UNIT - III

Regular Languages, Properties of Regular Languages

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Topics:

* Regular Languages
* Pumping Lemma
* Proving Languages not be Regular Language
* Closure properties of Regular Languages
* Decision properties of Regular Languages
* EQuivalence & Minimization of Automata

Regular Languages: The Languages accepted by DFA's or NFA's or ϵ-NFA's are regular languages.

Not every language is a regular language.

A powerful technique, known as "Pumping Lemma" is used to show certain languages not to be regular.

PUMPING LEMMA FOR REGULAR LANGUAGES

Let $M = (Q, \Sigma, \delta, q_0, F)$ be finite Automata & has 'n' number of states. Let L be the regular language accepted by M. Let for every string x in L, there exists a constant n such that $|x| \geq n$. The string x can be broken into three substrings u, v, w such that

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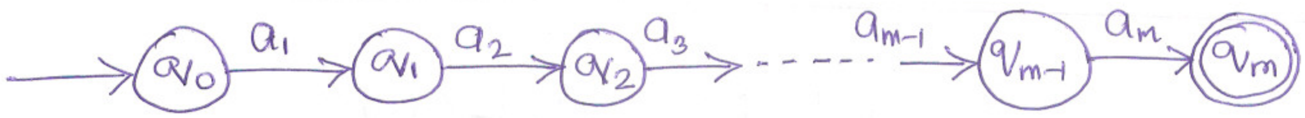
$$x = uvw$$

satisfying following constraints

1. $v \neq \epsilon$ ie $|v| \geq 1$
2. $|uv| \leq n$
3. For all $k \geq 0$, the string uv^kw is also in L

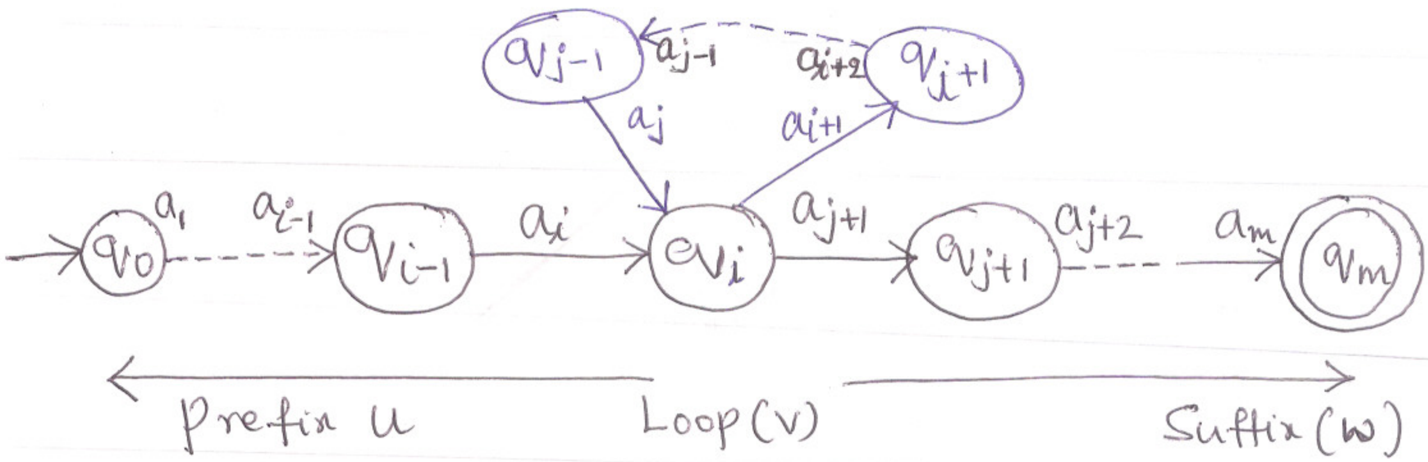
Proof: Let L is regular language, accepted by finite automata M. Let $x = a_1 a_2 a_3 \cdots a_m$ where $m \geq n$. The 'n' is states of DFA.

