

# Hardware for Machine Learning

## Lecture 5: DNN Kernel Computation

Sophia Shao



Ian Buck

I have completed my Ph.D. at Stanford and currently work at NVIDIA  
LinkedIn Profile: <http://www.linkedin.com/pub/ian-buck/15/13/192>  
Contact Info:



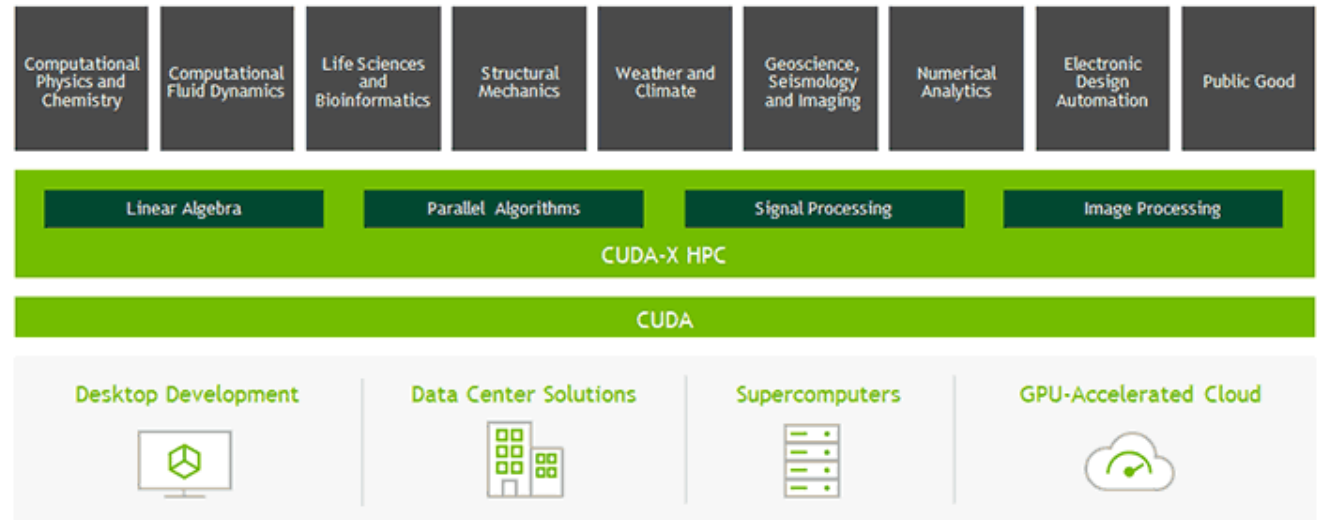
408-486-2000  
2701 San Tomas Expressway  
Santa Clara, CA 95050  
[ibuck@nvidia.com](mailto:ibuck@nvidia.com)

### BrookGPU



BrookGPU is a compiler and runtime implementation of the Brook stream programming language which provides an easy, C-like programming environment for today's GPU. As the programmability and performance of modern GPUs continues to increase, many researchers are looking to graphics hardware to solve problems previously performed on general purpose CPUs. In many cases, performing general purpose computation on graphics hardware can provide a significant advantage over implementations on traditional CPUs.

## CUDA (Compute Unified Device Architecture)



# Review

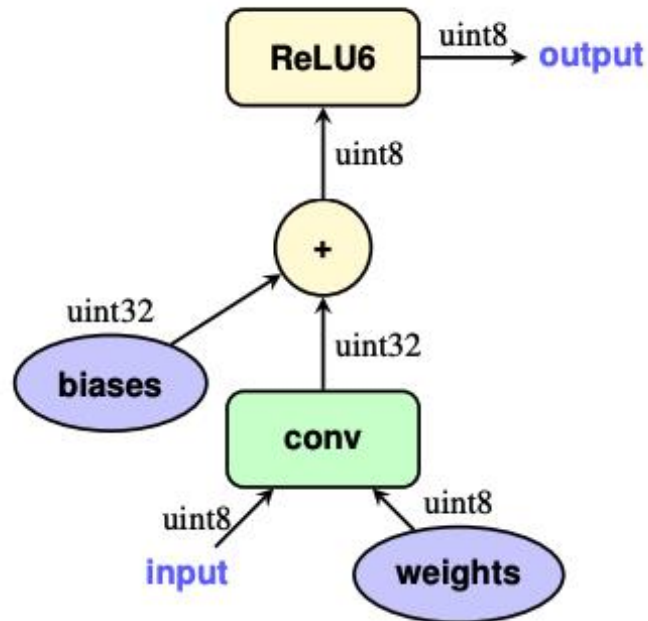
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- AlexNet's cost function and optimization function
- Floating-point and fixed-point representations
- Hardware implications:
  - Fewer # of bits -> Energy/storage efficiency
- DNN Quantization
  - Using the “slope and bias” of fixed-point representation:  $y = s \cdot x + z$ 
    - Scaling factor
      - How to scale? How to choose threshold value?
    - Zero point
  - Post-training quantization vs Quantization-aware training
  - State-of-the-art hardware support for low-precision DNNs

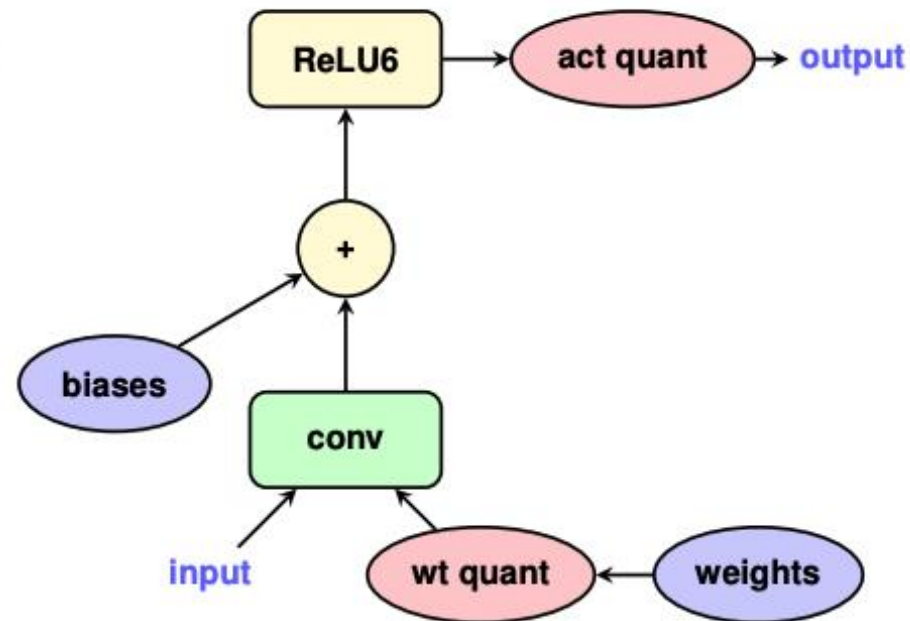


# Quantization-Aware Training

- Typically performs better than post-training quantization
- “Simulate” quantization effects in the forward pass
- Weights and biases are updated in floating point during backpropagation so that they can be nudged by small amounts.



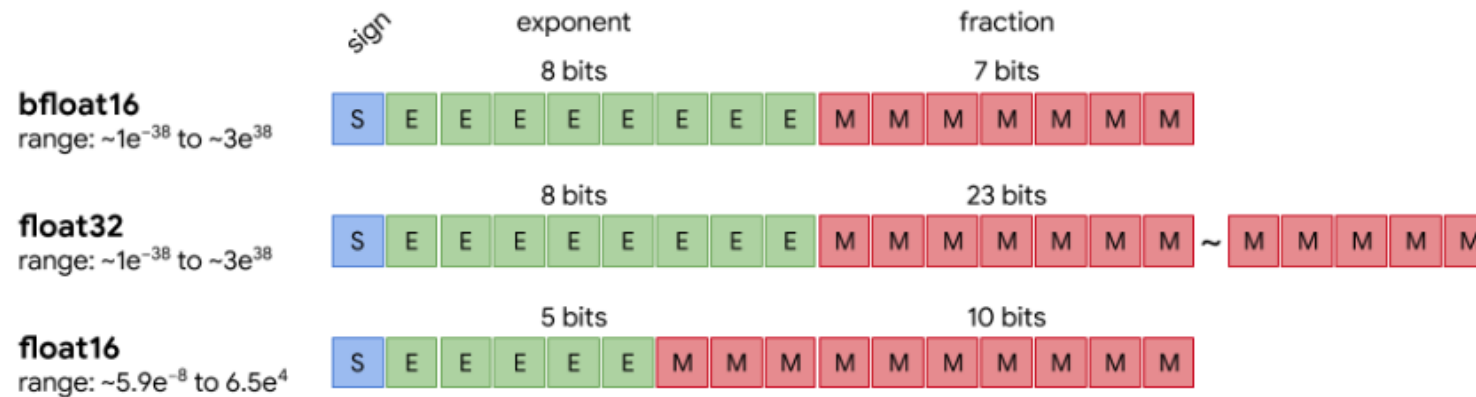
(a) Integer-arithmetic-only inference



(b) Training with simulated quantization

# Bfloat16 for Google's Tensor Processing Unit

- fp32 - IEEE single-precision floating-point
- fp16 - IEEE half-precision floating point
- bfloat16 - 16-bit *brain* floating point

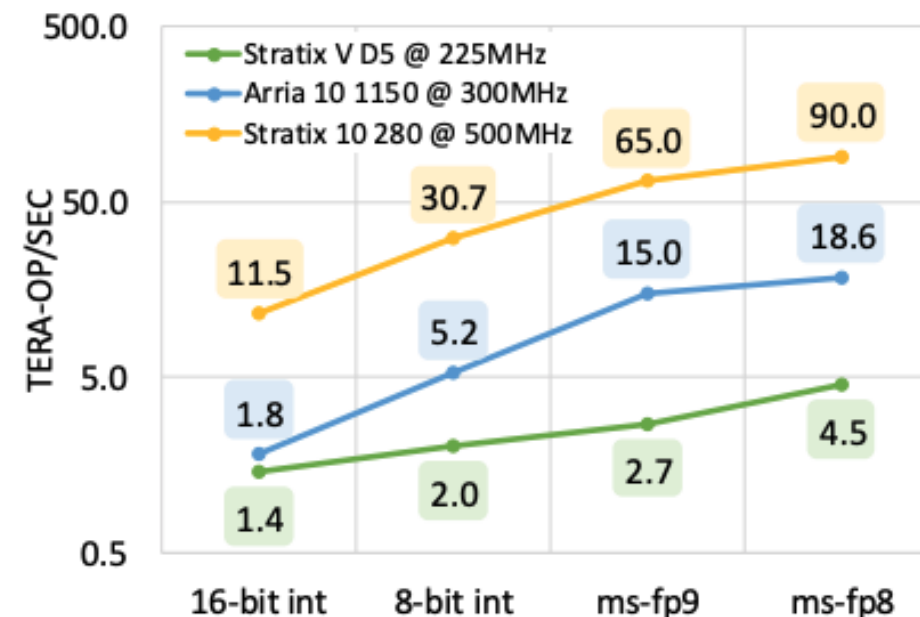


<https://cloud.google.com/tpu/docs/bfloat16>



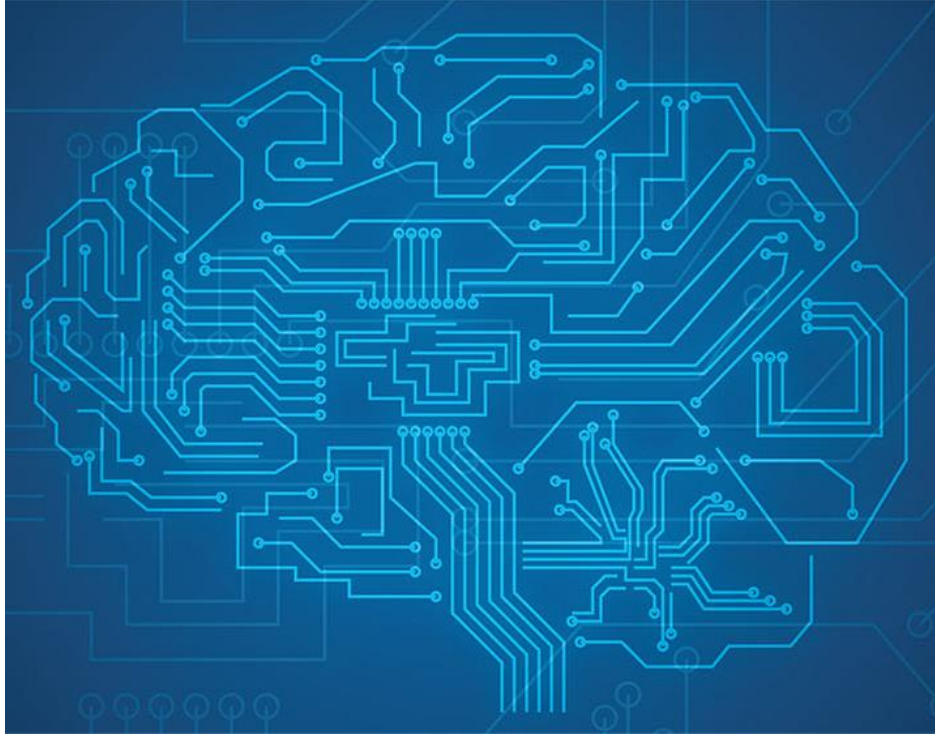
# MS-FP in Brainwave FPGA @ Microsoft

- “ ‘neural’-optimized data formats based on 8- and 9-bit floating point, where mantissas are trimmed to 2 or 3 bits. “
- “ These formats, referred to as ms-fp8 and ms-fp9, exploit efficient packing into reconfigurable resources and are comparable in FPGA area”



