FORMAL LANGUAGES & AUTOMATA THEORY

[FLAT 06CS56]

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

UNIT - 1

Introduction to Finite Automata

* Introduction to Finite Automata
* Central concepts of Automata Theory
* Deterministic Finite Automata
* Nondeterministic Finite Automata

Introduction to Automata

Automata theory is the study of abstract computing devices or machines. Theory of computation is based on mathematical computations. These computations are used to represent various mathematical models.

There are several reasons why study of Automata is important. Finite Automata is important kinds of hardware and software. Some of the most important kinds are

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

i) Software for designing and checking the behavior of digital circuits

ii) The Lexical Analyzer of typical compiler

iii) Software for scanning large bodies of text such as collection of webpages, to find occurance of words, phrases

IV) Automata theory deals with design of finite state m/c

v) Automata theory is base for design of formal languages

Central Concepts of Automata Theory

The most important definitions of terms that encompass the theory of automata includes

i) Alphabets

ii) Strings

iii) Languages

Alphabets: An alphabet is a finite, non empty set of symbols. Conventionally the symbol Σ is used to represent set of alphabets. Common alphabet includes

i) $\Sigma = \{0, 1\}$ the binary alphabet

ii) $\Sigma = \{a, b, \cdots z\}$ the set of lower case letters.

Strings: A string is a finite sequence of symbols chosen from some alphabet

For example 01101 is a string from binary alphabet $\Sigma = \{0, 1\}$

The empty string is the string with zero occurra-nce of symbols. This string is denoted by ε.

The length of the string is the number of positions for symbol in string. The standard notation for the length of string w is $|w|$. For example $|011| = 3$ and $|\epsilon| = 0$.

The power of the alphabet Σ is the set of all strings of certain length from that alphabet. Σ^K is used to define set of strings of length K. For example

$\Sigma = \{a, b, c\}$ then

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

$$\Sigma^0 = \varepsilon$$

$$\Sigma^1 = \{a, b, c\}$$

$$\Sigma^2 = \{aa, ab, ac, ba, bb, bc, ca, cb, cc\}$$

Σ^* – is used to represent set of strings over alphabet Σ including null. So $\Sigma^* = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cdots \cup \{\varepsilon\}$

Σ^+ is used to represent set of all string over alphabet Σ excluding null. So $\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \ldots\ldots$

Concatenation of strings is two strings are combin-ed together to form single string. For example Let $x = 1101$ and $y = 0011$. The $xy = 11010011$

Languages: A set of string, all of which are chosen from Σ^* where Σ is set of i/p symbols is called alphabet. If Σ is an alphabet, and $L \subseteq \Sigma^*$, then L is a language over Σ. The Language is collection of appropriate strings of alphabet.

For example (i) $\Sigma = \{ \varepsilon, 0, 00, 000, \ldots \}$

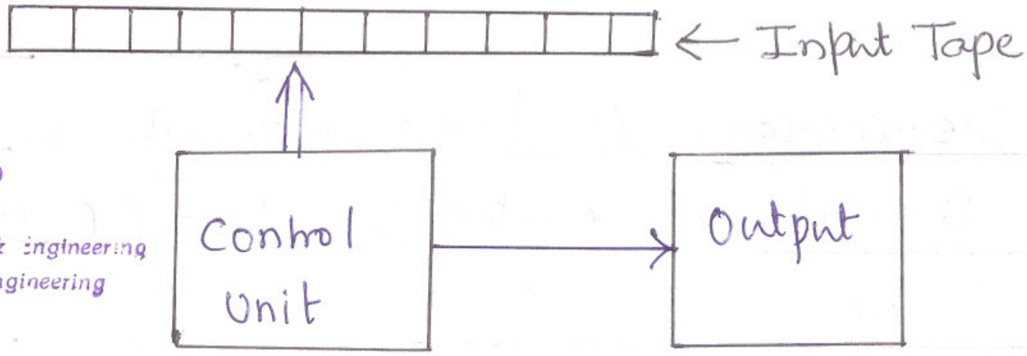
Here the Language L is defined by any number of zeros.

(ii) $\Sigma = \{ \varepsilon, 01, 10, 0011, 0101, 1001, \ldots \}$

Here the Language L is defined by set of strings of 0's and 1's with equal number of each.

FINITE AUTOMATA

A finite automaton is a mathematical or theoritical model which is used to study the abstract computing devices with inputs choosen from Σ, set of input symbols called alphabet. The machine may accept or reject string. Finite automaton is an abstract model of a digital computer represented as given below.



A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

The three components of Finite Automata are

i) Input tape : The tape divided into cells each of which can hold one input symbol. The string to be processed is stored in this tape

ii) Control Unit: The control decides the next state on receiving particular i/p from input tape.

iii) OutPut: Output may be acceptedor rejected

The notations used in finite automata design

One

○ Circle represent initial state or intermediate state

→ represents transition

◎ two concentric circles represents final state

Definition: A finite automata is a collection of 5-tuple represented by $M = (Q, q_0, F, \Sigma, \delta)$

where

Q = set of states $\{q_0, q_1, q_2, \ldots\ldots\}$

q_0 = Unique initial state

F = set of final states

Σ = Set of input symbols

δ = transitions represented by table.

The transition function δ is defined as

$$\delta : Q \times \Sigma \text{ to } Q$$

ie $\delta(q_0, a) = q_1$

q_0 is current state, a is input symbol and q_1 is resultant state.

From the transition table, the automaton can be represented diagramically, called as Transition diagram.

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

Types of Finite Automata

Based on the principle of transition on current input symbol from one state to other, Finite Automaton is differentiated as

i) Deterministic Finite Automata
ii) Non deterministic finite Automata
iii) Non deterministic finite Automata with ϵ moves

DETERMINISTIC FINITE AUTOMATA (DFA)

In DFA, there is exactly one transition for every input symbol from state. So it is possible to determine exactly to which state the machine enters after processing input symbol.

The DFA is defined as 5-tuple represented by $M = (Q, \Sigma, q_0, F, \delta)$

where

Q is set of states $\{q_0, q_1, \ldots q_n\}$

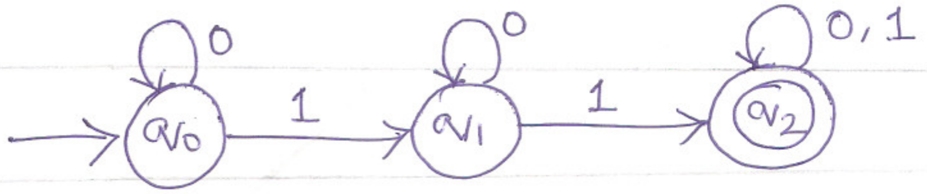
Σ is set of input symbols

q_0 is initial state

F is set of final states

δ is transition function

Consider the following transition diagram



In the above transition diagram, we can observe on given input symbol there is only one transition in all the transitions.

The transition table is

state	0	1
q_0	q_0	q_1
q_1	q_1	q_2
q_2	q_2	q_2

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

In transition table also we can observe, on given I/p symbol there is only one transition. So next state is deter-mined.

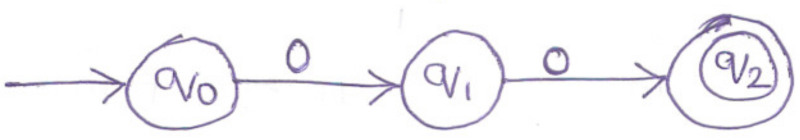
Problems.

1. Design DFA which accept the string ending with 00 over the $\Sigma = \{0, 1\}$.

Ans: DFA to accept the string ending with 00.

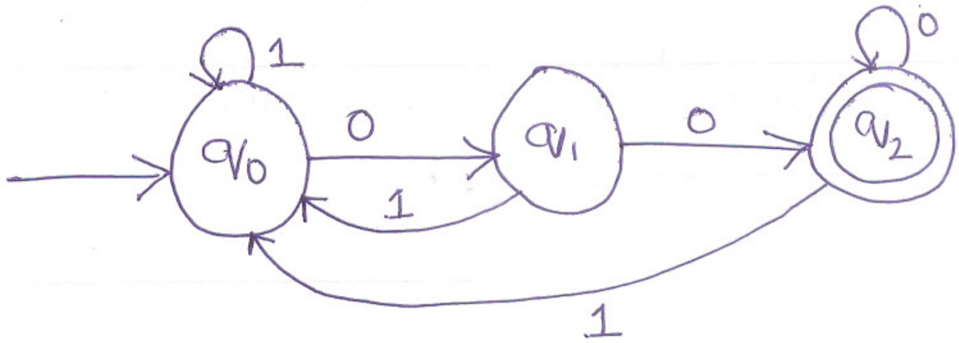
Eg: 100, 000, 1100, 1000 etc.

The minimum string is 00. First write DFA which accepts only minimum string 00.



Now, start adding rest of the transition according to question in transition table.

State	0	1
$\rightarrow q_0$	q_1	q_0
q_1	q_2	q_0
$* q_2$	q_2	q_0



Consider the string 10100, the transitions are

$q_0 \xrightarrow{1} q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_0 \xrightarrow{0} q_1 \xrightarrow{0} q_2 \leftarrow$ Final state

After all transitions, the last state is q_2. The q_2 is final state. Therefore string is accepted.

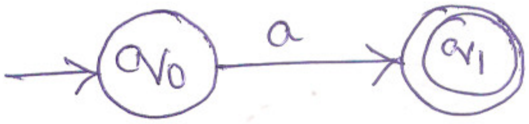
Consider the string 10101, the transition are

$q_0 \xrightarrow{1} q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_0 \leftarrow$ Non final state

Since q_0 is nonfinal state, the string is not accepted.

2. Draw DFA to accept string of a's and b's having at least one a.

Ans: DFA having atleast one a. The minimum string is a. Draw DFA for only a



Now start adding rest of transitions.

State	a	b
$\rightarrow q_0$	q_1	q_0
$* q_1$	q_1	q_1

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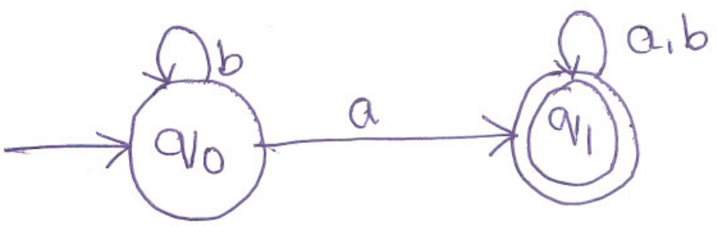
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

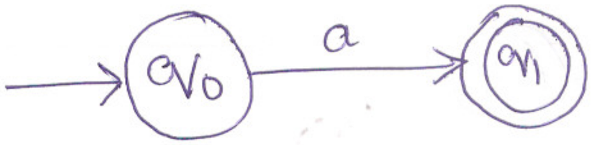


Consider the string babb

Consider the string bbb.

3. Draw DFA to accept string of a's & b's having exactly one a.

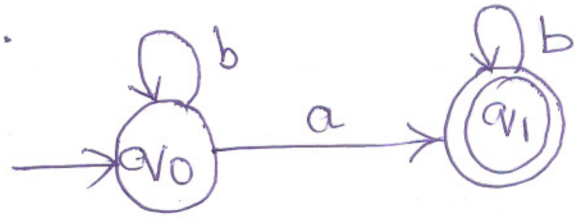
Ans: DFA to accept exactly one 'a'. The minimum string is a. Draw DFA for one a



Now add rest of the transition.

state	a	b
q_0	q_1	q_0
q_1	ϕ	q_1

We can observe q_1 on b transition cannot be given.



Consider abb

Consider aab

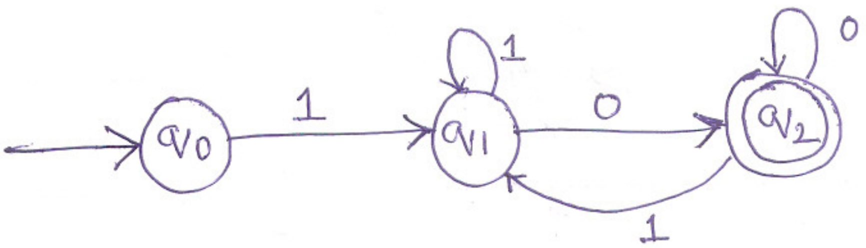
4. Design DFA which accepts those strings that starts with 1 and ends with 0.

