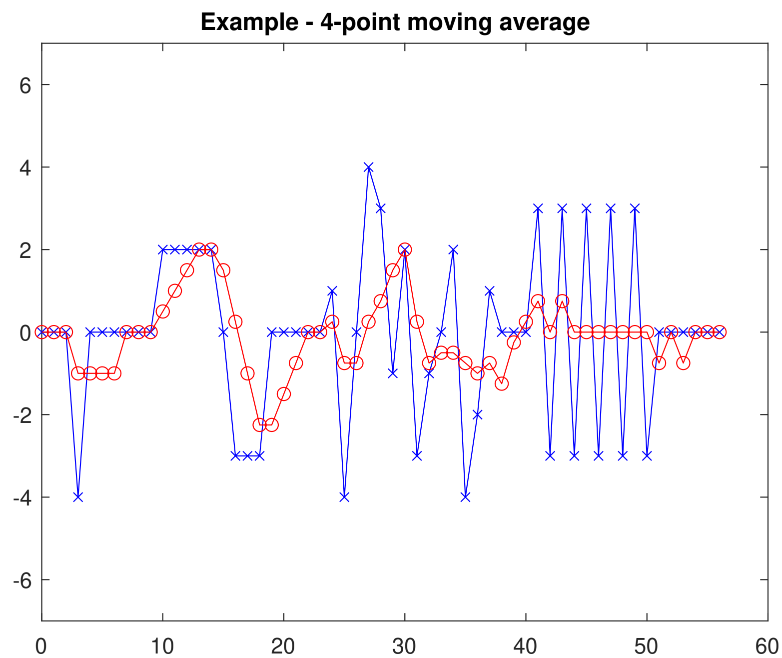
Contents

1.a

```
x = [ 0 0 0 -4 0 0 0 0 0 0 2 2 2 2 ...
2 0 -3 -3 -3 0 0 0 0 0 1 -4 0 4 ...
3 -1 2 -3 -1 0 2 -4 -2 1 0 0 0 3 ...
-3 3 -3 3 -3 3 -3 3 -3 0 0 0 0 0 0 ];
n = 0:length(x)-1;

a=[1];
b=[1 1 1 1]/4;
y = filter(b, a, x);
figure;
plot(n, x, 'bx-', n, y, 'ro-');
ylim([-7,7]);
title("Example - 4-point moving average");
```



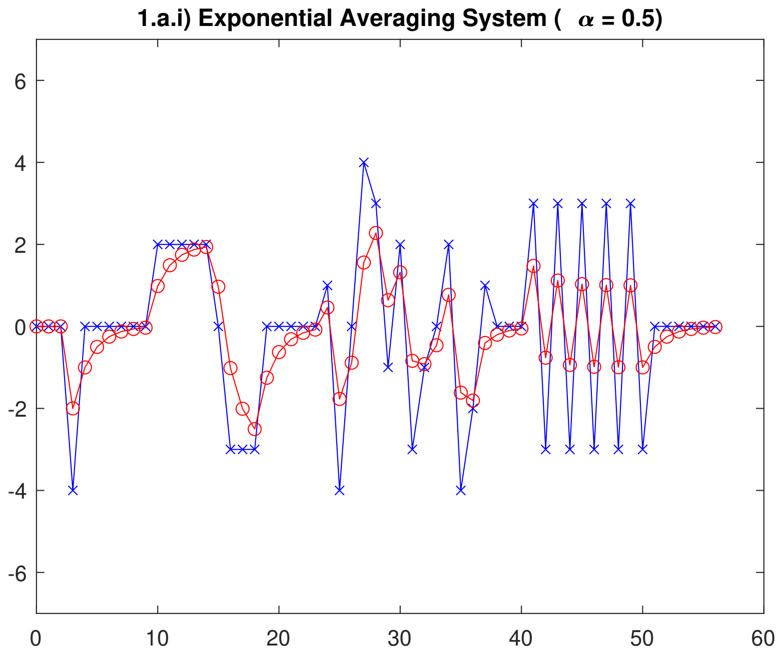
1.a.i) Exponential Averaging System

$y_n = alpha * x_n + (1 - alpha) * y_{n-1}$

$=> y_n - (1 - alpha) * y_{n-1} = alpha * x_n$

$=> [1, -(1 - alpha)].[y_n, y_{n-1}] = [alpha].[x_n]$

```
alpha = 0.5;
a=[1, -(1 - alpha)];
b=[alpha];
y = filter(b, a, x);
figure;
plot(n, x, 'bx-', n, y, 'ro-');
ylim([-7,7]);
title("1.a.i) Exponential Averaging System ({\alpha = 0.5})");
```



