Arrays: Quicksort

Agenda

- Explore the Quick sort algorithm

- Understand the following aspects
    - Algorithm mechanism and pseudocode
    - Algorithm iterations on varying input
    - Algorithm time and space complexity

Quick sort

- A “randomized” sort algorithm:
  - Addresses performance issues with Merge sort.
  - Performs well in most scenarios, badly very rarely.

- Algorithm scheme:
  1. Randomly shuffle the given array A[0...N-1].
  2. Partition the array into 2 pieces, as follows:
     - For some index j, A[j] is in-place.
     - No element to the left of j is larger than A[j].
     - No element to the right of j is smaller than A[j].
  3. Sort each piece recursively.

Quick sort

- Characteristics:
    - Quick sort is a comparison-based sort.
    - Quick sort can sort an array in-place.
    - Auxiliary array not required.
    - Starts by randomly shuffling the order of elements.

- Comparison operation:
        - Defined as required for the data type
            - Numbers
            - Strings
            - Objects: by attributes

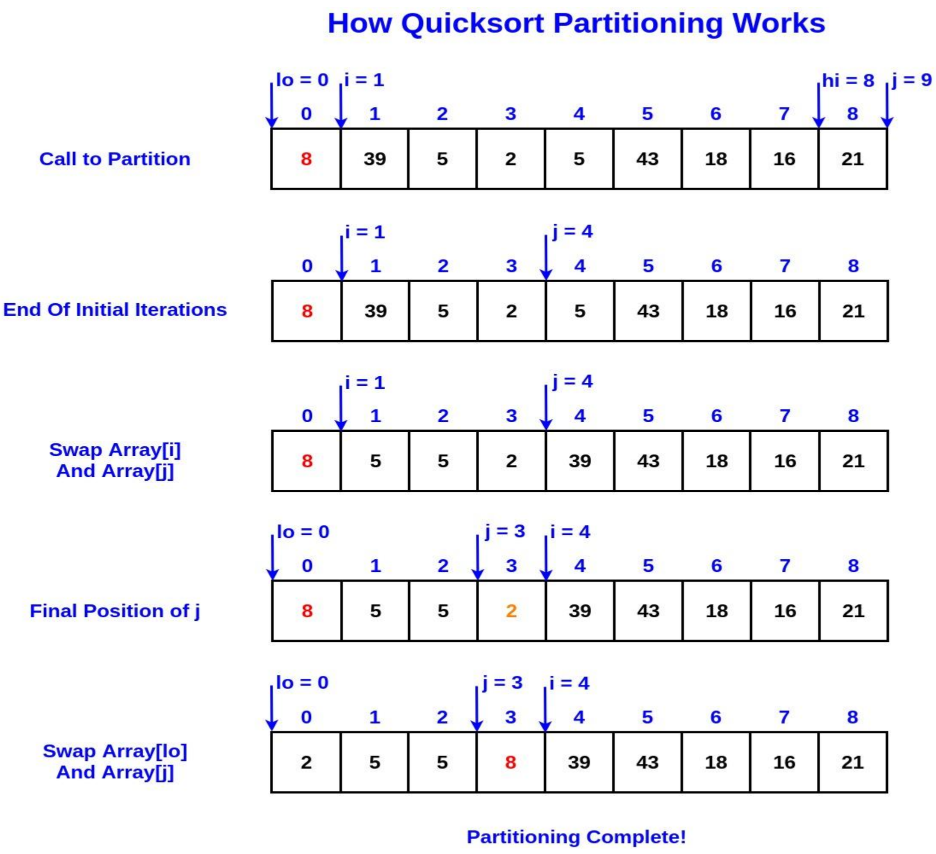
Quick sort

1. Shuffle the array.

2. Partition this array, so that, for some j:
    a. Array[j] is in place.
    b. All entries to the left of j are smaller.
    c. All entries to the right of j are larger.