Ans: The minimum string is 10. Draw DFA for 10

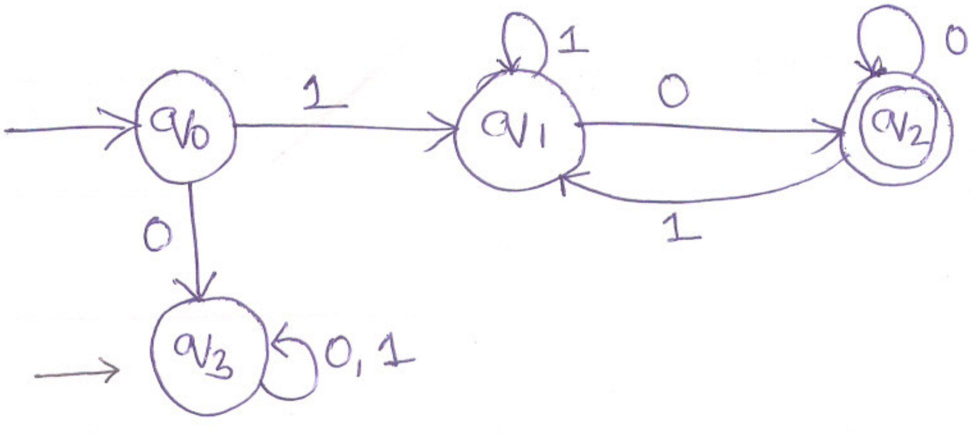
Add rest of the transitions one after the other

State	0	1
$\rightarrow q_0$	ϕ	q_1
q_1	q_2	q_1
$* q_2$	q_2	q_1

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078



Here we can observe $\delta(q_0, 0)$ is not possible. In this situation we can create one trap state. and given the transition to trap state. But it is optional



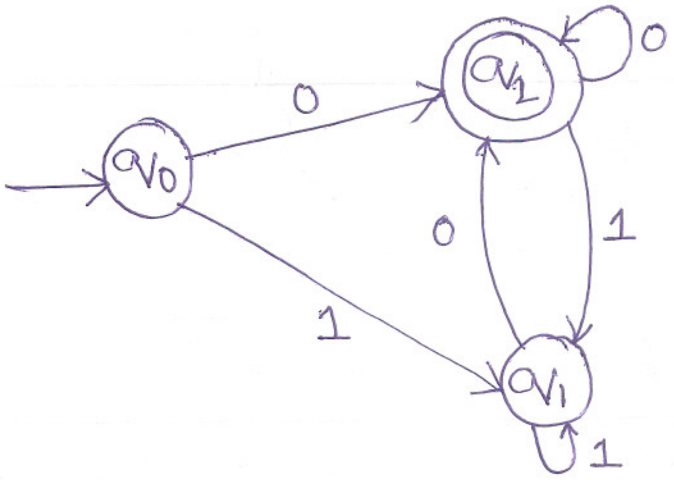
Trap state $\rightarrow$

Consider the string 100010

Consider the string 10011

5. Design DFA which checks whether the given binary number is even.

Ans: The minimum string is 0, 10, 100



State	0	1
$\rightarrow q_0$	q_2	q_1
q_1	q_2	q_1
$* q_2$	q_1	q_2

6. Design DFA which accept odd number of 1's and any no of 0's

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Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
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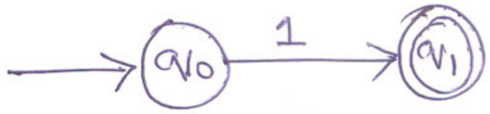
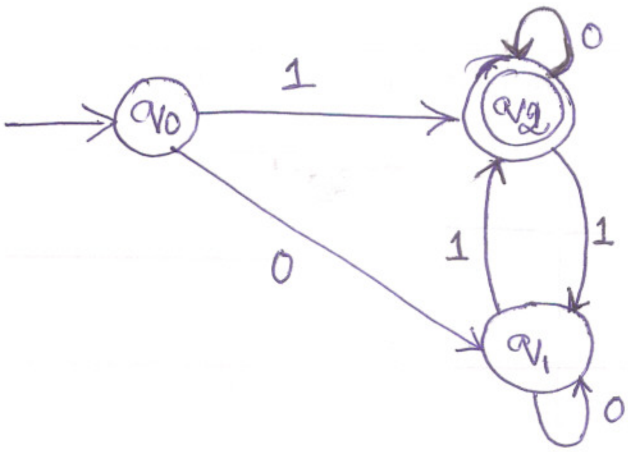
Ans: The minimum string is 1



Add rest of the transitions later

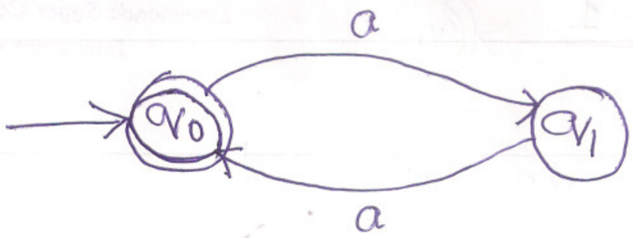


state	0	1
$\rightarrow q_0$	q_1	q_2
q_1	q_1	q_2
$* q_2$	q_2	q_1

7. Draw DFA to accept even number of a's over Σa

<u>Ans:</u> Since it is even number of a's (any even no)

In this type of problem, DFA should have loops.

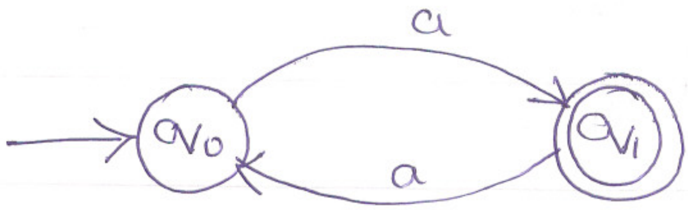


The transition table is

State	a
→ * q_0	q_1
- * q_1	q_0

8. Draw DFA to accept odd number of a's

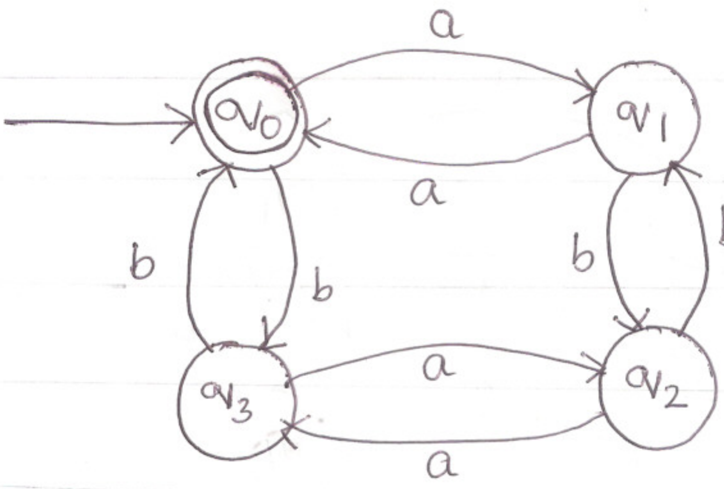
<u>Ans:</u>



State	a
→ * q_0	q_1
q_1	q_0

9. Draw DFA to accept even number of a's and even number of b's.

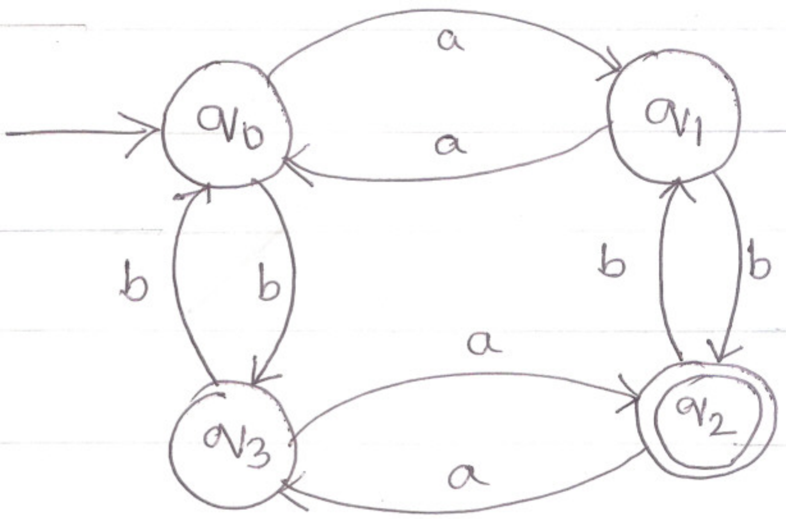
Ans:



State	a	b
→* q_0	q_1	q_3
q_1	q_0	q_2
q_2	q_3	q_1
q_3	q_2	q_0

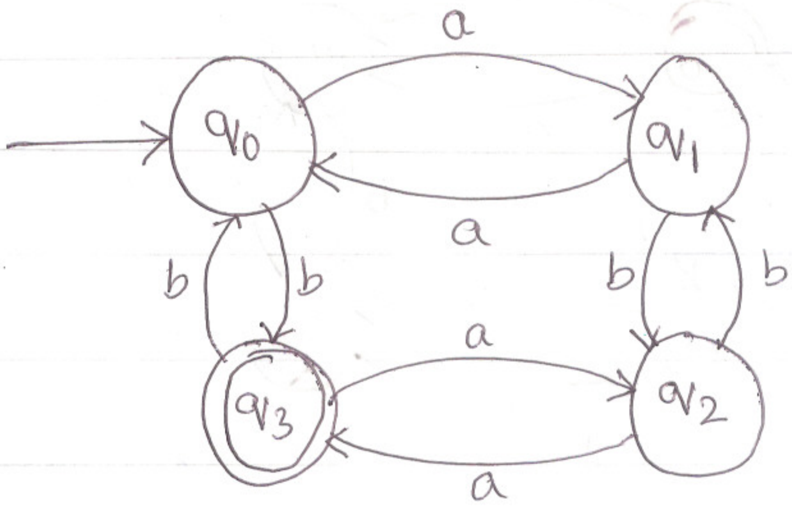
10. Draw DFA to accept odd no of a's and odd no of b's.

Ans:



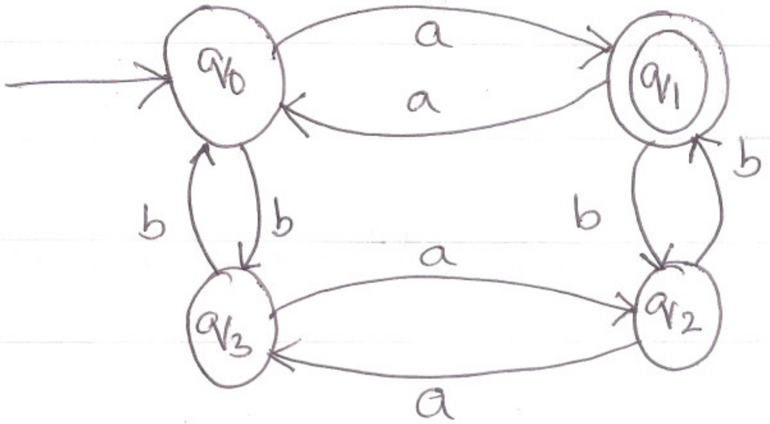
11. Draw DFA to accept even number of a's and odd number of b's.

Ans



12. Draw DFA to accept odd number of a's and even number of b's

Ans



13. Obtain a DFA to accept the language $L = \{w ; |w| \bmod 3 = 0\}$ over Σ a

Ans: Language $L = \{w, |w| \bmod 3 = 0\}$ is

$|w|$, length of the word mod 3 = 0

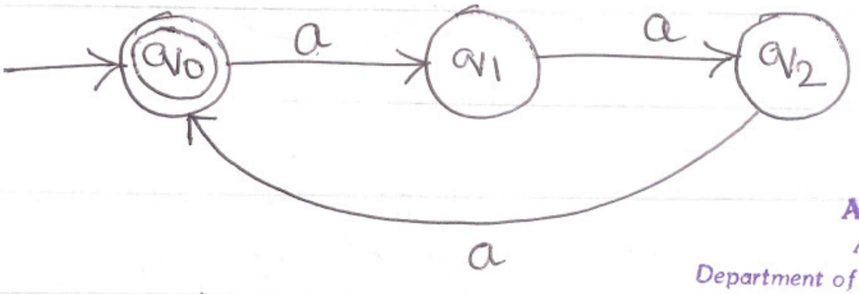
Example $0 \% 3 = 0$ $\quad$ $3 \bmod 3 = 0$

$1 \bmod 3 = 1$ $\quad$ $4 \bmod 3 = 1$

$2 \bmod 3 = 2$ $\quad$ $5 \bmod 3 = 2$ So on

The DFA should accept 0 a and multiple of 3.

The automata is

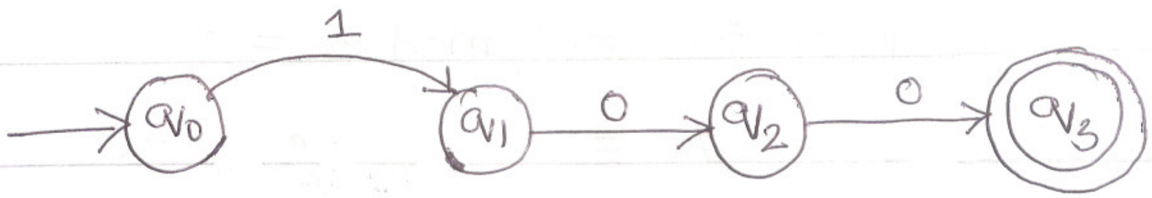


A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

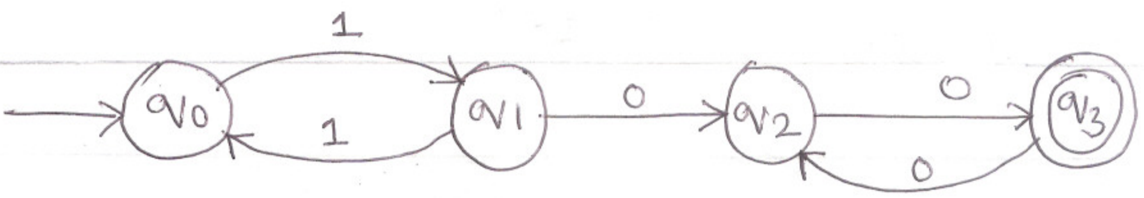
State	a
$\rightarrow * q_0$	q_1
q_1	q_2
q_2	q_0

14. Draw a DFA to accept the language odd no of 1's followed by even number of 0's. $\Sigma = \{0, 1\}$

Ans: The minimum string is 100



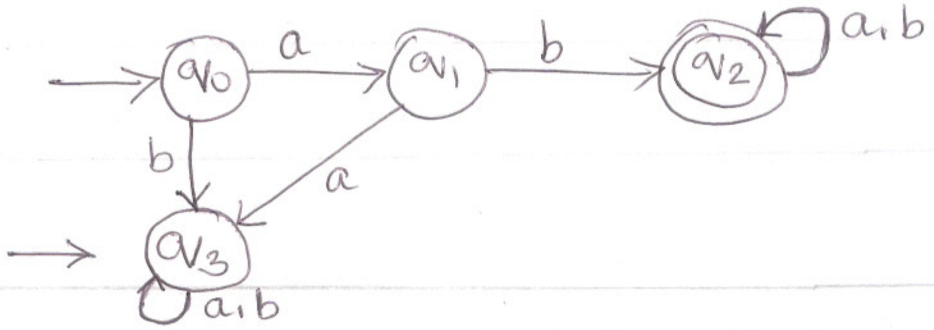
by making loops, DFA accepts any odd number of 1's and even 0's.



