DSP Assignment 2

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Table of Contents

2(a)

256-sample window produces a 256-sample DFT that is hermetian-symmetric (for a real-valued input signal) (see https://dsp.stackexchange.com/a/4843)

=> DFT[m] = DFT[256-m]

Bucket maximized when signal $\sin\left(2\pi f * \frac{i}{f_s}\right)$ matches $e^{-\mathrm{j}2\pi\frac{i}{n}k}$

$$2\pi \frac{i}{n} k = 2\pi f * \frac{i}{n} * \frac{n}{f_s}$$

$$k_{\text{single}} = \frac{f * n}{f_s}$$

For 941Hz $k_{\text{single941}} = \frac{941 * 256}{8000} = 30.1120$

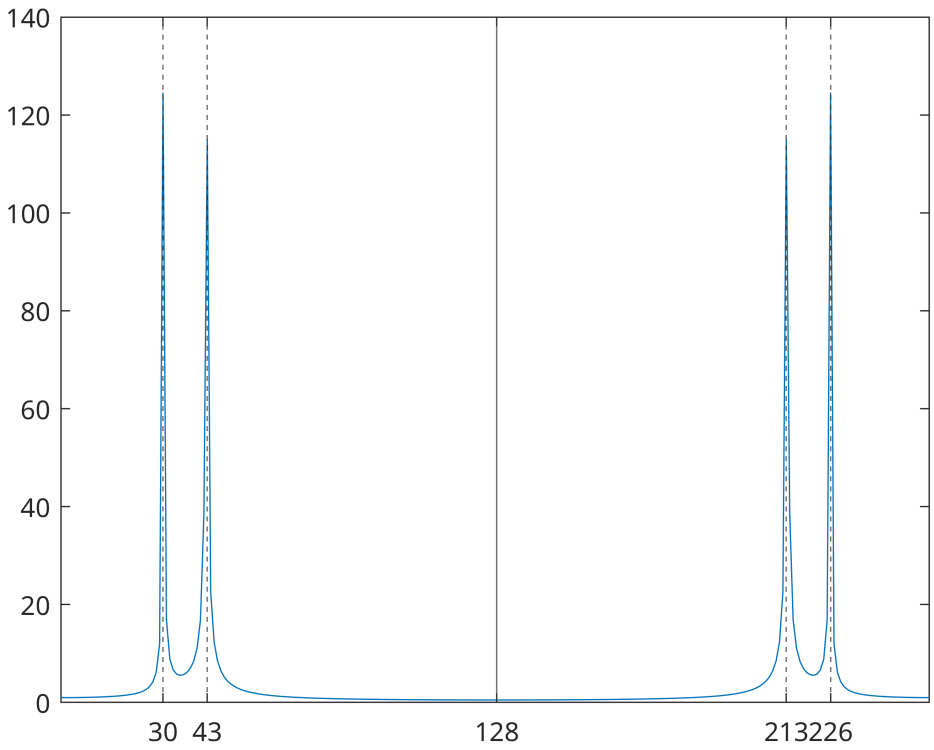
For 1336Hz $k_{\text{single1336}} = \frac{1336 * 128}{8000} = 42.7520$

=> Final peaks

$$k_{\text{double941}} = 30,\ 256 - 30 = 30, 226$$
$$k_{\text{double1336}} = 43, 256 - 43 = 43, 213$$

Example below:

```
Fs=8000;
n=256;
% n time samples spaced by 1/Fs
ts = (1/Fs):(1/Fs):(n/Fs);
x = sin(2*pi*941*ts)+sin(2*pi*1336*ts);
X = fft(x);
plot(0:255, abs(X))
xline(128)
xline([30 43 (256-43) (256-30)], '--')
xticks([30 43 128 (256-43) (256-30)])
xlim([0 255])
```



2(b)

```
% Determine sampling frequency
[y_original, Fs] = audioread('touchtone-synth.wav');
```