3. Recursively sort each piece.

Quick sort: Partition



How Quicksort Partitioning Works

Call to Partition

lo = 0, i = 1, hi = 8, j = 9

0	1	2	3	4	5	6	7	8
8	39	5	2	5	43	18	16	21

End Of Initial Iterations

i = 1, j = 4

0	1	2	3	4	5	6	7	8
8	39	5	2	5	43	18	16	21

Swap Array[i] And Array[j]

i = 1, j = 4

0	1	2	3	4	5	6	7	8
8	5	5	2	39	43	18	16	21

Final Position of j

lo = 0, j = 3, i = 4

0	1	2	3	4	5	6	7	8
8	5	5	2	39	43	18	16	21

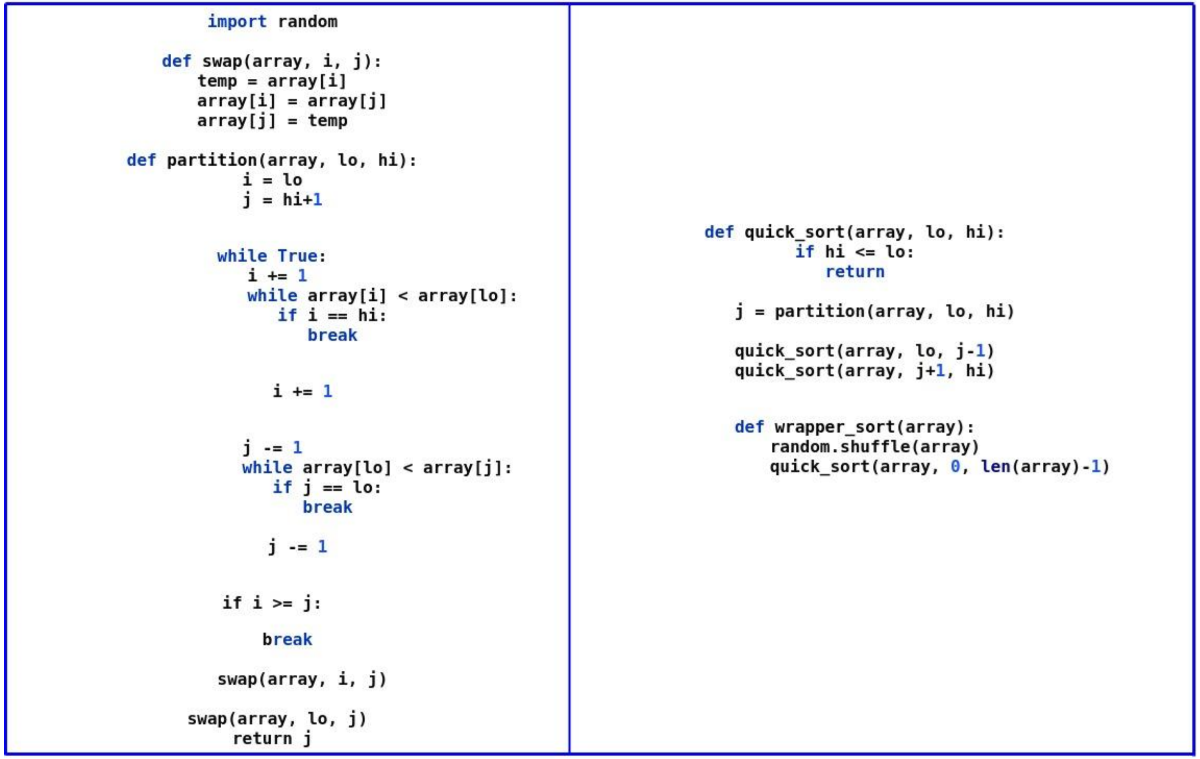
Swap Array[lo] And Array[j]

lo = 0, j = 3, i = 4

0	1	2	3	4	5	6	7	8
2	5	5	8	39	43	18	16	21

Partitioning Complete!

Quick sort pseudocode



Code For Quick Sort

```
        import random

    def swap(array, i, j):
       temp = array[i]
       array[i] = array[j]
       array[j] = temp

def partition(array, lo, hi):
            i = lo
            j = hi+1


         while True:
            i += 1
            while array[i] < array[lo]:
               if i == hi:
                  break


               i += 1


            j -= 1
            while array[lo] < array[j]:
               if j == lo:
                  break

              j -= 1


          if i >= j:

              break

         swap(array, i, j)

      swap(array, lo, j)
           return j
```