15. Obtain DFA to accept string of a's & b's starting with the string ab.

Ans: The minimum string is ab. The DFA for ab is

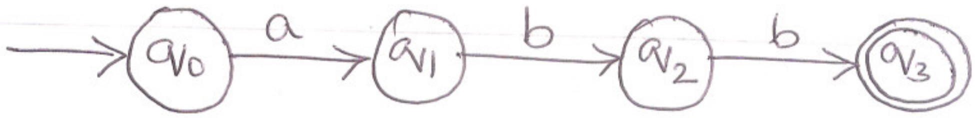
The q_2 can have both transition to itself



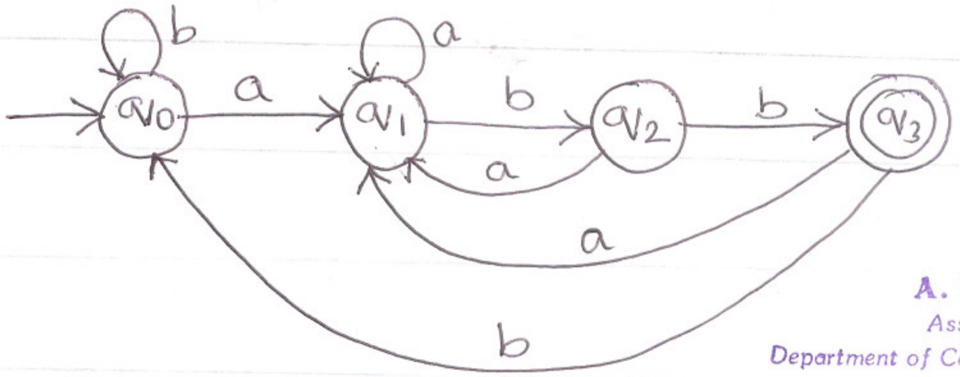
Trap state

16. Draw a DFA to accept string of a's & b's ending with the string abb.

Ans: The minimum string is abb. Draw DFA to accept abb only



Now add new transitions according to question

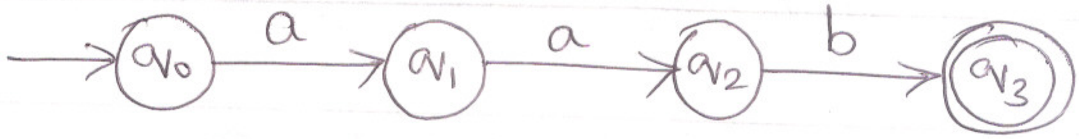


A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

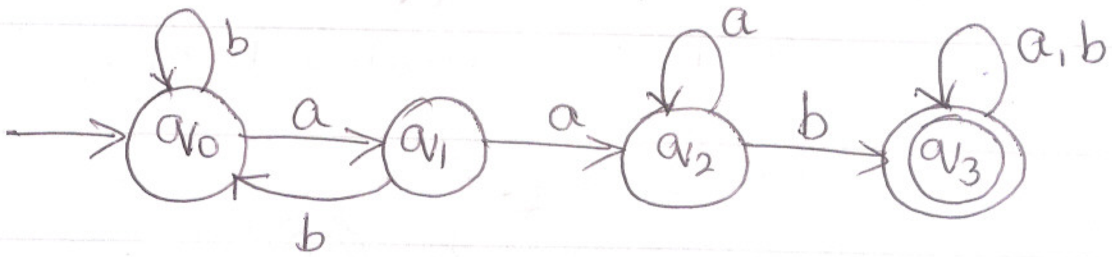
State	a	b
→ q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
* q_3	q_1	q_0

17. Draw a DFA to accept string of a's & b's having a substring aab.

Ans: The minimum string is aab

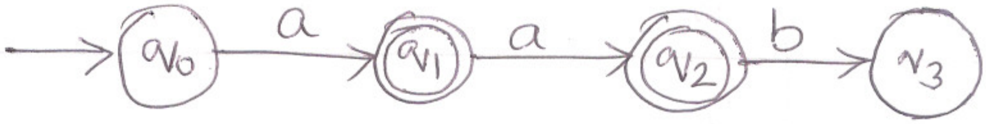


Add rest of the transition

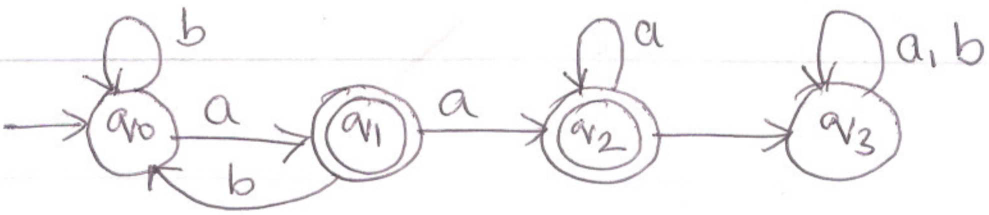


18. Draw DFA to accept string of a's & b's except those having substring aab

Ans: Minimum string is aab. Draw DFA for aab, and donot make last state as final



Add rest of the transitions, according to question



19. Construct DFA which accepts string of 0's and 1's where each string is represented as binary number,

$L = \{ w,\ w \bmod 5 = 0,\ \Sigma = \{0, 1\} \}$

Ans: For these problems, transition can be obtained using the formula

$\delta(q_i, a) = q_j$ where $j = (\underbrace{r}_{\text{radix}} * \underbrace{i}_{\text{remainder}} + \overbrace{d}^{\text{I/p symbol}}) \bmod \underbrace{k}_{\text{divisor}}$

$d = \{0, 1\}$

$r = 2$

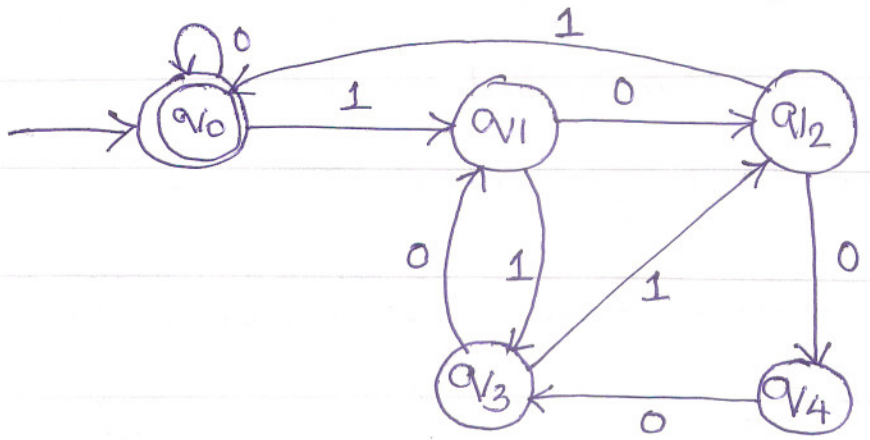
$i = 0, 1, 2, 3, 4$

$k = 5$

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

Remainder	d	$(2*i+d) \bmod 5$	$\delta(q_i, d) = q_j$
$i=0$	0	$(2*0+0) \bmod 5 = 0$	$\delta(q_0, 0) = q_0$
	1	$(2*0+1) \bmod 5 = 1$	$\delta(q_0, 1) = q_1$
$i=1$	0	$(2*1+0) \bmod 5 = 2$	$\delta(q_1, 0) = q_2$
	1	$(2*1+1) \bmod 5 = 3$	$\delta(q_1, 1) = q_3$
$i=2$	0	$(2*2+0) \bmod 5 = 4$	$\delta(q_2, 0) = q_4$
	1	$(2*2+1) \bmod 5 = 0$	$\delta(q_2, 1) = q_0$
$i=3$	0	$(2*3+0) \bmod 5 = 1$	$\delta(q_3, 0) = q_1$
	1	$(2*3+1) \bmod 5 = 2$	$\delta(q_3, 1) = q_2$
$i=4$	0	$(2*4+0) \bmod 5 = 3$	$\delta(q_4, 0) = q_3$
	1	$(2*4+1) \bmod 5 = 4$	$\delta(q_4, 1) = q_4$

Now, write the transition diagram for these transitions



State	0	1
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

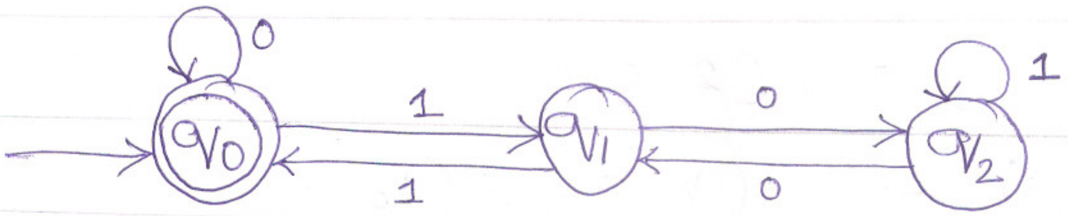
20. Draw DFA to accept string divisible by 3, $\Sigma_{0,1}$

Ans: The transitions are obtained by relation

$$\delta(q_i, a) = q_j, \quad q_j = (\underbrace{r}_{\text{radix}} * \underbrace{i}_{\text{Remainder}} + \underbrace{d}_{\text{i/p}}) \bmod \underbrace{k}_{\text{divisor}}$$

Remainder	d	$(2*i+d) \bmod 3$	$(q_i, d) = q_j$
$i=0$	0	$(2*0+0) \bmod 3 = q_0$	$(q_0, 0) = q_0$
	1	$(2*0+1) \bmod 3 = q_1$	$(q_0, 1) = q_1$
$i=1$	0	$(2*1+0) \bmod 3 = q_2$	$(q_1, 0) = q_2$
	1	$(2*1+1) \bmod 3 = q_0$	$(q_1, 1) = q_0$
$i=2$	0	$(2*2+0) \bmod 3 = q_1$	$(q_2, 0) = q_1$
	1	$(2*2+1) \bmod 3 = q_2$	$(q_2, 1) = q_2$

Now, draw transition diagram using these transitions

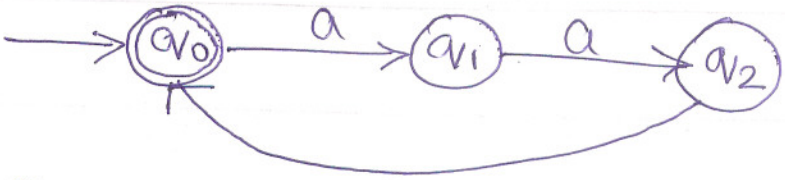


A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

21. Obtain DFA to accept the following

$L = |w| \bmod 3 \geq |w| \bmod 2$ over Σa

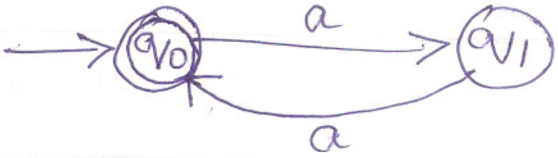
Ans: First, design DFA for $|w| \bmod 3$



— Machine M_1

$Q_1 = \{q_0, q_1, q_2\}$

Next, design DFA for $|w| \bmod 2$



— Machine M_2

$Q_2 = \{q_0, q_1\}$

Now compute $Q_1 \times Q_2$ (I Machine M_1 ↓, II Machine M_2 ↓)

$$\delta((q_0, q_0)a) = \delta(q_0, a), \delta(q_0, a)$$
$$= q_1 \quad , \quad q_1$$
$$= (1, 1) \text{ Consider Subscript}$$

$$\delta((q_1, q_1)a) = \delta(q_1, a), \delta(q_1, a)$$
$$= q_2, q_0$$
$$= (2, 0)$$

$$\delta((q_2, q_0)a) = \delta(q_2, a), \delta(q_0, a)$$
$$= q_0, q_1$$
$$= (0, 1)$$

$$\delta((q_0, q_1)a) = \delta(q_0, a), \delta(q_1, a)$$
$$= q_1, q_0$$
$$= (1, 0)$$

$$\delta((q_1, q_0)a) = \delta(q_1, a), \delta(q_0, a)$$
$$= q_2, q_1$$
$$= (2, 1)$$

$$\delta((q_2, q_1)a) = \delta(q_2, a), \delta(q_1, a)$$
$$= (q_0, q_0)$$
$$= (0, 0)$$