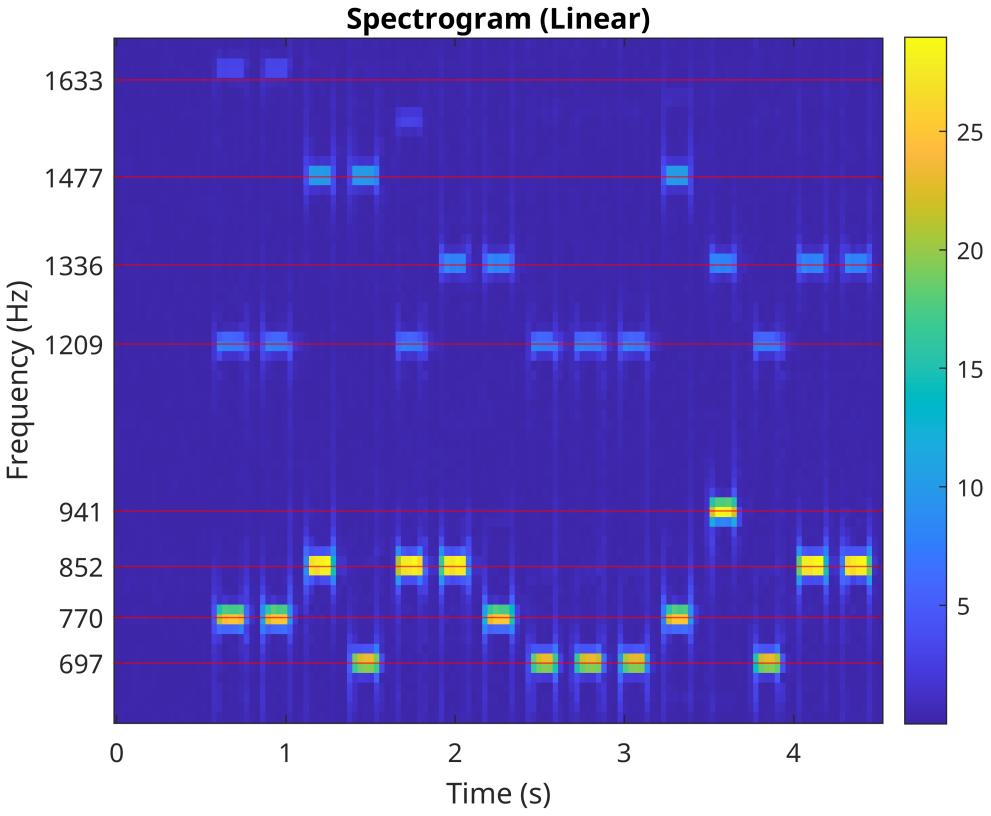
```
% Choose good window size
% Started with 320, to get 50Hz between buckets (16000/50 = 320)
% Didn't separate well enough, so tried 512 then 1024.
window_size = 1024;

% Each block has to overlap with another block by 50%
% => ranges are 1-512, 256-768...
% Pad the edges out by window_size/2 - this ensures all real samples are
% covered by two windows.
% Pad the far edge out a farther in case the length isn't divisible
% by window_size.
y = [zeros(window_size/2,1); y_original; zeros(window_size,1)];
block_starts = 1:(window_size/2):(length(y)-window_size);
N = length(block_starts);
blocks = zeros(window_size,N);
for i = 1:N
    data = y(block_starts(i):block_starts(i)+window_size-1);
    blocks(1:end,i) = data;
end
```

```
% Choose a window:
```

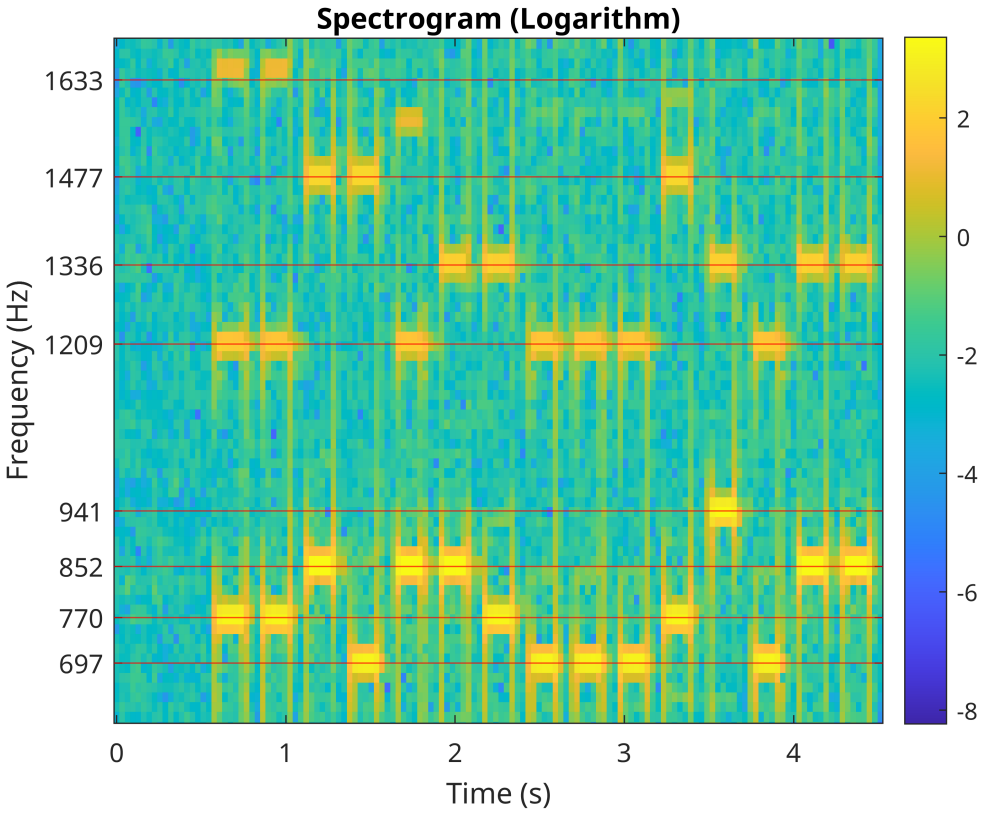
```
% Maximum frequency we need to find is 1633Hz, Fs is 16000Hz
% Choose Hamming, because it doesn't have as steep a falloff as Hann
w = hamming(window_size);
windowed_blocks = zeros(window_size,N);
for i = 1:N
    windowed_blocks(1:end,i) = blocks(1:end,i) .* w;
end
```

```
% Compute FFTs
dfts = fft(windowed_blocks, window_size);
```

```
% Discard negative frequencies, take amplitude
dft_posf_amp = zeros(window_size/2,N);
for i = 1:N
    % select first half of dft values => only the positive frequencies
    dft_posf_amp(1:end,i) = abs(dfts(1:(window_size/2), i));
end

% t = 0 to (final sample number / frequency)
image_xrange = [0, block_starts(end)/Fs];
% f = 0 to Fs/2
image_yrange = [0, Fs/2];

% Produce a spectrogram
image(image_xrange, image_yrange, ...
    dft_posf_amp, 'CDataMapping','scaled')
colorbar
% Limit to the frequency range we care about
ylim([600 1700])
% Mark important frequencies
yticks([697 770 852 941 1209 1336 1477 1633])
yline([697 770 852 941 1209 1336 1477 1633], 'r')
% Make the frequencies go bottom to top
set(gca, 'YDir','normal')
ylabel("Frequency (Hz)")
xlabel("Time (s)")
title("Spectrogram (Linear)")
```




```
% Logarithmic version
image(image_xrange, image_yrange, ...
    log(dft_posf_amp), 'CDataMapping','scaled')
colorbar
ylim([600 1700])
yticks([697 770 852 941 1209 1336 1477 1633])
yline([697 770 852 941 1209 1336 1477 1633], 'r')
set(gca, 'YDir','normal')
ylabel("Frequency (Hz)")
xlabel("Time (s)")
title("Spectrogram (Logarithm)")
```



