

# Homework 2

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## 1 Heterogeneity in Returns to Schooling

Using the link, download the data used in Koop and Tobias's [1] study of the relationship between wages and education, ability, and family characteristics. Their data set is a panel of 2,178 individuals with a total of 17,919 observations. Extract the first observations for the first 15 individuals in the sample.

A table for the first 15 individuals in the sample is given in the Appendix. It is also possible to click on links to be taken directly to the corresponding graphic/table in the Appendix.

Let  $X_1$  be a vector of education, experience, and ability (the individual's own characteristics). Let  $X_2$  contain the mother's education, the father's education, and the number of siblings (the household characteristics). Let  $y$  be the log wage.

- a. **Compute the least squares regression coefficients in the regression of  $y$  on  $X_1$ . Report and interpret the coefficients.**

The full regression table is available [here](#). The coefficients for *educ*, *potexp* and *ability* were all significant, and are estimated to be  $\beta_{educ} = 0.0737621$ ,  $\beta_{potexp} = 0.0394896$ , and  $\beta_{ability} = 0.0828907$  respectively.

Given the log-log form of the model, the coefficients can be interpreted as relative increases in the dependent variable given a unit increase in the independent variable: A 1 unit increase in education (in years), potential experience (parameterized as age - education - 5), or cognitive ability (proxied by a standardized ASVAB test score), would be expected to increase wages by 7.37% , 3.95% and 8.29% respectively, keeping all else equal.

- b. **Compute the least squares regression coefficients in the regression of  $y$  on  $X_1$  and  $X_2$ . Report and interpret the coefficients.**

The full regression table is available [here](#). All coefficients except for  $\beta_{mothered}$  were found to be significant at the  $\alpha = 0.05$  level, at  $\beta_{educ} = .0722035$ ,  $\beta_{potexp} = .0395093$ ,  $\beta_{ability} = 0.0774678$ ,  $\beta_{fathered} = 0.0054569$  and  $\beta_{siblings} = 0.0047656$  respectively. The coefficient estimate  $\beta_{mothered} = -0.000117$  was not found to be statistically significant from 0.

As above, a 1 unit increase in education, potential experience, cognitive ability, father's education (in years), and number of siblings, were expected to increase wages by 7.22%, 3.95%, 7.75%, 0.55%, and 0.48% respectively. Thus a fraction of the explanatory power of *educ* and *ability* were given to *fathered* and *siblings*, but it is unclear if this small statistical significance would translate into practical significance for decision-making.

- c. **Compute the  $R^2$  for the regression of  $y$  on  $X_1$  and  $X_2$  manually using the SSE and SST from the output. Repeat the computation for the case in which the constant term is omitted. You need use the *noconstant* option, which suppresses the constant in a regression model. What happens to  $R^2$ ?**

The Stata output from the previous table gives  $SSE = 4126.17535$ ,  $SST = 4999.72601$ . The  $R^2$  formula is

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{4126.17535}{4999.72601} = 0.17471$$

The Stata output for the no-intercept model is given here, where  $SSE = 4316.76885$ . However, an incorrect value of  $SST = 99529.3551$  is given, as a result of SST being implicitly calculated in the Stata backend assuming an intercept exists. The mathematical definition of  $SST = \sum_{i=1}^n (y_i - \bar{y})^2$  means it is constant, and so we can use the previous SST.

$$R_0^2 = 1 - \frac{4316.76885}{4999.72601} = 0.13659$$

- d. **Compute the adjusted  $R^2$  for the full regression with and without the constant term. Interpret your results. Do we need the constant term? (Hint: Make sure to refer to the economic theory to discuss whether one should have the constant term regardless of statistical significance)**

With the constant term, there are  $n = 17919$  observations and  $p = 6$ , and the adjusted  $R^2$  is

$$\bar{R}^2 = 1 - (1 - R^2) \frac{n-1}{n-p-1} = 1 - (1 - 0.17471) \frac{17919}{17919 - 6 - 1} = 0.17438$$

And similarly in the no-intercept model, we have

$$\bar{R}_0^2 = 1 - (1 - 0.13659) \frac{17919}{17919 - 6 - 1} = 0.13625$$

In this particular case, the intercept term is statistically significant in the full regression, and helps to explain more of the variance in the dependent variable. More generally, the intercept term helps in reducing some of the restrictions in the model. By including the intercept term, the mean of the least squares residual is always zero. This is assumption MR2 of the multiple regression model, and without the intercept term, MR2 is not guaranteed.<sup>1</sup>