The set is

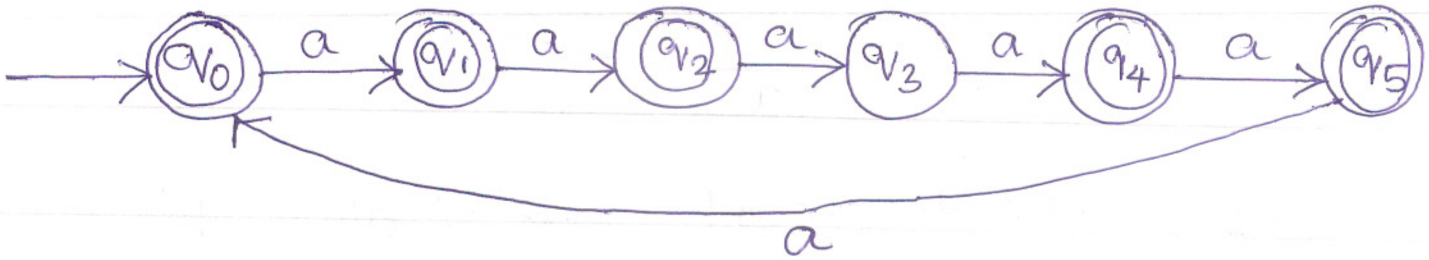
$$F = \{(0,0)\ (1,1), (2,0), (1,0), (2,1), (0,1)\}$$

27

$|w| \bmod 3 \geq |w| \bmod 2$

In the set, select the pair where I value $\geq$ II value for final state

So, $F = \{(0,0)\ (1,1)\ (2,0)\ (1,0)\ (2,1)\}$



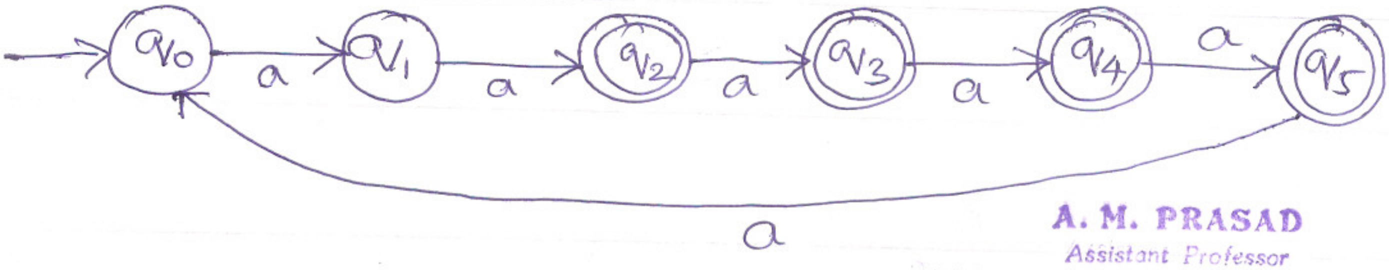
22. Obtain DFA to accept the language

$L = \{ |w| \bmod 3 \neq |w| \bmod 2,\ \Sigma = a\}$

Ans: Repeat the same procedure of problem (21).

In set F, select the pair where first value is not equal to second value as final states

$= \{(2,0), (0,1), (1,0), (2,1)\}$

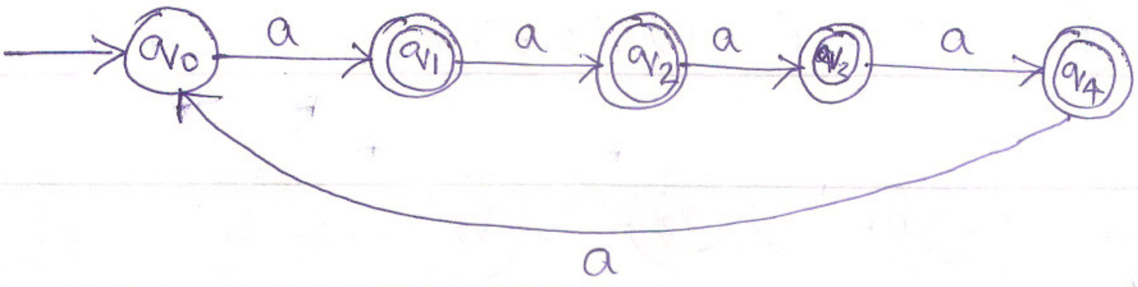


A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

23. Obtain DFA to accept $L = \{w : |w| \bmod 5 \neq 0\}$

Ans: The remainders of $|w| \bmod 5 = 0, 1, 2, 3, 4$,

make states 1, 2, 3, 4 as final states in $|w| \bmod 5 = 0$



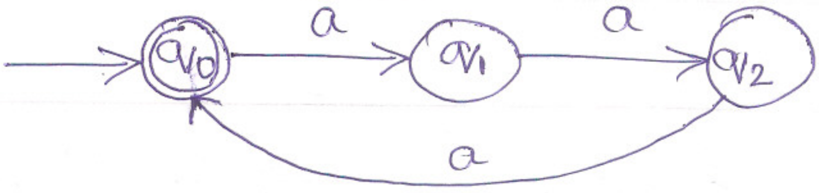
24. Obtain DFA to accept string of a's & b's such that $L = \{ W \mid N_a(W) \bmod 3 = 0$ and $N_b(W) \bmod 2 = 0 \}$

<u>Ans:</u> $N_a(W) \bmod 3$, the remainders are 0, 1, 2

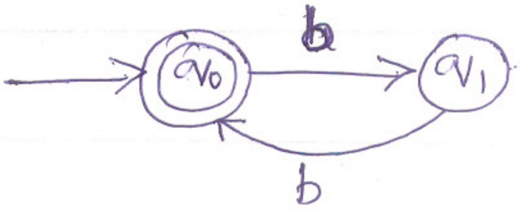
$N_b(W) \bmod 2$, the remainders are 0, 1

(A_0, A_1, A_2) (B_0, B_1) are states of I & II m/c.

$Q_1 \times Q_2 = (A_0, B_0)\ (A_0, B_1)\ (A_1, B_0)\ (A_1, B_1)$



M1

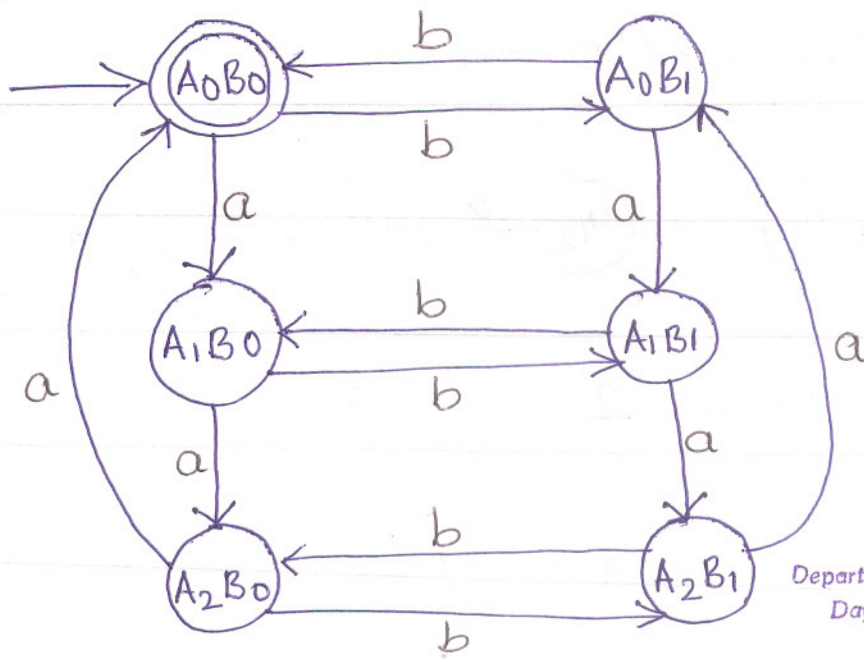


M2

$\delta(A_0, a) = A_1$ $\qquad$ $\delta(B_0, b) = B_1$

$\delta(A_1, a) = A_2$ $\qquad$ $\delta(B_0, a) = B_0$

$\delta(A_2, a) = A_0$



A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

25. Obtain DFA to accept string of a's and b's such that $N_a(W) \bmod 5 = 0$ and $N_b(W) \bmod 3 = 0$

Ans. $N_a(W) \bmod 5 = 0$, the remainders 0, 1, 2, 3, 4

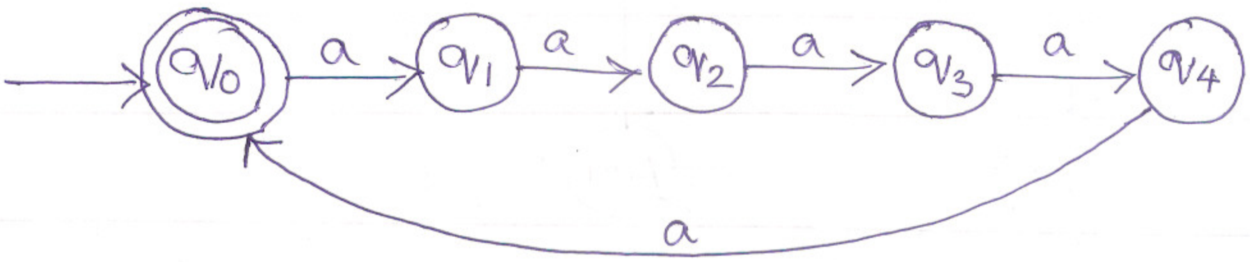
$Q_1 = \{A_0, A_1, A_2, A_3, A_4\}$

$N_b(W) \bmod 3 = 0$, the remainders 0, 1, 2

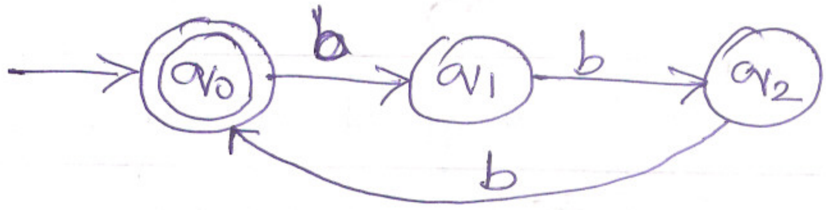
$Q_2 = \{B_0, B_1, B_2\}$

$$
\begin{aligned}
Q_1 \times Q_2 = \{ & (A_0, B_0), (A_0, B_1), (A_0, B_2) \\
& (A_1, B_0)\ (A_1, B_1)\ (A_1, B_2) \\
& (A_2, B_0)\ (A_2, B_1)\ (A_2, B_2) \\
& (A_3, B_0)\ (A_3, B_1)\ (A_3, B_2) \\
& (A_4, B_0)\ (A_4, B_1)\ (A_4, B_2) \\
\}
\end{aligned}
$$

DFA for $N_a(W) \bmod 5 = 0$



DFA for $N_b(w) \bmod 3 = 0$



$\delta(q_0, a) = q_1$ — A_0

$\delta(q_1, a) = q_2$ — A_1

$\delta(q_2, a) = q_3$ — A_2

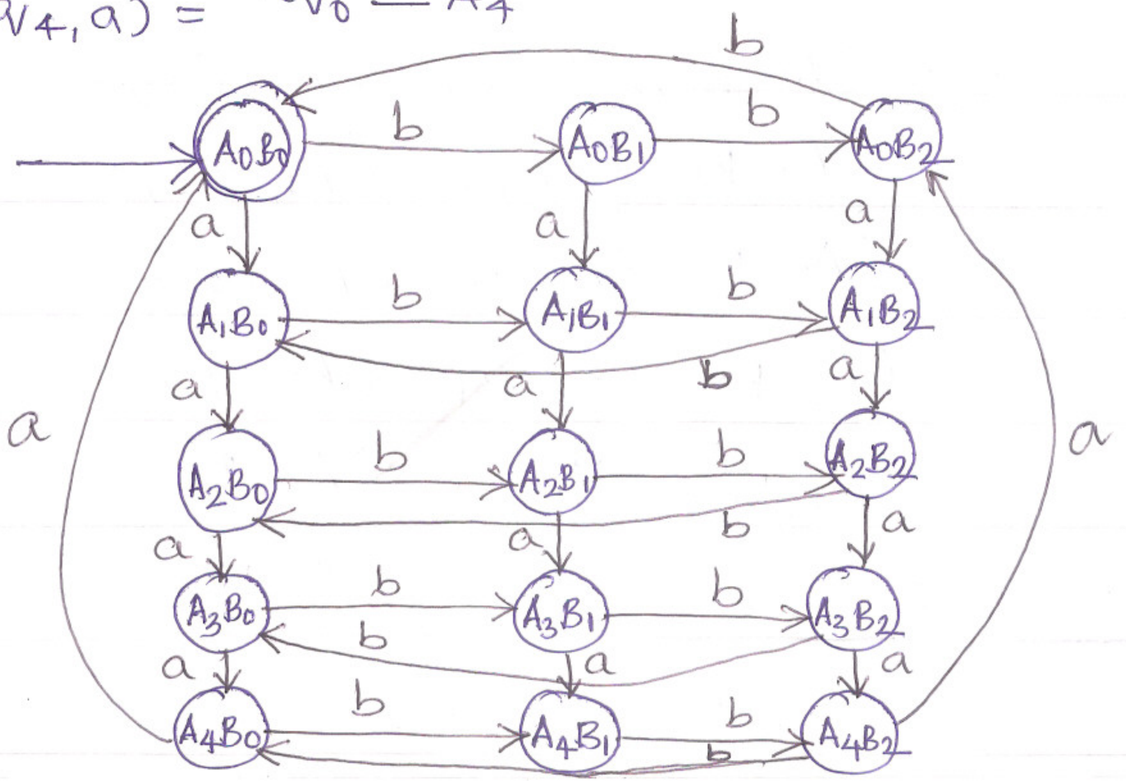
$\delta(q_3, a) = q_4$ — A_3

$\delta(q_4, a) = q_0$ — A_4

$\delta(q_0, b) = q_1$ — B_0

$\delta(q_1, b) = q_2$ — B_1

$\delta(q_2, b) = q_0$ — B_2



26. Construct DFA to accept the following language

$$L = \{ w : n_a(w) \bmod 5 \neq n_b(w) \bmod 3 \}$$

Ans: Same as previous problem, but while deciding final state, select the state where both are not of same subscript. Eg $A_1 B_2$ → Subscript.