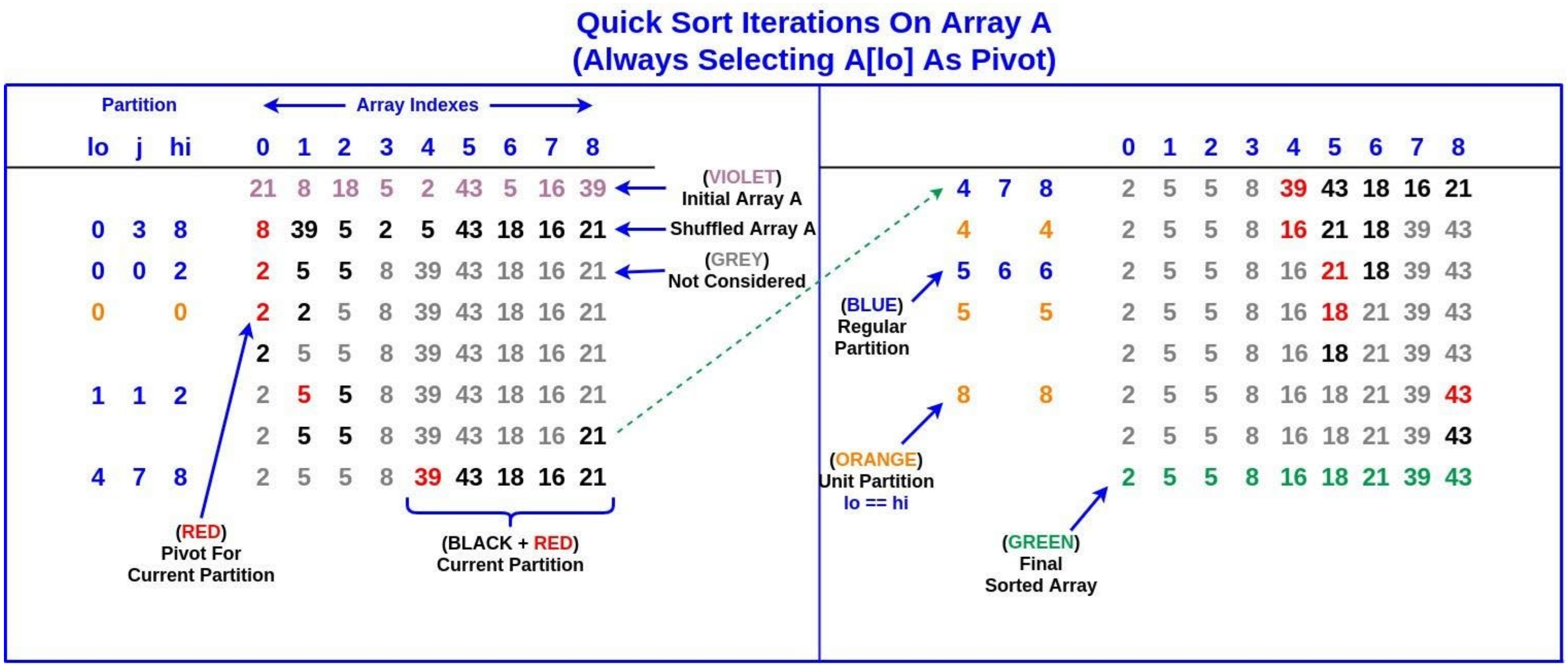
```
def quick_sort(array, lo, hi):
         if hi <= lo:
            return

   j = partition(array, lo, hi)

   quick_sort(array, lo, j-1)
   quick_sort(array, j+1, hi)


   def wrapper_sort(array):
       random.shuffle(array)
       quick_sort(array, 0, len(array)-1)
```

Quick sort iterations



Quick Sort Iterations On Array A
(Always Selecting A[lo] As Pivot)

Partition			Array Indexes									
lo	j	hi	0	1	2	3	4	5	6	7	8	
			21	8	18	5	2	43	5	16	39	← (VIOLET) Initial Array A
0	3	8	8	39	5	2	5	43	18	16	21	← Shuffled Array A
0	0	2	2	5	5	8	39	43	18	16	21	← (GREY) Not Considered
0		0	2	2	5	8	39	43	18	16	21	
			2	5	5	8	39	43	18	16	21	
1	1	2	2	5	5	8	39	43	18	16	21	
			2	5	5	8	39	43	18	16	21	
4	7	8	2	5	5	8	39	43	18	16	21	