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<sup>1</sup>#econometrictheory

e. **Are any of the classical assumptions violated in part a or part b? Refer to the assumptions MR1, MR2, MR5, and MR6.**

- MR1 states that the model linearly depends on the values of the explanatory variables, and the unknown parameters. This is a matter of model specification, and a log-linear model, such as the one in this problem, conforms to this assumption.
- MR2 states that each random errors has a probability distribution with zero mean. If there is omitted variable bias, or otherwise some significant specification error, this assumption is violated. Specifically, omitted variable bias can be detected through the Ramsay RESET test, which gives  $p = 0.019 < \alpha = 0.05$  and  $p = 0.0056 < \alpha = 0.05$  respectively. Thus there is likely some misspecification and this assumption is violated.
- MR5 states that there is no exact collinearity or multicollinearity among the explanatory variables. We can also check the VIF tables, given in the appendix. The mean VIF for the two regressions are 1.3 and 1.54, indicating that there is not a problematic amount of multicollinearity.
- MR6 states that the residuals are normally distributed, in other words  $e_i \sim N(0, \sigma^2)$ . While a histogram of the residuals would suggests the assumption of normality to be more or less reasonable, we can calculate the Jarque-Bera statistic, taking the values of skewness and kurtosis from Stata's summarize.

$$JB = \frac{N}{6} \left( S^2 + \frac{(K-3)^2}{4} \right)$$

$$JB_1 = \frac{17919}{6} \left( (-0.4579588)^2 + \frac{(4.392981 - 3)^2}{4} \right) = 2075.09 > 5.99$$

$$JB_2 = \frac{17919}{6} \left( (-0.4593643)^2 + \frac{(4.394376 - 3)^2}{4} \right) = 2081.85 > 5.99$$

With 5.99 being the critical value for the corresponding  $\chi^2_2$  distribution, we can conclude at the 5% significance level that the residuals do not show follow a normal distribution. However, to what extent this statistical significance achieves a practical significance is to be determined.

## 2 The U.S. Gasoline Market

- a. **Compute the multiple regression of per capita consumption of gasoline on per capita income, the price of gasoline, all the other prices and a time trend. Report all results. Do the signs of the estimates agree with your expectations?**

The signs of the estimates make sense. A rise in (disposable) income, for example, is unsurprisingly correlated with increased demand and therefore consumption. An increase in prices means a reduction in consumption. A positive sign for the price index of new cars could make sense; newer cars are more fuel efficient, so an increase in their prices incentivizes people to use their used cars, which requires more gasoline consumption. An increase in the price of used cars means people switch to newer, more fuel efficient cars, and thus consume less gasoline. An increase in the price of public transport means people switch to private cars and use more gasoline. I had no expectations on the impact of durables and non-durables. The final coefficient for  $p_s$  makes sense: a more expensive services sector means it is more expensive to use gasoline for cars, so the negative sign makes sense.

- b. **Test the hypothesis that at least in regard to demand for gasoline, consumers do not differentiate between changes in the prices of new and used cars.**

Here, the null and alternative hypotheses are:

$$H_0 : \frac{\partial \text{demand}}{\partial P_{nc}} = \frac{\partial \text{demand}}{\partial P_{uc}} \iff \beta_{pnc} - \beta_{puc} = 0, \quad H_A : \frac{\partial \text{demand}}{\partial P_{nc}} \neq \frac{\partial \text{demand}}{\partial P_{uc}} \iff \beta_{pnc} - \beta_{puc} \neq 0$$

We obtain the test statistic as follows, and obtain estimates for variances and covariance from Stata:

$$\begin{aligned} t = \frac{\bar{x} - \mu}{SE} &= \frac{\hat{\beta}_{pnc} - \hat{\beta}_{puc}}{\sqrt{\text{Var}(\hat{\beta}_{pnc} - \hat{\beta}_{puc})}} = \frac{\hat{\beta}_{pnc} - \hat{\beta}_{puc}}{\sqrt{\text{Var}(\hat{\beta}_{pnc}) + \text{Var}(\hat{\beta}_{puc}) + 2\text{Cov}(\hat{\beta}_{pnc}, \hat{\beta}_{puc})}} \sim t_{42} \\ &= \frac{(0.0005774) - (-0.0058746)}{\sqrt{.00016497 + .00002372 + 2(0.000009359)}} = 0.448 < t_{42}^* = 2.02 \end{aligned}$$

Thus we fail at 5% significance level ( $p = 0.656 > 0.05$ ) to reject the null hypothesis that there is a difference between the two coefficients.