Serving DNNs in Real Time at Datacenter Scale with Project Brainwave





# DNN Kernels

- Overview
- Convolution
  - Basics
  - Transformation
- Pooling
- BatchNorm



# AlexNet Model

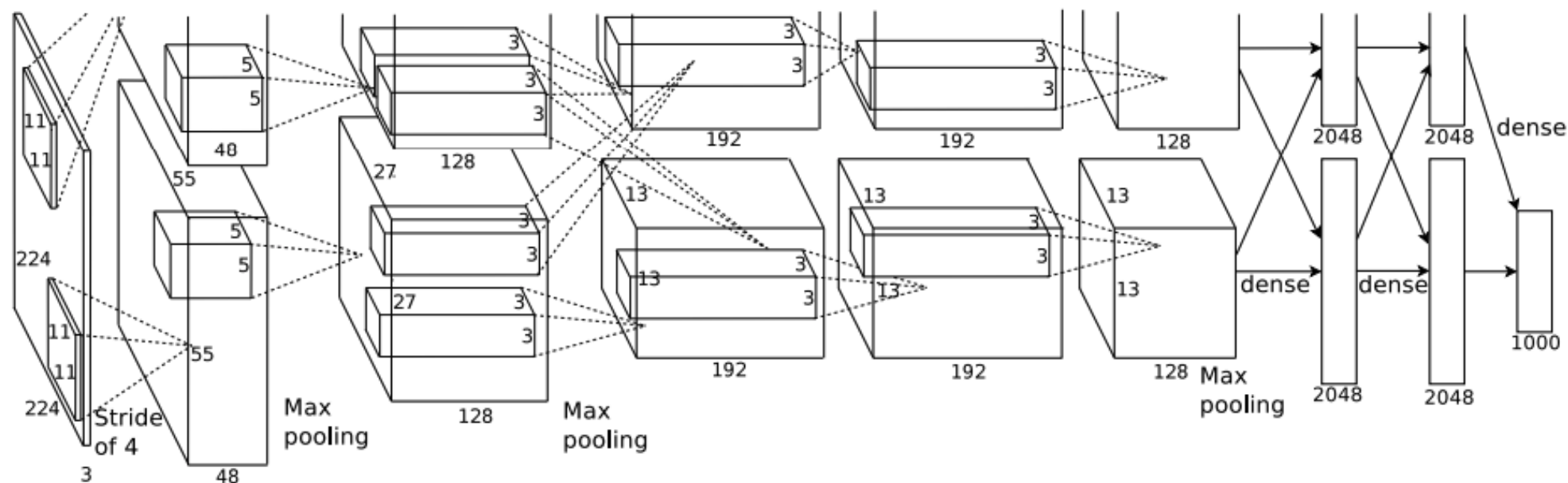


Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network's input is 150,528-dimensional, and the number of neurons in the network's remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.



# Convolutional Neural Networks Everywhere

Classification



Figures copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton. 2012. Reproduced with permission.

Retrieval



No errors

Minor errors

Somewhat related

## Image Captioning

[Vinyals et al., 2015]  
[Karpathy and Fei-Fei, 2015]



A white teddy bear sitting in the grass



A man in a baseball uniform throwing a ball



A woman is holding a cat in her hand



A man riding a wave on top of a surfboard



A cat sitting on a suitcase on the floor

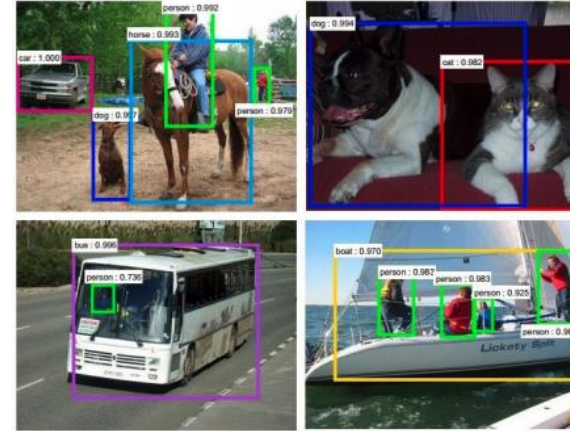


A woman standing on a beach holding a surfboard

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Captions generated by Justin Johnson using [NeuralTalk2](#)

Detection



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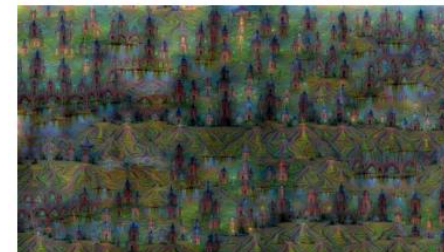
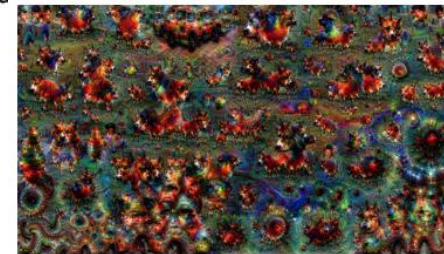
[Faster R-CNN: Ren, He, Girshick, Sun 2015]

Segmentation



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[Farabet et al., 2012]



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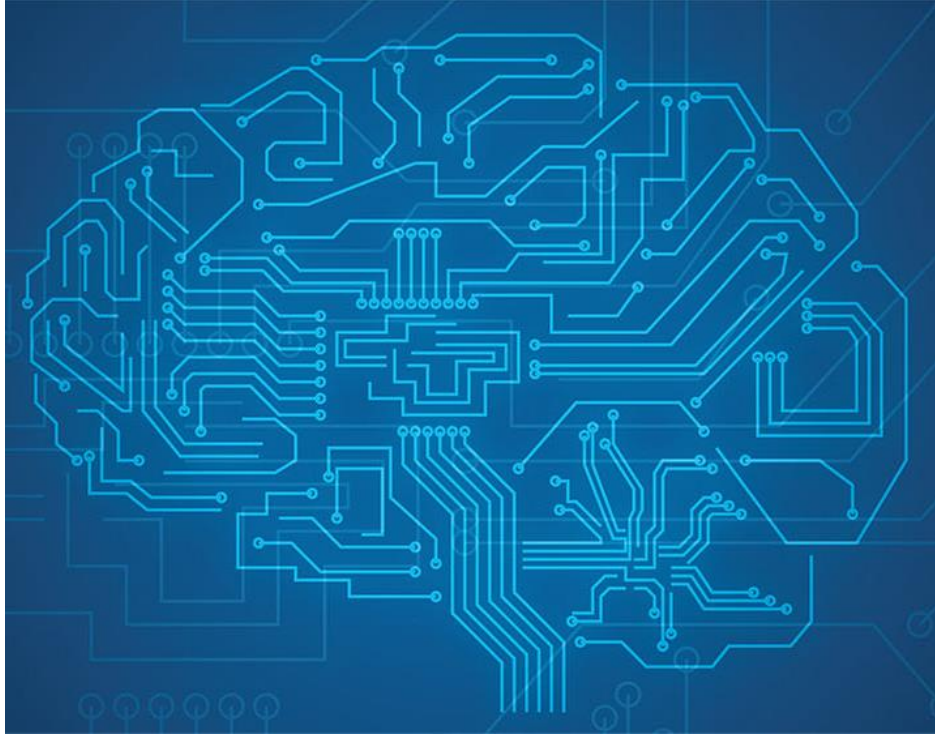


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Gatys et al. "Image Style Transfer using Convolutional Neural Networks", CVPR 2016  
Gatys et al. "Controlling Perceptual Factors in Neural Style Transfer", CVPR 2017

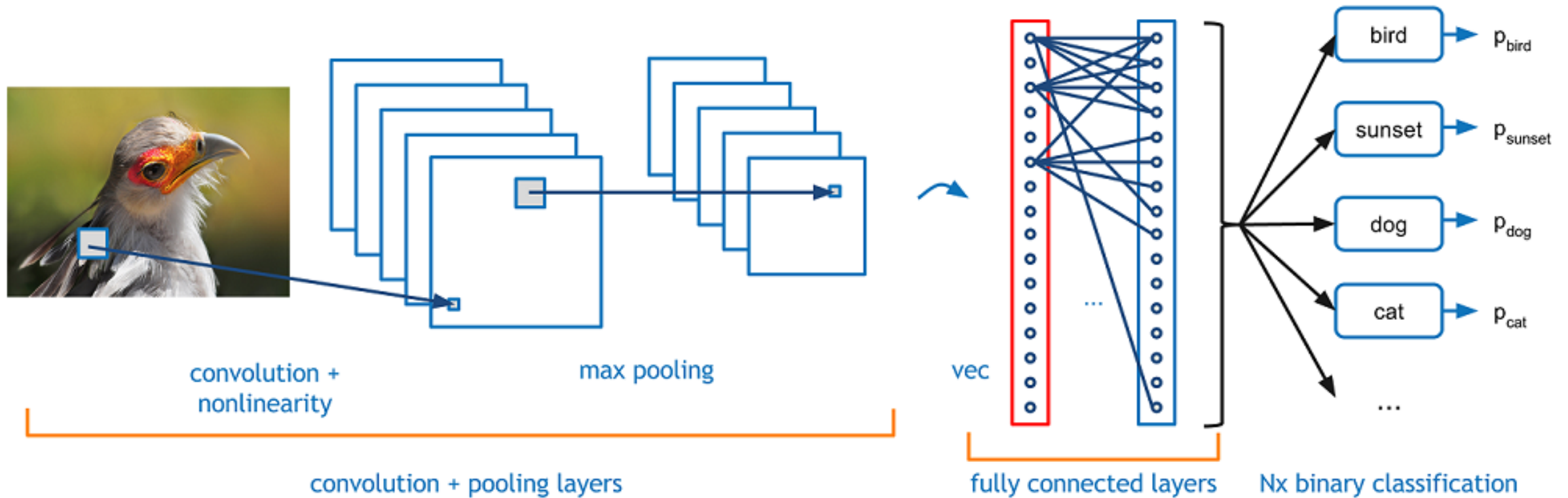




# DNN Kernels

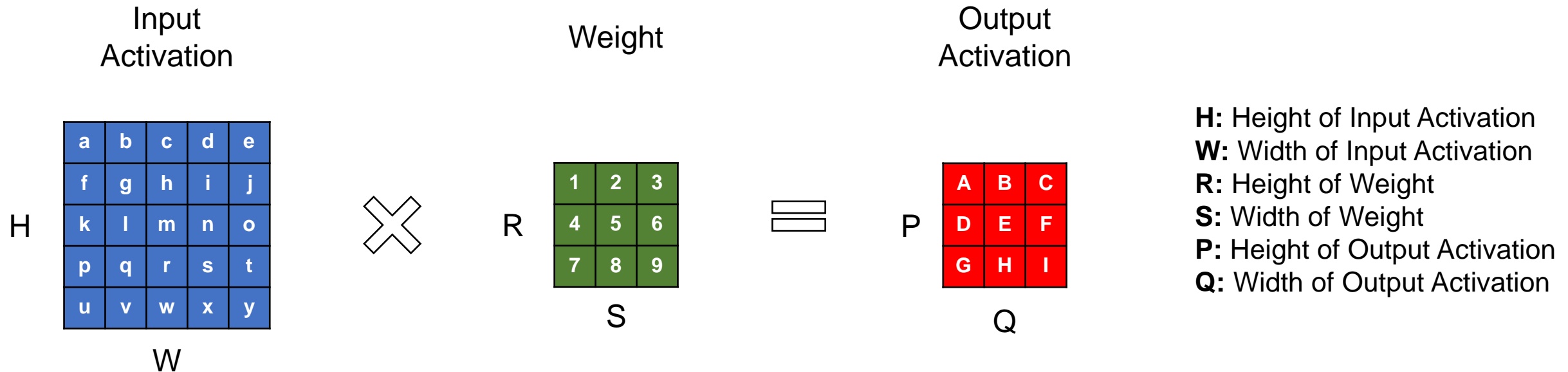
- Overview
- **Convolution**
  - Basics
  - Transformation
- Pooling
- BatchNorm