If 'm' inputs are there then number of states are $m+1$, shown below



The string x can be divided into three substrings $x = uvw$ as follows

1. $u = a_1 a_2 \cdots a_i$ (Prefix)
2. $v = a_{i+1} \cdots a_j$ (Loop)
3. $w = a_{j+1} \cdots a_m$ (Suffix)



The prefix string u takes the machine from q_0 to q_i,

the loop string v takes the machine from q_i to q_i

the suffix string w takes machine from q_j to q_f.

With $i=0$, the minimum string is uw is accepted by DFA

With $i=1$, the string is uvw is accepted by DFA.

With $i=2$, the string is $uvvw$ is accepted by DFA.

In general $uv^iw \in L$

Problems:

1. Show that $L = \{a^n b^n, n \geq 0\}$ is not regular.

Ans: Let L be regular & n be number of states of FA and string $x = a^n b^n$.

The length of x is $n+n = 2n$, $|x| = 2n \geq n$.

Split x into uvw such that $|uv| \leq n$ and $|v| \geq 1$

$$\underbrace{aaaaaa}_{u}\underbrace{a}_{v}\underbrace{bbbbbbb}_{w}$$

$|u| = n-1$

$|v| = 1$

$|uv| = |u| + |v| = n-1+1 = n$

$|w| = n.$

If $i=0$, the v string doesnot appear and so number of a's will be less than number of b's. So the language L $= \{a^n b^n, n \geq 0\}$ is not regular.

2. Show that $L = \{ww^R, \Sigma = \{0,1\}\}$ is not regular.

Ans: Consider the string

$$X = \underbrace{\underbrace{1\cdots1}_{n}\underbrace{0\cdots0}_{n}}_{w}\underbrace{\underbrace{0\cdots0}_{n}\underbrace{1\cdots1}_{n}}_{w^R}$$

Since $|x| = 4n \geq n$. Now split $x = uvw$ such that $|uv| \leq n$ and $|v| \geq 1$.

$$x = \underbrace{1 \ldots\ldots}_{u}\underbrace{1}_{v}\underbrace{0 \ldots\ldots 0\ 0 \ldots\ldots 0\ 1 \ldots\ldots 1}_{w}$$

$|u| = n-1$, $|v| = 1$, so $|uv| = n-1+1 = n$

If $i = 0$, v doesnot appear, so string is of not the for ww^R. So $uv^iw \notin L$. So $L = \{ww^R, \Sigma = 0,1\}$ is not regular.

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3. Show that $L = \{a^i b^j, i > j\}$ is not regular.

Ans: Let L is regular, n be number of states of FA

Let $x = a^{n+1} b^n$. $|x| = 2n+1 \geq n$.

Split $x = uvw$, such that $|uv| \leq n$, $|v| \geq 1$

$$x = \underbrace{a^j}_{u}\ \underbrace{a^k}_{v}\ \underbrace{a\, b^n}_{w}$$

$|u| = j$, $|v| = k \geq 1$, $|u| + |v| = j + k \leq n$

According to Pumping lemma $uv^i w = \underbrace{a^j}_{u} \underbrace{(a^k)^i}_{v} \underbrace{ab^n}_{w}$

If $i = 0$, no of a's in string u will not be more than the number of b's in w. So $L = \{a^i b^j, i > j\}$ is not regular.

4. Show that $L = \{a^{n!}, n \geq 0\}$ is not regular:

Ans: $n! = 1 \times 2 \times 3 \times \ldots n$

Let $x = a^{n!}$, $|x| \geq n$. So we can split x into uvw such that $|uv| \leq n$ and $|v| \geq 1$ as shown

$$x = \underbrace{a^j}_{u} \ \underbrace{a^k}_{v} \ \underbrace{a^{n!-j-k}}_{w}$$

$|u| = j$, $|v| = k \geq 1$ $|uv| = j + k \leq n$

According to lemma $uv^i w \in L$

ie

$$a^j (a^k)^i a^{n!-j-k}$$

B. With $i=0$, $x = a^j a^{n!-j-k}$

$$x = a^{n!-k}$$

But $n! > n!-k$, with $k=1$

$n! > n!-1$. So L is not regular.

So $L = \{a^{n!}, n \geq 0\}$ is not regular.