- c. **Estimate the own price elasticity of demand, the income elasticity, and the cross-price elasticity with respect to changes in the price of public transportation. Do the computations at the 2004 point in the data, which means that the covariates should take the values corresponding to 2004 (i.e., use the "if" command instead of "at").**

The tables for the elasticities are given in the appendix. The estimated proportional elasticities of gasoline consumption with respect to proportion changes in price, income, and price of public transportation in 2004 can also be computed manually via:

$$\begin{aligned} \text{PED}_{2004} &= \frac{\partial Q}{\partial P} \frac{P_{2004}}{Q_{2004}} = \beta_{\text{GasP}} \left( \frac{\text{GasP}_{2004}}{\text{Demand}_{2004}} \right) = (-.0110838) \frac{123.901}{6.164056} = -0.2225 \\ \text{YED}_{2004} &= \frac{\partial Q}{\partial I} \frac{I_{2004}}{Q_{2004}} = \beta_{\text{Income}} \left( \frac{\text{Income}_{2004}}{\text{Demand}_{2004}} \right) = (0.0002157) \frac{27113}{6.164056} = 0.9488 \\ \text{XED}_{2004} &= \frac{\partial Q}{\partial X} \frac{X_{2004}}{Q_{2004}} = \beta_{ppt} \left( \frac{ppt_{2004}}{\text{Demand}_{2004}} \right) = (0.0069073) \frac{209.1}{6.164056} = 0.2343 \end{aligned}$$

This matches the computer-generated elasticities.

- d. **Reestimate the regression in logarithms so that the coefficients are direct estimates of the elasticities. (Do not use the log of the time trend). How do your estimates compare with the results in the previous question? Which specification do you prefer?**

Elasticities are closely linked with partial derivatives: When using a log-log model, as follows

$$\ln \text{demand} = \beta_0 + \beta_{\text{GasP}} \ln(\text{GasP}) + \beta_{\text{income}} \ln(\text{Income}) + \beta_{\text{ppt}} \ln(\text{PPT}) + \dots$$

Partial-deriving both sides with respect to GasP, Income and PPT gives

$$\begin{aligned} \frac{\partial \text{demand}}{\partial \text{GasP}} \frac{1}{\text{demand}} &= \frac{\beta_{\text{GasP}}}{\text{GasP}} \iff \beta_{\text{GasP}} = \frac{\partial \text{demand}}{\partial \text{GasP}} \frac{\text{GasP}}{\text{demand}} = \widehat{\text{PED}}_{(\text{SE})} = 0.0605177_{(.0540101)} \\ \frac{\partial \text{demand}}{\partial \text{Income}} \frac{1}{\text{demand}} &= \frac{\beta_{\text{Income}}}{\text{Income}} \iff \beta_{\text{Income}} = \frac{\partial \text{demand}}{\partial \text{Income}} \frac{\text{Income}}{\text{demand}} = \widehat{\text{YED}}_{(\text{SE})} = 0.9929907_{(0.2503763)} \\ \frac{\partial \text{demand}}{\partial \text{PPT}} \frac{1}{\text{demand}} &= \frac{\beta_{\text{PPT}}}{\text{Income}} \iff \beta_{\text{PPT}} = \frac{\partial \text{demand}}{\partial \text{PPT}} \frac{\text{PPT}}{\text{demand}} = \widehat{\text{XED}}_{(\text{SE})} = .0192726_{(0.136449)} \end{aligned}$$

Thus the coefficient estimates can similarly be used to estimate the elasticities. The tables for the regression output is given in the appendix. However, a key issue with doing this is assumes constant elasticity (isoelasticity), which is not necessarily the case, and has clear implications, such as in the calculation for PED or YED. The price elasticity of demand should be negative (increases in price shower lower demand), and was previously estimated to be -0.22 in 2004, but has difficulty establishing even the non-positiveness of the price elasticity - most likely because of the prominence of oil crises, such as the oil crises of 1973 and 1979. During such crises, the shortage of oil, real or perceived, led to concerns (and sometimes panic). The price of oil endogenously affects, and is affected by demand, creating a feedback loop in which higher prices cause panic and greater demand, and thus there were periods in which the price elasticity of demand was positive.

Similarly, regression did not find the estimate of the cross-price elasticity significant from 0. In c, the cross-price elasticity with respect to the price of transport in 2004 is positive, as would be expected of substitutes. Thus the specification in d. fails to capture and take into account fluctuations within the elasticities themselves over time, and the specification in a. is preferred, as it allows for more accurate point-price elasticities to be computed.