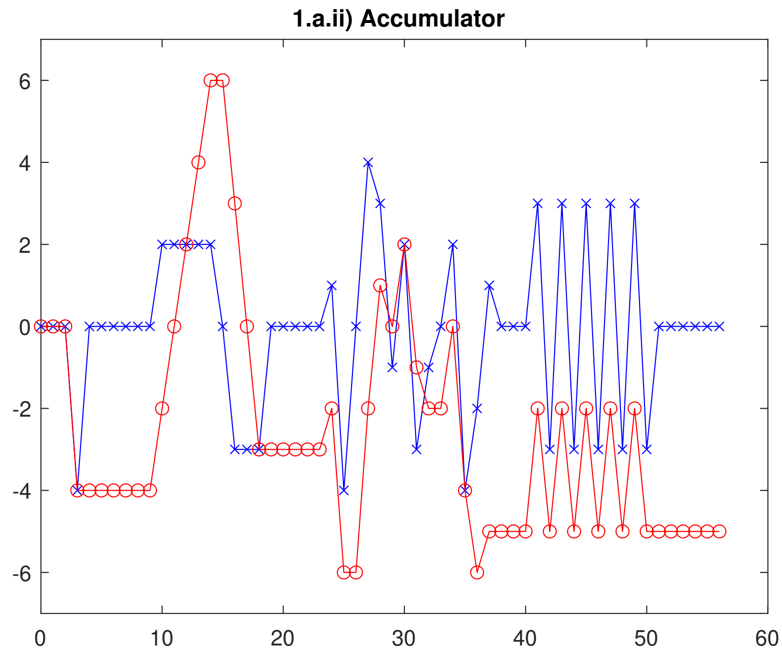
1.a.ii) Accumulator

$$y_n = x_n + y_{n-1}$$

$$=> y_n - y_{n-1} = x_n$$

$$=> [1, -1].[y_n, y_{n-1}] = [1].[x_n]$$

```
a=[1, -1];
b=[1];
y = filter(b, a, x);
figure;
plot(n, x, 'bx-', n, y, 'ro-');
ylim([-7,7]);
title("1.a.ii) Accumulator");
```

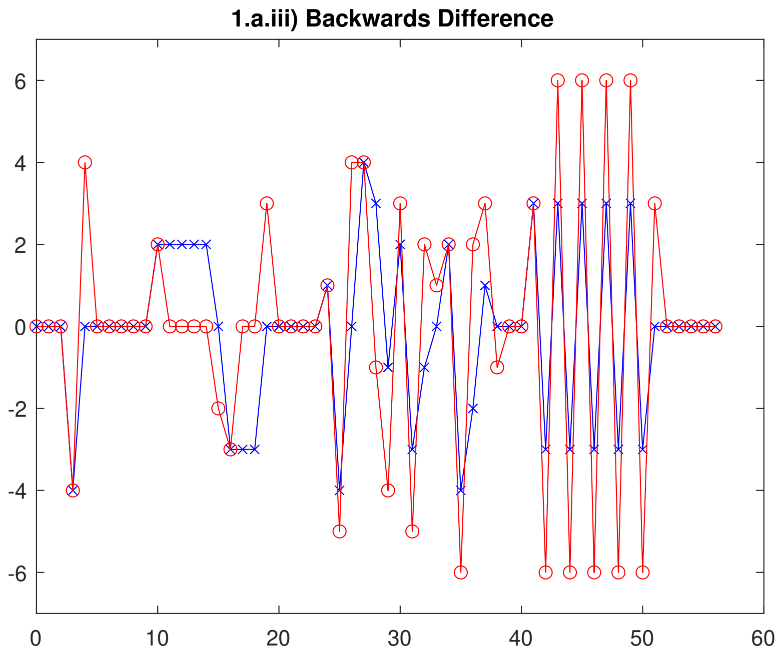


1.a.iii) Backwards Difference System

$$y_n = x_n - x_{n-1}$$

$$[1].[y_n] = [1, -1].[x_n, x_{n-1}]$$

```
a=[1];
b=[1, -1];
y = filter(b, a, x);
figure;
plot(n, x, 'bx-', n, y, 'ro-');
ylim([-7,7]);
title("1.a.iii) Backwards Difference");
```



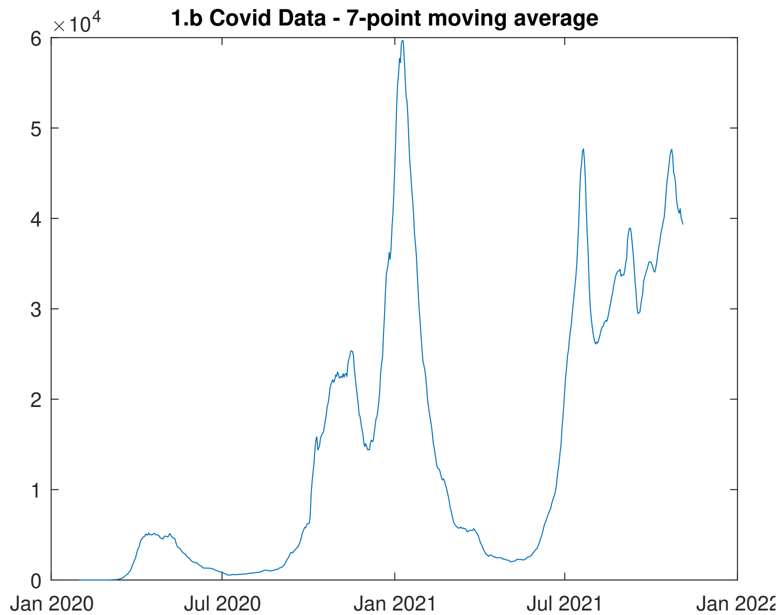
1.b

Get table date, taking only the 'date' and 'newCasesByPublishDate' columns

```
newcases = readtable("covid_new_cases_2021_11_04.csv", 'Range', 'D:E');
% Sort table by 'date'
newcases = sortrows(newcases, 1);
% Take x = newcases.newCasesByPublishDate
x = newcases.newCasesByPublishDate;
```