# Convolutional Neural Nets

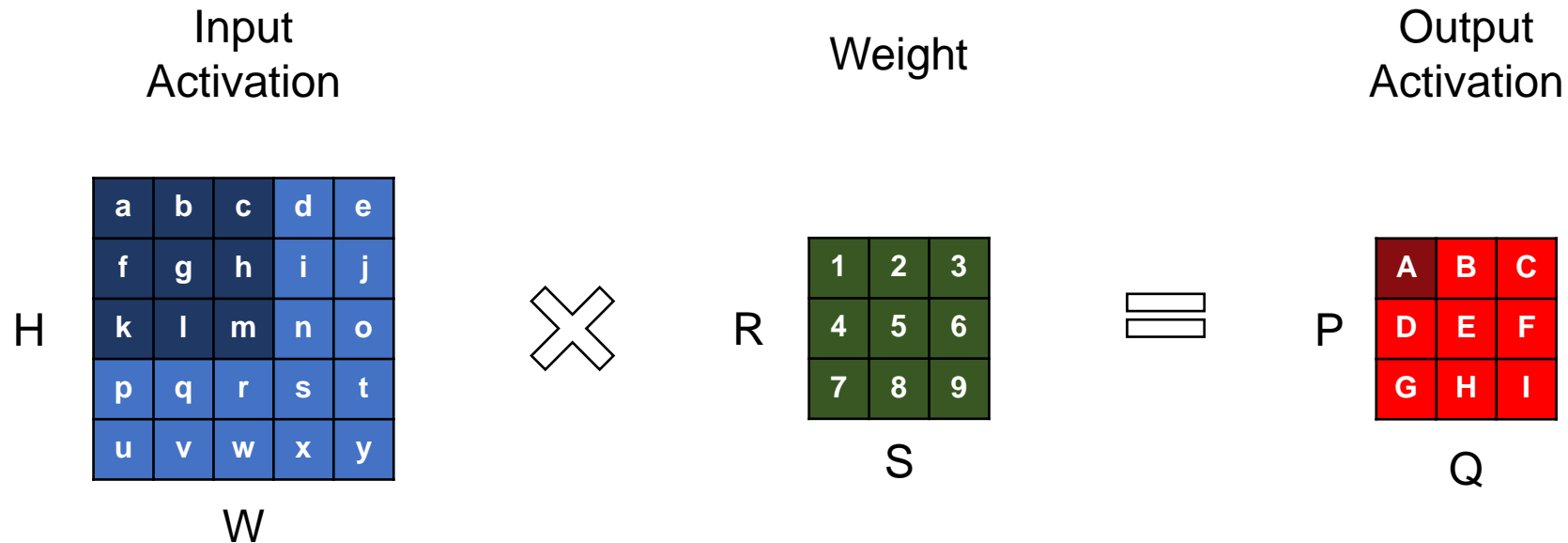


[https://github.com/vdumoulin/conv\\_arithmetic](https://github.com/vdumoulin/conv_arithmetic)

# 2-D Convolution



# 2-D Convolution



**H:** Height of Input Activation

**W:** Width of Input Activation

**R:** Height of Weight

**S:** Width of Weight

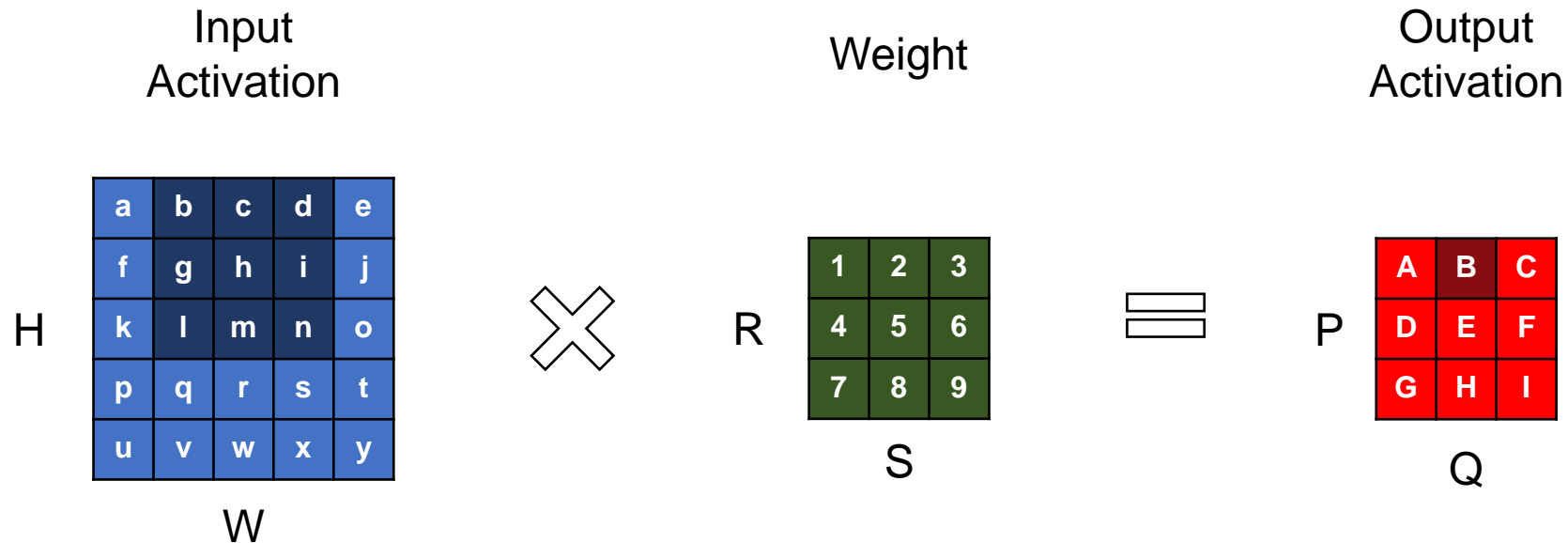
**P:** Height of Output Activation

**Q:** Width of Output Activation

$$\begin{aligned} A &= a * 1 + b * 2 + c * 3 \\ &\quad + f * 4 + g * 5 + h * 6 \\ &\quad + k * 7 + l * 8 + m * 9 \end{aligned}$$

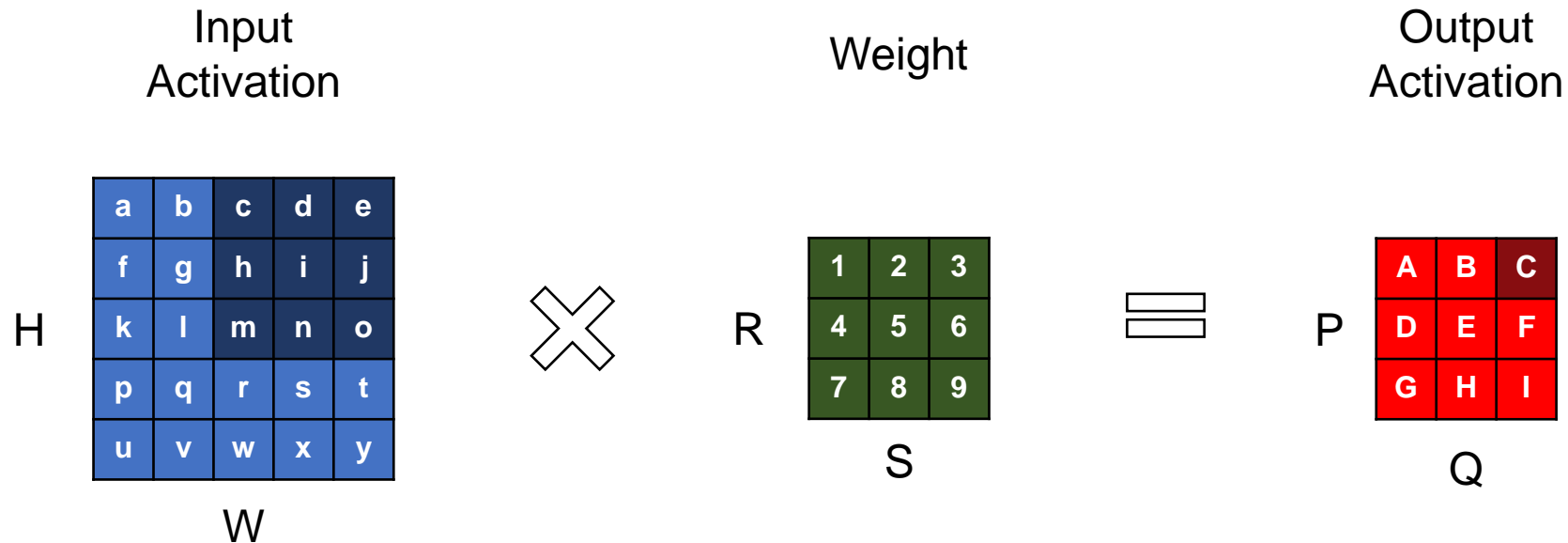


# 2-D Convolution (stride = 1)



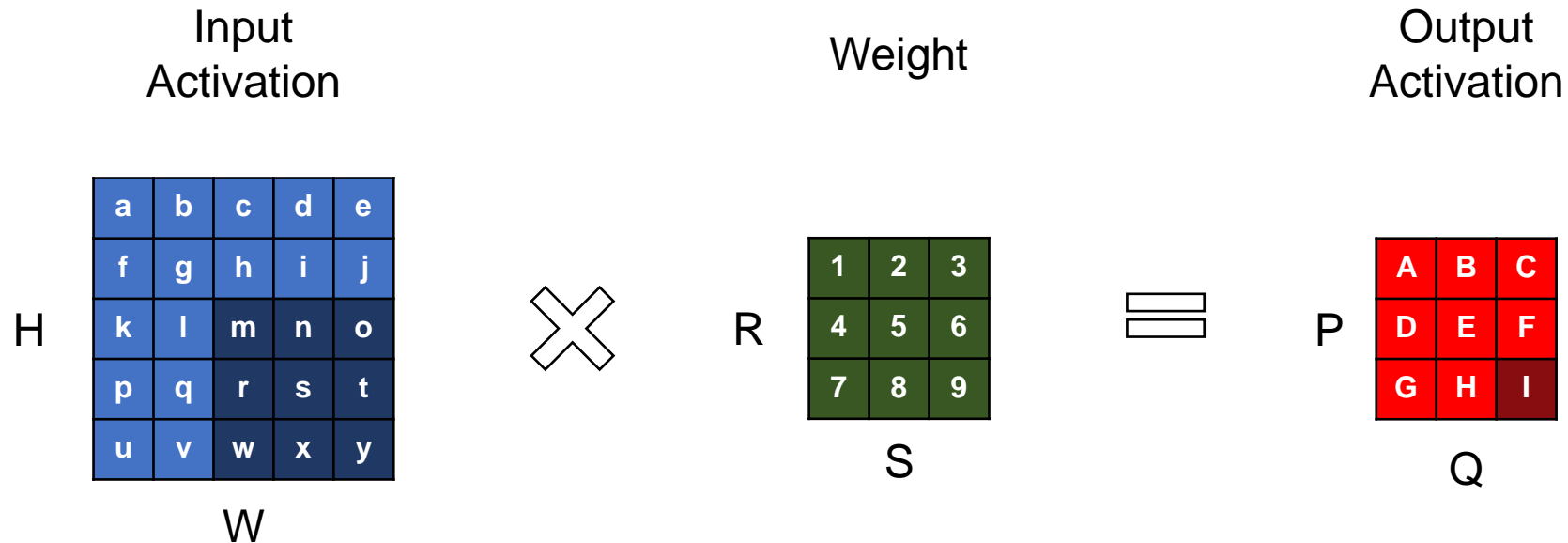
**H:** Height of Input Activation  
**W:** Width of Input Activation  
**R:** Height of Weight  
**S:** Width of Weight  
**P:** Height of Output Activation  
**Q:** Width of Output Activation  
**stride:** # of rows/columns traversed per step

# 2-D Convolution (stride = 1)



**H:** Height of Input Activation  
**W:** Width of Input Activation  
**R:** Height of Weight  
**S:** Width of Weight  
**P:** Height of Output Activation  
**Q:** Width of Output Activation  
**stride:** # of rows/columns traversed per step

