Rubric - Quiz 1

Advanced Statistics: Theory and Methods- Quiz 1

Name ______________________ ID Number ______________________

1. An analyst working for a city's transportation department is looking at traffic patterns and accident data at a specific intersection over a given period. Data include the following: (a) X: The number of accidents, (b) Y: for each accident, whether or not a Seatbelt/ Helmet were used. (c) Z: waiting time at the intersection, (d) R: traffic density (number of vehicles per kilometre).

For each random variable, define the domain and an appropriate probability distribution.

(a) $X \in \{0, 1, 2, 3, \dots\}$ (discrete)
$X \sim \text{Poiss}(\lambda)$, where λ is the avg no. of accidents obtained for data. 1

(c) $Z \in [0, \infty)$ (continuous)
$Z \sim$ Gamma (Exponential) (λ_0), where λ_0 is the avg no. of vehicles waiting at the intersection. 1

(b) $Y \in \{0, 1\}$, $Y \sim \text{Ber}(p)$
p is the prob. that one wear seatbelt / helmet. 1
(discrete)

(d) (i) $R \in \{0, 1, 2, 3, \dots\}$ (discrete)
$R \sim \text{Poiss}(\tilde{x})$, where $\tilde{x}$ is the average traffic density. 1
(ii) $R \in [0, \infty)$, $R \sim$ log-normal if R is right-highly skewed
(iii) $R \in [0, \infty)$, $R \sim$ Normal (iv) $R \in [0, \infty)$, $R \sim \exp$

2. The cumulative distribution function of a discrete random variable X is given by

$$F(x) = \begin{cases} 0, & x < -2\,; \\ 0.1, & -2 \le x < -1; \\ 0.4, & -1 \le x < 1; \\ 0.9, & 1 \le x < 2; \\ 1, & x \ge 2. \end{cases}$$

Find the corresponding probability mass function.

$$p(x) = F(x) - \lim_{h \to 0} F(x-h)$$

$$p(x) = \begin{cases} 0.1, & x = -2 \\ 0.3, & x = -1 \\ 0.5, & x = 1 \\ 0.1, & x = 2 \end{cases}$$

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3. The proportion of impurities X in certain copper ore samples is a continuous random variable with pdf given by

$$f(x) = \begin{cases} Cx^2(1-x) & 0 \le x \le 1 \\ 0 & o.w. \end{cases}$$

(a) For what value of C is $f(x)$ a valid probability distribution function?

(b) Find the corresponding cumulative distribution function.

(c) Find the mean of X.

(a) $\int_0^1 f(x)\,dx = 1 \Rightarrow c\int_0^1 (x^2 - x^3)\,dx = 1$

$\Rightarrow c\left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 = 1 \Rightarrow c\left(\frac{1}{3} - \frac{1}{4}\right) = 1$ 1

$\Rightarrow \boxed{C = 12} \Rightarrow f(x) = 12(x^2 - x^3), \quad 0 \le x \le 1$

(b) $F_X(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$

$$= \begin{cases} \int_{-\infty}^{x} 0 \cdot dx' , & x < 0 \\ \int_0^x 12(x^2 - x^3)\,dx , & x \in [0,1] \\ \int_0^x 12(x^2 - x^3)\,dx , & x \in (1,\infty) \end{cases}$$

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$$= \begin{cases} 0 , & x < 0 \\ 12\left(\frac{x^3}{3} - \frac{x^4}{4}\right) , & x \in [0,1] \\ \int_0^1 12(x^2 - x^3)\,dx + \int_1^x 0\,dx , & x \in (1,\infty) \end{cases}$$

$$F_X(x) = \begin{cases} 0 , & x < 0 \\ x^3(4 - 3x) , & x \in [0,1] \\ 1 , & x \in (1,\infty) \end{cases}$$

(c) $E(X) = \int_0^1 x f(x)\,dx = 12\int_0^1 (x^3 - x^4)\,dx = 12\left(\frac{x^4}{4} - \frac{x^5}{5}\right)\Big|_0^1 = 12\left(\frac{1}{4} - \frac{1}{5}\right)$

$\Rightarrow \boxed{\mu = \frac{12}{20} = \frac{3}{5} = 0.6}$ 1