1.b - 7 Point Moving Average

```
a=[1];
b=[1 1 1 1 1 1 1]/7;
y = filter(b, a, x);
figure;
plot(newcases.date, y)
title("1.b Covid Data - 7-point moving average");
```



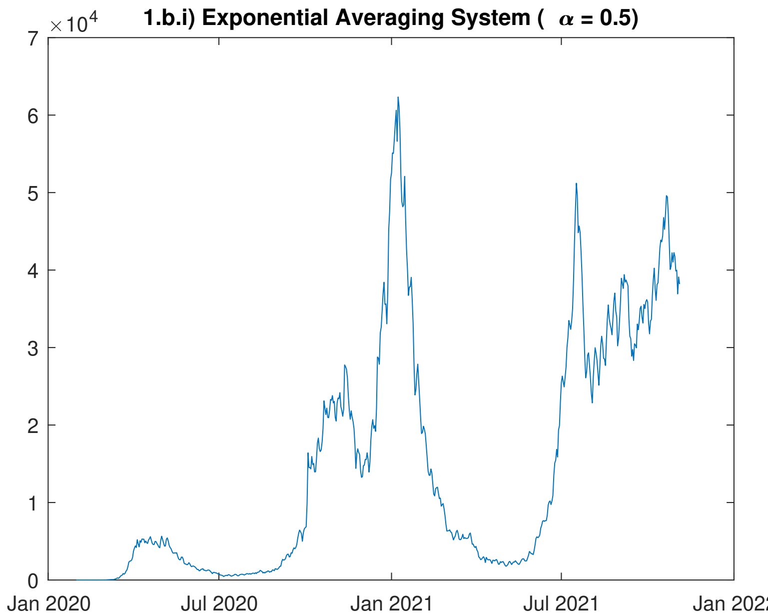
1.b.i) Exponential Averaging System

$y_n = alpha * x_n + (1 - alpha) * y_{n-1}$

$=> y_n - (1 - alpha) * y_{n-1} = alpha * x_n$

$=> [1, -(1 - alpha)].[y_n, y_{n-1}] = [alpha].[x_n]$

```
alpha = 0.5;
a=[1, -(1 - alpha)];
b=[alpha];
y = filter(b, a, x);
figure;
plot(newcases.date, y)
title("1.b.i) Exponential Averaging System ({\alpha = 0.5})");
```



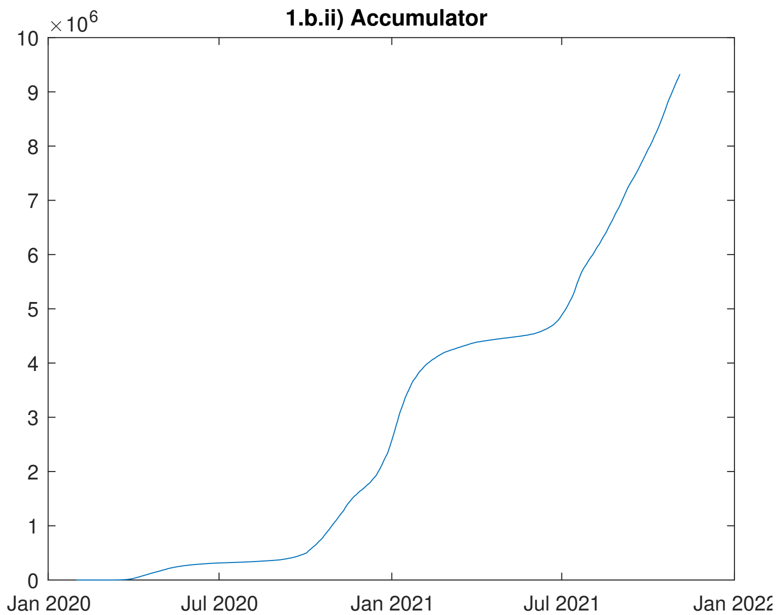
1.b.ii) Accumulator

$$y_n = x_n + y_{n-1}$$

$$=> y_n - y_{n-1} = x_n$$

$$=> [1, -1].[y_n, y_{n-1}] = [1].[x_n]$$

```
a=[1, -1];
b=[1];
y = filter(b, a, x);
figure;
plot(newcases.date, y)
title("1.b.ii) Accumulator");
```