- e. **Compute the simple correlations of the price variables. Would you conclude that multicollinearity is a “problem” for the regression in part a or part d?**

Looking at the correlation table in the appendix, there clearly is a high degree of collinearity in between the variables, with multiple pairwise correlations being above 0.9. While there is no exact collinearity, and calculations can be done, the model will have trouble allocating explanatory power amongst the independent variables.

- f. **Notice that the price index for gasoline is normalized to 100 in 2000, whereas the other price indices are anchored at 1983 (roughly). If you were to renormalize the indices so that they were all 100.00 in 2004, then how would the results of the regression in part a change? How would the results of the regression in part d change?**

Price indices represent relative changes with respect to a given base year. In order to renormalize the index to a different year, divide each data point by the price of the desired base year, and multiply by 100. For example, the new index number for the price of public transport for the year 1983 (the previous base year) when using 2004 as the new base is:

$$ppt_{1983}^* = \frac{ppt_{1983}}{ppt_{2004}} \times 100 = \frac{100}{209.1} = 47.824$$

Performing the same regression in part a. using the newly normalized index gives similar results. The  $R^2$ , both normal and adjusted are identical. There is no effect on the fit of the regression, and the coefficient estimates of the other explanatory variables (income, year, and intercept) remain identical. The only difference –that of the coefficient estimates of the corresponding price index variables– comes from an inverse proportional scaling of the original estimates in a. by the price index in 2004. For example, in the coefficient estimate for the price of public transport is originally  $\beta_{ppt} = 0.0069073$  in a., and when using 2004 prices as the base, becomes

$$\beta_{ppt}^* = \beta_{ppt} \times \frac{ppt_{2004}}{100} = (0.0069073) \times \frac{(209.1)}{100} = 0.0144431$$

The results of the regression in d. change in a similar fashion. The fit of the regression, as well as the coefficient estimate for income, year, and the intercept remain identical. The estimates for the coefficient for price index variables are again scaled according to the corresponding price index for 2004. For example, in the estimate for  $\beta_{ppt} = 0.0019642$  in d. and when using 2004 prices as the base, becomes

$$\beta_{ppt}^* = \beta_{ppt} \times \frac{ppt_{2004}}{100} = (0.0019642) \times \frac{(209.1)}{100} = 0.004107$$

Word Count (excluding questions, tables, charts, graphs and appendix):

## References

- [1] Gary Koop and Justin L Tobias. Learning about heterogeneity in returns to schooling. *Journal of Applied Econometrics*, 19(7):827–849, 2004.

### 3 Appendix

#### First 15 Observations

personid	educ	logwage	potexper	timetrnd	ability	mothered	fathered	brknhome	siblings
1	13	1.82	1	0	1	12	12	0	1
2	15	2.14	4	6	1.5	12	12	0	1
3	10	1.56	1	2	-0.36	12	12	1	1
4	12	1.85	1	3	0.26	12	10	1	4
5	15	2.41	2	3	0.3	12	12	1	1
6	15	1.83	2	2	0.44	12	16	0	2
7	15	1.78	3	8	0.91	12	12	0	1
8	13	2.12	4	0	0.51	12	15	1	2
9	13	1.95	2	0	0.86	12	12	0	2
10	11	2.19	5	3	0.26	12	12	0	2
11	12	2.44	1	2	1.82	16	17	1	2
12	13	2.41	4	2	-1.3	13	12	0	5
13	12	2.07	3	0	-0.63	12	12	1	4
14	12	2.2	6	4	-0.36	10	12	1	2
15	12	2.12	3	4	-0.28	10	12	1	3

#### Regression Table for $y$ on $X_1$

Source	SS	df	MS	Number of obs	=	17,919
Model	867.088558	3	289.029519	F(3, 17915)	=	1252.94
Residual	4132.63745	17,915	.230680293	Prob > F	=	0.0000
				R-squared	=	0.1734
				Adj R-squared	=	0.1733
Total	4999.72601	17,918	.279033709	Root MSE	=	.48029

logwage	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
educ	.0737621	.0022143	33.31	0.000	.0694219	.0781022
potexper	.0394896	.0008984	43.96	0.000	.0377287	.0412504
ability	.0828907	.0046	18.02	0.000	.0738744	.0919071
_cons	1.027229	.0300415	34.19	0.000	.968345	1.086113

#### Regression Table for $y$ on $X_1$ and $X_2$

Source	SS	df	MS	Number of obs	=	17,919
Model	873.550652	6	145.591775	F(6, 17912)	=	632.02
Residual	4126.17535	17,912	.23035816	Prob > F	=	0.0000
				R-squared	=	0.1747

-----+-----				Adj R-squared	=	0.1744
Total		4999.72601	17,918 .279033709	Root MSE	=	.47996

