DS3001: ADVANCED STATISTICS

PRACTICE PROBLEMS (UNGRADED)

1. **Click-Through Rate:** An online advertising campaign has a click-through rate of 5%. If the campaign is shown to 1,000,000 users, what is the probability that exactly 80,000 users will click on the ad? Compute the same probability using the Normal and Poisson approximations to the binomial.

2. **Loan Default Rate:** A lending platform approves 30% of loan applications. If they receive 100 loan applications on a given day, what is the probability that exactly 10 of them will be approved? What assumptions are you making about the underlying process?

3. Twelve prototype rockets have been developed by an aerospace company. The probability of a successful lift-off for any rocket is assumed to be 0.9.

   (a) Is this a binomial experiment? Justify.

   (b) Find the probability that at least 9 rockets successfully lift-off.

   (c) If the experiment does result in only 1 successful launch, what would you infer about the company or the product?

4. **Inventory Management:** A tech company has 100 software engineers, of which 40 are skilled in Python. If they randomly select 10 engineers to form a Python project team, what is the probability that a majority of the team are skilled in Python?

5. The percentage of individuals in the population possessing a rare blood type is 2 %. Individuals arrive at a blood bank and are tested. The moment a match is located, the testing stops.

   a. Find the probability that exactly three individuals are tested.

   b. Suppose twenty individuals arrive on a particular day. What is the probability that the blood bank fails to find a donor?

6. The number of alpha particles emitted by barium-133 is known to have a Poisson distribution with an average of 8 particles in 1/10-th of a second.

   a. Find the probability that a sample of barium-133 emits 10 alpha particles in 1/10-th of a second.

b. Find the probability that in a time interval of 1/10-th of a second, no alpha particles are emitted.

7. One of the major contributors to air pollution is hydrocarbons emitted from the exhaust system of automobiles. Let X denote the number of grams of hydrocarbons emitted by an automobile per mile. Assume that X is normally distributed with a mean of 2 grams, and a standard deviation of 0.5 grams.

   a. Find the probability that a randomly selected automobile will emit between 1.9 and 3.5 gms. of hydrocarbons per mile.

   b. Automobiles whose emissions fall in the top 5 % will have to undergo repairs. Find the level of emission that would require an automobile to undergo repairs.

   c. Automobiles whose emissions fall in the bottom 10 % receive the status of "environmentally friendly" vehicles. Find the level of emission that guarantees this status.

8. Weights of paper discarded by households each week are normally distributed with a mean of 5.4 lbs and a standard deviation of 2.2 lbs. [This is paper that should have been recycled.]

   (a) What percentage of households discard less than 3 lbs per week?

   (b) The Municipal Corporation of Mohali wants to impose a penalty on the 10 % of households who discard high amounts of paper in their garbage. Determine the appropriate cut-off weight for the city.

9. In a certain city, the daily consumption of electric power in millions of kilowatt-hours is modeled as a gamma random variable with $\alpha = 3$ and $\beta = 2$. If the power plant has a daily capacity of 12 kilowatt hours, what is the probability that the supply will be inadequate on any given day?

10. The response time of a web server is exponentially distributed with a mean of 2 seconds. Calculate the probability that a randomly selected request will be processed in less than 1 second. What is the median response time?

11. The annual proportion of erroneous IT returns can be modeled as a beta distribution with $\alpha = 2$ and $\beta = 9$. Find the probability that inany given year, there will be fewer than 10% erroneous returns.