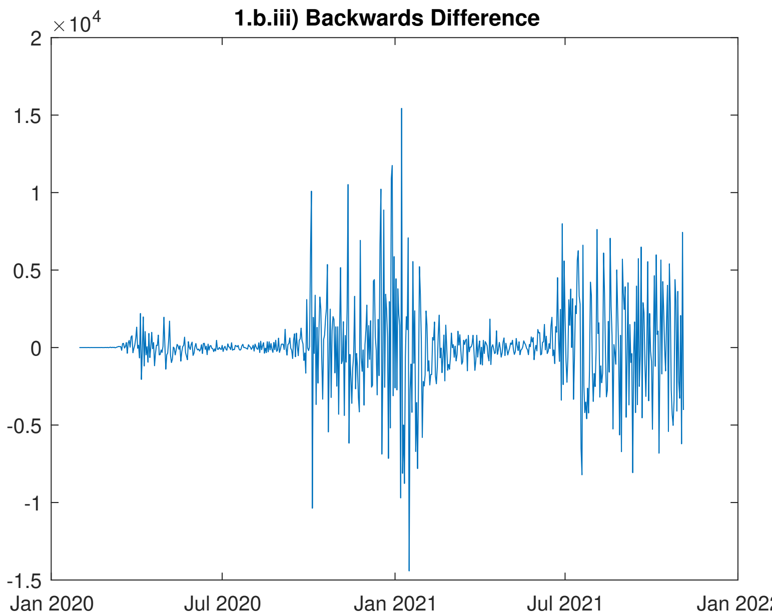


1.b.iii) Backwards Difference System

$$y_n = x_n - x_{n-1}$$

$$[1].[y_n] = [1, -1].[x_n, x_{n-1}]$$

```
a=[1];
b=[1, -1];
y = filter(b, a, x);
figure;
plot(newcases.date, y)
title("1.b.iii) Backwards Difference");
```



1.c)

Generate a one second long Gaussian noise sequence r with a sampling rate of 300Hz

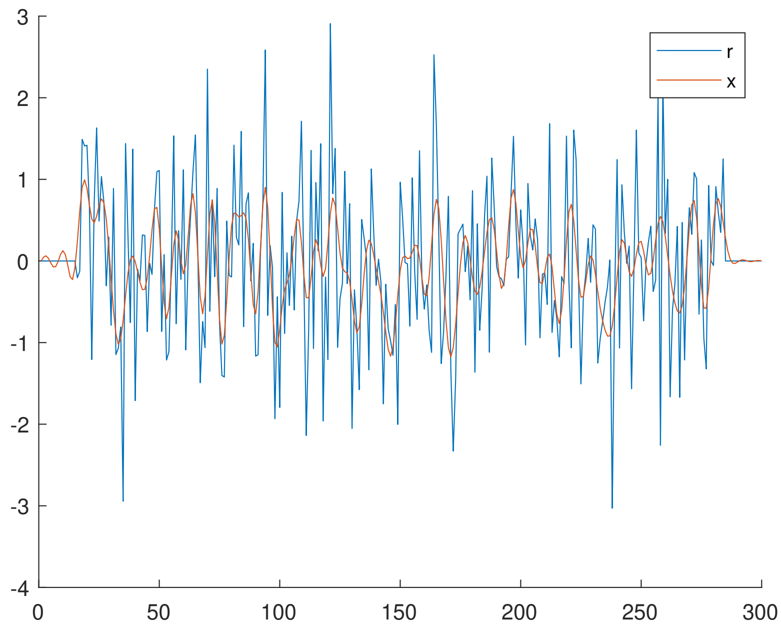
```
f_s = 300;
t = 1; % 1 second long
N = f_s * t;
ts = linspace(1/f_s, t, N);
r = randn(N,1);
% Taper $r$ by setting its first and last 15 samples to zero.
r(1:15) = 0;
r(end-15:end) = 0;
```

Make a Finite-Impulse Response low-pass filter with cut-off frequency

```
f_c = 45; % cutoff frequency
a = [1]; % FIR has no y-terms
b = fir1(50, f_c/(f_s/2));
% Use the filtfilt function in order to apply that filter to the
% generated noise signal, resulting in the filtered noise signal x
x = filtfilt(b, a, r);
```

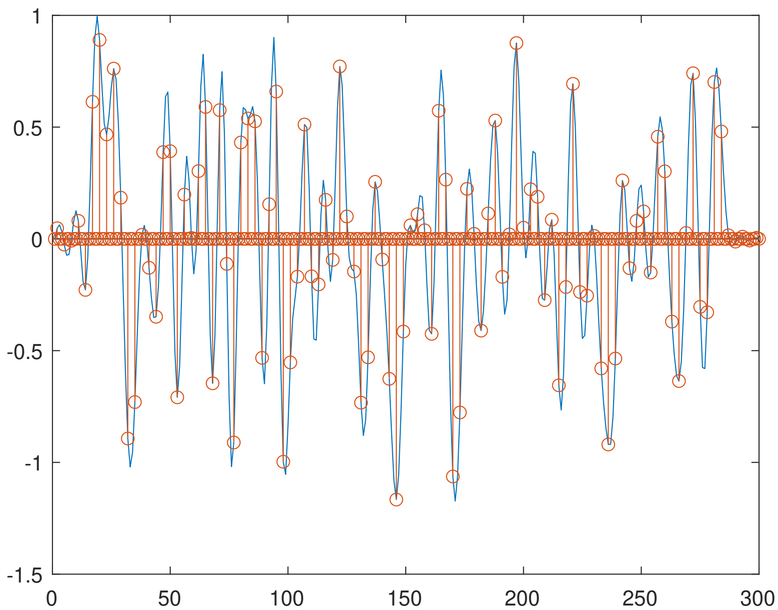
Plot r, x together.

```
figure;
hold on;
plot(r);
plot(x);
legend("r","x");
hold off;
```



Sample x at 100Hz by setting all but every third sample value to zero

```
y = x;
% element 1, 4, 7... = 0
y(1:3:end) = 0;
% element 2, 5, 8... = 0
y(3:3:end) = 0;
% element 3, 6, 9... = unchanged

figure;
plot(x);
hold on;
stem(y);
hold off;
```