(RED)
Pivot For
Current Partition

(BLACK + RED)
Current Partition

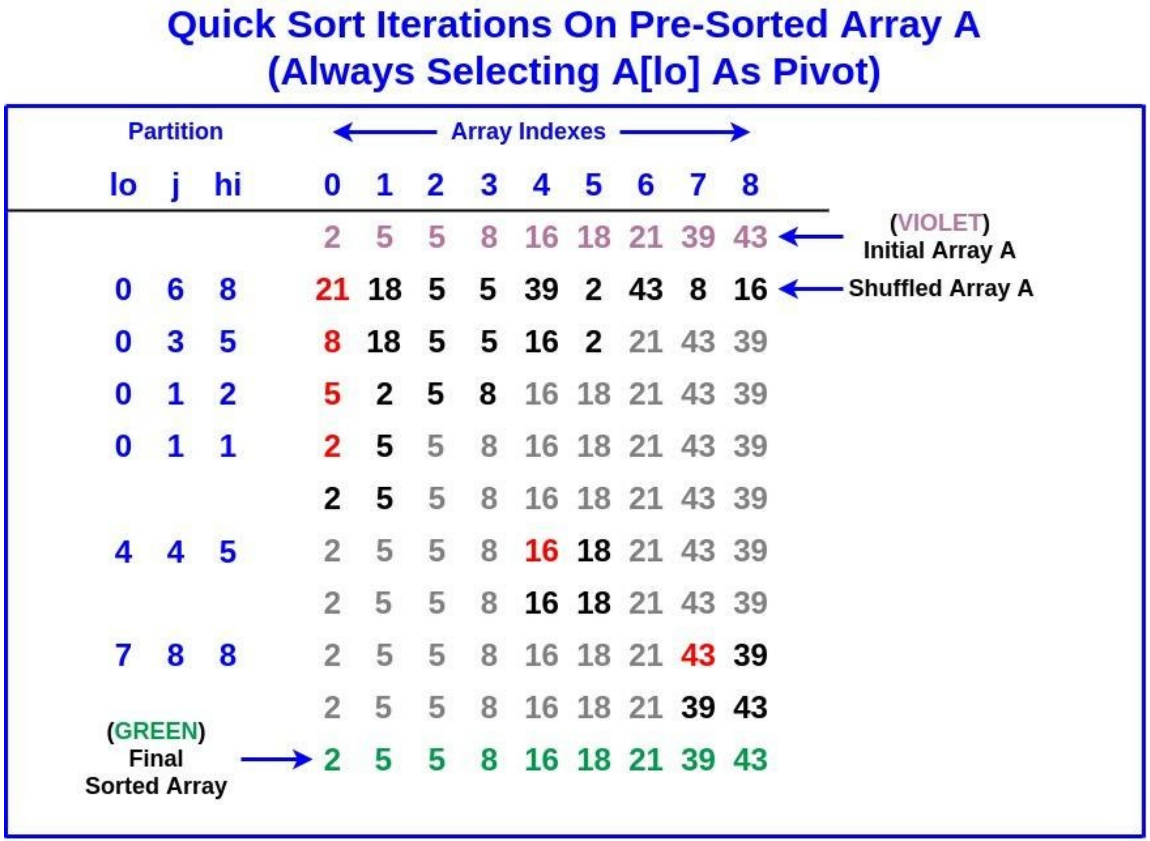
			0	1	2	3	4	5	6	7	8
4	7	8	2	5	5	8	39	43	18	16	21
4		4	2	5	5	8	16	21	18	39	43
5	6	6	2	5	5	8	16	21	18	39	43
5		5	2	5	5	8	16	18	21	39	43
			2	5	5	8	16	18	21	39	43
8		8	2	5	5	8	16	18	21	39	43
			2	5	5	8	16	18	21	39	43
			2	5	5	8	16	18	21	39	43