Numbers:

770x1209, 770x1209, 852x1477, 697x1477, 852x1209, 852x1336, 770x1336, 697x1209, 697x1209, 697x1209, 770x1477, 941x1336, 697x1209, 852x1336, 852x1336

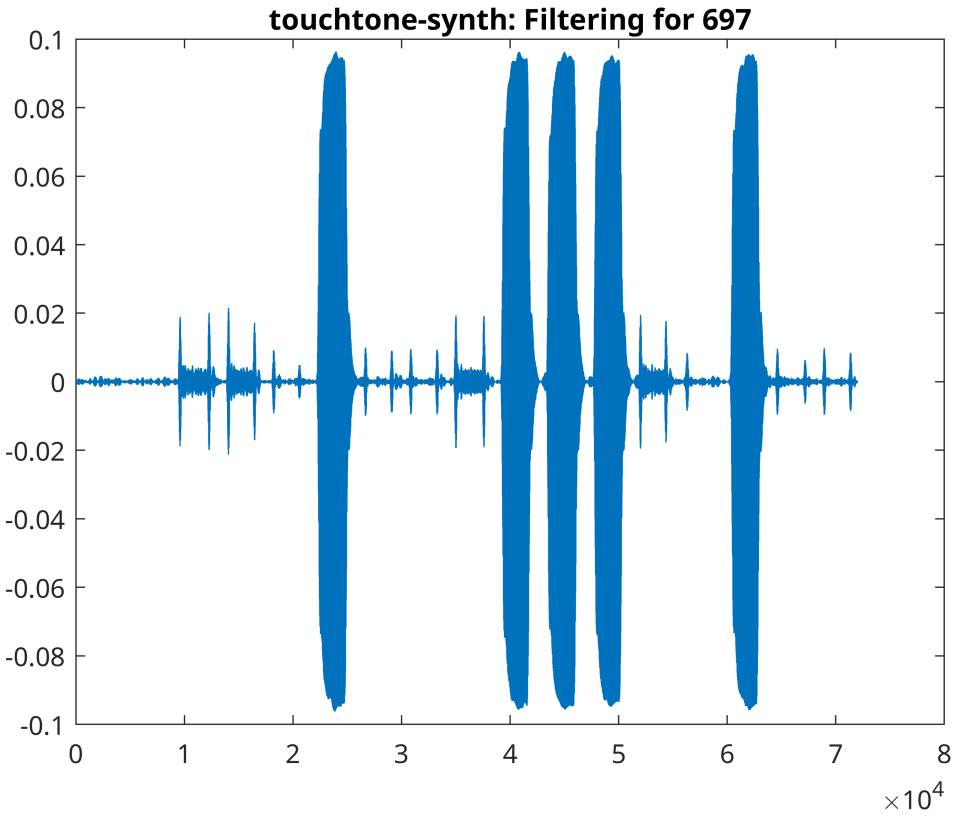
= 449378511160188

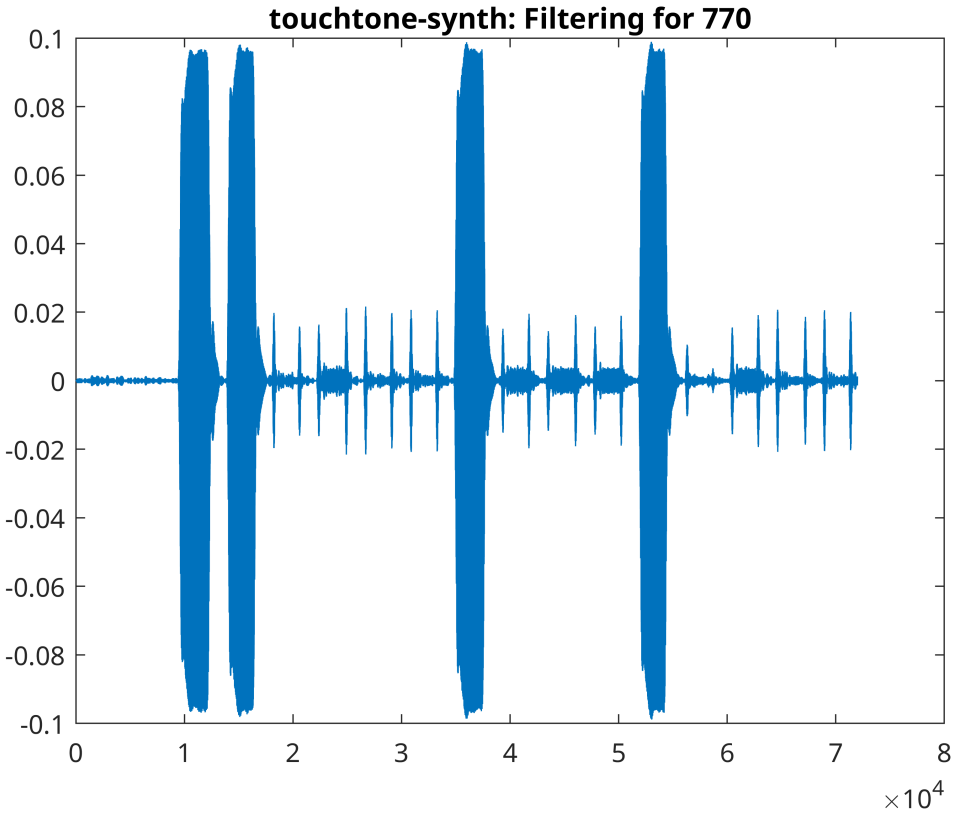
2(c)