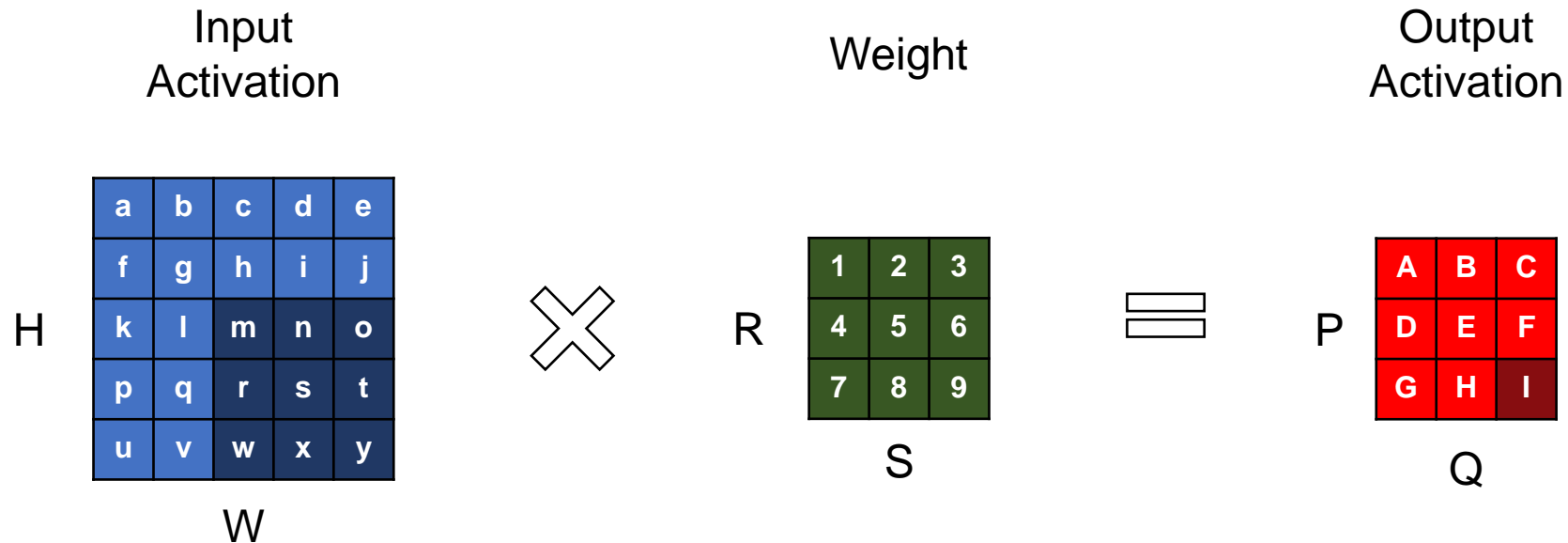
# 2-D Convolution (stride = 1)



**H:** Height of Input Activation  
**W:** Width of Input Activation  
**R:** Height of Weight  
**S:** Width of Weight  
**P:** Height of Output Activation  
**Q:** Width of Output Activation  
**stride:** # of rows/columns traversed per step

$$I = m * 1 + n * 2 + o * 3 \\ + r * 4 + s * 5 + t * 6 \\ + w * 7 + x * 8 + y * 9$$

# 2-D Convolution (stride = 1, valid conv.)



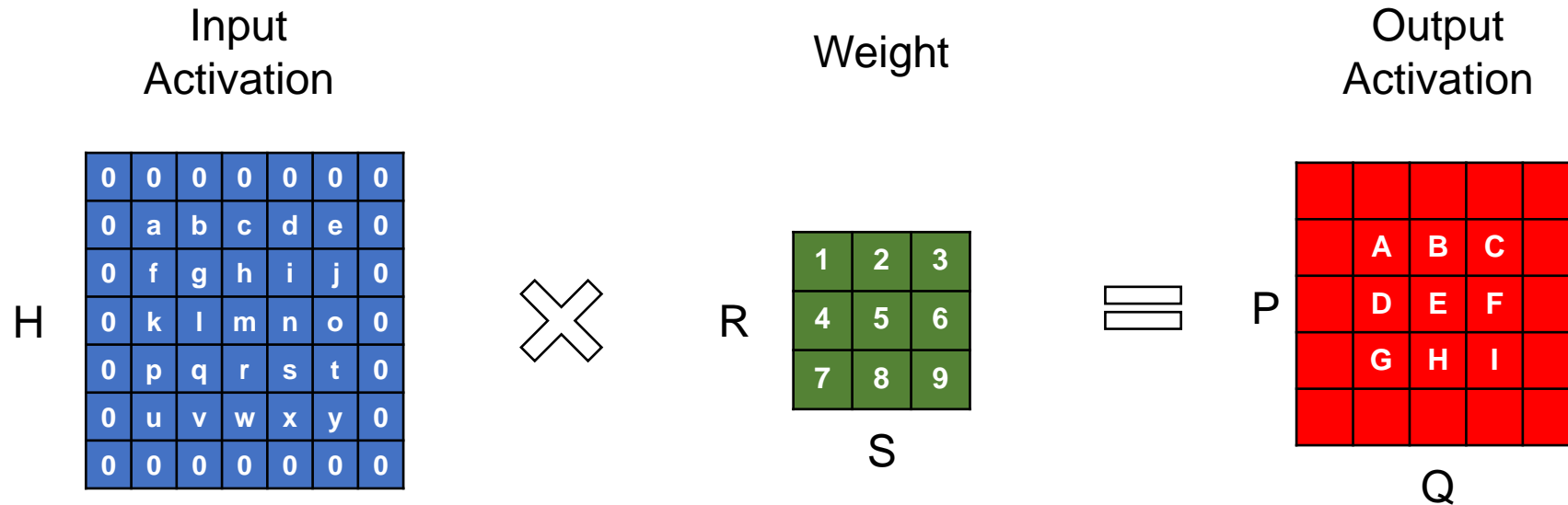
**H:** Height of Input Activation  
**W:** Width of Input Activation  
**R:** Height of Weight  
**S:** Width of Weight  
**P:** Height of Output Activation  
**Q:** Width of Output Activation  
**stride:** # of rows/columns traversed per step

$$P = \frac{(H - R)}{\text{stride}} + 1$$

$$Q = \frac{(W - S)}{\text{stride}} + 1$$



# 2-D Convolution (stride = 1, padding = 1)

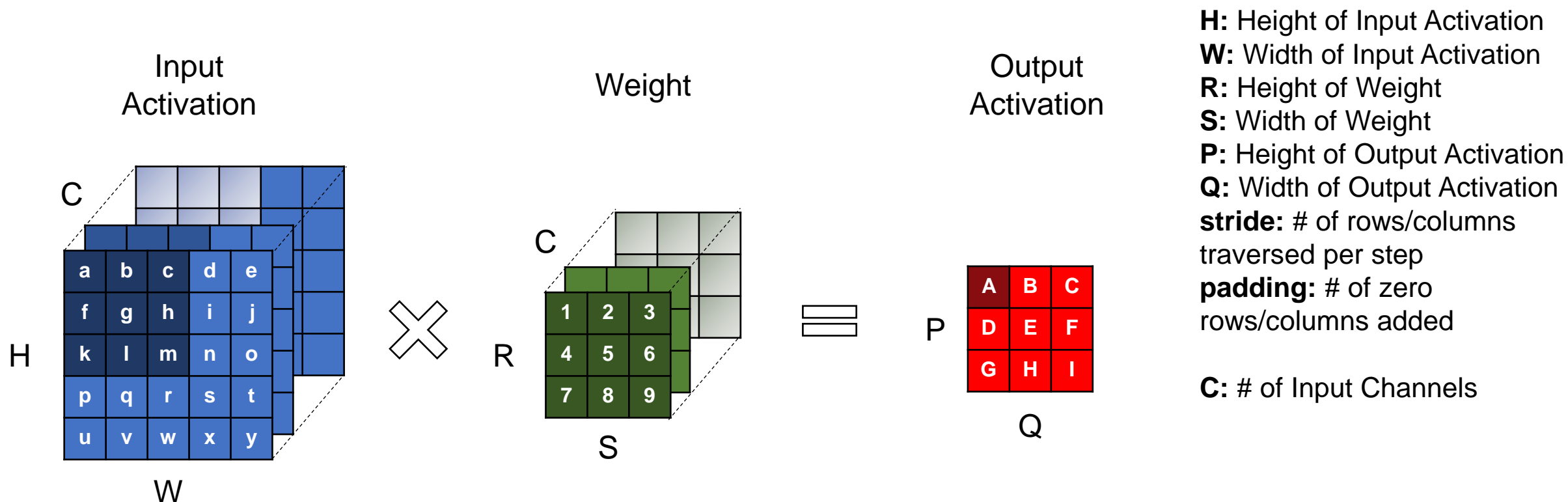


$$P = \frac{(H - R + 2 * pad)}{stride} + 1$$

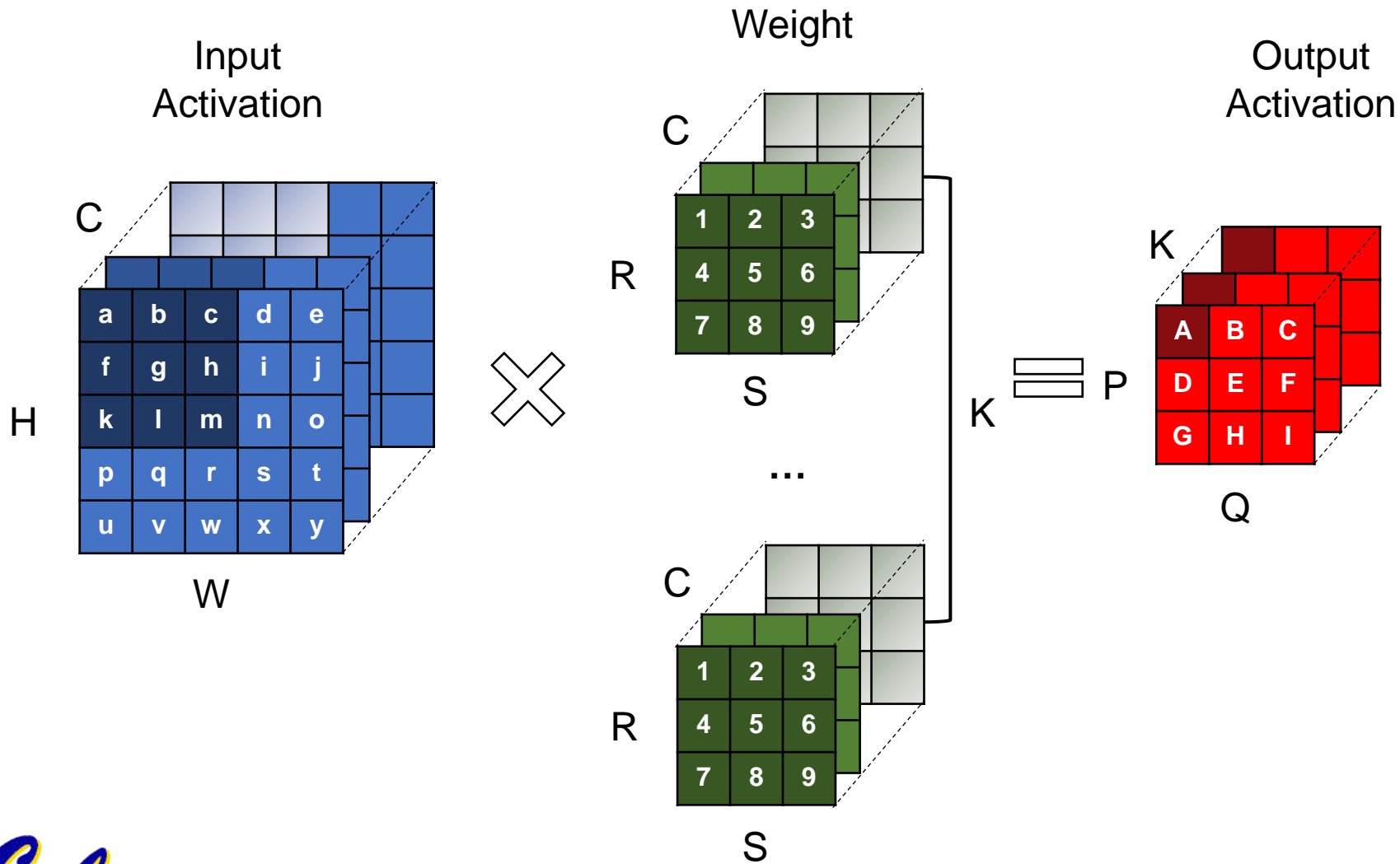
$$Q = \frac{(W - S + 2 * pad)}{stride} + 1$$