(BLUE)
Regular
Partition

(ORANGE)
Unit Partition
lo == hi

(GREEN)
Final
Sorted Array

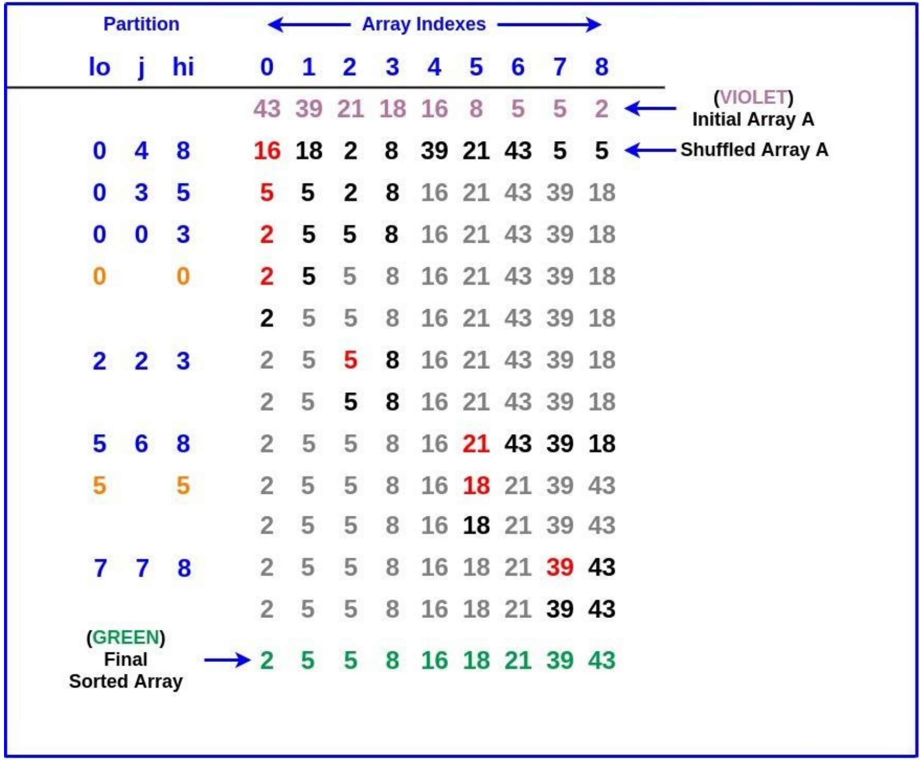
Quick sort iterations



Quick Sort Iterations On Pre-Sorted Array A (Always Selecting A[lo] As Pivot)

Partition			← Array Indexes →									
lo	j	hi	0	1	2	3	4	5	6	7	8	
			2	5	5	8	16	18	21	39	43	← (VIOLET) Initial Array A
0	6	8	21	18	5	5	39	2	43	8	16	← Shuffled Array A
0	3	5	8	18	5	5	16	2	21	43	39	
0	1	2	5	2	5	8	16	18	21	43	39	
0	1	1	2	5	5	8	16	18	21	43	39	
			2	5	5	8	16	18	21	43	39	
4	4	5	2	5	5	8	16	18	21	43	39	
			2	5	5	8	16	18	21	43	39	
7	8	8	2	5	5	8	16	18	21	43	39	
			2	5	5	8	16	18	21	39	43	
(GREEN) Final Sorted Array →			2	5	5	8	16	18	21	39	43	

Quick sort iterations



Quick Sort Iterations On Reverse-Sorted Array A (Always Selecting A[lo] As Pivot)

Partition			← Array Indexes →									
lo	j	hi	0	1	2	3	4	5	6	7	8	
			43	39	21	18	16	8	5	5	2	← (VIOLET) Initial Array A
0	4	8	16	18	2	8	39	21	43	5	5	← Shuffled Array A
0	3	5	5	5	2	8	16	21	43	39	18	
0	0	3	2	5	5	8	16	21	43	39	18	
0		0	2	5	5	8	16	21	43	39	18	
			2	5	5	8	16	21	43	39	18	
2	2	3	2	5	5	8	16	21	43	39	18	
			2	5	5	8	16	21	43	39	18	
5	6	8	2	5	5	8	16	21	43	39	18	
5		5	2	5	5	8	16	18	21	39	43	
			2	5	5	8	16	18	21	39	43	
7	7	8	2	5	5	8	16	18	21	39	43	
			2	5	5	8	16	18	21	39	43	
(GREEN) Final Sorted Array →			2	5	5	8	16	18	21	39	43	

Quick sort: Complexity

- Analyzing time complexity:
  - Note that the input is always randomly shuffled.
  - This does not eliminate worst behaviour of Quicksort, but greatly reduces its probability.

- Worst case: The total number of compares and entry swaps (during partitioning) is:

$C(N) = N + (N-1) + (N-2) + \ldots + 1$: proportional to N^2 steps

- Average case: Proportional to $N\log_2 N$ steps

Quick sort: Complexity

- Analyzing space complexity:
  - Quicksort is an in-place sorting algorithm.
    - Extra space is not used during the sorting.

  - Best case: Constant space
  - Worst case: Constant space
  - Average case: Constant space

Summary

We explored the Quick sort algorithm as follows:

- Algorithm mechanism and pseudocode
- Algorithm iterations on varying input
- Time and space complexity

Thank You