

Extended essay May 2018

Effects of Diffraction on Solar cells

Research question: **How does the current output of a solar cell depend upon the amount of Fraunhofer diffraction through a single rectangular slit?**

Word count:3946

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Introduction

Aim and rationale

With the increased use in fossil fuels, there is an almost a certain chance of an energy crisis happening in 50 years. One of the most promising solutions to this potential crisis are Photovoltaic cells. On an average, the earth receives 1000W/m^2 ¹ each second. However, the problem is with harnessing that energy which is due to the inefficiency of solar panels. They are not nearly as efficient as fossil fuels. There are many reasons for this lack of efficiency: manufacturing defects, technology, heat loss, impurities and so on. Unfortunately, not much research has been done on how optical effects affects the output of solar cells which is one of the reasons why I decided to pursue this topic. Taking into account availability and feasibility issues, my research question in this topic is:

How does the current output of a solar cell depend upon the amount of Fraunhofer diffraction through a single rectangular slit?

¹ The Suns Energy. Accessed Feb. & march 2018.

<https://ag.tennessee.edu/solar/Pages/What%20Is%20Solar%20Energy/Suns%20Energy.aspx>

Scope of research

This research is only based on Fraunhofer diffraction and how the photoelectric current depends on the variation of the diffraction. The research dives into some of the characteristics of solar cells but mostly the research is based on the intensity of Fraunhofer diffraction and using Huygens principles for qualitative examination. A lot of research papers and forums were consulted to research the information. Mathematics was used to support as much of the relationships as possible to the extent of my understanding.

Background

Fraunhofer diffraction

Diffraction is the spreading or the bending of a wave when encountering an obstacle or of a size that is comparable to the wavelength of the wave. The most basic principle in the concept of diffraction which is used by the Fraunhofer theory is the Huygens principle: *“Every point on a wave-front may be considered a source of secondary spherical wavelets which spread out in the forward direction at the speed of light. The new wave-front is the tangential surface to all of these secondary wavelets.”*² This is shown on figure 1 as seen below.

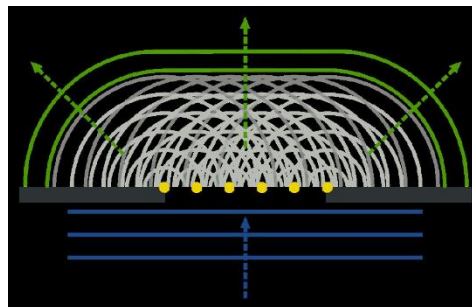


Figure 1 Huygens principle in diffraction

² Huygens' principle, accessed March 01, 2018, <http://farside.ph.utexas.edu/teaching/302l/lectures/node150.html>

There are two types of diffraction: near field and far field. My research question is centered around Fraunhofer diffraction which is solely for the purposes of far field diffraction. Far field diffraction can happen in one of two ways³. One of the ways this can occur is, in the case of diffraction, is by adhering to the following formula where W is the aperture width, L distance from aperture and λ the wavelength of the incident light.

$$\frac{W^2}{L\lambda} \ll 1$$

Intensity is an aspect of any wave that gives the energy transferred by the wave. Intensity is defined generally as $\frac{\text{power}}{\text{area}}$ and in the case of waves, it can also be found through the direct proportion that exists with the square of the amplitude of the wave.

In the case of electromagnetic waves, the intensity is determined by both the square of the electric field strength and the magnetic field strength. This is because of both of those vectors being the amplitude of an electromagnetic wave. Through the use of Poynting vector and a few other derivations⁴, the average intensity of the wave can be mathematically defined as

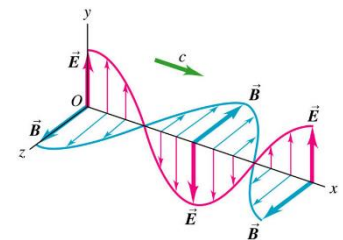


Figure 2 Electromagnetic wave with vectors

$$I_{ave} = \frac{c\epsilon_0 E_0^2}{2}$$

³ Only one of the way is covered in this research as the other is not relevant.

⁴ OpenStax. "Physics." Lumen. Accessed Feb. & march 2018.
<https://courses.lumenlearning.com/physics/chapter/24-4-energy-in-electromagnetic-waves/>.

Where c is the speed of light, ϵ_0 refers to vacuum permittivity and E_0 refers to the peak electric field strength.

Theory of solar cells

Solar cells are made from semiconductors which are materials that, under certain circumstances can be conductors of electricity. The most common form of solar cell semiconductor is silicon. It has a property when light strikes the silicon atoms, the electrons in it gain energy and thereby get excited. These excited electrons get knocked off from their shells and become free electrons that flow leading to a photoelectric current. They have junctions known as p-n junctions. Every solar cell has a band gap i.e. the minimum amount of energy required for an electron to break free of its bound state. Only when the incident photon has more energy than the bandgap will there be electricity and any excess energy is given off as heat. When this happens, the free electrons move from the p side of the solar cell to the n side causing a potential difference. These sides are connected with a circuit using metal contacts to trigger a flow of current. This flow of current is called photoelectric current or light generated current.

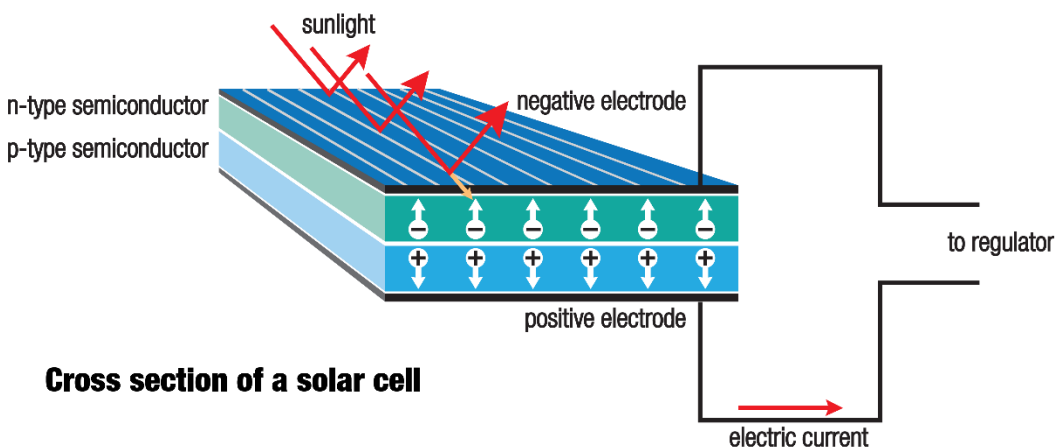


Figure 3 Solar cell cross section

Hypothesis

My hypothesis for this particular research is that with increasing diffraction, there will be lesser current output from the solar cell. I believe this will happen because diffraction is the spreading of wave therefore it is reasonable to deduce that the intensity of the central maxima would decrease with more diffraction. Subsequently, from the photoelectric effect we know that with decreasing intensity, there is a decrease in the number of ejected electrons which directly translates into less light-generated current.

Approach

In order to test my hypothesis while taking into account feasibility and precision, my research was divided into 2 approaches. An experimental one to see how my particular solar cell's current output depended on the intensity of the incident light. A theoretical simulation of my own to see how the intensity of the central maxima was affected by the slit width. Consequently, the data was analyzed to show qualitatively how intensity is affected by the slit width using the Fraunhofer diffraction theory mentioned above. The data was then consolidated to find the final relationship between slit width and photoelectric current.