Implement sinc interpolation to reconstruct the zeroed samples of y

$$x(t) = \sum_{n=-\infty}^{\infty} x_n * sinc(t/t_s - n)$$

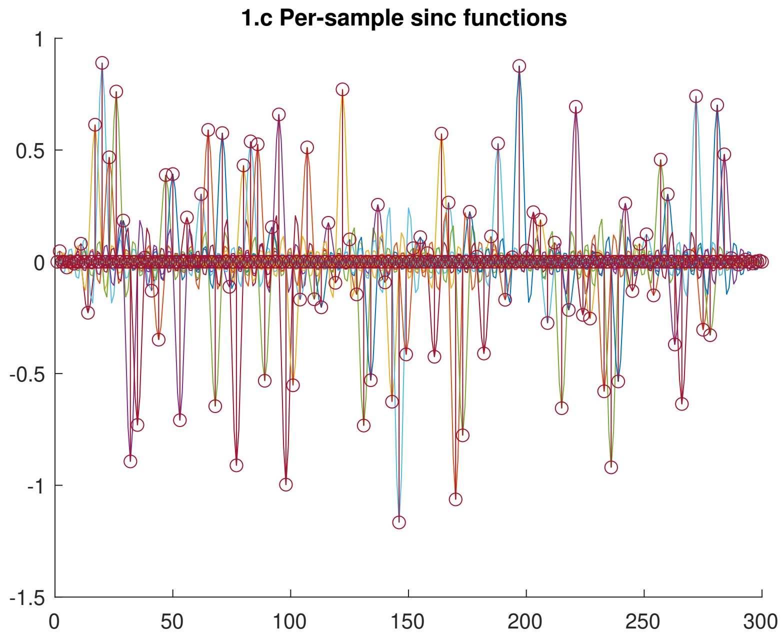
MATLAB version:

$$z = \sum_{i_y=1}^{N} y(i_y) * sinc((ts/t - i_y/N) * f_s/3)$$

Translate the sinc by i_y/N to align it with the sample, and scale it by f_s over 3 to align the zero crossings with the other sample points.

```
z = zeros(N,1);
figure;
hold on;
for i_y = 1:N
    data = y(i_y) * sinc((ts/t - (i_y)/N)* f_s/3);
    plot(data);
    z = z + data';
end
stem(y)
```

```
hold off;
title("1.c Per-sample sinc functions")
```



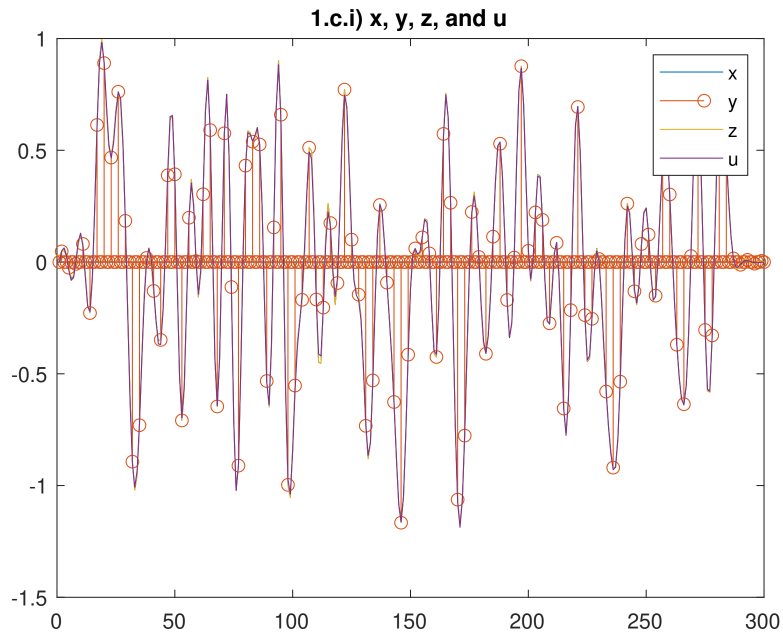
Generate another low-pass filter with fir1, cut-off frequency 50Hz

```
f_c = 50; % cutoff frequency
a = [1]; % FIR has no y-terms
b = fir1(50, f_c/(f_s/2));
% Apply it to y, resulting in interpolated sequence u.
% multiply by 3 to compensate for energy lost during sampling.
u = 3 * filtfilt(b, a, y);
```