**H:** Height of Input Activation  
**W:** Width of Input Activation  
**R:** Height of Weight  
**S:** Width of Weight  
**P:** Height of Output Activation  
**Q:** Width of Output Activation  
**stride:** # of rows/columns traversed per step  
**padding:** # of zero rows/columns added

# 3-D Convolution



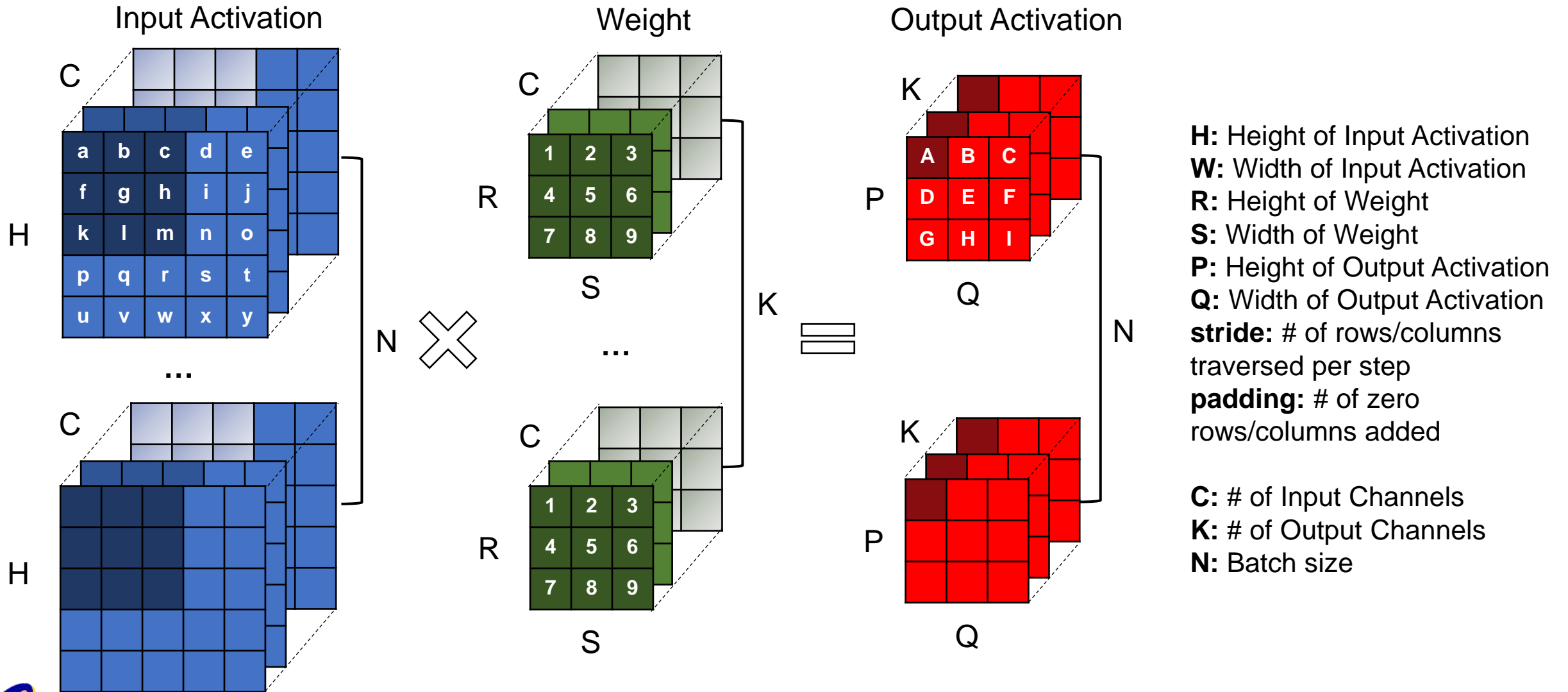
# 3-D Convolution



**H:** Height of Input Activation  
**W:** Width of Input Activation  
**R:** Height of Weight  
**S:** Width of Weight  
**P:** Height of Output Activation  
**Q:** Width of Output Activation  
**stride:** # of rows/columns traversed per step  
**padding:** # of zero rows/columns added

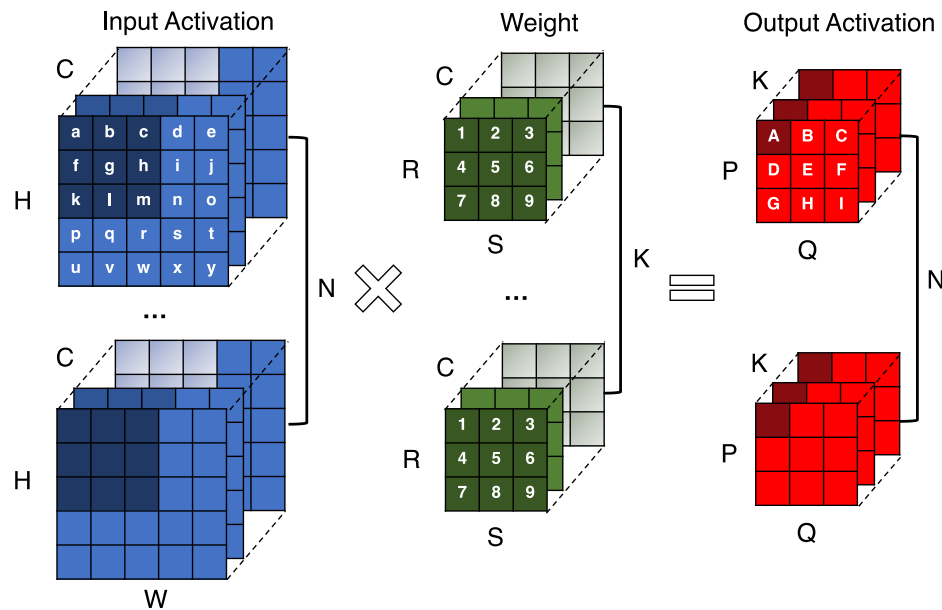
**C:** # of Input Channels  
**K:** # of Output Channels

# 3-D Convolution





# Convolution Loop Nest



```

for (n=0; n<N; n++) {
    for (k=0; k<K; k++) {
        for (p=0; p<P; p++) {
            for (q=0; q<Q; q++) {
                OA[n][k][p][q] = 0;
                for (r=0; r<R; r++) {
                    for (s=0; s<S; s++) {
                        for (c=0; c<C; c++) {
                            h = p * stride - pad + r;
                            w = q * stride - pad + s;
                            OA[n][k][p][q] +=
                                IA[n][c][h][w]
                                * W[k][c][r][s];
                        }
                    }
                }
                OA[n][k][p][q] = Activation(OA[n][k][p][q]);
            }
        }
    }
}

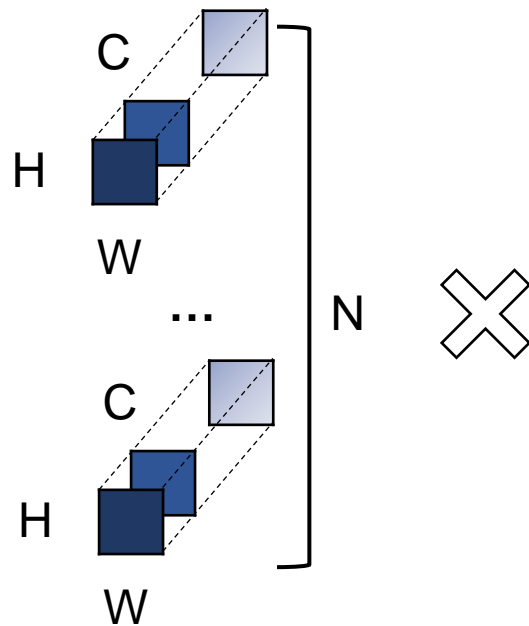
```

for each output activation

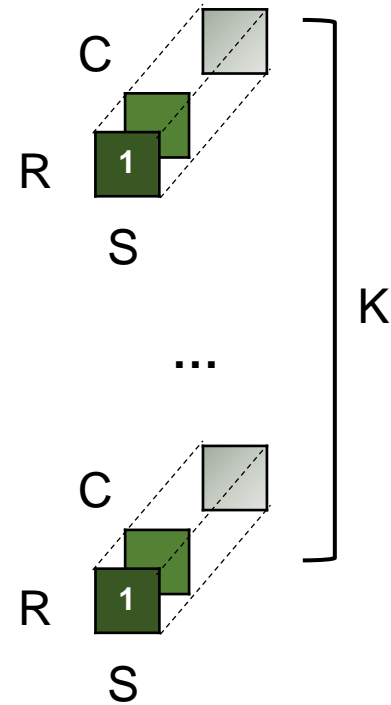
convolution window

# Fully-Connected Layer

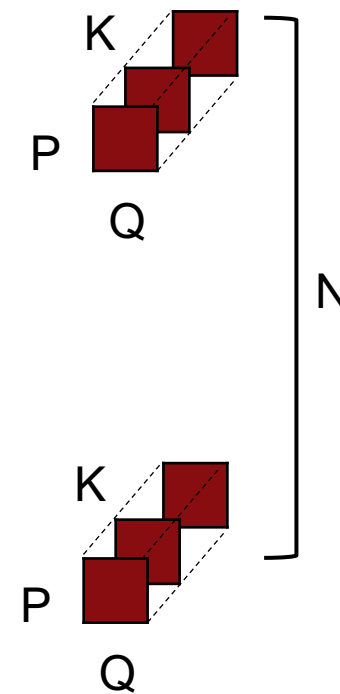
Input Activation



Weight



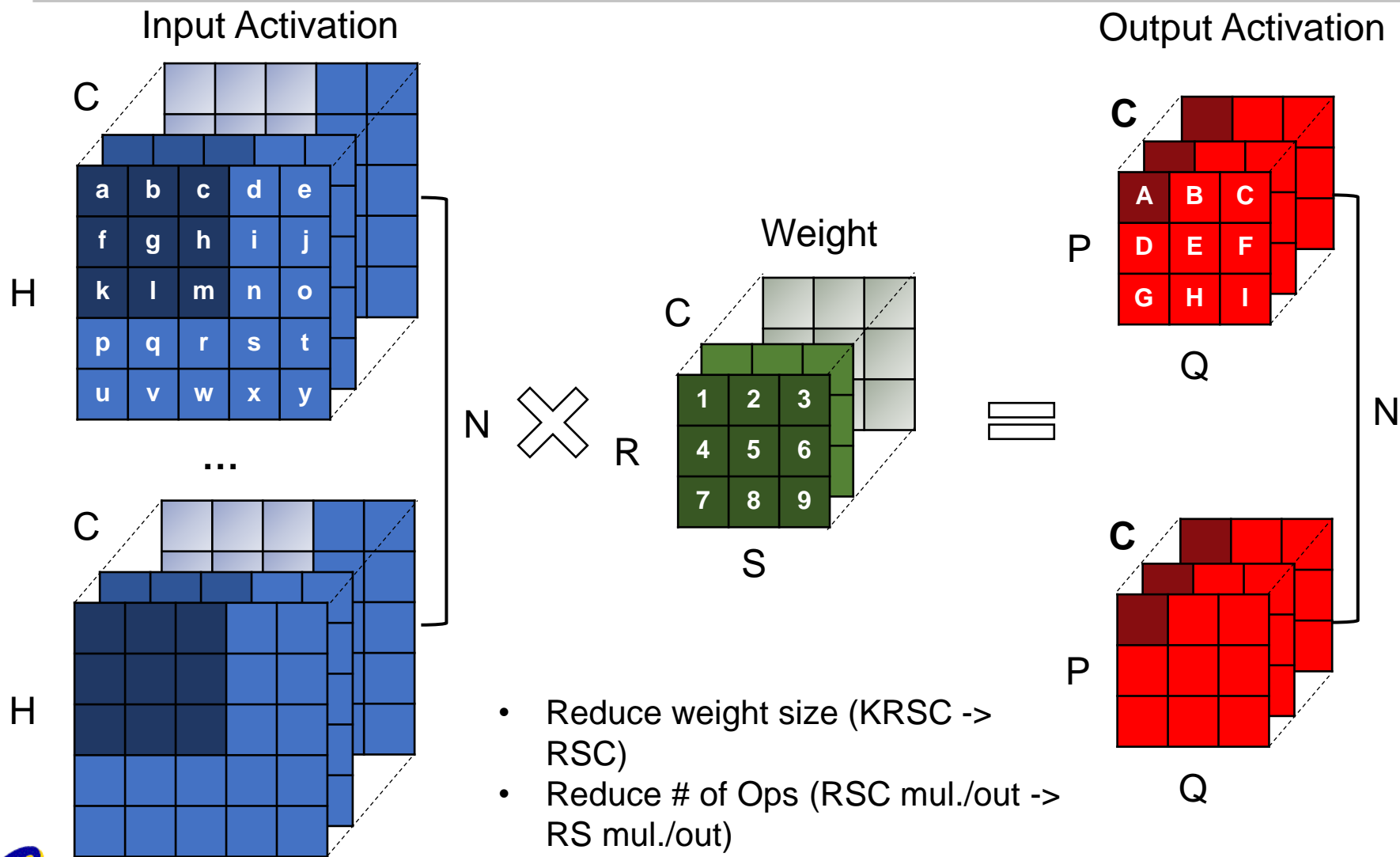
Output Activation



$H = 1$   
 $W = 1$   
 $R = 1$   
 $S = 1$   
 $P = 1$   
 $Q = 1$   
**stride = 1**  
**padding = 0**