RESEARCH

Data collection-Experiment

Objective of experiment: To find relationship between current and intensity.

Independent variable:

Distance of source from solar cell-This variable was chosen because the intensity of the light source couldn't be varied therefore by the inverse square law, the distance was varied to change the intensity.

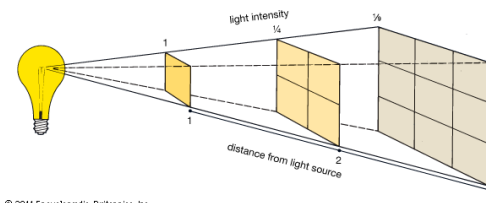


Figure 4 Inverse square law

Dependent variable:

Light-generated Current produced from the solar cell.

Controlled variables:

Color of light and light source: white light from the same light source was used for all of the readings so it also implies that the wavelengths were roughly the same for the entirety of the experiment.

Angle of incidence: always at the same level as the solar cell. There was no difference in the angle of incidence. Kept at 5° .

Temperature- as temperature is a factor in determining the efficiency of the solar cell, the entire experiment was done at the same room temperature on the same day in a duration of 2 hours and due to that the temperature variation and the efficiency variations can be **neglected**.

Surface area of solar cells: as the research question specifies only one solar cell, I taped over the other parts of the solar cell with opaque black tape. Through this, only the desired part of the solar cell was shown to the light.

Solar cell- kept as the same solar cell throughout the experiment. In this case, it was an amorphous solar cell thereby the silicon structure's properties were conserved throughout the experiment.

Resistance: The circuit from which the photocurrent was measured from was kept the same with the same load throughout the experiment.

Preparation for the experiment.

Apparatus

1. solar cell
2. Blocks of wood, plywood sheets and cardboard box
3. Meter rulers
4. Roll of Black insulation tape
5. Scissors
6. Light source-tungsten incandescent bulb
7. Intensity meter connected to data logger
8. Alligator clips and few wires
9. Digital Panel meter (mA)
10. Small load

Procedure to performing the experiment:

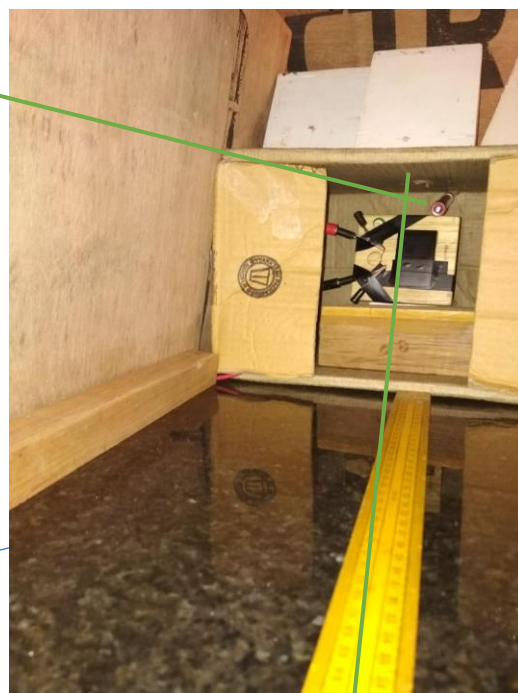
- 1) Cut as much electrical tape as needed to cover all of the solar cell except the desired area.
- 2) Place the taped solar cell inside the cardboard box. Use the blocks of wood as a stand for the solar cell
- 3) Use the plywood sheets to create a cave-like structure outside the box in order to minimize background light.
- 4) Tape the Meter Rulers one after the other starting from the end of the cardboard box.
- 5) Keep the source of light and its stand on the rulers
- 6) Poke a hole through the back of the box to insert the luminosity probe and connect it to a computer using the interface
- 7) Let the probe rest on top of the solar cell as shown in the picture
- 8) Using alligator clips, connect the solar cell to a series circuit with a small load and the milliammeter. The circuit should look as shown below.
- 9) Measure the distance from the source of light and the solar cell and record it in the table under d/cm
- 10) Record the luminosity in the probe under “Background intensity”
- 11) Switch on the source of light from the given distance
- 12) Record the current given in the milliammeter
- 13) Repeat steps 10-12 twice more to minimize the effects of random errors
- 14) Switch off the source and move it 10 cm more using the ruler under it.
- 15) Repeat steps 9-14 until the milliammeter stops at 0.



Figure 5 Experimental setup

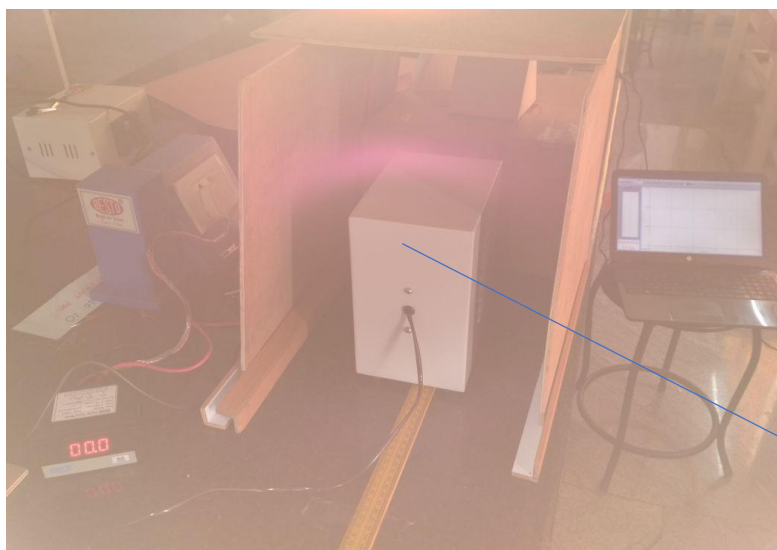
Intensity probe

Digital Panel meter(mA)



Solar cell

Figure 6 Inside the cave structure. The wires connected are to the circuit with the load