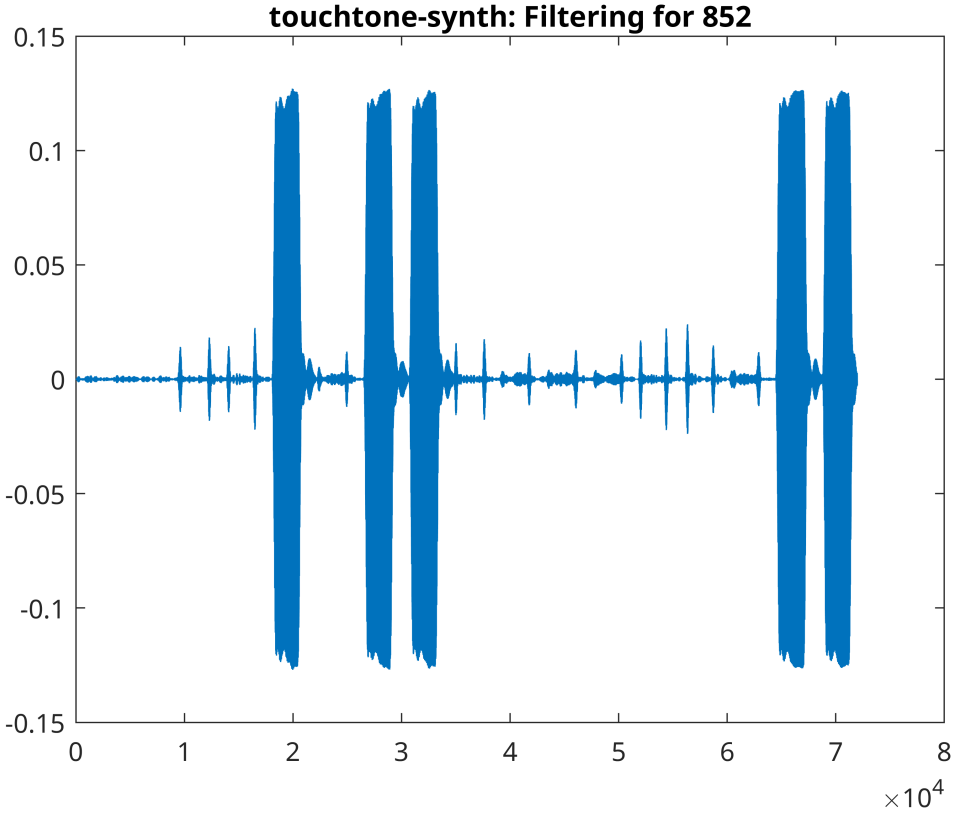
Design band-pass filters

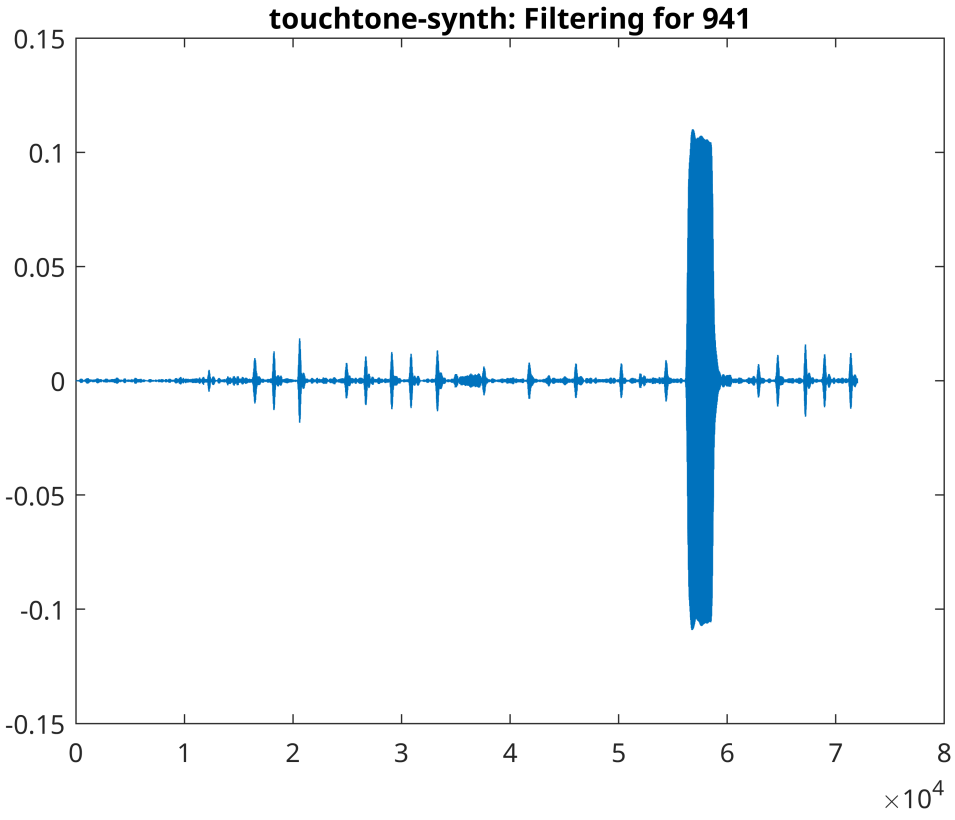
```
freqs = [697 770 852 941 1209 1336 1477]; % Exclude 1633
% Band should be f +- 1.8%
freqs_pass_low = freqs * (1.0 - 0.018);
freqs_pass_high = freqs * (1.0 + 0.018);

fir_order = 400;
% 1 for yes, 0 for no
show_freq_response = 0;
show_filtered_y = 1;
stack_all_tones = 1;
y = y_original;

for i = 1:length(freqs)