27. Construct DFA to accept the following language

$$L = \{ w : n_a(w) \bmod 5 > n_b(w) \bmod 3 \}$$

Ans: Same as problem 25. But the final states are those pair in which subscript of A greater than subscript of B. Eg: $A_4 B_1$

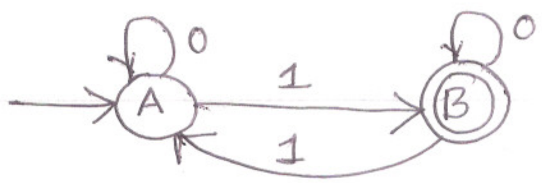
A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

28. Consider the DFA

δ	0	1
$\rightarrow A$	A	B
$* B$	B	A

Informally describe the language accepted by DFA.

Ans. The transition diagram is



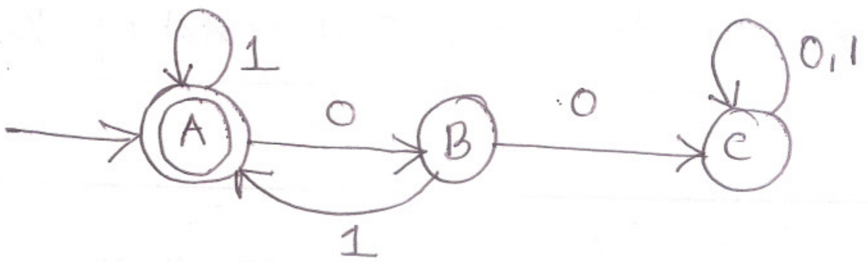
In the diagram we can observed the DFA accepts odd number of 1's with any number of 0's.

So the given DFA accepts string 0's and 1's with odd number of 1's.

29. Consider DFA with following transition table

δ	0	1
$\rightarrow$ * A	B	A
B	C	A
* C	C	C

The diagram is



We can observe (i) the diagram accepts the string ending with 1 only.

(ii) the diagram accepts ε

(iii) Can have consecutive 1's but not 0's.

Nondeterministic Finite Automata (NFA)

The disadvantages of DFA are

(i) DFA design is difficult

(ii) Donot have power to be in several state at once.

The above disadvantages can be overcome by using NFA. The advantages of NFA are

(i) Power to be in several state at once

(ii) Very easy to construct

(iii) Efficient mechanism to solve difficult problem.

Definition: The NFA is 5-tuple represented by

$$M = \{ Q, q_0, F, \Sigma, \delta \}$$

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

where Q – set of states

q_0 – unique initial state

F – set of final states

Σ – set of input symbols

δ – transition function represented as transition table.

Consider the following transition diagram

The transition table for above diagram is

State	0	1
$\rightarrow q_0$	$q_0 q_1$	q_1
q_1	q_1	q_2
$* q_2$	q_2	q_2

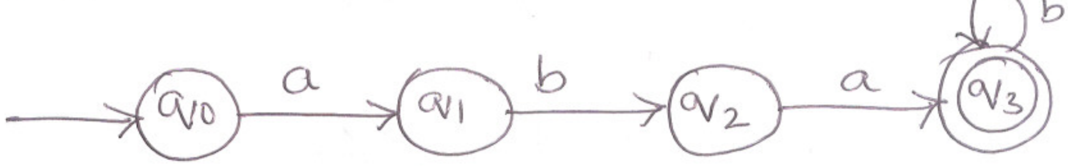
In transition table / diagram, we can observe, on given i/p symbol 0, q_0 changes to q_0 and q_1 also. In finite Automata, if there are more than one transition on given i/p symbol, such machines are called Nondeterministic finite Automata.

Problems.

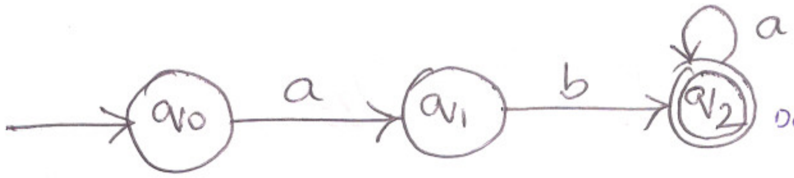
1. Obtain an NFA which accepts the following language $L = \{ abab^n \text{ or } aba^n, n \geq 0 \}$

Solution:- The machine has to accept both $abab^n$ or aba^n. One approach is to design each seperately and combine them.

$abab^n$

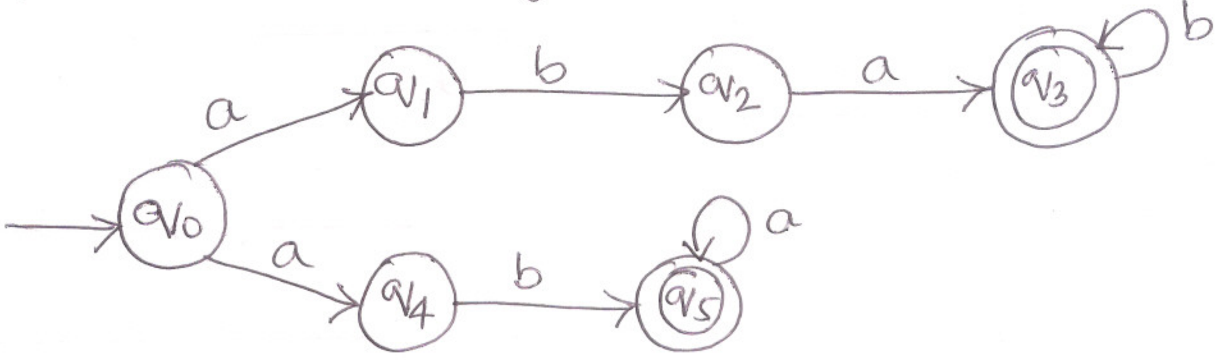


aba^n



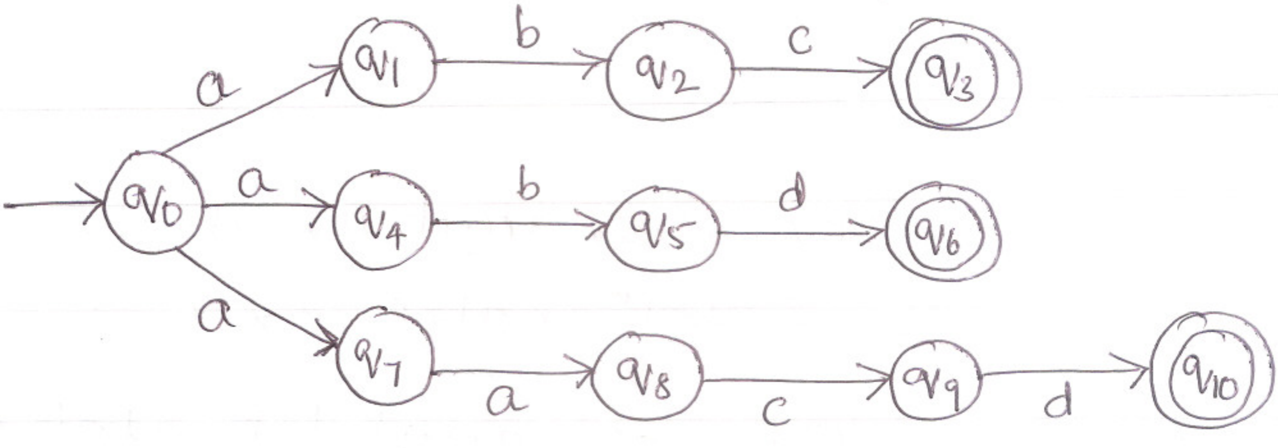
A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

$abab^n$ or aba^n together, the machine is



On i/p symbol 'a' q_0 changes to q_1 as well q_4 also. So it is NFA

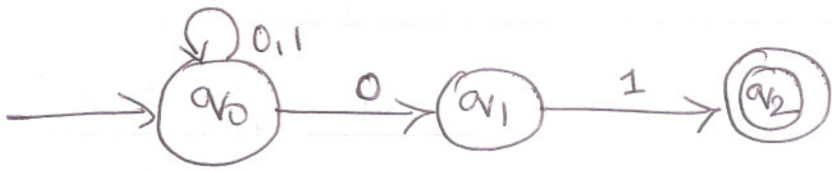
2. Design NFA to accept the following strings abc, abd, aacd.



3. Design NFA which accepts all the strings ending with 01. over $\Sigma = \{0, 1\}$.

Soln: Draw automata for 01

Now add rest of the transitions.



Conversion of NFA to DFA

Using DFA only for some of the problems automaton can be constructed. For some problems DFA cannot be constructed. In this case, for those problems NFA can be constructed. But practically Nondeterministic machines will not exists. So NFA designed to be converted to DFA. The NFA can be converted to DFA using two methods.

⟨i⟩ Subset Construction method.

⟨ii⟩ Lazy Evaluation.

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

Subset Construction Method

The subset construction starts from an NFA $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$. Its goal is the description of a DFA $D = (Q_D, \Sigma, \delta_D, q_0, F_D)$ such that $L(D) = L(N)$. The components of D is constructed as follows.

⟨i⟩ q_0 is start state of DFA, the q_0 is initial state of NFA also.

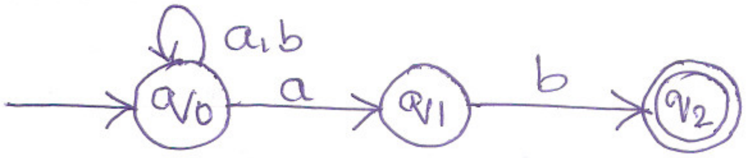
ii) The states of DFA, Q_D are the set of subsets of Q_N. So if Q_N has 'n' states then Q_D will have 2^n states.

iii) The final state of NFA are those states where the final state of NFA are present

Thus DFA can be obtained using subset construction method.

Problem: 1. Obtain DFA for the following NFA using subset construction method.

Ans:



The states of DFA are subsets of states of NFA

$$Q_D = \{ \phi, q_0, q_1, q_2, \{q_0, q_1\} \{q_0, q_2\} \{q_1, q_2\} \{q_0, q_1, q_2\} \}$$ Totally eight states

(i) For state ϕ:

$\delta(\phi, a) = \phi \qquad \delta(\phi, b) = \phi$

(ii) For state q_0: $\delta(q_0, a) = \{q_0 q_1\} \qquad \delta(q_0, b) = \{q_0\}$

(iii) For state q_1: $\delta(q_1, a) = \phi \qquad \delta(q_1, b) = \{q_2\}$

(iv) For state q_2: $\delta(q_2, a) = \phi \qquad \delta(q_2, b) = \phi$

(v) For state $\{q_0, q_1\}$: $\delta(\{q_0 q_1\}, a) = \{q_0 q_1\}$

$\delta(\{q_0 q_1\}, b) = \{q_0 q_2\}$

(vi) For state $\{q_0, q_2\}$: $\delta(\{q_0 q_2\}\ a) = \{q_0, q_1\}$

$\delta(\{q_0 q_2\}\ b) = \{q_0\}$

(vii) For state $\{q_1 q_2\}$: $\delta(\{q_1 q_2\}\ a) = \phi$

$\delta(\{q_1 q_2\}\ b) = q_2$

(viii) For state $\{q_0, q_1 q_2\}$: $\delta(\{q_0, q_1 q_2\} a) = \{q_0 q_1\}$

$\delta(\{q_0, q_1, q_2\}\ b) = \{q_0, q_2\}$

state	a	b
ϕ	ϕ	ϕ
q_0	$\{q_0 q_1\}$	q_0
q_1	ϕ	q_2
q_2	ϕ	ϕ
$q_0 q_1$	$\{q_0 q_1\}$	$\{q_0 q_2\}$
$q_0 q_2$	$\{q_0 q_1\}$	$\{q_0\}$
$q_1 q_2$	ϕ	$\{q_2\}$
$q_0 q_1 q_2$	$\{q_0 q_1\}$	$\{q_0 q_2\}$

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

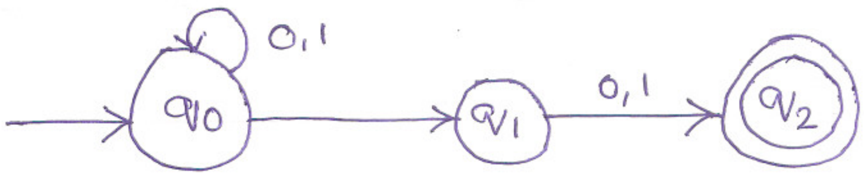
The eight states are A, B, C, D, E, F, G, H.

The transition table is

	state	a	b
	A	A	A
→	B	E	B
	C	A	D
*	D	A	A
	E	E	F
*	F	E	B
*	G	A	D
*	H	E	F

The states which are reachable from B are states of DFA, the states unreacble can be eliminated.