light source

Figure 7 First reading

Results

S.no	d/cm \pm 0.1 cm	I/mA \pm 0.1mA			Intensity/lx \pm 0.2lx			Background intensity/lx \pm 0.2lx			Avg Background intensity/lx \pm 0.2 lx	Average current/mA \pm 0.1mA	Real Avg Intensity/lx \pm 0.4 lx
1	11.0	1.2	1.1	1.2	770.0	769.3	769.8	0.3	0.4	0.7	0.5	1.2	769.2
2	21.0	0.9	1.0	1.0	743.5	745.4	741.3	1.1	5.3	2.6	3.0	1.0	740.4
3	31.0	0.8	0.9	1.0	724.7	719.9	720.1	3.6	6.2	4.9	4.9	0.9	716.7
4	41.0	0.7	0.8	0.8	710.5	713.4	711.2	5.6	5.3	6.4	5.8	0.8	705.9
5	51.0	0.7	0.8	0.7	701.5	703.8	702.0	7.8	9.8	8.2	8.6	0.7	693.8
6	61.0	0.7	0.7	0.7	689.1	684.3	695.6	9.4	9.8	10.6	9.9	0.7	679.7
7	71.0	0.7	0.6	0.6	654.1	672.2	660.2	10.8	11.1	12.2	11.4	0.6	650.8
8	81.0	0.6	0.6	0.6	639.0	627.0	599.7	11.5	9.7	12.3	11.2	0.6	610.7
9	91.0	0.6	0.5	0.5	543.4	539.8	512.7	10.5	11.0	13.3	11.6	0.5	520.4
10	101.0	0.5	0.5	0.6	412.2	440.6	435.7	10.7	11.1	12.2	11.3	0.5	418.2
11	111.0	0.4	0.5	0.4	343.0	348.0	352.0	16.8	13.0	12.8	14.2	0.4	333.5
12	121.0	0.4	0.4	0.5	301.0	297.3	267.4	12.8	15.1	14.3	14.1	0.4	274.5
13	131.0	0.3	0.4	0.4	252.0	280.0	258.0	17.5	10.4	16.5	14.8	0.4	248.5
14	141.0	0.4	0.3	0.3	209.0	229.0	201.0	17.1	14.3	9.9	13.8	0.3	199.2
15	151.0	0.3	0.3	0.3	185.0	200.9	175.0	12.2	11.0	13.0	12.1	0.3	174.9
16	161.0	0.3	0.2	0.3	169.4	163.7	176.1	11.7	12.5	14.5	12.9	0.3	156.8
17	171.0	0.2	0.2	0.3	158.3	156.7	149.7	10.4	11.5	12.3	11.4	0.2	143.5
18	181.0	0.2	0.2	0.2	124.3	136.7	135.4	10.5	9.9	9.8	10.1	0.2	122.1
19	191.0	0.2	0.2	0.2	130.0	131.0	127.0	9.7	12.3	11.5	11.2	0.2	118.2
20	201.0	0.1	0.2	0.2	112.3	110.9	103.8	11.9	11.9	12.4	12.1	0.2	96.9
21	211.0	0.1	0.1	0.2	101.6	104.5	102.5	8.7	11.9	13.2	11.3	0.1	91.6
22	221.0	0.1	0.1	0.2	97.1	99.2	100.0	15.8	13.2	11.5	13.5	0.1	85.3
23	231.0	0.1	0.1	0.1	91.2	93.4	95.4	16.3	14.8	13.7	14.9	0.1	78.4
24	241.0	0.1	0.1	0.1	92.6	89.5	94.6	9.8	10.5	11.9	10.7	0.1	81.5
25	251.0	0.0	0.0	0.1	86.7	84.0	86.2	14.2	15.1	13.9	14.4	0.0	71.2

Table 1 Observation table

Analysis and discussion

Distance and Intensity

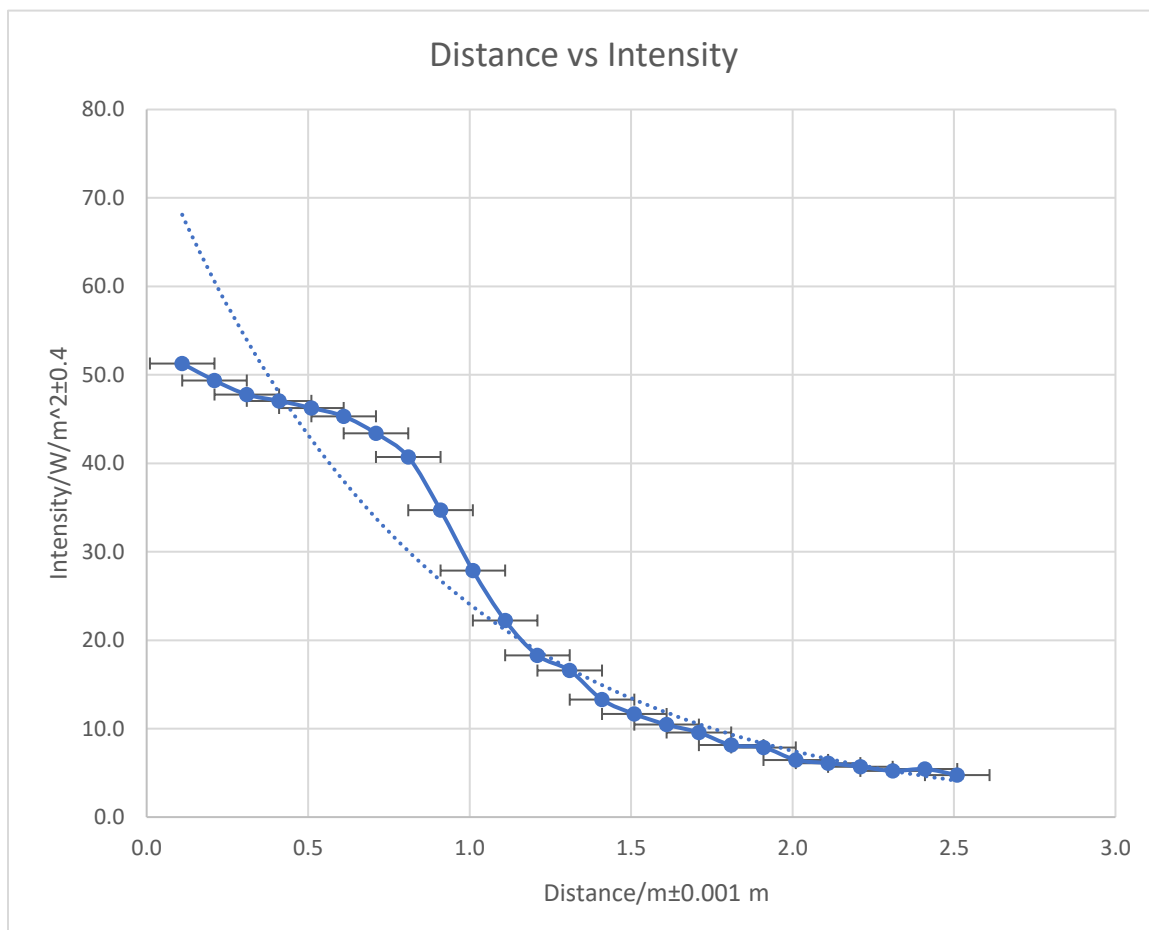
In order to have the real value of the intensity, the average of the background intensities was subtracted from the average of the recorded intensity from the luminous source. This averaged data is shown in the table below. As each reading is independent of each other, the uncertainties stay the same even in the averaged values. However, the subtraction, through error propagation, leads to an increase to 0.4 in the uncertainty of the real average intensity. As for the units, they are not standard therefore, for consistency to be maintained, the distance values were converted to meters and intensity from lux was converted to watts per square meter⁵⁶. The processed data is shown below.

⁵ Lux is a measure of the intensity as seen by the human eye. The conversion from lux to watts/m² for tungsten incandescent bulb is done by dividing the $E_{v(lx)}$ / 15. (15 is the typical efficacy of incandescent bulbs).

⁶ Conversion taken from "Lux to watts calculator." Lux to watts (W) conversion calculator. Accessed Feb. & march 2018. <https://www.rapidtables.com/calc/light/lux-to-watt-calculator.html>.