    b = fir1(fir_order, (2/Fs)*[freqs_pass_low(i) freqs_pass_high(i)]);
    a = [1];
```

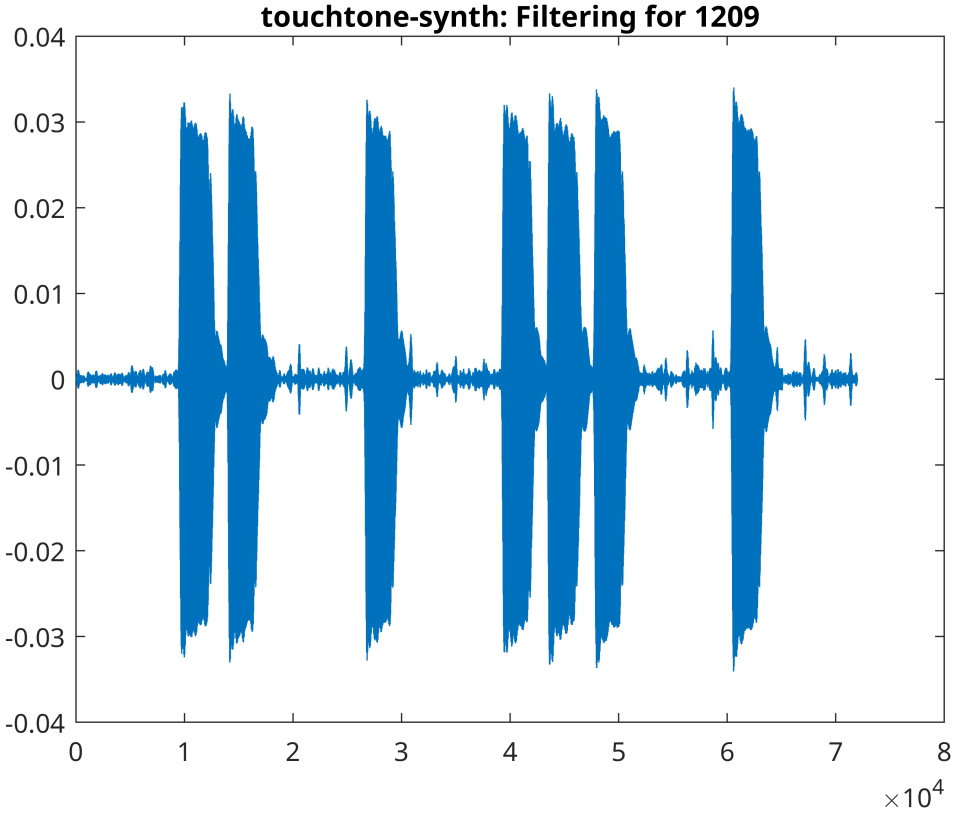
```
    if show_freq_response
        figure;
        title("Response for filter for " + freqs(i));
        freqz(b, a, 512, Fs);
    end

    if show_filtered_y
        figure;
        plot(filter(b, a, y));
        title("touchtone-synth: Filtering for " + freqs(i));
    end
end
```