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logwage		Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
-----+-----						
educ		.0722035	.0022508	32.08	0.000	.0677918 .0766152
potexper		.0395093	.0008993	43.94	0.000	.0377467 .0412719
ability		.0774678	.0049373	15.69	0.000	.0677903 .0871453
mothered		-.000117	.0016963	-0.07	0.945	-.003442 .003208
fathered		.0054569	.0013387	4.08	0.000	.002833 .0080809
siblings		.0047656	.0017924	2.66	0.008	.0012523 .0082789
_cons		.9695096	.0337054	28.76	0.000	.9034437 1.035575
-----+-----						



### Regression Table for $y$ on $X_1$ and $X_2$ , no intercept

Source		SS	df	MS	Number of obs	=	17,919
-----+							
Model		873.550652	6	145.591775	F(6, 17912)	=	632.02
Residual		4126.17535	17,912	.23035816	Prob > F	=	0.0000
-----+							
Total		4999.72601	17,918	.279033709	R-squared	=	0.1747
					Adj R-squared	=	0.1744
					Root MSE	=	.47996

logwage	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
educ	.0722035	.0022508	32.08	0.000	.0677918	.0766152
potexper	.0395093	.0008993	43.94	0.000	.0377467	.0412719
ability	.0774678	.0049373	15.69	0.000	.0677903	.0871453
mothered	-.000117	.0016963	-0.07	0.945	-.003442	.003208
fathered	.0054569	.0013387	4.08	0.000	.002833	.0080809
siblings	.0047656	.0017924	2.66	0.008	.0012523	.0082789
_cons	.9695096	.0337054	28.76	0.000	.9034437	1.035575

### Ramsay RESET in the Regression of $y$ on $X_1$

Ramsay RESET test using powers of the fitted values of logwage

Ho: model has no omitted variables

F(3, 17912) = 3.32

Prob > F = 0.0190

### Ramsay RESET in the Regression of $y$ on $X_1$ and $X_2$

Ramsay RESET test using powers of the fitted values of logwage

Ho: model has no omitted variables

F(3, 17909) = 4.20

Prob > F = 0.0056

### Variance Inflation Factor in the Regression of $y$ on $X_1$

		educ	potexper	ability
-----+				
educ		1.0000		
potexper		-0.2187	1.0000	
ability		0.5279	-0.2222	1.0000

### Variance Inflation Factor in the Regression of $y$ on $X_1$ and $X_2$

Variable	VIF	1/VIF
mothered	2.00	0.500127
fathered	1.98	0.505565
ability	1.63	0.614889
educ	1.46	0.686681
siblings	1.12	0.889541
potexper	1.07	0.933196
Mean VIF	1.54	

### Regression Table for Gasoline Per Capita Consumption on other variables

Source	SS	df	MS	Number of obs	=	52
Model	56708303.2	9	6300922.57	F(9, 42)	=	530.82
Residual	498548.982	42	11870.2139	Prob > F	=	0.0000
				R-squared	=	0.9913
				Adj R-squared	=	0.9894
Total	57206852.1	51	1121702.98	Root MSE	=	108.95

pc QUAN	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
income	.2157495	.0517618	4.17	0.000	.1112899	.3202091
gasp	-11.08386	3.978121	-2.79	0.008	-19.11203	-3.055683
pnc	.5773703	12.84414	0.04	0.964	-25.34316	26.4979
puc	-5.874637	4.87032	-1.21	0.234	-15.70334	3.954067
ppt	6.907265	4.836129	1.43	0.161	-2.852439	16.66697
pd	1.228872	11.88175	0.10	0.918	-22.74947	25.20722
pn	12.69048	12.59799	1.01	0.320	-12.73329	38.11425
ps	-28.0278	7.996249	-3.51	0.001	-44.16488	-11.89072
year	72.5037	14.1828	5.11	0.000	43.88165	101.1257
_cons	-140421.3	27199.85	-5.16	0.000	-195312.9	-85529.82

### Price Elasticities for 2004

margins if year == 2004, eyex(gasp income ppt)

Average marginal effects                      Number of obs        =            1  
 Model VCE        : OLS

Expression        : Linear prediction, predict()  
 ey/ex w.r.t.     : income gasp ppt

		Delta-method					
		ey/ex	Std. Err.	t	P> t	[95% Conf. Interval]	
	income	.9476599	.2262966	4.19	0.000	.4909749	1.404345
	gasp	-.2224796	.0809311	-2.75	0.009	-.3858053	-.059154
	ppt	.2339837	.1644132	1.42	0.162	-.0978155	.565783