Table 2 Distance and intensity

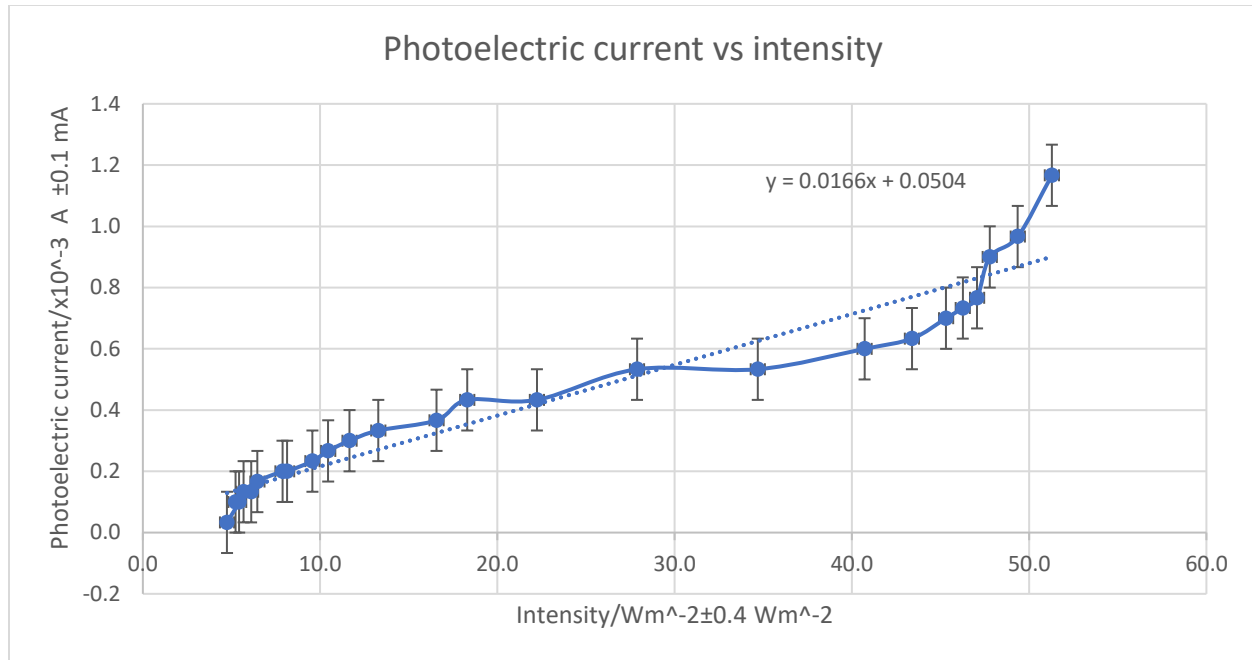
d/m \pm 0.001 m	Real avg intensity/Wm ⁻² \pm 0.4Wm ⁻²
0.110	51.3
0.210	49.4
0.310	47.8
0.410	47.1
0.510	46.3
0.610	45.3
0.710	43.4
0.810	40.7
0.910	34.7
1.010	27.9
1.110	22.2
1.210	18.3
1.310	16.6
1.410	13.3
1.510	11.7
1.610	10.5
1.710	9.6
1.810	8.1
1.910	7.9
2.010	6.5
2.110	6.1
2.210	5.7
2.310	5.2
2.410	5.4
2.510	4.7



Graph 1 Intensity vs Distance

The data was then taken and put into a graph as shown above. As expected, the data shows an inverse correlation between distance and intensity. The trend line shows an exponential decrease, representing the inverse square law. However, it is not seen to be completely followed by the data due to random errors. As for the error bars, only the horizontal is seen because the order of magnitude of the uncertainty is same as the axis intervals however, for vertical error bars 0.4 is too small of an uncertainty to be seen in this graph with the given axis intervals.

Intensity and photoelectric current relationship



Graph 2 Photoelectric current vs Intensity

As seen above in the graph, there is a direct positive correlation between intensity and the photoelectric current. The correlation also is linear because of the curve of best fit is a straight line. This gives us the relationship:

$$P_e = jI$$

Where P_e is photoelectric current, I is the incident intensity of light and j is a constant of proportionality⁷ which is equal to $\frac{\eta}{V_{oc} \cdot FF}$

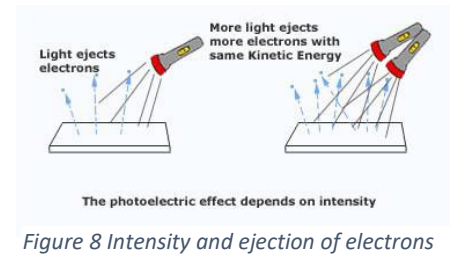
Where V_{oc} is the open circuit voltage⁸, FF is the fill factor⁹, η is the conversion efficiency

⁷ Solar cell Parameters and Equivalent circuit. Delfts. Accessed Feb. & march 2018. https://ocw.tudelft.nl/wp-content/uploads/solar_energy_section_9_1_9_3.pdf.

⁸ The maximum voltage a solar cell can produce.

⁹ Fill factor is the maximum possible power a solar cell can produce.

This is understandable because according to the photoelectric effect, the intensity increases the ejection of electrons from the solar cell as shown in Figure 5. However, this only happens as long as the incident light that hits the surface of the solar cell is in the band spectrum of the solar cell as the photocells have a frequency range for them to work. The graph is not showing an exact straight-line curve because of random errors in the experimental procedure which will be explained in the next section.



Intensity of central maxima and slit width

This part of the analysis is theoretical due to feasibility issues therefore a simulation was used to model the relationship between the intensity of the central maxima and the slit width.

Theory

As slit width increases, the total energy of the light passing in increases. This can be explained using the Huygens principle, the number of secondary sources would increase as the slit width increases as shown in the figure below.

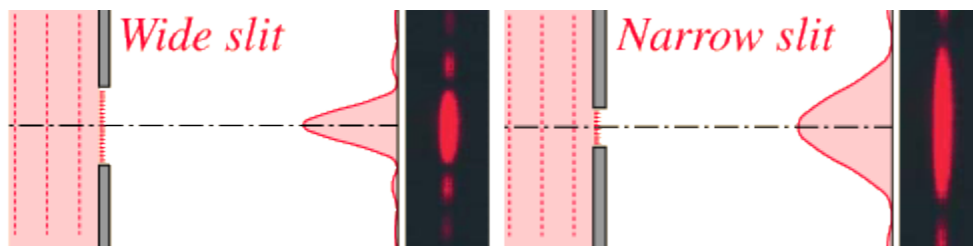


Figure 9 Fraunhofer varying slit width

The energy of a wave is determined by the amplitude of the wave and also in the case of electromagnetic wave, the amplitude is an electric field. Therefore, mathematically we can show this relationship as

$$E_{tot} = \sum_N E_N$$

Where N= number of Huygens sources/ electric field vectors passing through the slit

Therefore, it can be deduced that slit width is directly proportional to the total number of electric vectors passing through it.

$$b \propto E_{tot}$$

Where b is the slit width and E_{tot} is the sum of all the electric field vectors passing through the slit.

Subsequently, intensity at $\theta = 0$ is proportional to the square of the amplitude of the ingoing light wave and so thereby, in this case:

As seen from the background, the relationship can be shown mathematically:

$$I_{ave} = \frac{c\epsilon_0 E_0^2}{2}$$

So, for a double in slit width, there would be double in the number of Huygens sources, leading to 4 times the initial intensity at the central maxima because of the proportionality.