2. Convert the following NFA to DFA using subset construction method.



Ans: The states of the DFA are subsets of states of NFA $\{q_0, q_1, q_2\}$.

$\{\emptyset, q_0, q_1, q_2, \{q_0 q_1\}, \{q_0 q_2\}, \{q_1, q_2\}, \{q_0 q_1, q_2\}\}$

Now, write the transition for all subsets

i) $\delta(\phi, a) = \phi$ $\quad$ $\delta(\phi, 1) = \phi$

ii) $\delta(q_0, 0) = q_0$ $\quad$ $\delta(q_0, 1) = q_0 q_1$

iii) $\delta(q_1, 0) = q_2$ $\quad$ $\delta(q_1, 1) = q_2$

iv) $\delta(q_2, 0) = \phi$ $\quad$ $\delta(q_2, 1) = \phi$

v) $\delta(q_0 q_1, 0) = q_0 q_2$ $\quad$ $\delta(q_2, 1) = q_0 q_1 q_2$

vi) $\delta(q_0 q_2, 0) = q_0$ $\quad$ $\delta(q_0 q_2, 1) = q_0 q_1$

vii) $\delta(q_1 q_2, 0) = q_2$ $\quad$ $\delta(q_1 q_2, 1) = q_2$

viii) $\delta(q_0 q_1 q_2, 0) = q_0 q_2$ $\quad$ $\delta(q_0 q_1 q_2) = q_0 q_1 q_2$

The eight states are A, B, C, D, E, F, G, H respectively

δ	0	1
A	A	A
B	B	E
C	D	D
* D	A	A
E	F	H
* F	B	E
* G	D	D
* H	F	H

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

Of these states only reachable states are B, E, F & H.
The other four states are not accessible and can be ignored.

Conversion From NFA to DFA Using Lazy Evaluation Method.

In subset construction method there are 2^n states and from each state we have the transition from input symbol in Σ and hence the time complexity to convert an NFA to DFA is $\Sigma * 2^n$.

Since the time complexity is exponential, the procedure takes very long time to construct the table. This exponential time complexity can be avoided using this technique called "Lazy Evolution" on the subsets.

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

Algorithm:

Given an NFA $M_N = (Q_N, \Sigma, \delta_N, q_0, F_N)$ which accepts the language $L(M_N)$, we can find an equivalent DFA $M_D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$ such that $L(M_D) = L(M_N)$

Proof:

Step 1: Identify the start state of DFA : $\{q_0\}$ is the start state of DFA.

Step 2: Identify the alphabets of DFA : The input alphabets of DFA are the input alphabets of NFA. So, $\Sigma = \{a, b\}$

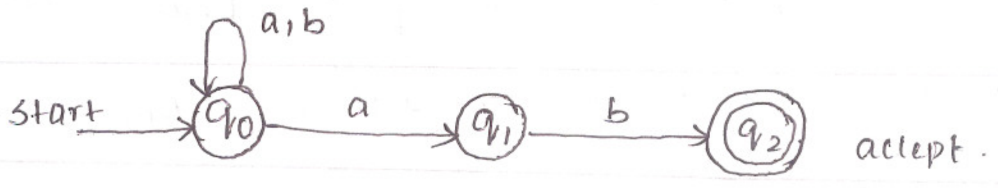
Step 3: Identify the transition of DFA : for each state $\{q_i, q_j \cdots q_k\}$ in Q_D and for each input symbol a in Σ, the transition can be obtained as shown below :

$$\delta_D(\{q_i, q_j \ldots q_k\}, a) = \delta_N(q_i, a) \cup \delta_N(q_j, a) \cup \ldots \delta_N(q_k, a)$$
$$= [q_l, q_m, \ldots q_n]$$

- Add the state $[q_l, q_m, \ldots q_n]$ to Q_D, if it is not already in Q_D.
- Add the transition from $[q_i, q_j \ldots q_k]$ to $[q_l, q_m \ldots q_n]$ on the input symbol a.

Step 4 : Identify the final states of DFA : If $\{q_i, q_j \ldots q_k\}$ is a state in Q_D and if one of $q_i, q_j \ldots q_k$ is the final state of NFA, then $\{q_i, q_j \ldots q_k\}$ will be the final state of DFA.

EXAMPLE :



Sol^n : The transition table for the above DFA can be written as

δ	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	ϕ	$\{q_2\}$
$* q_2$	ϕ	ϕ

Step 1 : Identify the start state of DFA : $\{q_0\}$

Step 2 : Identify the alphabets of DFA : $\Sigma = \{a, b\}$

Step 3 : Identify the transition of DFA, start from the start state q_0 and find the transition as shown below,

For state $\{q_0\}$:

Input symbol = a ; $\delta(q_0, a) = \{q_0, q_1\}$

Input symbol = b ; $\delta(q_0, b) = \{q_0\}$

For state $\{q_0, q_1\}$:

Input symbol = a ; $\delta((q_0, q_1), a) = \delta(\{q_0, q_1\}, a)$

$= \delta(q_0, a) \cup \delta(q_1, a)$

$= \{q_0, q_1\} \cup \phi$

$= \{q_0, q_1\}$

Input symbol = b ; $\delta(\{q_0, q_1\}, b) = \delta(\{q_0, q_1\}, b)$

$= \delta(q_0, b) \cup \delta(q_1, b)$

$= \{q_0\} \cup \{q_2\}$

$= \{q_0, q_2\}$

For state $\{q_0, q_2\}$:

Input symbol = a ; $\delta(\{q_0, q_2\}, a) = \delta(\{q_0, q_2\}, a)$

$= \delta(q_0, a) \cup \delta(q_2, a)$

$= \{q_0, q_1\} \cup \phi$

$= \{q_0, q_1\}$

Input symbol = b ; $\delta(\{q_0, q_2\}, b) = \delta(\{q_0, q_2\}, b)$

$$= \delta[q_0, b] \cup \delta(q_2, b)$$

$$= \{q_0\} \cup \phi$$

$$= \{q_0\}$$

Since, no new state is generated this step is terminated.

Step 4 : Identify the final states of DFA ; Since q_2 is the final state of NFA in the above set, wherever q_2 is present as an element, the corresponding set is the final state of DFA. So, the final state is $\{q_0, q_2\}$.

So, Transition Table,

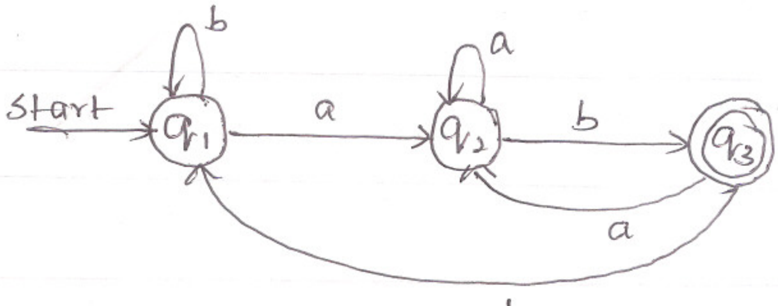
δ	a	b
$\rightarrow \{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$* \{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0\}$

By renaming the states $\longrightarrow$

δ	a	b
A	B	A
B	B	C
$*$C	B	A

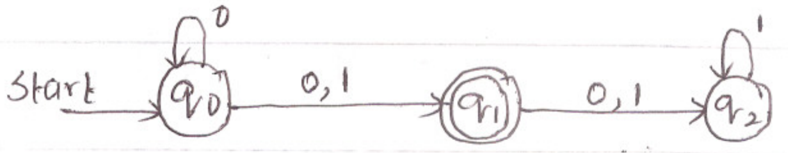
So, the final DFA is given by $M = (Q, \Sigma, \delta, q_0, F)$ where

- $Q = \{A, B, C\}$
- $\Sigma = \{a, b\}$
- $q_0 = A$
- $F = \{C\}$



PROBLEMS :

1. Convert the following NFA to its Equivalent DFA



Soln : The transition table for the above DFA is given by

δ	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$* q_1$	$\{q_2\}$	$\{q_2\}$
q_2	ϕ	$\{q_2\}$

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

Step 1 : Identify the start state of DFA ; $Q = \{q_0\}$

Step 2 : Identify the alphabets of DFA ; $\Sigma = \{0, 1\}$

Step 3 : Identify the transitions of DFA ;

For state $\{q_0\}$:

Input symbol = 0 ; $\delta(q_0, 0) = \delta(q_0, 0)$
$= \{q_0, q_1\}$

Input symbol = 1 ; $\delta(q_0, 1) = \delta(q_0, 1)$
$= \{q_1\}$

For state $\{q_0, q_1\}$:

Input symbol = 0 ; $\delta(\{q_0, q_1\}, 0) = \delta((q_0, q_1), 0)$
$= \delta(q_0, 0) \cup \delta(q_1, 0)$
$= (q_0, q_1) \cup (q_2)$
$= (q_0, q_1, q_2)$

Input Symbol = 1 ; $\delta(\{q_0, q_1\}, 1) = \delta(\{q_0, q_1\}, 1)$

$= \delta(q_0, 1) \cup \delta(q_1, 1)$

$= \{q_1\} \cup \{q_2\}$

$= \{q_1, q_2\}$

For state $\{q_1\}$:

Input symbol = 0 ; $\delta(\{q_1\}, 0) = \delta(\{q_1, 0\}) = \{q_2\}$

Input symbol = 1 ; $\delta(\{q_1\}, 1\} = \delta(\{q_1, 1\}) = \{q_2\}$

For state $\{q_0, q_1, q_2\}$:

Input symbol = 0 ; $\delta[\{q_0, q_1, q_2\}, 0) = \delta[\{(q_0, q_1, q_2), 0)$

$= \delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0)$

$= \{q_0, q_1\} \cup \{q_2\} \cup \{\phi\}$

$= \{q_0, q_1, q_2\}$

Input symbol = 1 ; $\delta(\{q_0, q_1, q_2\}, 1\} = \delta(\{q_0, q_1, q_2\}, 1)$

$= \delta(q_0, 1) \cup \delta[q_1, 1) \cup \delta(q_2, 1)$

$= \{q_1\} \cup \{q_2\} \cup \{q_2\}$

$= \{q_1, q_2\}$

For state $\{q_1, q_2\}$:

Input symbol = 0 ; $\delta(\{q_1, q_2\}, 0) = \delta(\{q_1, q_2\}, 0)$

$= \delta(q_1, 0) \cup \delta(q_2, 0)$

$= \{q_2\} \cup \phi$

$= \{q_2\}$

Input symbol = 1 ; $\delta(\{q_1, q_2\}, 1) = \delta(\{q_1, q_2\}, 1)$

$= \delta(q_1, 1) \cup \delta(q_2, 1)$

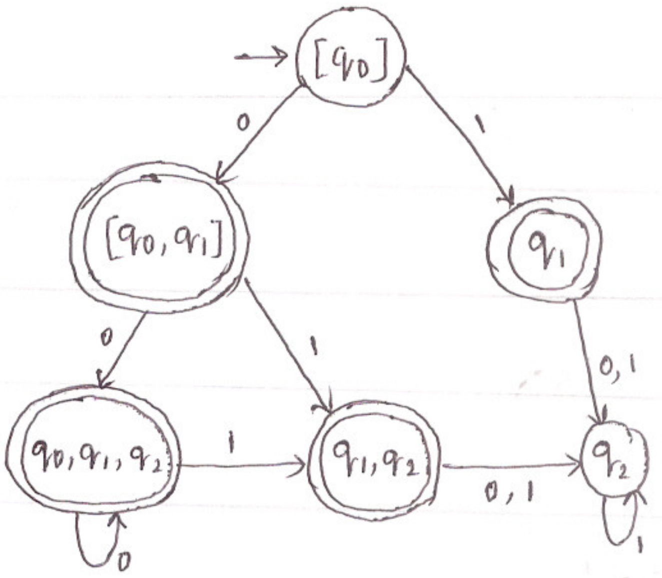
$= \{q_2\} \cup \{q_2\} = \{q_2\}$

For state $\{q_2\}$:

Input symbol $= 0$; $\delta(\{q_2\}, 0) = \phi$

Input symbol $= 1$; $\delta(\{q_2\}, 1) = q_2$

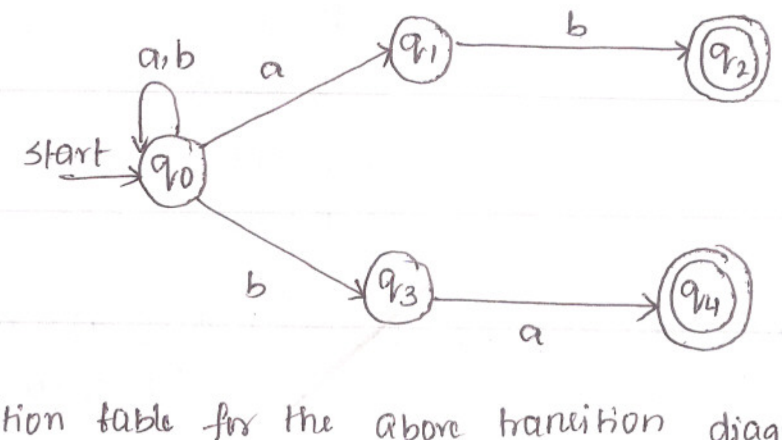
The final transition table along with transition diagram is shown below :



δ	0	1
$\rightarrow \{q_0\}$	$\{q_0, q_1\}$	$\{q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_1\}$	$\{q_2\}$	$\{q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
$* \{q_1, q_2\}$	$\{q_2\}$	$\{q_2\}$
$\{q_2\}$	ϕ	$\{q_2\}$