1.c.i) Comparisons

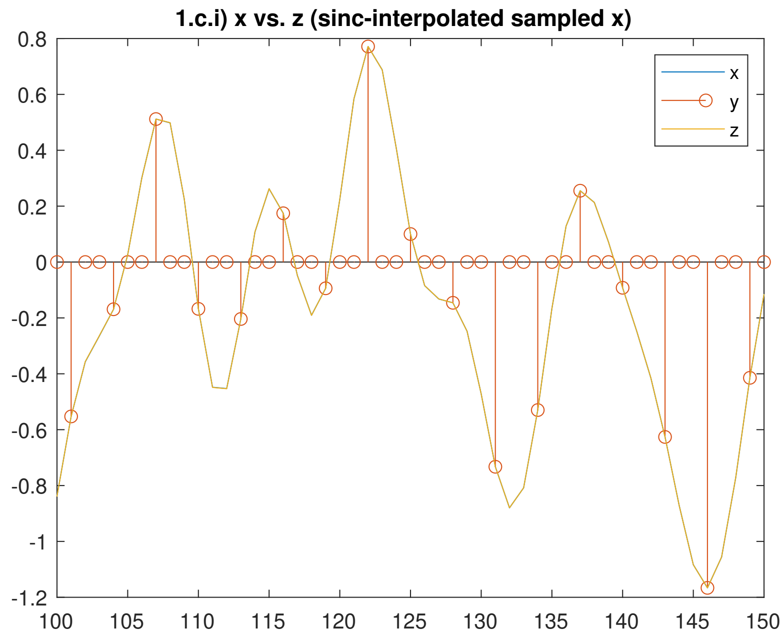
Plot x, y, z, and u on top of each other in one figure

```
figure;
plot(x);
hold on;
stem(y);
plot(z);
plot(u);
legend("x", "y", "z", "u");
title("1.c.i) x, y, z, and u")
hold off;
```



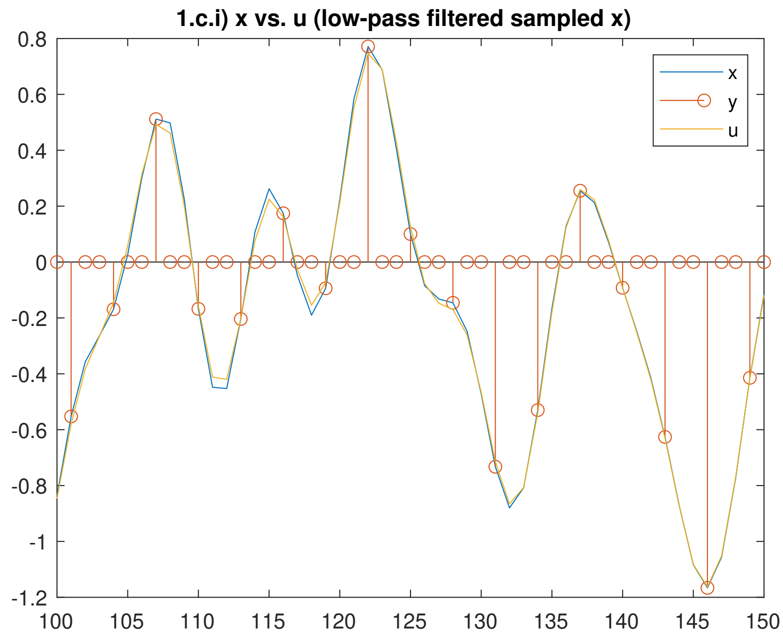
x and z are equal

```
figure;
plot(x);
hold on;
stem(y);
plot(z);

legend("x", "y", "z");
xlim([100 150]);
title("1.c.i) x vs. z (sinc-interpolated sampled x)");
hold off;
```



x and u are similar, but not equal. TODO - is this to do with aliasing introduced by sampling?

```
figure;
plot(x);
hold on;
stem(y);
plot(u);

legend("x", "y", "u");
xlim([100 150]);
title("1.c.i) x vs. u (low-pass filtered sampled x)");
hold off;
```



1.c.ii)

Q: Why should the first filter have a lower cut-off frequency than the second?

If the second low-pass filter had a lower cut-off frequency than the first filter, it would discard higher-frequency information and not reconstruct it.%% 1.d) Simulate the reconstruction of a sampled band-pass signal

Generate a 1 s noise sequence r, as in part (c)(i), but this time use a sampling frequency of 3 kHz. Set the first and last 500 samples to zero.

```
f_s = 3000;
t = 1; % 1 second long
N = f_s * t;
ts = linspace(1/f_s, t, N);
r = randn(N,1);
r(1:500) = 0;
r(end-500:end) = 0;
```

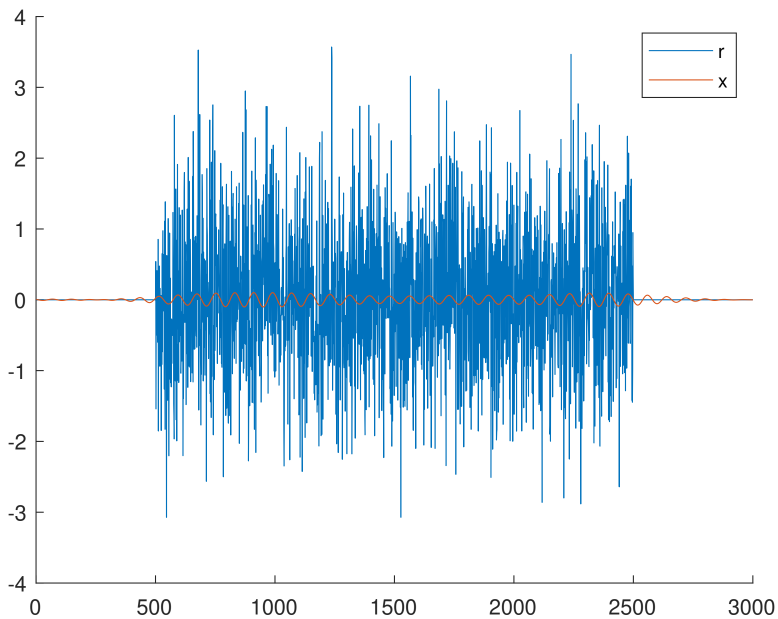
Apply a band-pass filter that attenuates frequencies outside 31-44Hz

```
f1 = 31;
f2 = 44;
% 3 = 3rd order filter
% 30 = -30dB for frequencies outside the range
```

```
[b, a] = cheby2(3, 30, [f1 f2]/(f_s/2));
% Apply the filter
x = filtfilt(b, a, r);
```