This implies that: $I \propto b^2$

Using the inverse Fourier transform and paraxial approximation, this relationship's constant of proportionality was found. This derivation was found from a secondary source¹⁰ and is found in appendix A.

¹⁰ Fraunhofer diffraction. Lab report. Physics, University of Calgary. Accessed Feb. & march 2018. <http://people.ucalgary.ca/~lvov/471/labs/fraunhofer.pdf>.

Constant of proportionality was found to be: $\frac{2I_{in}}{4\pi^2 c \epsilon_0}$

Where I_{in} refers to the incident intensity on the aperture, c refers to the speed of light in vacuum, ϵ_0 refers to the dielectric permittivity of free space.

Simulation¹¹

Using this relationship, the simulation was built:

As the experiment is based on the Fraunhofer model, it is of the highest importance that the diffraction is kept far field. So, the following condition was added to the code.

```
while ( (w**2) < (l*wlength) ) :
```

The condition is such that the formula given in the background part of this essay is always true.

$$\frac{W^2}{L\lambda} \ll 1$$

For the condition to be true, the maximum value W^2 can take is $L\lambda$

As the code is supposed to show how the intensity of central maxima is affected by slit width, it needs a set of parameters such as incident intensity on the aperture and distance of source from aperture should be already set to a constant value. This was done by the following steps.

¹¹ The full code of the simulation is found in Appendix C

The length was taken to be 131 cm because that was the average of all the readings taken in the experiment. This was done in order to make there is an easier consolidation of the data from the experiment and the simulation.

$$l=131.e-02$$

For appreciable diffraction to occur, there is a condition that the wavelength of the incident light is of the same order of magnitude as the opening or bigger, $\lambda \geq w$ therefore, as white light has wavelengths in the order of magnitude 10^{-7} which is the reason why the code has the lines.

```
a=0
const=1.e-7
w=a*const
```

The incident intensity on the aperture was found from the experiment done above and as the distance from the aperture was taken to be 131 cm, the corresponding real average intensity was 16.6 lux.

The wavelength was taken to be 550 nm because that is the wavelength at which a typical silicon solar cell absorbs the most intensity. This is supported by this graph from a secondary source¹².

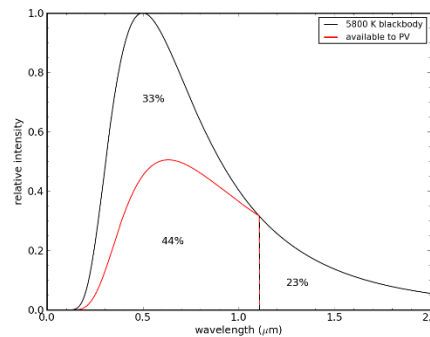


Figure 10 Absorption efficiency vs wavelength for a PV cell, given in red line

¹² Do the Math. Accessed Feb. & march 2018. <https://dothemath.ucsd.edu/2011/09/dont-be-a-pv-efficiency-snob/>.

A loop was created to increment the slit width until the Fraunhofer limit and the intensity of the central maxima for each slit width was recorded in an array. This is shown by this part of the code:

```
while ( (b**2)<(l*wlength)) :
    intensity=(2*I_naught/(c*epsilon_naught))/(4*((np.pi)**2))*(b**2)
    tab.add_row([b,intensity])
    i.insert(counter,b)
    j.insert(counter,intensity)
    a=a+1
    if a%10==0:
        a=1
        const=const*10

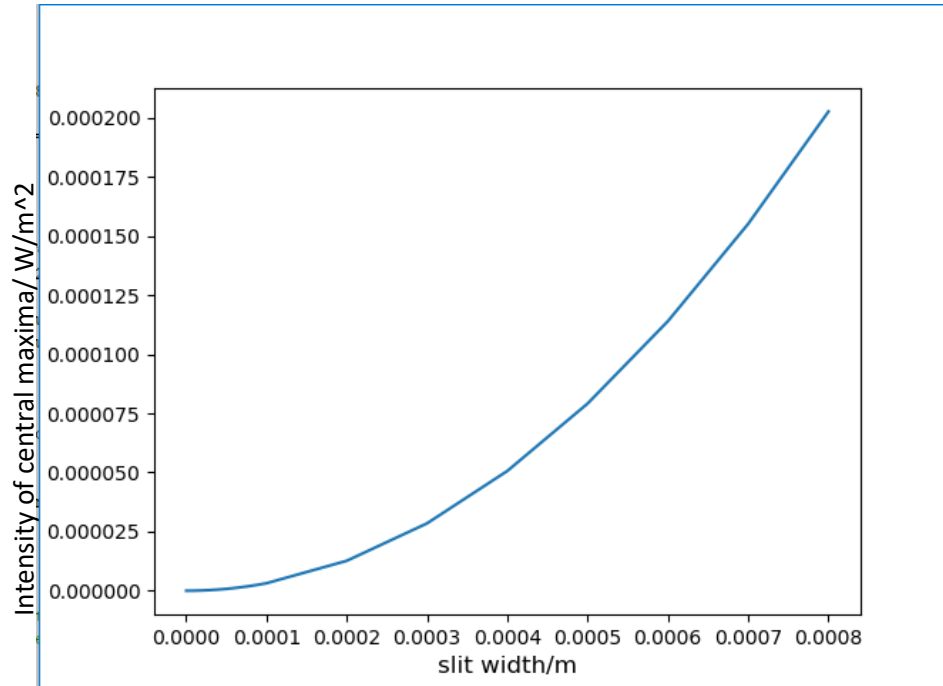
    b=a*const
    counter=counter+1

print(tab)
plt.plot(i,j)
plt.show()
```

The data collected was put into a tabulated column and also a graph.

Table 3 slit width and intensity of central maxima

slit width/m	intensity of central maxima/W/m ²
1e-07	3.16604858155e-12
2e-07	1.26641943262e-11
3e-07	2.8494437234e-11
4e-07	5.06567773048e-11
5e-07	7.91512145388e-11
6e-07	1.13977748936e-10
7e-07	1.55136380496e-10
8e-07	2.02627109219e-10
9e-07	2.56449935106e-10
1e-06	3.16604858155e-10
2e-06	1.26641943262e-09
3e-06	2.8494437234e-09
4e-06	5.06567773048e-09
5e-06	7.91512145388e-09
6e-06	1.13977748936e-08
7e-06	1.55136380496e-08
8e-06	2.02627109219e-08
9e-06	2.56449935106e-08
1e-05	3.16604858155e-08
2e-05	1.26641943262e-07
3e-05	2.8494437234e-07
4e-05	5.06567773048e-07
5e-05	7.91512145388e-07
6e-05	1.13977748936e-06
7e-05	1.55136380496e-06
8e-05	2.02627109219e-06
9e-05	2.56449935106e-06
0.0001	3.16604858155e-06
0.0002	1.26641943262e-05
0.0003	2.8494437234e-05
0.0004	5.06567773048e-05
0.0005	7.91512145388e-05
0.0006	0.000113977748936
0.0007	0.000155136380496
0.0008	0.000202627109219



Graph 3 Intensity of central maxima vs slit width

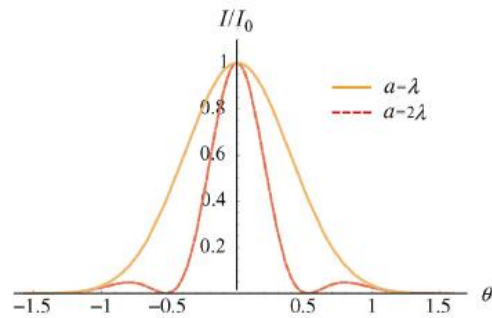


Figure 11 Intensity profile for varying slit width

From an observation perspective, the central maxima would become brighter and narrower as the slit width increases as shown in the intensity profile above. The narrowing is because of the linear

width decreasing with more slit width. This can easily be shown using the equation for linear width¹³:

$$W=2\lambda D/b$$

Where D refers to the distance from the aperture and the other symbols are already defined above.

