MA106

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1 Lecture 1: Introduction

- Matrices are a new universe of Numbers

- Visualizing the matrices as a column vector of row vectors or a row vector of column vectors, is an important thing

- Outer Product is called so, as its sort of doing the inner product/scalar product(or dot product!), the other way round!

- Going over the various ways to write the Product of Two Matrices

- Exercise: Proving Trivial Results like $\left(\mathbf{AB}\right)^T = \mathbf{B^T}\,\mathbf{A^T}$

- The jth row of $\mathbf{AB}$ is a linear combination of the jth row of $\mathbf{A}$ with coefficient of some common, and analogically in case of kth column of $\mathbf{AB}$ would be

- Really Nice Question: Justifying the different cases of solutions to system of linear equations using concepts from matrices

2 Lecture 2: Linear Systems

- General Linear system will include homogeneous as well as non-homogeneous.

- **Deducing Connections:** How to relate $\mathbf{Ax} = \mathbf{b}$ to $\mathbf{Ax} = \mathbf{0}$. If $\mathbf{Ax} = \mathbf{0}$ has non-trivial solutions, than that would mean infinitely many solutions if we know just one solution exists.

- Extending the past concepts to more general cases: Using the above thing to solve any general system of m equations in n variables.

3 Lecture 3: Gaussian Elimation

Nothing as such apart from Lecture Notes introduced, Just a very nice and thoughtful question: Let $\mathbf{A} \in \mathbb{R}^{9x4}$ and $\mathbf{B} \in \mathbb{R}^{7x3}$. Is there $\mathbf{X} \in \mathbb{R}^{4x7}$ such that $\mathbf{X} \neq \mathbf{O}$ but $\mathbf{AXB} = \mathbf{O}$

4 In General Observations/Queries which people posted on Whatsapp Group

- From LEC2, where first we identify the pivot points in REF, identify the free and non-free variables and then set the non-free ones to zero, following which we also identify the basis vectors by setting each one of them to one in and getting separate solutions, so that the overall solution is a linear combination of these.

- In the REF of an inconsistent System, we can get different REFs there is no unique one, but we do have a unique Reduced REF or Row-canonical Form.

- Another way to progress, after we've identified the Free variables in REF, would be to simply substitute the Free variables $x_i = \alpha_i$, but we don't do this, Why?

Let $\mathbf{s} := \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}^{\mathsf{T}} \in \mathbb{R}^{n \times 1}$ be any solution of the homogeneous system, that is, $\mathbf{As} = \mathbf{0}$. Then $\mathbf{s}$ is a linear combination of the $n - r$ basic solutions $\mathbf{s}_{\ell_1}, \ldots, \mathbf{s}_{\ell_{n-r}}$. To see this, let $\mathbf{y} := \mathbf{s} - x_{\ell_1} \mathbf{s}_{\ell_1} - \cdots - x_{\ell_{n-r}} \mathbf{s}_{\ell_{n-r}}$. Then $\mathbf{Ay} = \mathbf{As} - x_{\ell_1} \mathbf{As}_{\ell_1} - \cdots - x_{\ell_{n-r}} \mathbf{As}_{\ell_{n-r}} = \mathbf{0}$, and moreover, the kth entry of $\mathbf{y}$ is 0 for each $k \in \{\ell_1, \ldots, \ell_{n-r}\}$. It then follows that $\mathbf{y} = \mathbf{0}$, that is, $\mathbf{s} = x_{\ell_1} \mathbf{s}_{\ell_1} + \cdots + x_{\ell_{n-r}} \mathbf{s}_{\ell_{n-r}}$. Thus we find that the general solution of the homogeneous system is given by

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Why is $\mathbf{y} = 0$ in the above paragraph?

- Try proving: For all $\mathbf{A} \in \mathbb{R}^{n \times n}$ if $\mathbf{AE} = \mathbf{EA}$, for some $\mathbf{E} \in \mathbb{R}^{n \times n}$ such that $\mathbf{E} = \mathbf{I}$.

- Let $I \subset \mathbb{R}^{nn}$ be a nonempty set closed under addition such that $\mathbf{MN}, \mathbf{NM} \in I$ whenever $\mathbf{N} \in I$ and $\mathbf{M} \in \mathbb{R}^{n \times n}$. Show that either $I = 0$ or $I = \mathbb{R}^{n \times n}$.

5 Lecture 4: Inverses and its usage in Linear Equations

- Try to prove: Let $\mathbf{A} \in \mathbb{R}^{n \times n}$.if $\mathbf{Ax} = \mathbf{b}$ has only zero solution, then $\mathbf{A}$ is invertible

- Prove that for $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{n \times n}$ such that $AB = I$, then $BA = I$.

- Lwt $\mathbf{A}$ and $\mathbf{B}$ be square matrices. Then $\mathbf{AB}$ is invertible iff $\mathbf{A}$ and $\mathbf{B}$ are invertible, and then $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.

- Prove that every matrix has a unique Row-canonical form.

- Row Echelon Forms are never unique.