**C:** # of Input Channels  
**K:** # of Output Channels  
**N:** Batch size

# Depth-wise Convolution



**H:** Height of Input Activation  
**W:** Width of Input Activation  
**R:** Height of Weight  
**S:** Width of Weight  
**P:** Height of Output Activation  
**Q:** Width of Output Activation  
**stride:** # of rows/columns traversed per step  
**padding:** # of zero rows/columns added

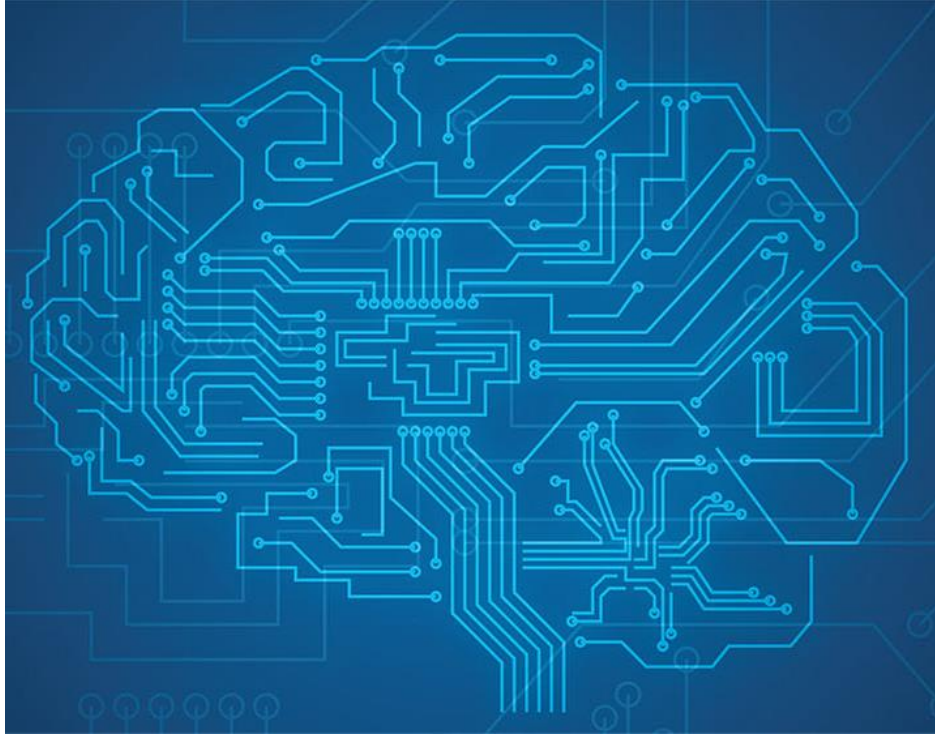
**C:** # of Input Channels  
**K:** # of Output Channels  
**N:** Batch size

# Administrivia

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- Lab 1 due this Friday.
- Lab 2 will release next Monday.
- Week 3 reading is posted.

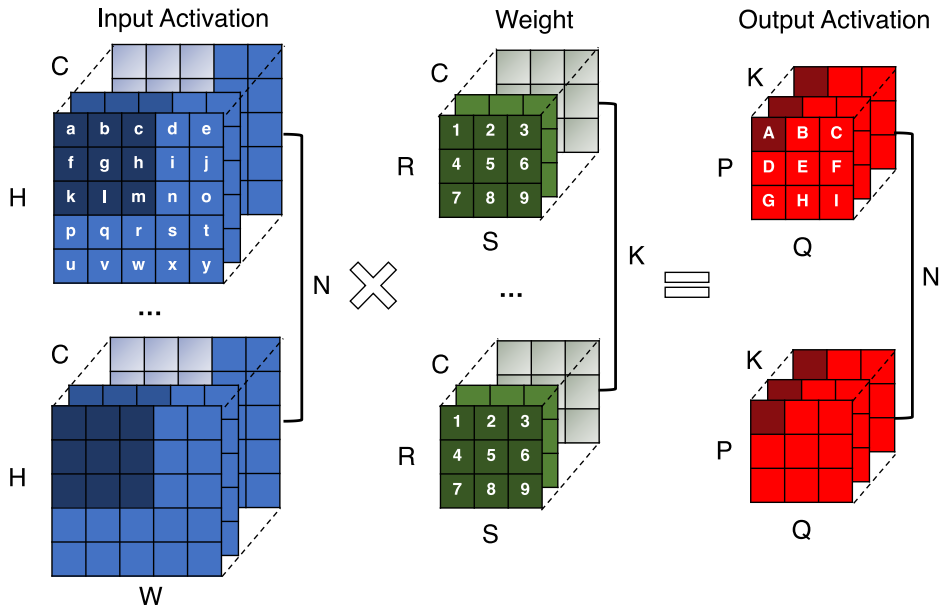




# DNN Kernels

- Overview
- **Convolution**
  - Basics
  - Transformation
- Pooling
- BatchNorm

# Option 1: Direct Convolution



```

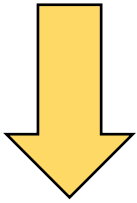
for (n=0; n<N; n++) {
    for (k=0; k<K; k++) {
        for (p=0; p<P; p++) {
            for (q=0; q<Q; q++) {
                OA[n][k][p][q] = 0;
                for (r=0; r<R; r++) {
                    for (s=0; s<S; s++) {
                        for (c=0; c<C; c++) {
                            h = p * stride - pad + r;
                            w = q * stride - pad + s;
                            OA[n][k][p][q] +=
                                IA[n][c][h][w]
                                * W[k][c][r][s];
                        }
                    }
                }
                OA[n][k][p][q] = Activation(OA[n][k][p][q]);
            }
        }
    }
}

```

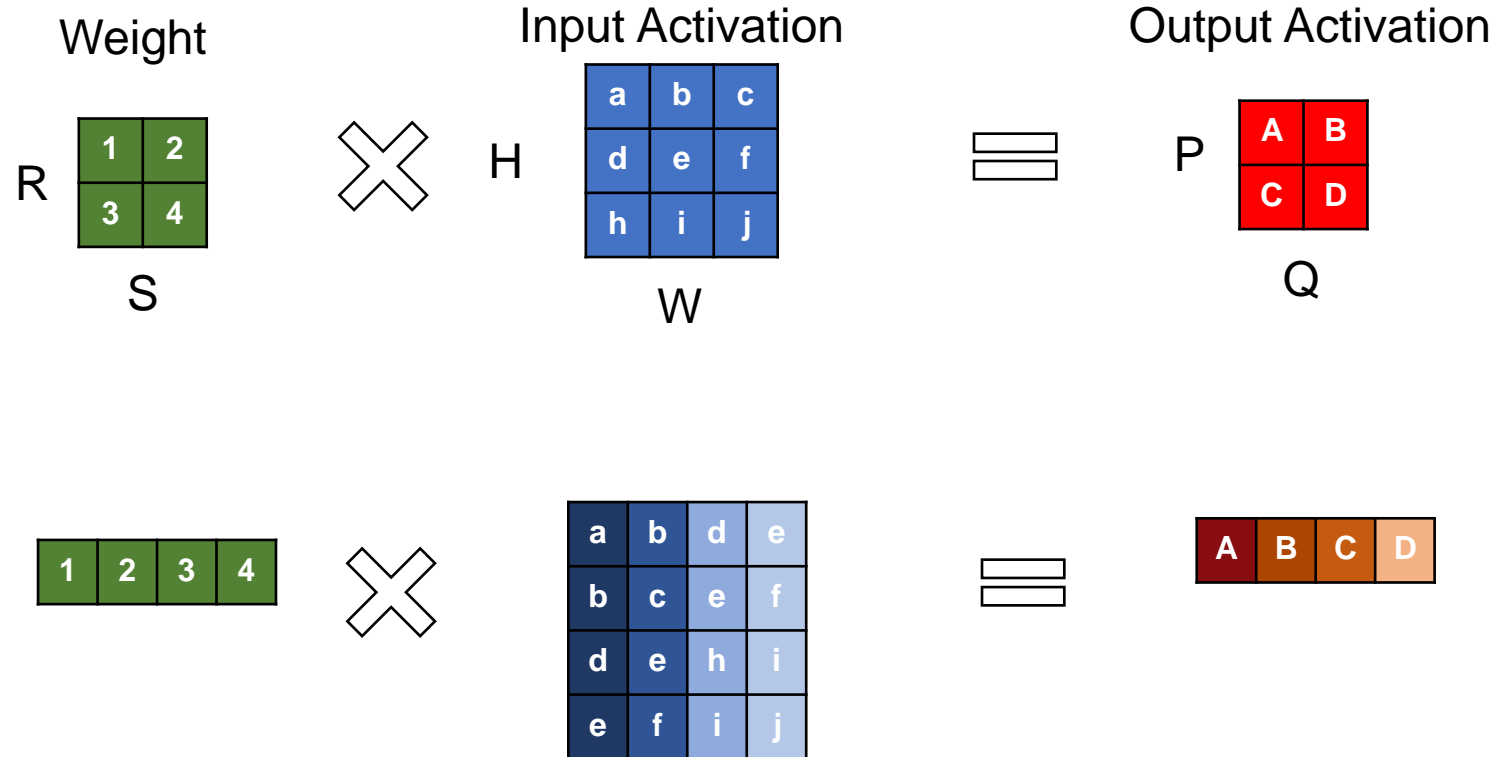
# Option 2: GEMM

- Converting convolution to GEMM via **im2col**

Convolution



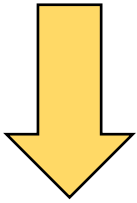
GEMM



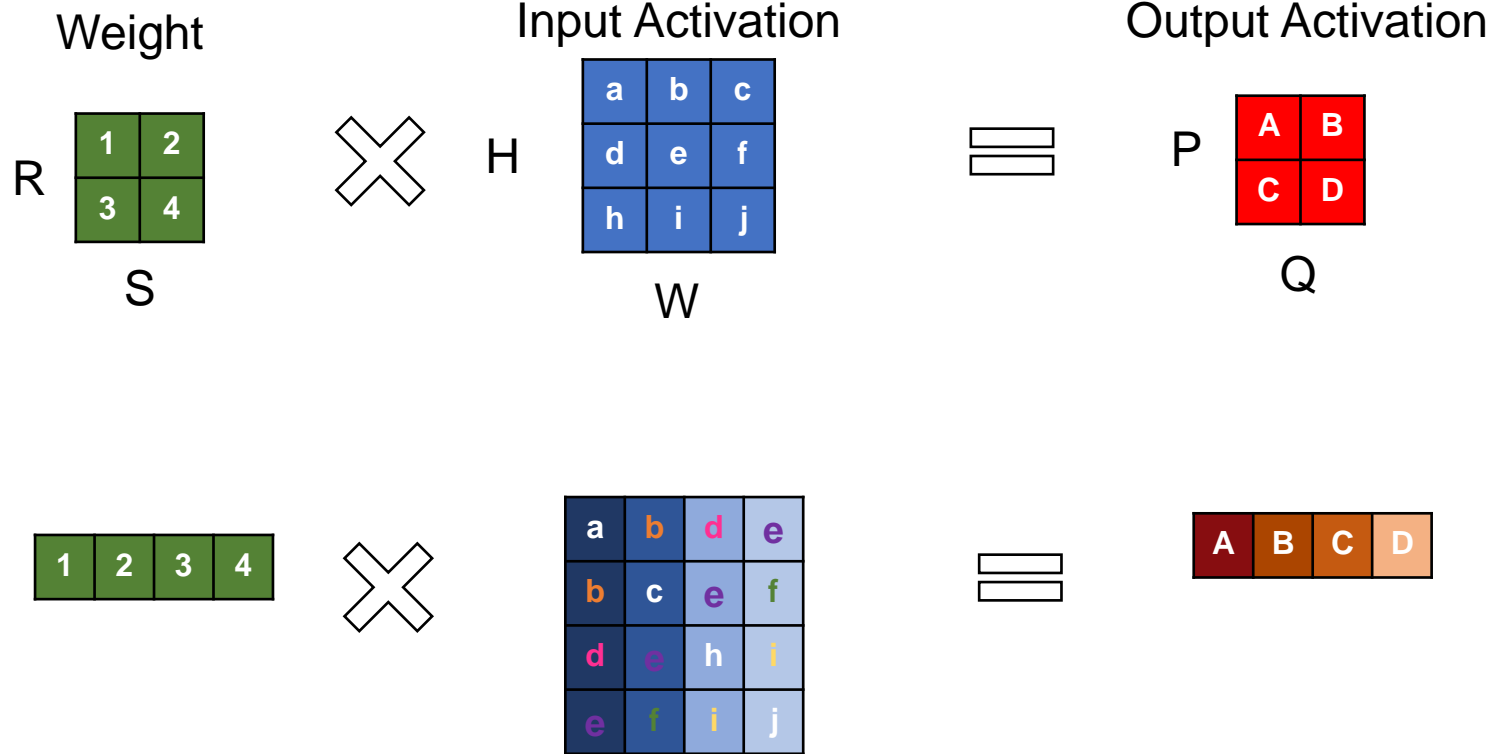
# Option 2: GEMM

- Converting convolution to GEMM via **im2col**

Convolution



GEMM  
(w/ data  
duplication)