The numerator being a constant, it is clear from the equation that linear width will decrease with increase in slit width.

Consolidating the above information, intensity of the central maxima increases exponentially with slit width. Qualitatively, this is because as the slit width increases, the energy passing through is linearly increasing with it and at the same time as seen above, the linear width of the central maxima is decreasing with increasing slit width and as intensity is directly proportional to power and inversely proportional to the area, therefore, intensity changes two-fold in variations with slit width in single slit diffraction.

Slit width and photoelectric current:

This relationship can be derived through consolidating the above two relationships. However, creating a data table for these two variables would be tricky considering the difference in the orders of magnitude. Therefore, Using the equation of the trendline from graph 2, it is possible to approximate the photoelectric current for a given intensity.

¹³ Derivation in the appendix B

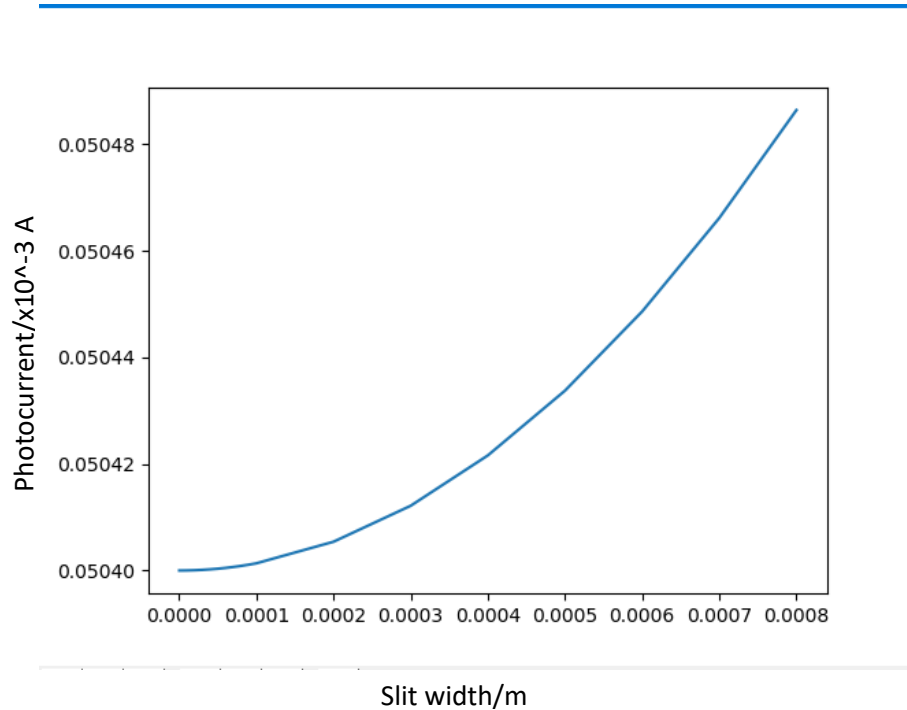
The equation of the trendline is: $y = 0.0166x + 0.0504$

where x is the intensity of the central maxima and y is the photoelectric current.

Table 4 Photoelectric current and slit width

slit width/m	Photoelectric current/ $\times 10^{-3}$ A
1e-07	0.0504000000014
2e-07	0.0504000000054
3e-07	0.0504000000122
4e-07	0.0504000000216
5e-07	0.0504000000338
6e-07	0.0504000000486
7e-07	0.0504000000662
8e-07	0.0504000000864
9e-07	0.0504000001094
1e-06	0.050400000135
2e-06	0.0504000005401
3e-06	0.0504000012153
4e-06	0.0504000021605
5e-06	0.0504000033758
6e-06	0.0504000048612
7e-06	0.0504000066166
8e-06	0.050400008642
9e-06	0.0504000109376
1e-05	0.0504000135032
2e-05	0.0504000540128
3e-05	0.0504001215288
4e-05	0.0504002160512
5e-05	0.0504003375799
6e-05	0.0504004861151
7e-05	0.0504006616567
8e-05	0.0504008642046
9e-05	0.050401093759
0.0001	0.0504013503197
0.0002	0.0504054012789
0.0003	0.0504121528775
0.0004	0.0504216051155
0.0005	0.050433757993
0.0006	0.0504486115099
0.0007	0.0504661656663
0.0008	0.0504864204621

Graph 4 Photocurrent vs Slit width



As seen from the graphs and the information above, slit width increase leads to an exponential increase in intensity of the central maxima. At the same time an increase in intensity of the central maxima leads to a linear increase in photoelectric current. Therefore, slit width and the photoelectric current are positively correlated.

Mathematically, these relationships can be shown as:

$$I = \frac{2I_{in}}{4\pi^2 c \epsilon_0} b^2 \quad P_e = \frac{\eta}{V_{oc} FF} I$$

NOTE: I_{in} is intensity of light incident on the aperture while I is the intensity of the central maxima and P_e is the photoelectric current.

This implies that,

$$P_e = Qb^2$$

Where Q is the product of the two constants of proportionality that equals: $\frac{2I_{in} \eta}{V_{oc} FF 4\pi^2 c \epsilon_0}$

Therefore, the graph between photoelectric current and slit width is same as the graph between slit width and intensity as shown in figure 5.

Conclusion

In conclusion, my hypothesis was true in this scenario because as slit width increase implies decrease in the amount of diffraction, and as seen in graph 3, current output increases with slit width therefore, less the amount of diffraction, the higher the current output. This is evidently because the light's energy is spread over a large area and as photocurrent depends on intensity shown by the data, there is a reduce in current output. This answers the research question but however, there are a couple of limitations (mainly because of availability issues) that could be avoided to make this research better and lead to a more in-depth analysis.

Evaluation

Assumptions

Experiment

1. Only the central maxima was assumed to fall on the solar cell. The secondary maximas were neglected for the sake of simplicity but despite that, the negligence would not have much of an effect on the final relationships because nearly 80% of the total incident intensity is concentrated in the central maxima. Moreover, as the research question is based only one solar cell, not all the maximas fall on the solar cell's area therefore, the effects of other maximas are further decreased.
2. The solar cell was assumed to be ideal where the conversion efficiency is 100%, fill factor is 1 and the short circuit current= the light generated current

Simulation

1. Light shone was monochromatic.
2. Assumed to be vacuum- no further diffraction by air particles

Errors and its impacts

- 1) **Measurement of background intensity:** while measuring there were constant fluctuations in the background intensity. This could have been mainly because of unwanted reflections from the cave like structure. The readings that were taken into the table were the most frequent values seen on the screen. This explains the inverse square law not being completely followed by the intensity vs distance graph. However, 3 readings were taken to reduce this error and was averaged to find a mean value minimizing the error to a comfortable extent. One way this could have been avoided is if a laser was used, it would have a unidirectional wave and therefore would not led to unwanted reflections.
- 2) **Design of the experiment:** Due to the size of the light source, from 41 cm to 121 cm there are some random errors happening. This is shown by the sudden change in the trend of the graph in graphs 1 and 2. The light source blocked the solar cell and the probe from all background light from 11 to 41 cm but at 41 cm, the source was far enough to let more light come in from outside which resulted in bumps and more fluctuation in the background intensity until in 121 cm where the fluctuations stabilize to a constant value.
- 3) **Apparatus-** the area of the intensity probe was small compared to that of the solar cell therefore, there would be systematic errors in the correspondence of the intensity and

the photoelectric current. However, this would be negligible because the area of the probe was enough to project the central maxima onto it and because of my primary assumption of only investigating the central maxima, this error, while having an effect, can be neglected.

