46	27	27.96	25	27.22	**20**	**26.72**	28	28.00
path475	202	214.50	206	212.86	112	132.24	**102**	**112.94**	189	217.56
494_bus	220	230.76	222	228.72	135	165.74	**128**	**138.62**	216	233.38
tree21x2	199	212.08	203	208.96	139	171.34	**124**	**140.84**	200	210.68
caterpillar44	481	493.28	479	491.24	340	400.78	**281**	**321.70**	480	495.60
impcol_d	207	209.60	207	208.80	190	202.98	**159**	**169.74**	208	209.80
tree2x9	475	489.08	478	485.86	296	330.14	**257**	**276.60**	472	491.84
Average	308.90	316.38	310.50	315.75	239.20	262.26	**207.30**	**224.50**	307.30	317.47
p-value	6.71e-14									

Table: Experimental results of MA using 5 different recombination operators.

Computational results

Graph	MA_{OX2} Best	MA_{OX2} Avg	TS_{CB} Best	TS_{CB} Avg	CC
nos6	216	227.84	22	23.50	-194
path1000	226	247.54	8	8.90	-218
nos4	28	34.48	10	10.00	-18
tree10x2	28	29.26	28	28.00	0
cycle1000	226	246.94	8	8.50	-218
mesh2D8x25	57	62.94	8	8.20	-49
caterpillar29	100	127.32	24	25.80	-76
mesh3D6	73	78.26	31	31.00	-42
hypercube11	952	1010.48	570	582.20	-382
cycle475	99	110.76	5	5.80	-94
mesh2D28x30	270	287.46	30	174.00	-240
mesh3D11	507	522.82	336	336.80	-171
can__715	293	316.70	60	65.80	-233
impcol_b	20	26.72	17	17.00	-3
path475	102	112.94	5	5.60	-97
494_bus	128	138.62	46	56.10	-82
tree21x2	124	140.84	116	116.00	-8
caterpillar44	281	321.70	39	54.00	-242
impcol_d	159	169.74	38	43.10	-121
tree2x9	257	276.60	63	64.20	-194
Average	207.30	224.50	**73.20**	**83.23**	
p-value	1.31e-4				

Table: Comparison between MA_{OX2} and TS_{CB} [9].

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Distance and population entropy

Distance of population

Average solution distance $D_{avg}(P)$ of the population.

$$D_{avg}(P) = \frac{2}{|P|(|P|-1)} \sum_{i=1}^{|P|} \sum_{j=i+1}^{|P|} d_{ij} \tag{1}$$

where d_{ij} is the distance between two solutions γ_i and γ_j of P, which is defined as the number of the adjacent connections that are contained in γ_i but not in γ_j. This is described in Section of crossover DPX.

Population entropy

Population entropy $E_p(P)$ is introduced in [2] to describe population diversity:

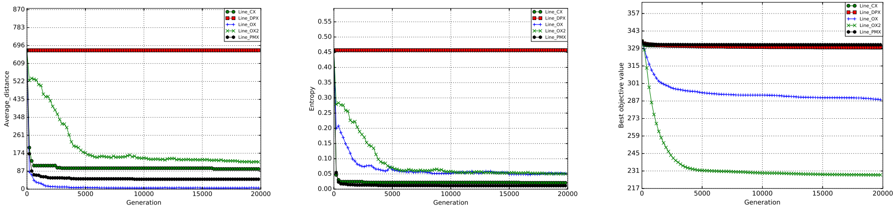
$$E_p(P) = \frac{-\sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{n_{ij}}{|P|}\right) \log \left(\frac{n_{ij}}{|P|}\right)}{n \log n} \tag{2}$$

where n_{ij} represents the number of times that variable i is set to value j in all solutions in P. One notices that $E_p(P)$ varies in the interval [0,1]. When $E_p(P)$ equals 0, all the solutions of P are identical. A large $E_p(P)$ value indicates a more diverse population.

Distance and population entropy

Distance and population entropy for the MAs with different crossovers

- Under 5000 generations, the population of the MA with OX2 has a high average distance and entropy, leading to much better solutions. From generations 5000 to 20000, the entropy is identical to that of OX, and the best average objective found stops decreasing.
- The recombination operator with a higher entropy and average distance of the population generally helps to find solutions of good quality.
- The DPX operator fails to reach good solutions even if the entropy and average distance of population under the MA with DPX always stay at a high level.



(a) Average distance of the population in 20000 generations

(b) Entropy of the population in 20000 generations

(c) Best objective value found in 20000 generations

Figure: Distance and population entropy applied to the instance nos6.

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Interaction between crossover and problem characteristics

Differences of characteristics between TSP and CBP

- For TSP, only the two linked edges of each vertex are taken into consideration. The DPX is designed to preserve the common connections between adjacent vertices.
- However for CBP, we need to consider for each vertex all the edges linked to the vertex in the graph (that is not necessarily adjacent vertices), such that the objective value relies on the largest cyclic bandwidth. The key point for CBP is the relative position for the pairs of nodes linked by an edge.

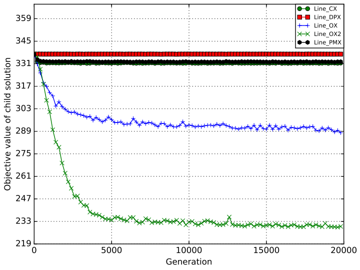

Figure: Average objective value of the child solution over 50 independent executions.

Summary

- We have investigated the memetic framework for solving the $\mathcal{NP}$-hard Cyclic Bandwidth problem.
- We have compared five permutation recombination operators (OX, OX2, CX, PMX and DPX) within a basic memetic algorthm which uses a simple descent procedure for local optimization. The experimental results indicate that OX2 achieves the best performance for the test instances. But it is not competitive compared to the state-of-the-art CBP algorithms.
- We have also explored the correlation between the population diversity and the performance of the studies MA variants. A higher entropy and average distance of the population generally helps to find solutions of good quality.
- We have also investigated the reason why DPX failed to find a good solution with a high entropy and average distance of population. Thus, there exists a need for developing meaningful recombination operators which are suitable for the CBP.
- In the future, we could integrate such a meaningful recombination operator and a powerful local optimization procedure into a MA to achieve state-of-the-art performances.

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Thanks & Questions!