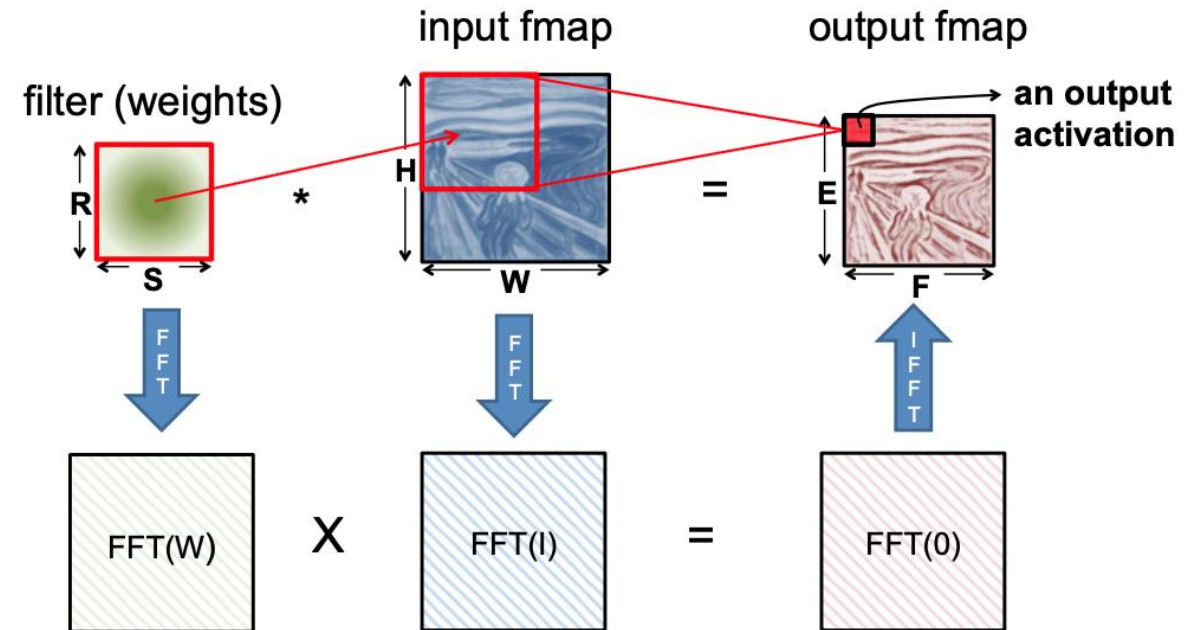


# Option 3: FFT-based Convolution

- **Convolution theorem:**  
convolution in the time domain is equivalent to point-wise multiply in the frequency domain.

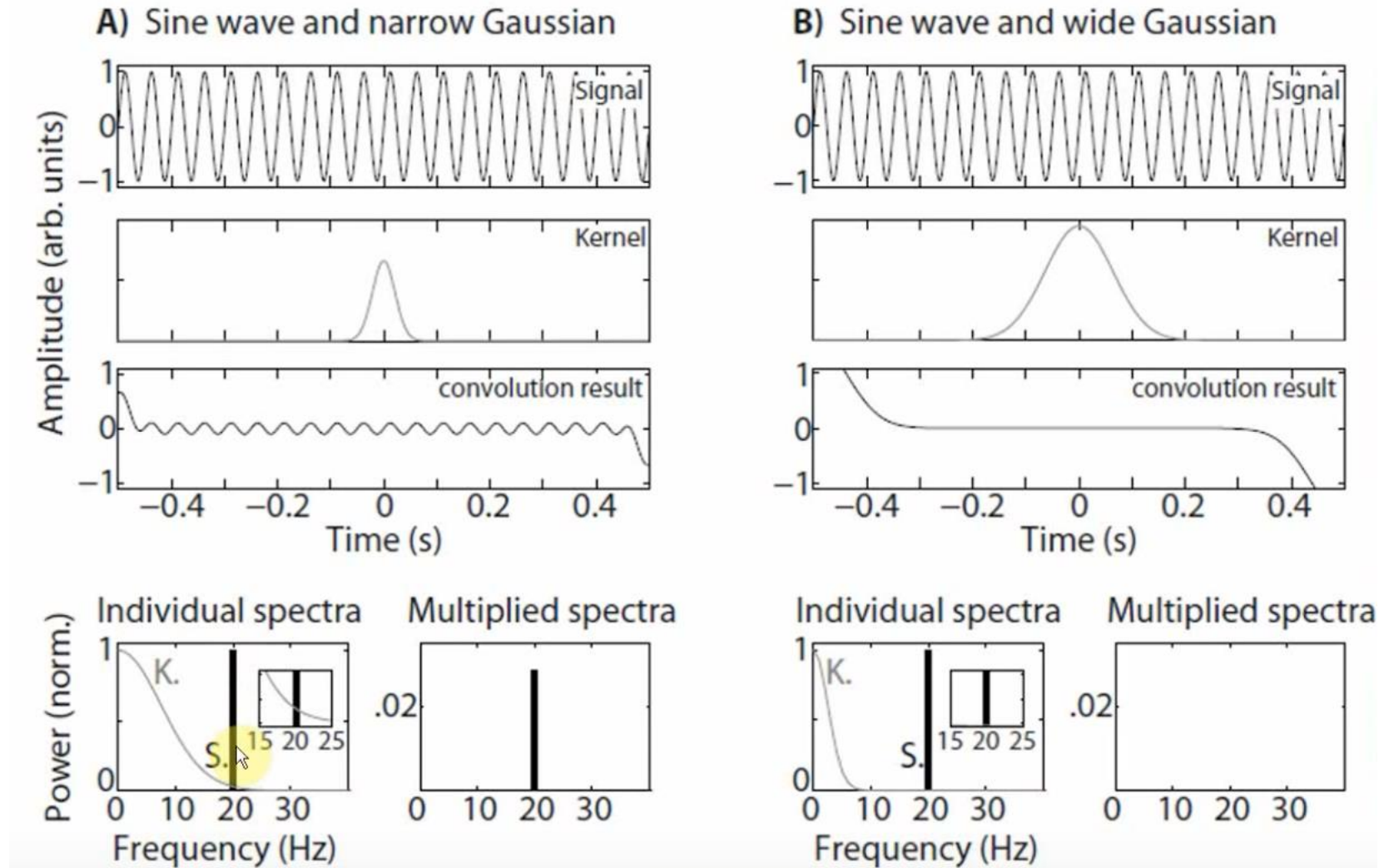
$$f * g = \mathcal{F}^{-1} \{ \mathcal{F}\{f\} \cdot \mathcal{F}\{g\} \}$$

$\mathcal{F}\{f\}$  and  $\mathcal{F}\{g\}$  are the Fourier transforms of  $f$  and  $g$   
The asterisk denotes convolution, not multiplication.



Eyeriss tutorial

# Option 3: FFT-based Convolution





# Option 3: FFT-based Convolution

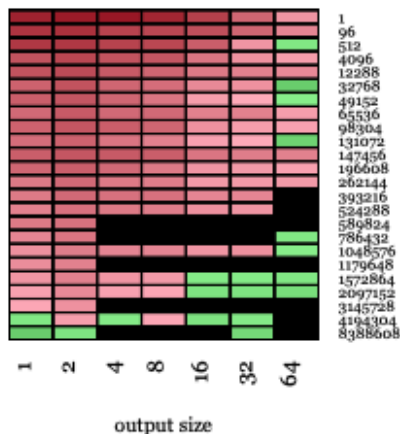


Figure 1: 3 × 3 kernel (K40m)

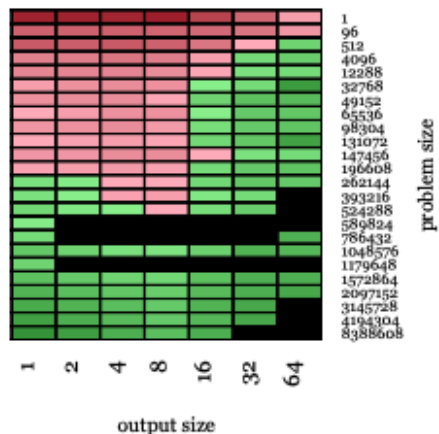


Figure 2: 5 × 5 kernel (K40m)

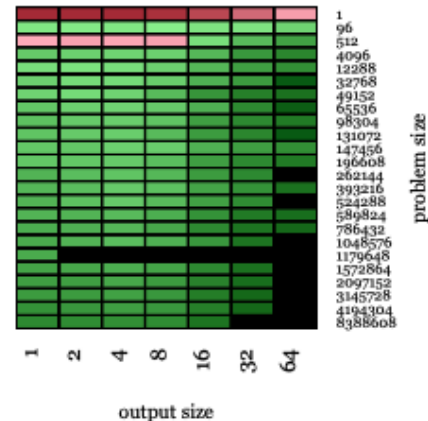


Figure 5: 11 × 11 kernel (K40m)

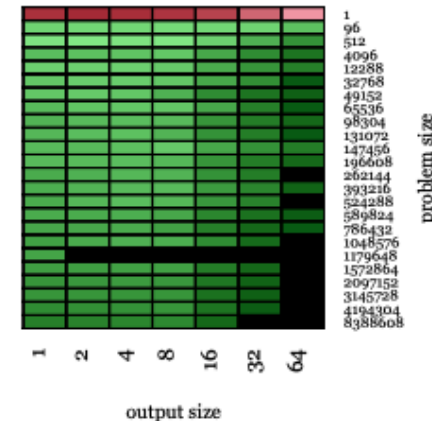


Figure 6: 13 × 13 kernel (K40m)

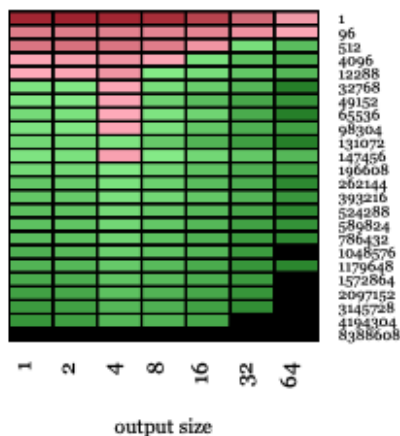


Figure 3: 7 × 7 kernel (K40m)

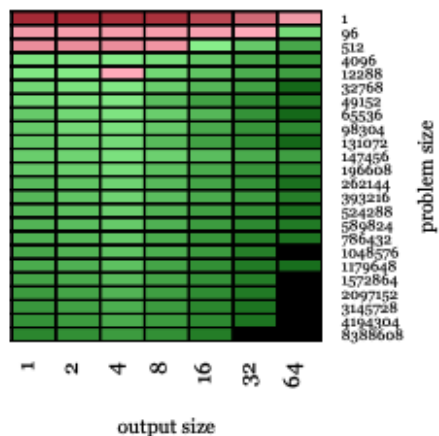
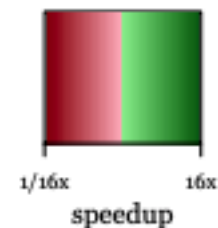


Figure 4: 9 × 9 kernel (K40m)