- 4) **Staebler-Wronski Effect**- as the solar cell used was an amorphous cell, this effect could have led to slight variations in the photo electric current production. The effect states that when intense light is shone on an amorphous cell, the efficiency of the solar cell tends to decrease with time. Therefore, due to random errors in the delay of measurement of the current there could be errors due to this effect. This error could have been avoided by using a crystalline solar cell or using a data logger to reduce those delays in measurement.
- 5) **Shunt and series resistance**: the shunt and series resistance of the solar cell could have interfered with the current readings which might partially be the reason why the data points had some anomalies in graph 2.
- 6) **Negligence of 6 wavelengths**: white light has 7 wavelengths, which means that the output of the simulation would not be entirely applicable as the simulation only made use of one wavelength. Even though, the wavelength set was the wavelength at which the solar cell is the most efficient, this error would affect the correspondence between slit width and the photoelectric current. This is because one variable was from the results of the simulation and the other from the experiment. This could have been avoided if a laser was used as it would be monochromatic.
- 7) **External influences**: temperature was a huge influence as during the experiment there were some weather changes that could have led to some temperature variations which

while still negligible could have led to some variations in the efficiency of the solar cell. This could have been avoided if the experiment was done in a more temperature-controlled environment.

Limitations

1. Different slit widths could not be acquired, thereby the simulation was used however using the slits and doing the same experiment would give way more accurate and true results. Though the relationships shown in the study are true, the data is still for an ideal solar cell and ideal diffraction setup.
2. Experiment could not be done in the dark. If it had been, the data would be free of background intensity.
3. Laser could not be used because of the high constant multiplier it has. This is supported by the gaussian beam theory. This effect basically shows that lasers have a large constant of proportionality when it comes to the inverse square law and as my research was partly based on it, I could not use it.

Further investigation

In addition to avoiding the limitations and the sources of error shown above the research can be improved in the following ways:

1. This research only covered single slit diffraction. The study can be extended to double slits and diffraction grating which can give more insight into how solar cells are affected by diffraction as a whole.
2. Furthermore, more types of apertures can be used such as circular to see how the aperture is affecting the relationships shown above.
3. Finally, instead of a solar cell, solar panels can be used along with data on how clouds diffract the sunlight which would give the most practical application to this research which would show how solar panels are affected by diffraction.

Appendices

Appendix A

Derivation of formula between Intensity of central maxima and central width from secondary source.

Consider a general aperture illuminated by light as in figure 3. In the plane of the aperture, perpendicular to the plane-wave propagation axis \hat{z} , the electric field is given by:

$$E(x, y) = E_0 f(x, y) e^{-i\omega t}, \quad (1)$$

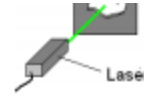


Figure 3

where $f = (x, y)$ reflects the geometry of the aperture. We treat this field according to Huygens' principle as an antenna (extended oscillator) that generates secondary waves that form the diffraction pattern. To calculate this pattern, we re-express the oscillator as a linear combination of infinite plane antennas:

$$E(x, y) = E_0 \iint_{k_x, k_y} f(k_x, k_y) e^{i(k_x x + k_y y) - i\omega t} dk_x dk_y, \quad (2)$$

where each antenna is characterized by a pair of wave vectors (k_x, k_y) and $f(k_x, k_y)$ is the **inverse Fourier transform** of the aperture function:

$$f(k_x, k_y) = \mathfrak{T}^{-1} \{f(x, y)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-ik_x x - ik_y y} f(x, y) dx dy. \quad (3)$$

Each antenna in Eq. (2) generates a plane wave

$$e^{i(k_x x + k_y y) - i\omega t} \rightarrow e^{i(k_x x + k_y y + k_z z) - i\omega t}$$

with $k_y^2 + k_x^2 + k_z^2 = (2\pi / \lambda)^2 = \omega^2 / c^2$, so the field behind the aperture can be written as

$$E(x, y, z) = E_0 \iint_{k_x, k_y} f(k_x, k_y) e^{i(k_x x + k_y y + k_z z) - i\omega t} dk_x dk_y.$$

The field produced by each plane wave antenna propagates in the directions defined by angles (θ_x, θ_y) with respect to the z axis, such that

$$\begin{aligned} \sin \theta_x &= \frac{k_x}{k} \\ \sin \theta_y &= \frac{k_y}{k}. \end{aligned} \quad (4)$$

We continue our treatment within the so-called **paraxial approximation**, where we assume that $k_z \gg k_x, k_y$, so $k_z \approx k = \frac{2\pi}{\lambda}$, independent of k_x and k_y . The paraxial approximation also implies that $\theta_x \approx \sin \theta_x$ and $\theta_y \approx \sin \theta_y$. If the diffraction pattern is observed on a screen very far away from the aperture (i.e. in the far-field approximation relevant to the Fraunhofer diffraction), we can assume that the irradiance at some point (x', y') on the screen will be determined by the electric field of the plane wave propagating at angles

$$\begin{aligned}\theta_x &= \frac{x'}{L} \\ \theta_y &= \frac{y'}{L}\end{aligned}\tag{5}$$

with respect to the z axis. Combining Eqs. (5) and (6) we find for the far field amplitude

$$E(x', y') = f\left(k \frac{x'}{L}, k \frac{y'}{L}\right)\tag{6}$$

In other words, in the paraxial approximation **the far-field diffraction pattern is just a scaled inverse Fourier transform of the aperture**. The intensity of the diffraction pattern is found according to

$$I(x', y') = \left\langle E(x', y') \right\rangle^2,\tag{8}$$

Consider now a single slit, with the aperture function as described in figure 4. The vertical dimension of the slit is sufficiently large so that we may treat the problem as one-dimensional. The aperture function here is the “top-hat” function:

$$\begin{aligned}f(x) &= E_0, \quad x \in \left[-\frac{a}{2}, \frac{a}{2}\right] \\ f(x) &= 0, \quad x \notin \left[-\frac{a}{2}, \frac{a}{2}\right].\end{aligned}\tag{9}$$

