Rubric – Quiz 2

Advanced Statistics: Theory and Methods - Quiz 2

Name ______________________ ID Number ______________________

1. Let U be a random variable that follows a Uniform distribution $U(0, 1)$. Let us define the random variable Y as follows:

$f_U(u) = 1 \quad , u \in (0,1)$

$$Y = -\frac{1}{\lambda}\ln(1-U)$$

$F_U(u) = \begin{cases} 0 & ; u \le 0 \\ u & ; u \in (0,1) \\ 1 & ; u \ge 1 \end{cases}$

where λ is a positive constant. Use any method to find the distribution of Y.

for $y > 0$,

$F_Y(y) = P(Y \le y) = P\left(-\frac{1}{\lambda}\ln(1-U) \le y\right) = P\left(\ln(1-U) \ge -\lambda y\right) = P\left(1-U \ge e^{-\lambda y}\right)$

$\Rightarrow F_Y(y) = P(U \le 1-e^{-\lambda y}) = F_U(1-e^{-\lambda y}) = 1-e^{-\lambda y}$

$f_Y(y) = F_Y'(y) = -e^{-\lambda y} \times (-\lambda) = \lambda e^{-\lambda y} \quad , y > 0$

$\Rightarrow Y \sim \exp(\lambda)$

3

2. Mahua must decide on the courses she wishes to take for the Monsoon semester. She must choose from upper division courses in CS and Statistics. Mahua has to register for (**exactly**) three upper division courses to complete her major requirements. There are 4 CS and 3 Statistics courses being offered.

Let X represent the number of CS courses and Y represent the number of Statistics courses in her final selection. (Assume that selections are made at random.)

$\max(X) = 4, \max(Y) = 3$

Find the joint pmf of X and Y.

choose r from 4 and $3-r$ from 3, whenever $r \ge 0, 3-r \ge 0$.

X \ Y	Y = 0	Y = 1	Y = 2	Y = 3
X = 0	0	0	0	$\frac{{}^4C_0\,{}^3C_3}{35} = \frac{1}{35}$
X = 1	0	0	$\frac{{}^4C_1\,{}^3C_2}{35} = \frac{12}{35}$	0
X = 2	0	$\frac{{}^4C_2\,{}^3C_1}{35} = \frac{18}{35}$	0	0
X = 3	$\frac{{}^4C_3\,{}^3C_0}{35} = \frac{4}{35}$	0	0	0
X = 4	0	0	0	0

$\left({}^4C_3\,{}^3C_0 + {}^4C_2\,{}^3C_1 + {}^4C_1\,{}^3C_2 + {}^4C_0\,{}^3C_3 = 4+18+12+1 = 35 = \text{Total cases}\right)$

$\binom{7}{3} = 35$

3

3. In Reliability and Survival Analysis, the hazard function specifies the instantaneous rate of failure at time t, **given** that the individual/ item has survived up till that time. It is also called the conditional failure rate, force of mortality, or age-specific failure rate.

For continuous random variables, the hazard rate is given by

$$h(t) = \frac{f(t)}{1 - F(t)}$$

where f and F denote the density and cumulative distribution function respectively.

Consider the Weibull distribution with cumulative distribution function given by

$$F(t) = 1 - e^{-(\lambda t)^{\gamma}}$$

where $\lambda, \gamma > 0$.

Find the hazard function. Provide a **rough sketch** of the hazard function for $\gamma = 0.5, 1, 2$.

Sol^n: $F(t) = 1 - e^{-(\lambda t)^{\gamma}} \Rightarrow f(t) = F'(t) = -e^{-(\lambda t)^{\gamma}} \times (-\gamma(\lambda t)^{\gamma-1}) \times \lambda$

$\Rightarrow f(t) = \lambda\gamma(\lambda t)^{\gamma-1} e^{-(\lambda t)^{\gamma}}$ and $1 - F(t) = e^{-(\lambda t)^{\gamma}}$

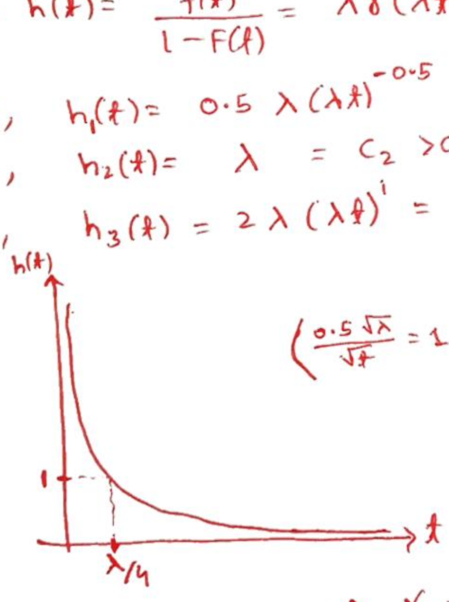
1 $\Rightarrow h(t) = \dfrac{f(t)}{1-F(t)} = \lambda\gamma(\lambda t)^{\gamma-1}$, $\lambda, \gamma > 0$

1 a) $\gamma_1 = 0.5$, $h_1(t) = 0.5\lambda(\lambda t)^{-0.5} = \dfrac{0.5\sqrt{\lambda}}{\sqrt{t}} = \dfrac{c_1}{\sqrt{t}} \geq 0$

1 b) $\gamma_2 = 1$, $h_2(t) = \lambda = c_2 > 0$

1 c) $\gamma_3 = 2$, $h_3(t) = 2\lambda(\lambda t)^1 = 2\lambda^2 t = c_3 t \geq 0$

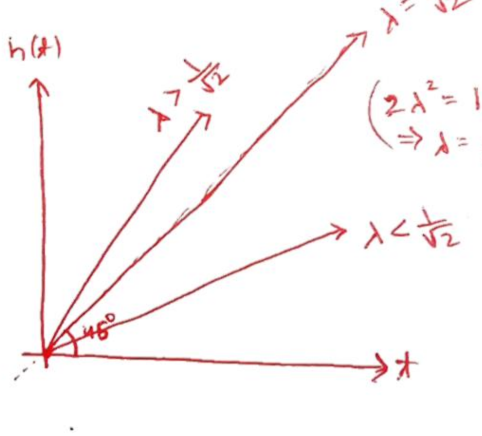
a) γ_1:



$\left(\dfrac{0.5\sqrt{\lambda}}{\sqrt{t}} = 1 \Rightarrow t = \dfrac{\lambda}{4}\right)$

b) γ_2:

c) γ_3:



$\left(2\lambda^2 = 1 \Rightarrow \lambda = \dfrac{1}{\sqrt{2}}\right)$