2. Obtain an NFA to accept strings of a's and b's ending with ab or ba. From this NFA obtain an equivalent DFA.

Sol^n: The NFA to accept strings of a's and b's ending with ab or ba is shown below;



A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

The transition table for the above transition diagram is shown below

δ	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0, q_3\}$
q_1	$\{\phi\}$	$\{q_2\}$
$* q_2$	ϕ	ϕ
q_3	q_4	ϕ
$* q_4$	ϕ	ϕ

Step 1 : Identify the start state of DFA = $\{q_0\}$

Step 2 : Identify the alphabets of DFA : $\Sigma = \{a, b\}$

Step 3 : Identify the transition of DFA :

For state $\{q_0\}$:

Input symbol = a ; $\delta(q_0, a) = \{q_0, q_1\}$

Input symbol = b ; $\delta(q_0, b) = \{q_0, q_3)$

For state $\{q_0, q_1\}$:

Input symbol = a ; $\delta(\{q_0, q_1\}, a\} = \delta(q_0, a) \cup \delta(q_1, a)$

$= \{q_0, q_1\} \cup \phi$

$= \{q_0, q_1\}$

Input symbol = b ; $\delta(\{q_0, q_1\}, b\} = \delta(q_0, b) \cup \delta(q_1, b)$

$= \{q_0, q_3) \cup \{q_2\}$

$= \{q_0, q_2, q_3\}$

For state $\{q_0, q_3\}$:

Input symbol = a ; $\delta(\{q_0, q_3\}, a) = \delta(q_0, a) \cup \delta(q_3, a)$

$= \{q_0, q_1\} \cup \{q_4\}$

$= \{q_0, q_1, q_4\}$

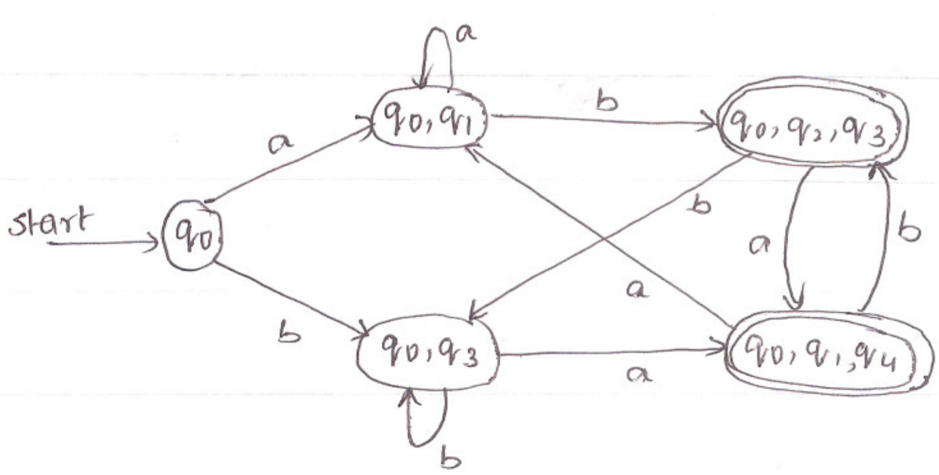
Input symbol = b ; $\delta(\{q_0, q_3\}, b) = \delta(q_0, b) \cup \delta(q_3, b)$

$= \{q_0, q_3) \cup \phi$

$= \{q_0, q_3)$

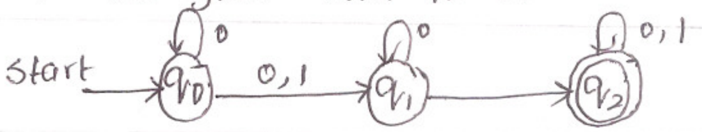
The final DFA obtained along with transition diagram and transition table is shown below

δ	a	b
$\rightarrow \{q_0\}$	$\{q_0, q_1\}$	$\{q_0, q_3\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2, q_3\}$
$\{q_0, q_3\}$	$\{q_0, q_1, q_4\}$	$\{q_0, q_3\}$
$* \{q_0, q_2, q_3\}$	$\{q_0, q_1, q_4\}$	$\{q_0, q_3\}$
$* \{q_0, q_1, q_4\}$	$\{q_0, q_1\}$	$\{q_0, q_2, q_3\}$



A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

3. Convert the given NFA to DFA.



Step 1: Identify the start state of DFA $= \{q_0\}$

Step 2: Identify the alphabets of DFA : $\Sigma = \{0, 1\}$

Step 3: Identify the transition of DFA :

For state $\{q_0\}$:

Input symbol $=0$; $\delta(q_0, 0) = \{q_0, q_1\}$

Input symbol $=1$; $\delta(q_0, 1) = \{q_1\}$

For state q_1 :

Input symbol = 0 ; $\delta(q_1, 0) = \{q_1\}$

Input symbol = 1 ; $\delta(q_1, 1) = \{q_2\}$

For state $\{q_0, q_1\}$:

Input symbol = 0 ; $\delta(\{q_0, q_1\}, 0) = \delta(q_0, 0) \cup (q_1, 0)$

$= \{q_0, q_1\} \cup \{q_1\}$

$= \{q_0, q_1\}$

Input symbol = 1 ; $\delta(\{q_0, q_1\}, 1) = \delta(q_0, 1) \cup \delta(q_1, 1)$

$= \{q_1\} \cup \{q_2\}$

$= \{q_1, q_2\}$

For state $\{q_1, q_2\}$:

Input symbol = 0 ; $\delta(\{q_1, q_2\}, 0) = \delta(q_1, 0) \cup \delta(q_2, 0)$

$= \{q_1\} \cup \{q_2\}$

$= \{q_1, q_2\}$

Input symbol = 1 ; $\delta(\{q_1, q_2\}, 1) = \delta(q_1, 1) \cup \delta(q_2, 1)$

$= \{q_2\} \cup \{q_2\}$

$= \{q_2\}$

For state $\{q_2\}$:

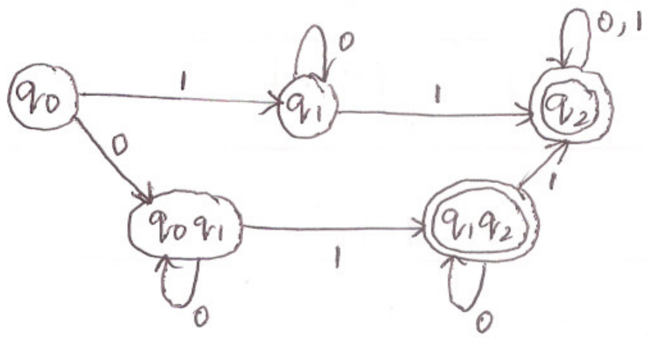
Input symbol = 0 ; $\delta(q_2, 0) = \{q_2\}$

Input symbol = 1 ; $\delta(q_2, 1) = \{q_2\}$

Transition table ;

δ	0	1
$\rightarrow q_0$	q_0q_1	q_1
q_0q_1	q_0q_1	q_1q_2
q_1	q_1	q_2
$* \; q_1q_2$	q_1q_2	q_2
$* \; q_2$	q_2	q_2

Transition diagram,



4. convert the following NFA to DFA

state	0	1
$\rightarrow$ p	q, s	q
* q	r	q, r
r	s	p
* s	ϕ	p

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

Step 1 : Identify the start state of DFA = {p}

Step 2: Identify the alphabets of DFA = {0, 1}

Step 3: Identify the transition of DFA :

For state {p}:

Input symbol = 0 : $\delta(p, 0) = \{q, s\}$

Input symbol = 1 : $\delta(p, 1) = \{q\}$

For state {q, s}:

Input symbol = 0 : $\delta(\{q, s\}, 0) = \delta(q, 0) \cup \delta(s, 0)$

$= \{r\} \cup \{\phi\}$

$= \{r\}$

Input symbol = 1 : $\delta(\{q, s\}, 1) = \delta(q, 1) \cup \delta(s, 1)$

$= \{q, r\} \cup \{\phi\}$

$= \{q, r\} \cup \{p\} = \{p, r, q\}$

For state {q}:

Input symbol = 0 ; $\delta(q,0) = \{r\}$

Input symbol = 1 ; $\delta(q,1) = \{q,r\}$

For state {r}:

Input symbol = 0 ; $\delta(r,0) = \{s\}$

Input symbol = 1 ; $\delta(r,1) = \{p\}$

For state {p,q,r}:

Input symbol = 0 ; $\delta(\{p,q,r\},0) = \delta(p,0) \cup \delta(q,0) \cup \delta(r,0)$

$= \{q,s\} \cup \{r\} \cup \{s\}$

$= \{q,r,s\}$

Input symbol = 1 ; $\delta(\{p,q,r\},1) = \delta(p,1) \cup \delta(q,1) \cup \delta(r,1)$

$= \{q,\cancel{s}\} \cup \{q,r\} \cup \{p\}$

$= \{p,q,r\}$

For state {q,r}:

Input symbol = 0 : $\delta(\{q,r\},0) = \delta(q,0) \cup \delta(r,0)$

$= \{r\} \cup \{s\}$

$= \{r,s\}$

Input symbol = 1 : $\delta(\{q,r\},1) = \delta(q,1) \cup \delta(r,1)$

$= \{q,r\} \cup \{p\}$

$= \{p,q,r\}$

For state {s}:

Input symbol = 0 : $\delta(s,0) = \{\phi\}$

Input symbol = 1 : $\delta(s,1) = \{p\}$

For state {q,r,s}:

Input symbol = 0 : $\delta(\{q,r,s\},0) = \delta(q,0) \cup \delta(r,0) \cup \delta(s,0)$

$= \{r\} \cup \{s\} \cup \{\phi\}$
$= \{r, s\}$

Input symbol = 1 : $\delta(\{q, r, s\}, 1) = \delta(q, 1) \cup \delta(r, 1) \cup \delta(s, 1)$
$= \{q, r\} \cup \{p\} \cup \{p\}$
$= \{p, q, r\}$

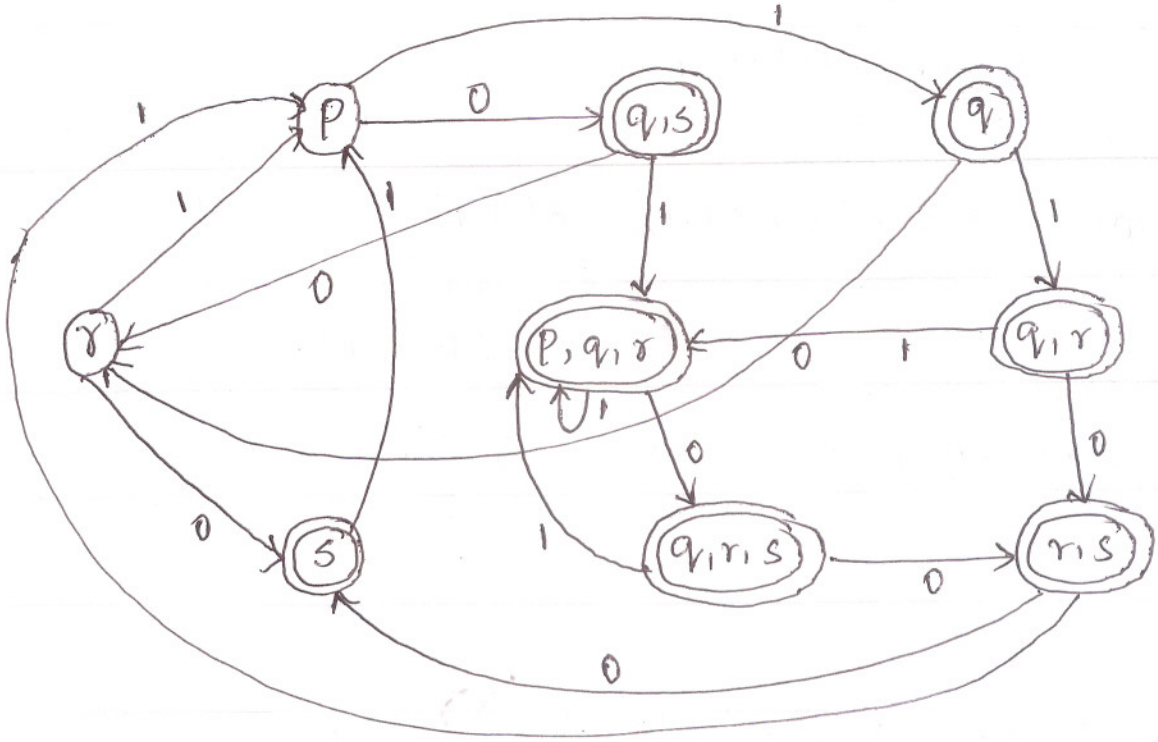
For state $\{r, s\}$:

Input symbol = 0 : $\delta(\{r, s\}, 0) = \delta(r, 0) \cup \delta(s, 0)$
$= \{s\} \cup \{\phi\}$
$= \{s\}$

Input symbol = 1 : $\delta(\{r, s\}, 1) = \delta(r, 1) \cup \delta(s, 1)$
$= \{p\} \cup \{p\}$
$= \{p\}$

The transition table and transition diagram is shown below

δ	0	1
$\rightarrow p$	$\{q, s\}$	$\{q\}$
$* \{q, s\}$	$\{r\}$	$\{p, q, r\}$
$* \{q\}$	$\{r\}$	$\{q, r\}$
$\{r\}$	$\{s\}$	$\{p\}$
$* \{p, q, r\}$	$\{q, r, s\}$	$\{p, q, r\}$
$* \{q, r\}$	$\{r, s\}$	$\{p, q, r\}$
$* \{s\}$	$\{\phi\}$	$\{p\}$
$* \{q, r, s\}$	$\{r, s\}$	$\{p, q, r\}$
$* \{r, s\}$	$\{s\}$	$\{p\}$



5.