# Option 4: Winograd Transform

- Re-association of intermediate values to reduce # of multiplications.
- Works well for 3x3 convolution.

$$F(2,3) = \begin{bmatrix} d_0 & d_1 & d_2 \\ d_1 & d_2 & d_3 \end{bmatrix} \begin{bmatrix} g_0 \\ g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} m_1 + m_2 + m_3 \\ m_2 - m_3 - m_4 \end{bmatrix} \quad (5)$$

where

$$\begin{aligned} m_1 &= (d_0 - d_2)g_0 & m_2 &= (d_1 + d_2)\frac{g_0 + g_1 + g_2}{2} \\ m_4 &= (d_1 - d_3)g_2 & m_3 &= (d_2 - d_1)\frac{g_0 - g_1 + g_2}{2} \end{aligned}$$

Before: 6 MULs, 4 ADDs

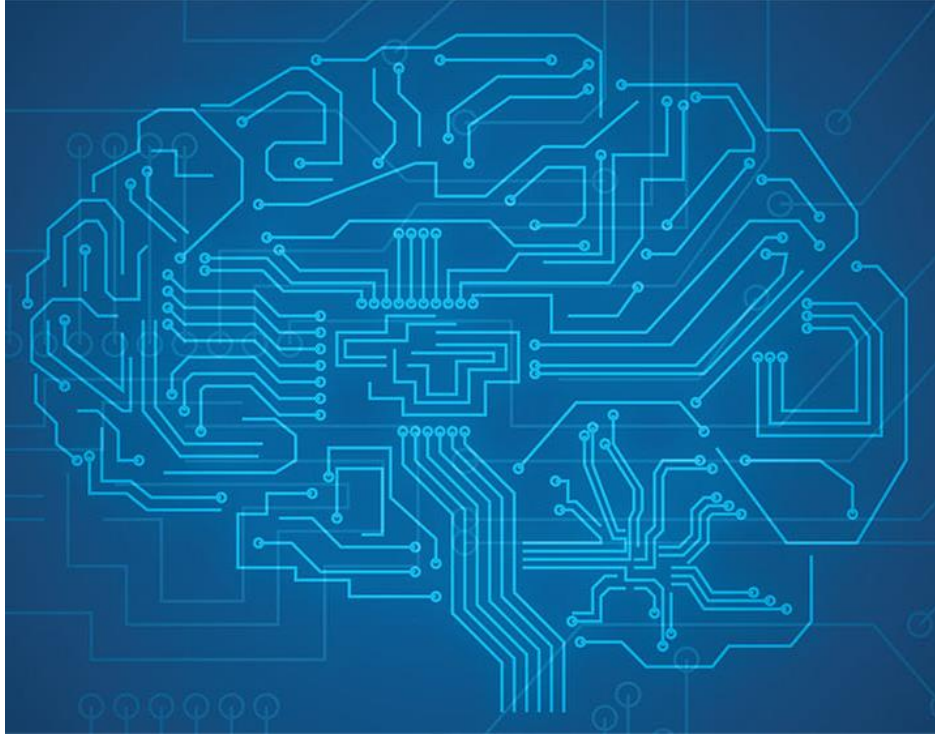
After:

- IA (d): 4 ADDs
- W (g): 3 ADDs, 2 MULs
- OA (m): 4 MULs, 4 ADDs

$$Y = A^T [(Gg) \odot (B^T d)] \quad (6)$$

$$\begin{aligned} B^T &= \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \\ G &= \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \\ A^T &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & -1 \end{bmatrix} \\ g &= [g_0 \ g_1 \ g_2]^T \\ d &= [d_0 \ d_1 \ d_2 \ d_3]^T \end{aligned} \quad (7)$$



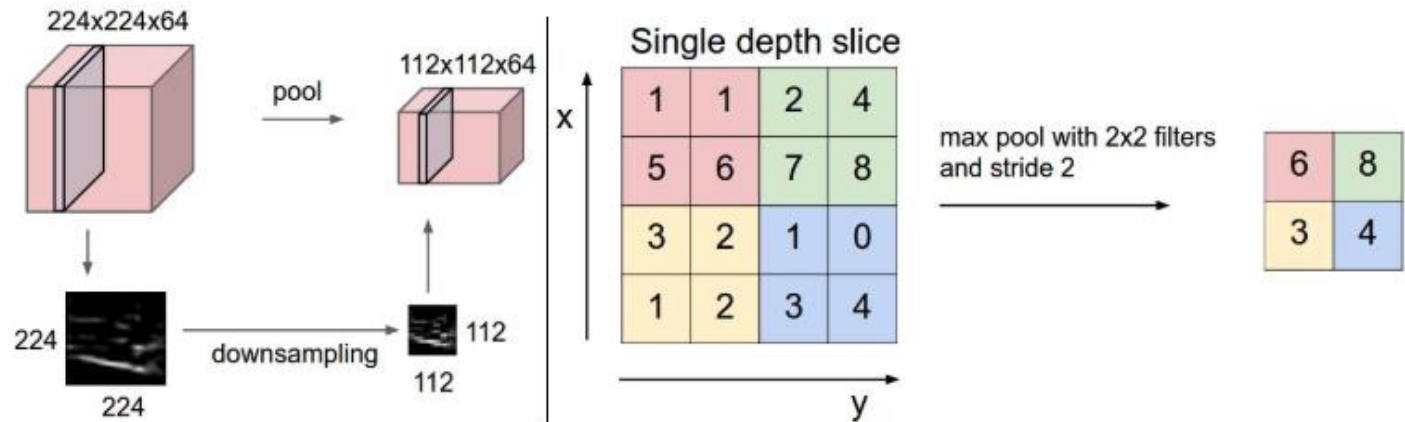


# DNN Kernels

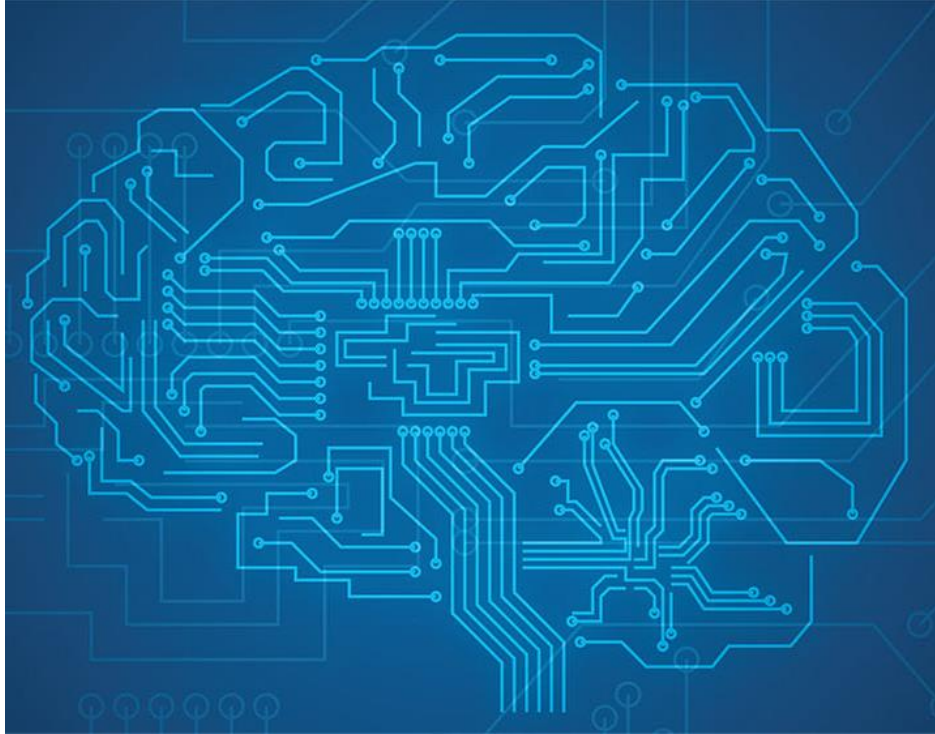
- Overview
- Convolution
  - Basics
  - Transformation
- **Pooling**
- BatchNorm

# Pooling Layer

- Progressively reduce the spatial size of the representation to reduce the amount of parameters and computation in the network, and hence to also control overfitting.
- Parameters:
  - Type: MAX and AVG
  - Pooling kernel size
  - Pooling stride



Pooling layer downsamples the volume spatially, independently in each depth slice of the input volume. Left: In this example, the input volume of size  $[224 \times 224 \times 64]$  is pooled with filter size 2, stride 2 into output volume of size  $[112 \times 112 \times 64]$ . Notice that the volume depth is preserved. Right: The most common downsampling operation is max, giving rise to max pooling, here shown with a stride of 2. That is, each max is taken over 4 numbers (little  $2 \times 2$  square).

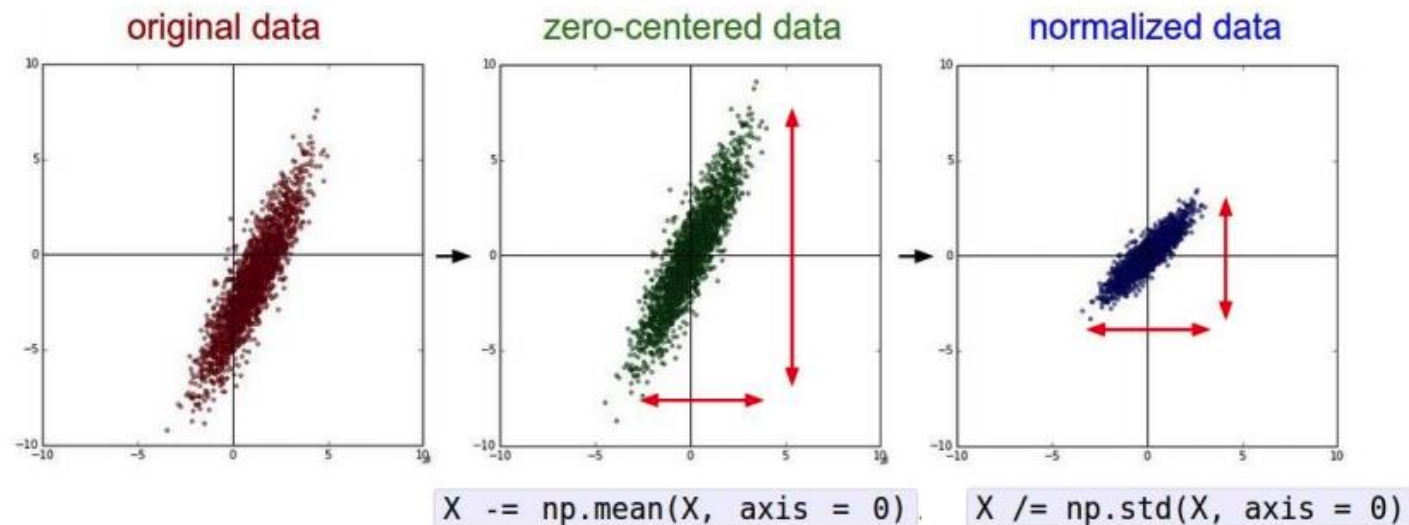


# DNN Kernels

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- Pooling
- BatchNorm

# BatchNorm Layer

- Goal: make it easier to train by providing zero-mean, unit-variance activations.
  - The training is complicated by the fact that the inputs to each layer are affected by the parameters of all preceding layers – so that small changes to the network parameters amplify as the network becomes deeper.





# BatchNorm Layer

## Training Time

**Input:**  $x : N \times D$

**Learnable scale and shift parameters:**

$$\gamma, \beta : D$$

Learning  $\gamma = \sigma$ ,  
 $\beta = \mu$ , will recover the  
identity function!

$$\mu_j = \frac{1}{N} \sum_{i=1}^N x_{i,j} \quad \text{Per-channel mean, shape is D}$$

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2 \quad \text{Per-channel var, shape is D}$$

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}} \quad \text{Normalized x, Shape is N x D}$$

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j \quad \text{Output, Shape is N x D}$$

## Test Time

$$\mu_j = \text{(Running) average of values seen during training} \quad \text{Per-channel mean, shape is D}$$

$$\sigma_j^2 = \text{(Running) average of values seen during training} \quad \text{Per-channel var, shape is D}$$

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}} \quad \text{Normalized x, Shape is N x D}$$

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j \quad \text{Output, Shape is N x D}$$



# Review

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- Deep neural networks typically have a sequence of convolutional, fully-connected, pooling, batch normalization, and activation layers.
- Convolution is one of the fundamental kernel in DNNs.
  - 2-D convolution
  - Stride and padding
  - 3-D convolution with input/output channels
  - Batch size
- Convolution can be calculated in different ways.
  - Direct, GEMM, FFT-based, Winograd-based.
- Pooling and Batch Normalization layers.