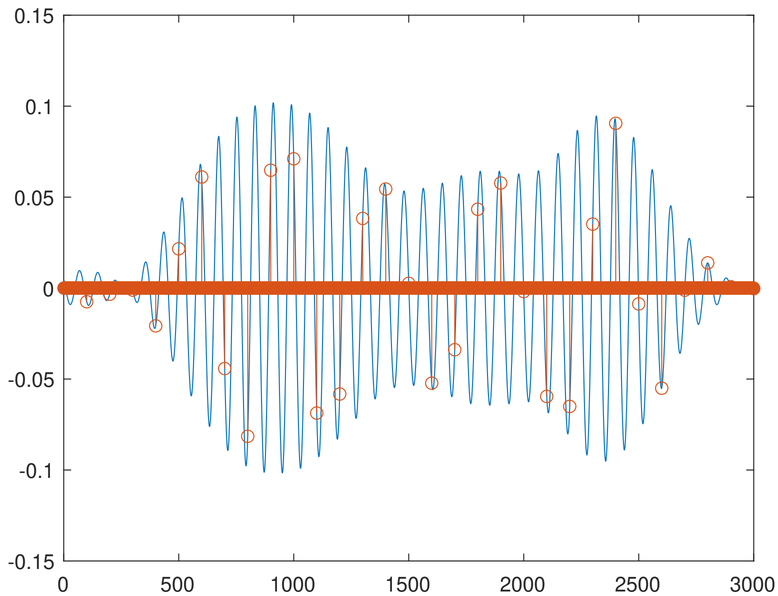
Plot r, x together. r is very high frequency, so the band-pass will remove a lot and x will have a much lower amplitiude

```
figure;
hold on;
plot(r);
plot(x);
legend("r","x");
hold off;
```



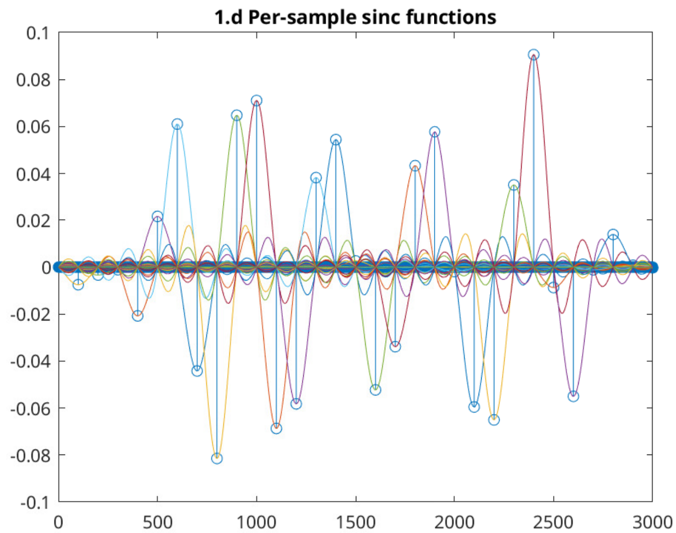
Sample x at 30Hz, set all but every 100th value to 0

```
y = x;
for i = 1:99
    y(i:100:end) = 0;
end

figure;
plot(x);
hold on;
stem(y);
hold off;
```



Reconstruct y with sinc interpolation (see 1.c.i for working) Change the scaling factor to 1/100 instead of 1/3, because we sampled every 100th value

```
z = zeros(N,1);
figure;
stem(y)
hold on;
for i_y = 1:N
    data = y(i_y) * sinc((ts/t - i_y/N) * f_s/100);
    plot(data);
    z = z + data';
end
hold off;
title("1.d Per-sample sinc functions")
```



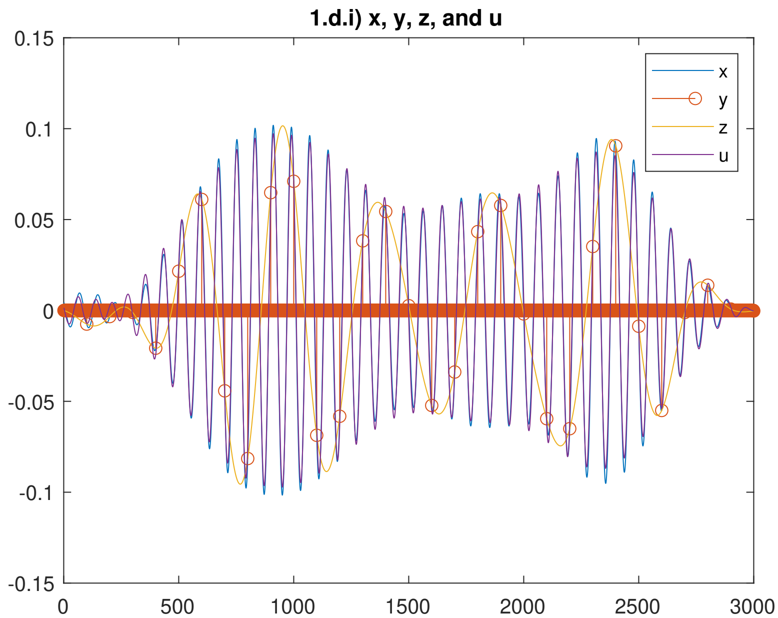
Generate another band-pass filter for 30-45Hz, apply to y to reconstruct as u. Multiply by 100 to compensate for energy lost during sampling.

```
f1 = 30;
f2 = 45;
% 3 = 3rd order filter
% 30 = -30dB for frequencies outside the range
[b, a] = cheby2(3, 30, [f1 f2]/(f_s/2));
u = 100 * filtfilt(b, a, y);
```