State	0	1
→ p	{p,q}	{p}
q	{r}	{r}
r	{s}	ϕ
* s	{s}	{s}

Step 1 : Identify the start state of DFA : {p}

Step 2 : Identify the alphabets of DFA = {0,1}

Step 3 : Identify the transitions of DFA :

For state {p} :

Input symbol = 0 ; $\delta(p,0) = \{p,q\}$

Input symbol = 1 ; $\delta(p,1) = \{p\}$

For state {p,q}

Input symbol = 0 ; $\delta(\{p,q\},0) = \delta(p,0) \cup \delta(q,0)$

$= \{p,q\} \cup \{r\}$

$= \{p,q,r\}$

Input symbol = 1 ; $\delta(\{p,q\},1) = \delta(p,1) \cup \delta(q,1)$

$= \{p,\cancel{q}\} \cup \{r\}$

$= \{p,r\}$

For state $\{p,q,r\}$

Input symbol = 0 ; $\delta(\{p,q,r\},0) = \delta(p,0) \cup \delta(q,0) \cup \delta(r,0)$

$= \{p,q\} \cup \{r\} \cup \{s\}$

$= \{p,q,r,s\}$

Input symbol = 1 ; $\delta(\{p,q,r\},1) = \delta(p,1) \cup \delta(q,1) \cup \delta(r,1)$

$= \{p\} \cup \{r\} \cup \{\phi\}$

$= \{p,r\}$

For state $\{p,r\}$

Input symbol = 0 ; $\delta(\{p,r\},0) = \delta(p,0) \cup \delta(r,0)$

$= \{p,q\} \cup \{s\}$

$= \{p,q,s\}$

Input symbol = 1 ; $\delta(\{p,r\},1) = \delta(p,1) \cup \delta(r,1)$

$= \{p\} \cup \{\phi\}$

$= \{p\}$

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

For state $\{p,q,r,s\}$

Input symbol = 0 ; $\delta(\{p,q,r,s\},0) = \delta(p,0) \cup \delta(q,0) \cup \delta(r,0) \cup \delta(s,0)$

$= \{p,q\} \cup \{r\} \cup \{s\} \cup \{s\}$

$= \{p,q,r,s\}$

Input symbol = 1 ; $\delta(\{p,q,r,s\},1) = \delta(p,1) \cup \delta(q,1) \cup \delta(r,1) \cup \delta(s,1)$

$= \{p\} \cup \{r\} \cup \{\phi\} \cup \{s\}$

$= \{p,r,s\}$

For state pqs

Input symbol = 0 ; $\delta(\{P, q, s\}, 0) = \delta(P,0) \cup \delta(q,0) \cup \delta(s,0)$

$= \{P, q\} \cup \{r\} \cup \{s\}$

$= \{P, q, r, s\}$

Input symbol = 1 ; $\delta(\{P, q, s\}, 1) = \delta(P,1) \cup \delta(q,1) \cup \delta(s,1)$

$= \{P\} \cup \{r\} \cup \{s\}$

$= \{P, r, s\}$

For state {P, r, s}

Input symbol = 0 ; $\delta(\{P, r, s\}, 0) = \delta(P,0) \cup \delta(r,0) \cup \delta(s,0)$

$= \{P, q\} \cup \{s\} \cup \{s\}$

$= \{P, q, s\}$

Input symbol = 1 ; $\delta(\{P, r, s\}, 1) = \delta(P,1) \cup \delta(r,1) \cup \delta(s,1)$

$= \{P, \phi\} \cup \{s\} \cup \{s\}$

$= \{P, s\}$

For state {P, s}

Input symbol = 0 ; $\delta(\{P, s\}, 0) = \delta(P,0) \cup \delta(s,0)$

$= \{P, q\} \cup \{s\}$

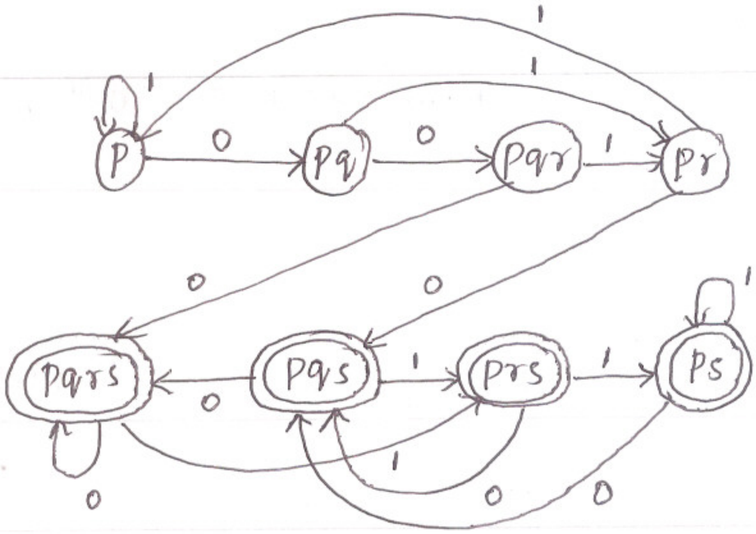
$= \{P, q, s\}$

Input symbol = 1 ; $\delta(\{P, s\}, 1) = \delta(P,1) \cup \delta(s,1)$

$= \{P\} \cup \{s\}$

$= \{P, s\}$

State	0	1
→ p	pq	p
pq	pqr	pr
pqr	pqrs	pr
pr	pqs	p
* pqrs	pqrs	prs
* pqs	pqrs	prs
* prs	pqs	ps
* ps	pqs	ps



6.

State	0	1
→ p	pq	p
q	ϕ	r
* r	pr	q

Soln: Step 1: Identify the starting state of DFA : {p}

Step 2: Identify the alphabets of DFA ; {0, 1}

Step 3: Identify the transition of DFA :

For state {p}:

Input symbol = 0; $\delta(P,0) = \{P,q\}$

Input symbol = 1; $\delta(P,1) = \{P\}$

For state $\{P,q\}$:

Input symbol = 0; $\delta(\{P,q\},0) = \delta(P,0) \cup \delta(q,0)$

$= \{P,q\} \cup \{\phi\}$

$= \{P,q\}$

Input symbol = 1; $\delta(\{P,q\},1) = \delta(P,1) \cup \delta(q,1)$

$= \{P\} \cup \{r\}$

$= \{P,r\}$

For state $\{P,r\}$:

Input symbol = 0; $\delta(\{P,r\},0) = \delta(P,0) \cup \delta(r,0)$

$= \{P,q\} \cup \{P,r\}$

$= \{P,q,r\}$

Input symbol = 1; $\delta(\{P,r\},1) = \delta(P,1) \cup \delta(r,1)$

$= \delta(P) \cup (q)$

$= \{P,q\}$

For state $\{P,q,r\}$:

Input symbol = 0; $\delta(\{P,q,r\},0) = \delta(P,0) \cup \delta(q,0) \cup \delta(r,0)$

$= \{P,q\} \cup \{\phi\} \cup \{P,r\}$

$= \{P,q,r\}$

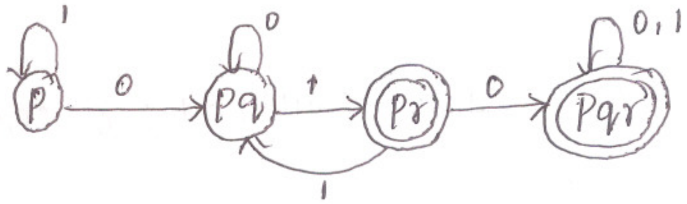
Input symbol = 1; $\delta(\{P,q,r\},1) = \delta(P,1) \cup \delta(q,1) \cup \delta(r,1)$

$= \{P\} \cup \{r\} \cup \{q\}$

$= \{P,q,r\}$

Transition diagram and Transition Table is shown below :

State	0	1
$\rightarrow$ p	pq	p
pq	pq	pr
* pr	pqr	pq
* pqr	pqr	pqr



7)

State	0	1
$\rightarrow$ p	pr	p
q	rs	p
* r	ps	r
* s	qr	ϕ

A. M. PRASAD
Assistant Professor
Department of Computer Science & Engineering
Dayananda Sagar College of Engineering
Bangalore - 560 078

Soln : Step 1 : Identify the starting state of DFA : $\{p\}$

Step 2 : Identify the alphabets of DFA : $\{0, 1\}$

Step 3 : Identify the transition state of DFA :

For state $\{p\}$:

Input symbol : 0 ; $\delta(p, 0) = \{p, r\}$

Input symbol = 1 ; $\delta(p, 1) = \{p\}$

For state $\{p, r\}$:

Input symbol = 0 ; $\delta(\{p, r\}, 0) = \delta(p, 0) \cup \delta(r, 0)$

$= \{p, r\} \cup \{p, s\}$

$= \{p, r, s\}$

Input symbol = 1 ; $\delta(\{p, r\}, 1) = \delta(p, 1) \cup \delta(r, 1)$

$= \{p\} \cup \{r\}$

$= \{p, r\}$.

For state $\{P, r, s\}$

Input symbol = 0 ; $\delta(\{P, r, s\}, 0) = \delta(P, 0) \cup \delta(r, 0) \cup \delta(s, 0)$

$= \{P, r\} \cup \{P, s\} \cup \{q, r\}$

$= \{P, q, r, s\}$

Input symbol = 1 ; $\delta(\{P, r, s\}, 1) = \delta(P, 1) \cup \delta(r, 1) \cup \delta(s, 1)$

$= \{P\} \cup \{r\} \cup \{\phi\}$

$= \{P, r\}$

For state $\{P, q, r, s\}$

Input symbol = 0 ; $\delta(\{P, q, r, s\}, 0) = \delta(P, 0) \cup \delta(q, 0) \cup \delta(r, 0) \cup \delta(s, 0)$

$= \{P, r\} \cup \{r, s\} \cup \{P, s\} \cup \{q, r\}$

$= \{P, q, r, s\}$

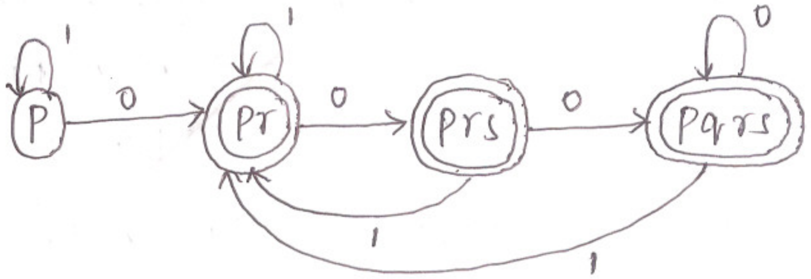
Input symbol = 1 ; $\delta(\{P, q, r, s\}, 1) = \delta(P, 1) \cup \delta(q, 1) \cup \delta(r, 1) \cup \delta(s, 1)$

$= \{P\} \cup \{P\} \cup \{r\} \cup \{\phi\}$

$= \{P, r\}$

The transition diagram and transition table is shown below

	δ	0	1
$\rightarrow$	P	Pr	P
*	Pr	Prs	Pr
*	Prs	Pqrs	Pr
*	Pqrs	Pqrs	Pr



8)

State	0	1
→ p	pq	p
q	rs	t
r	pr	t
* s	ϕ	ϕ
* t	ϕ	ϕ

Soln: Step 1: Identify the starting state of DFA : {p}

Step 2: Identify the alphabets of DFA : {0, 1}

Step 3: Identify the transition state of DFA :

For state {p} :

Input symbol = 0 ; $\delta(p,0) = \{p,q\}$

Input symbol = 1 ; $\delta(p,1) = \{p\}$

For state {p,q} :

Input symbol = 0 ; $\delta(\{p,q\},0) = \delta(p,0) \cup \delta(q,0)$
$= \{p,q\} \cup \{r,s\}$
$= \{p,q,r,s\}$

Input symbol = 1 ; $\delta(\{p,q\},1) = \delta(p,1) \cup \delta(q,1)$
$= \{p\} \cup \{t\}$
$= \{p,t\}$

For state {p,q,r,s} :

Input symbol = 0; $\delta(\{p,q,r,s\},0) = \delta(p,0) \cup \delta(q,0) \cup \delta(r,0) \cup \delta(s,0)$
$= \{p,q\} \cup \{r,s\} \cup \{p,r\} \cup \{\phi\}$
$= \{p,q,r,s\}$

Input symbol = 1 ; $\delta(\{p,q,r,s\},1)$
$= \delta(p,1) \cup \delta(q,1) \cup \delta(r,1) \cup \delta(s,1)$
$= \{p\} \cup \{t\} \cup \{t\} \cup \{\phi\}$
$= \{p,t\}$

For state {p,t}:

Input symbol = 0; $\delta(\{p,t\},0) = \delta(p,0) \cup \delta(t,0)$
$= \{p,q\} \cup \{\phi\}$
$= \{p,q\}$

Input symbol = 1; $\delta(\{p,t\},1) = \delta(p,1) \cup \delta(t,1)$
$= \{p\} \cup \{\phi\}$
$= \{p\}$

Transition Table;

State	0	1
→ p	pq	p
pq	pqrs	pt
* pqrs	pqrs	pt
* pt	pq	p

Transition diagram;