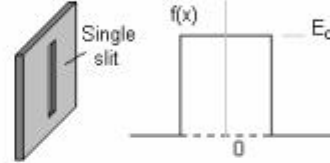


Figure 4

We can then evaluate the Fourier transformation of the aperture function to find the electric field at a point on a screen in the far-field regime

$$E(x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(k_x) e^{ik_x x'} dx = \frac{1}{2\pi} \int_{-a/2}^{a/2} E_0 e^{ik_x x} dx = E_0 \frac{e^{ik_x a/2} - e^{-ik_x a/2}}{2\pi i k} = \frac{2aE_0}{2\pi i k} \sin(k_x a/2)$$

or, using equation (4) for k_x and defining:

$$ak_x/2 = ak \sin \theta / 2 = \beta\tag{10}$$

we have:

$$E(\beta) = \frac{aE_0}{2\pi} \frac{\sin \beta}{\beta}.\tag{11}$$

Recalling that the irradiance at x' is just the time-averaged electric field at that point, we can write:

$$I(\theta) = I_0 [\text{sinc}(\beta)]^2,\tag{12}$$

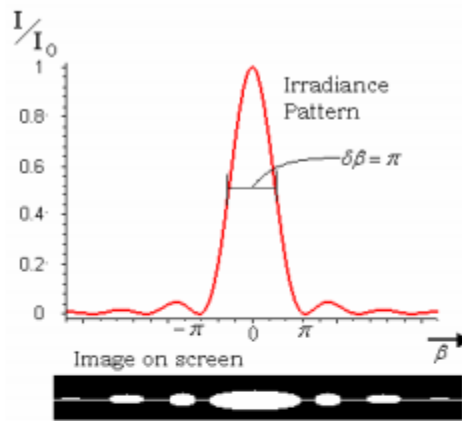


Figure 5

may be used, along with equation (10), to find the angular positions of the maxima of the refracted beams.

Fraunhofer Diffraction from a more complicated apparatus can be calculated by using the fact that the Fourier transform of the convolution two functions $f(x)$ and $g(x)$ is the product of the individual Fourier transforms:

$$\mathfrak{T}^{-1}\{f(x) * g(x)\} = \frac{1}{2\pi} \mathfrak{T}^{-1}\{f(x)\} \cdot \mathfrak{T}^{-1}\{g(x)\}, \quad (13)$$

where the convolution of functions $f(x)$ and $g(x)$ is given by

$$h(x) = f(x) * g(x) = \int_{-\infty}^{\infty} f(t)g(x-t)dt. \quad (14)$$

For example, the irradiance pattern for a two slits separated by a distance b , with each slit having width a may be evaluated using (13) as the convolution of two **Dirac delta functions** and a single slit (Figure 6.)

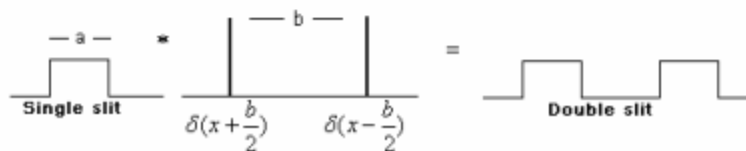


Figure 6

where we've defined

I_0 to be $\left(\frac{aE_0}{2\pi}\right)^2$ and

used $\text{sinc}(x) = \frac{\sin x}{x}$.

Figure 5 displays the resultant irradiance pattern. Note that equation (12) will have minima whenever $\beta = n\pi$, for all non-zero integers n

(recall that $\lim_{n \rightarrow 0} \frac{\sin n}{n} = 1$). Taking the derivative of (12) and setting it equal to zero, we see that $\text{sinc } \beta$ reaches maxima whenever $\beta = \tan \beta$. This is a transcendental equation, for which some roots are listed in Table 1. These values

Roots of $\sin(\beta) - \beta$	
n	β
1	1.43π
2	2.46π
3	3.47π
4	4.48π
5	5.48π
6	6.48π
7	7.49π

Table 1

As seen above, the equation for $I_0 = \left(\frac{aE_0}{2\pi}\right)^2$ but it is in terms of the incident energy but as it is more contextually useful if it was in terms of incident intensity therefore, using the substitution

$$I_{\text{ave}} = \frac{c\epsilon_0 E_0^2}{2}$$

for electric field from this formula:

The following relationship was derived: $I_0 = \frac{2I_{\text{in}}}{4\pi^2 c\epsilon_0} b^2$

Appendix B

Derivation of the linear width formula

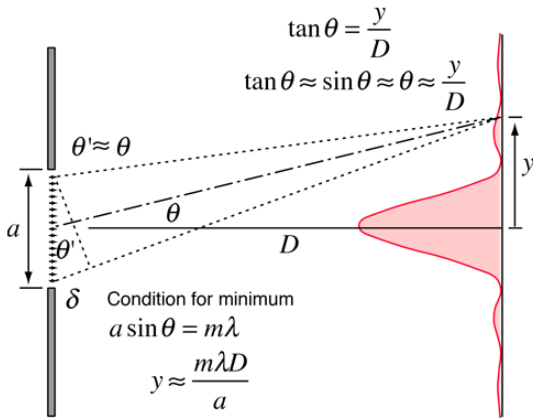


Figure 13 Single slit diffraction geometry

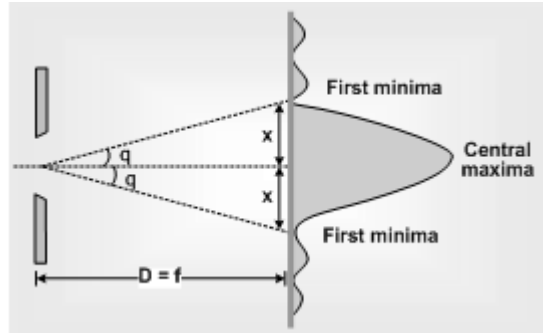


Figure 12 Linear width is shown by $2x$

As shown in figure 11, in single slit diffraction, $a \sin \theta = m\lambda$ and at the same time

$\tan \theta \approx \sin \theta \approx \theta = \frac{y}{D}$ when θ is small in value and m is the integral order of minimum. In this

context, we can take it as 1 as the linear width of the central maxima is two times the length between the first minima and the central maxima. Multiplication by 2 is because y is just the width from one minima to the center of the central maxima. This is shown in figure 11.

Consolidating both:

$$\sin \theta = \frac{m\lambda}{a} \approx \theta \text{ at small theta values.}$$

So

$$\frac{m\lambda}{a} \approx \frac{y}{D}$$

Transpose the variables and we get:

$$y \approx \frac{m\lambda D}{a}$$

Therefore, the final equation for the linear width is: $W=2\lambda D/b$

Appendix C

Simulation program code

```
import numpy as np
import matplotlib.pyplot as plt
from prettytable import PrettyTable

l=131.e-02
counter=0
wlength=55.e-08
const=1.e-07
I_naught=16.6
c=3.e8
epsilon_naught=8854.e-15
tab=PrettyTable()
tab.field_names=["slit width/m","intensity of central maxima/W/m^2"]
i=[]
j=[]
a=1
b=a*const
while ((b**2)<(l*wlength)):
    intensity=(2*I_naught/(c*epsilon_naught))/(4*((np.pi)**2))*(b**2)
    tab.add_row([b,intensity])
    i.insert(counter,b)
    j.insert(counter,intensity)
    a=a+1
    if a%10==0:
        a=1
        const=const*10

    b=a*const
    counter=counter+1

print(tab)
plt.plot(i,j)
plt.xlabel("slit width/m", fontsize=12)
plt.ylabel("intensity of central maxima/W/m^2", fontsize=12)
plt.show()
```

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Pictures

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