1.d.i) Comparisons

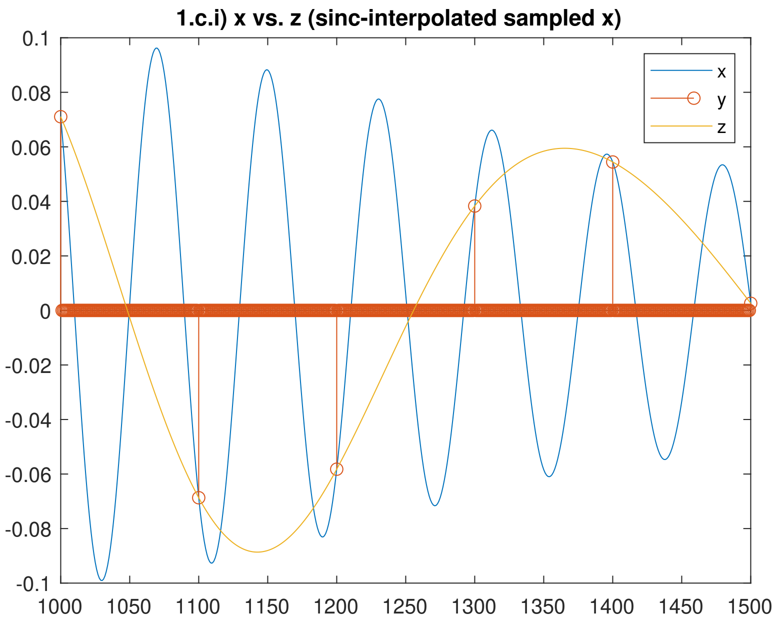
Plot x, y, z, and u on top of each other in one figure

```
figure;
plot(x);
hold on;
stem(y);
plot(z);
plot(u);
legend("x", "y", "z", "u");
title("1.d.i) x, y, z, and u")
hold off;
```



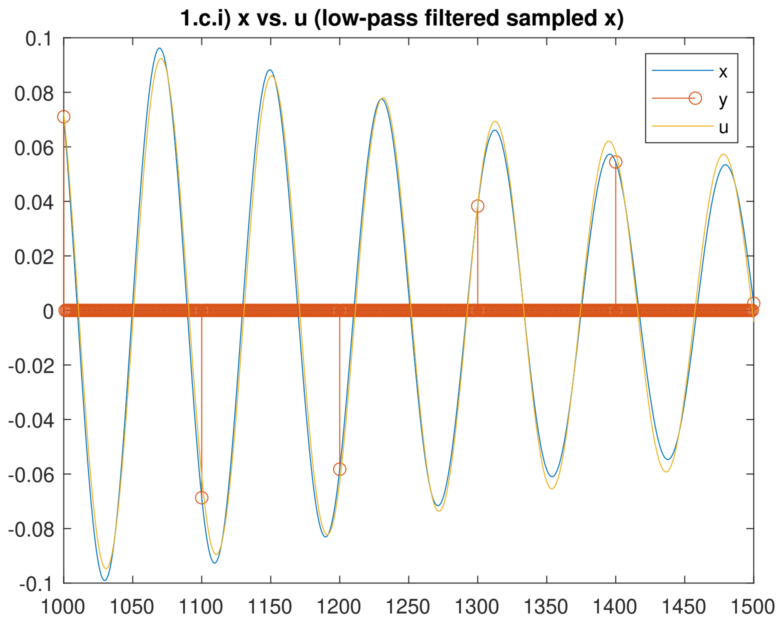
x and z are not equal - sinc interpolation can only reconstruct frequencies up to $f_{sample}/2 = 30/2 = 15Hz$, but x contains higher-frequency data.

```
figure;
plot(x);
hold on;
stem(y);
plot(z);

legend("x", "y", "z");
xlim([1000 1500]);
title("1.c.i) x vs. z (sinc-interpolated sampled x)");
hold off;
```



x and u are similar, but not equal. TODO - is this to do with aliasing introduced by sampling?

```
figure;
plot(x);
hold on;
stem(y);
plot(u);

legend("x", "y", "u");
xlim([1000 1500]);
title("1.c.i) x vs. u (low-pass filtered sampled x)");
hold off;
```



1.d.ii)

Q: Why does the reconstructed waveform differ much more from the original if you reduce the cut-off frequencies of all band-pass filters by 5 Hz?

TODO