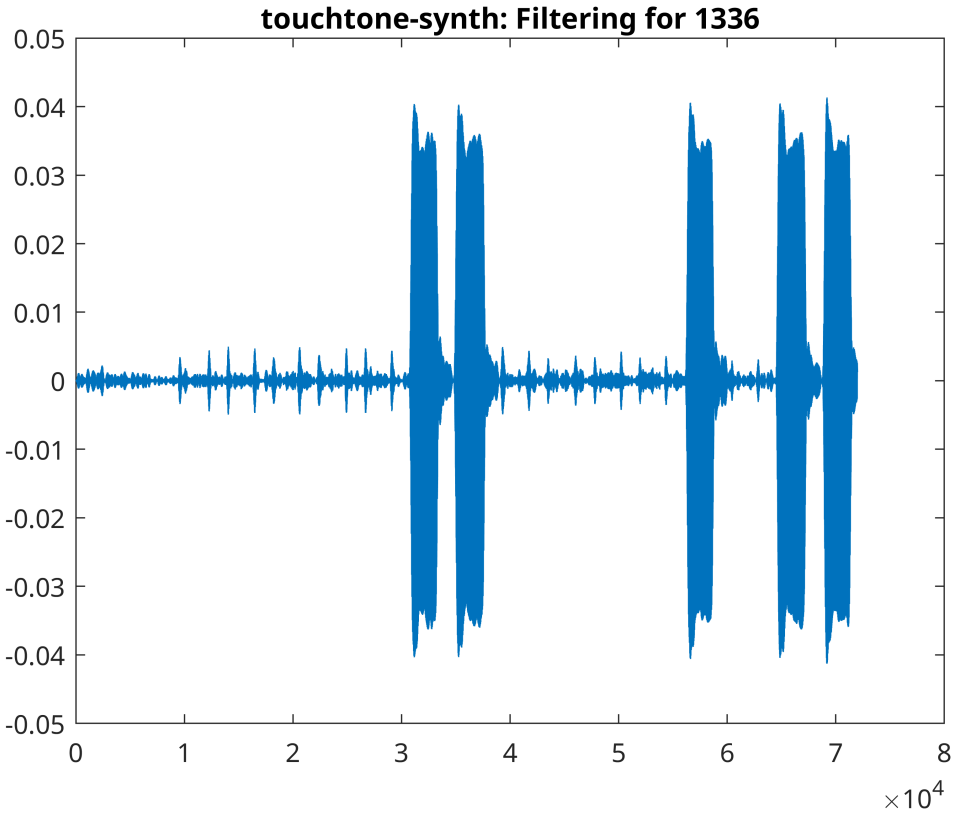


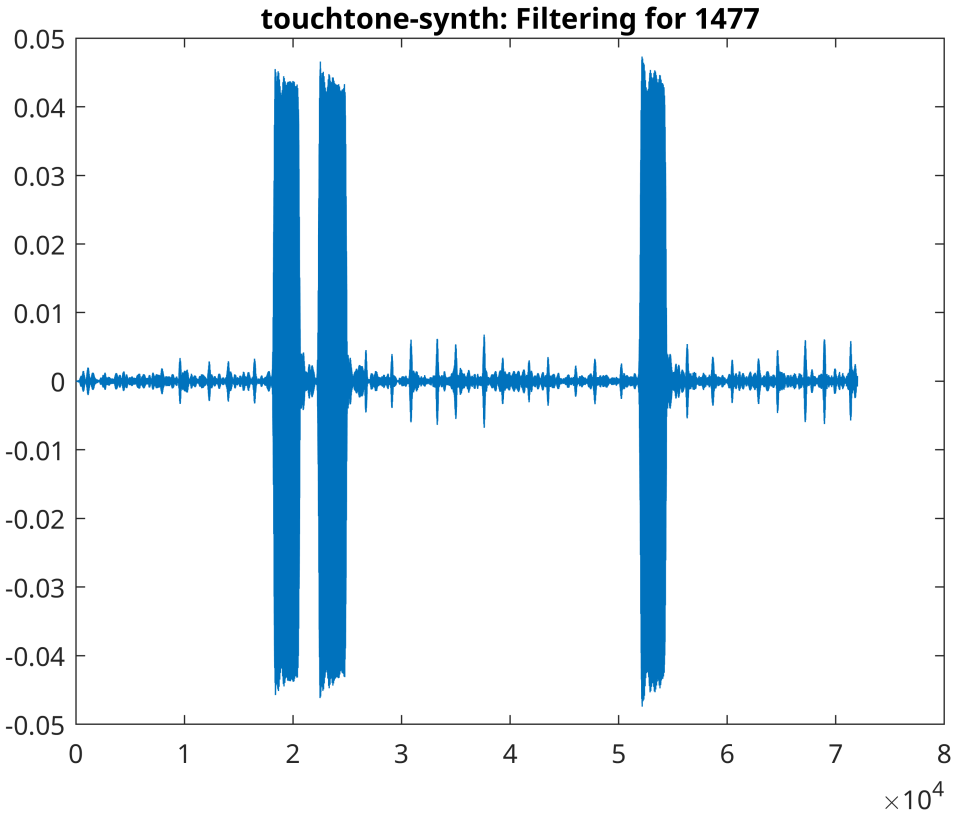

touchtone-synth: Filtering for 770
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
0
1
2
3
4
5
6
7
8
×10^4


touchtone-synth: Filtering for 852
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
0
1
2
3
4
5
6
7
8
×10^4


touchtone-synth: Filtering for 941
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
0
1
2
3
4
5
6
7
8
×10^4

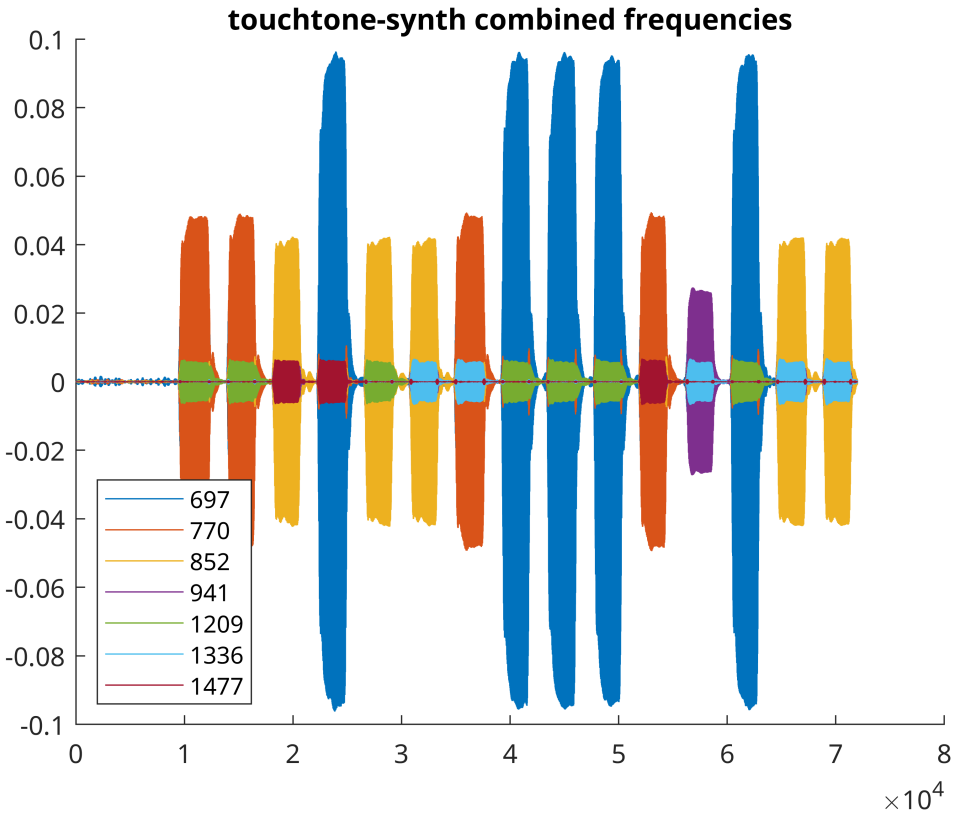

touchtone-synth: Filtering for 1209
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
0
1
2
3
4
5
6
7
8
×10^4






```
if stack_all_tones
    figure;
    hold on;
```

```
    for i = 1:length(freqs)
        b = fir1(fir_order, (2/Fs)*[freqs_pass_low(i) freqs_pass_high(i)]);
        a = [1];
        plot(filter(b, a, y)/i);
    end
    title(["touchtone-synth combined frequencies"]);
    legend(string(freqs), 'Location','southwest');
    hold off;
end
```

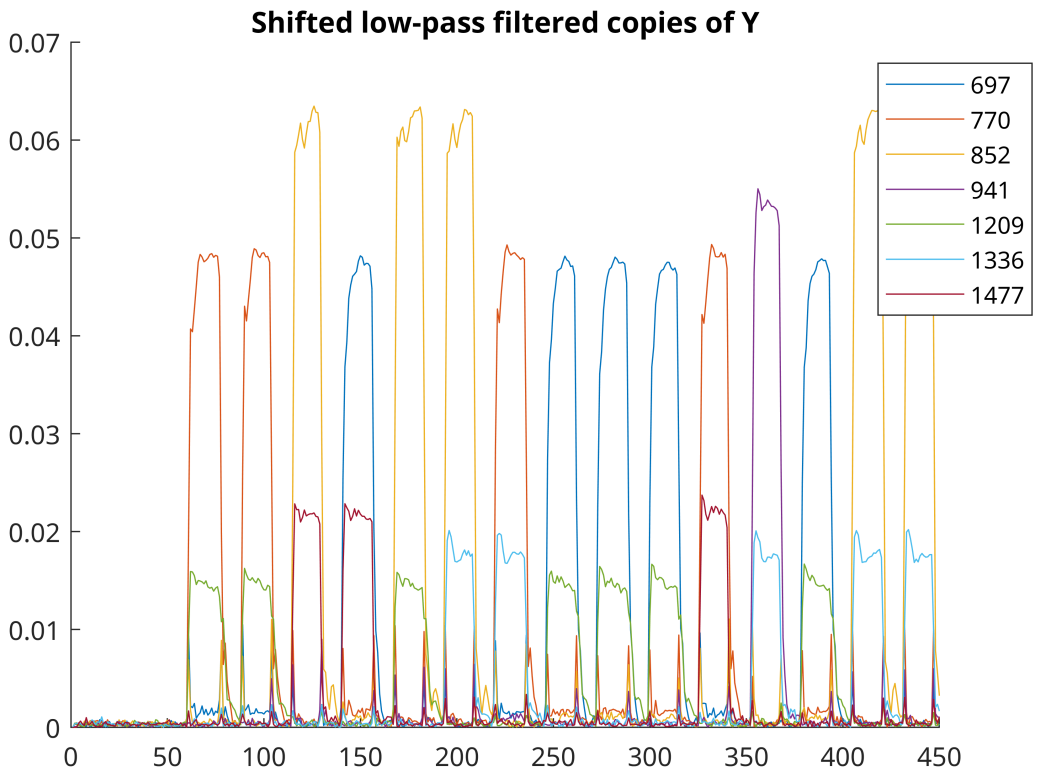


The final plot shows each frequency (scaled so that higher frequencies are smaller), and the colors on the combined graph show which tones were dialled.

2(d)

```
low_pass_fir_order = 400;
% Only enable one at a time
show_alias_check = 0;
show_complete_figure = 1;

if show_complete_figure
    figure;
    complete_figure = gca;
end

for i = 1:length(freqs)
    % Multiply each waveform by a complex phasor
    phasor = exp(-2*pi*1i*freqs(i)*(1:length(y_original))/Fs);
```

```
    shifted_y = y_original .* phasor';

    % Apply a low-pass filter to let in 0Hz +- 1.8% of freqs(i)
    % low-pass = pass frequencies lower than X
    % => low-pass filter for freqs(i) * 1.8%
    b = fir1(low_pass_fir_order, freqs(i) * 0.018 * 2/Fs);
    a = [1];
    shifted_filtered_y = filter(b, a, shifted_y);

    % Reduce the sampling rate to 100Hz
    % Original is 16000 => keep every 16000/100 = 160th sample
    %
    % I tried checking for aliasing by visually inspecting and comparing
    % waveforms, but I don't think that's the right way to do it.
    % The right way would probably involve looking at the FFTs, but I
    % don't have enough time to do that right now. Sorry!
    reduced_shifted_filtered_y = shifted_filtered_y(1:(Fs/100):end);

    if show_alias_check
        figure;
        hold on;
        % Plot both on the same axes
        plot(abs(shifted_filtered_y));
        plot(1:(Fs/100):length(shifted_filtered_y), ...
            abs(reduced_shifted_filtered_y));
        hold off;
    end

    if show_complete_figure
        hold (complete_figure, "on");
        plot(abs(reduced_shifted_filtered_y));
        hold (complete_figure, "off");
    end
end

if show_complete_figure
    hold (complete_figure, "on");
    % position legend so it doesn't block graph
    % From https://stackoverflow.com/a/3602614
    h = legend(string(freqs));
    pos = get(h, 'position');
    set(h, 'position', [0.85, 0.6, pos(3:4)]);
    % Set title
    title("Shifted low-pass filtered copies of Y")
    hold (complete_figure, "off");
end
```



As before, the colors on the combined graph show which frequencies are used for each press.

770x1209, 770x1209, 852x1477, 697x1477, 852x1209, 852x1336, 770x1336, 697x1209, 697x1209, 697x1209, 770x1477, 941x1336, 697x1209, 852x1336, 852x1336

= 449378511160188