5. Show that $L = \{0^n, n \text{ is Prime}\}$ is not regular.

Ans: $L = \{0^n, n \text{ is Prime}\}$, So L can be

$$L = \{00, 000, 00000, \cdots\cdots\}$$

$0^n = X$ and $|X| = n$. So split X into uvw

$|uv| \leq n$ and $|v| \geq 1$

$$X = 0^n = \underset{u}{0^j}\ \underset{v}{0^k}\ \underset{w}{0^{n-j-k}}$$

$|u| = j$, $|v| = k \geq 1$ $|uv| = j+k$.

According to Pumping lemma $uv^iw \in L$ ie

$$(0)^j (0^k)^i 0^{n-j-k} \in L$$

With $i = \cancel{0}\ n+1$,

$$0^j (0^k)^i 0^{n-j-k}.$$

$$= j + ki + n - j - k$$

$$= n + ki - k$$

$$= n + k(i-1), \text{ if } i = n+1$$

$$= n + k(n+1-1)$$

$$= n + kn = n(1+k)$$

$n(1+k)$ is prime for all $k \geq 1$. If $k=1$, it will not be prime. So the language $L = \{0^n, n \text{ is prime}\}$ is not regular.

6. Show that $L = \{w, n_a(w) = n_b(w)\}$ is not regular.

Ans: Let $x = a^n b^n \in L$, $|x| = 2n \geq n$. So split x into uvw.

$$X = a^n b^n = \underbrace{a^j}_{u}\ \underbrace{a^k}_{v}\ \underbrace{b^n}_{w}$$

$|u| = j$ and $|v| = k \geq 1$ $\quad |uv| = j + k$

According to pumping lemma $uv^iw \in L$ for $i=0,1, \ldots$

ie $a^j (a^k)^i b^n \in L$

if $i=0$, number of a's will be less than b's &

if $i=2$, number of a's will be more than b's.

So $L = \{ w \mid n_a(w) = n_b(w) \}$ is not regular.

7. Show that $L = \{ a^n, n = k^2$ for $k \geq 0 \}$ is not regular.

Ans: Let $x = a^m$, where $m = n^2$

So $|x| \geq n$, split $x = uvw$ such that

$|uv| \leq n$ and $|v| \geq 1$

$$X = a^m = \underset{u}{a^j}\ \underset{v}{a^k}\ \underset{w}{a^{m-j-k}}$$

$|u| = j$ and $|v| = k \geq 1$, so that $|uv| = j+k$

According to pumping lemma $uv^iw \in L$

$a^j (a^k)^i a^{m-j-k} \in L$, for $i = 0, 1, 2 \ldots$

if $i=2$ $$uv^2w \in L$$

$$a^j a^{2k} a^{m-j-k} \in L$$

$$a^{m+k} \in L$$

$$|a^{m+k}| = m+k = n^2+k$$

So $n^2 < n^2+k$ and is not perfect Square. So $L = \{a^n \mid n = k^2, \text{ for } k \geq 0\}$ is not regular.

Closure Property of Regular Languages

The closure property is a useful tool for building complex automata. The Various closure properties of Regular languages are

i) Union of two Regular languages is Regular

ii) Intersection —〃— —〃— —〃— ——

iii) Complement of Regular language is Regular

iv) Closure (Star) of Regular language is Regular

v) Concatenation of two Regular language is Regular

vi) Difference —〃— —〃— —〃— ——

vii) Reverse of Regular language is Regular

viii) Homomorphism of —〃— —〃— ——

ix) Inverse of Homomorphism of Regular language is regular.

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Regular Languages are closed under

i) Union

ii) Concatenation

iii) Star

If L_1 and L_2 are regular, then

$L_1 \cup L_2$ is regular language

$L_1 \cdot L_2$ —— " —— " ——

L_1^* —— " —— " ——

Closure under Complementation

Theorem: If L is regular language, then the complement of $\bar{L}$ is also regular. (The set of regular languages is closed under complementation)

Proof: Let $M1 = (Q, \Sigma, \delta, q_0, F)$ be a DFA which accepts the language L. Let $M_2 = (Q, \Sigma \delta, q_0, Q-F)$ which accepts $\bar{L}$. There is no final between M_1 & M_2 expect the final states. The non final states of M_1 are final states of M_2 and the nonfinal states of M_2 are final states of M_1. The language which is rejected by M1 is accepted by M_2 and Vice-Versa. So regular languages are closed under complementation.

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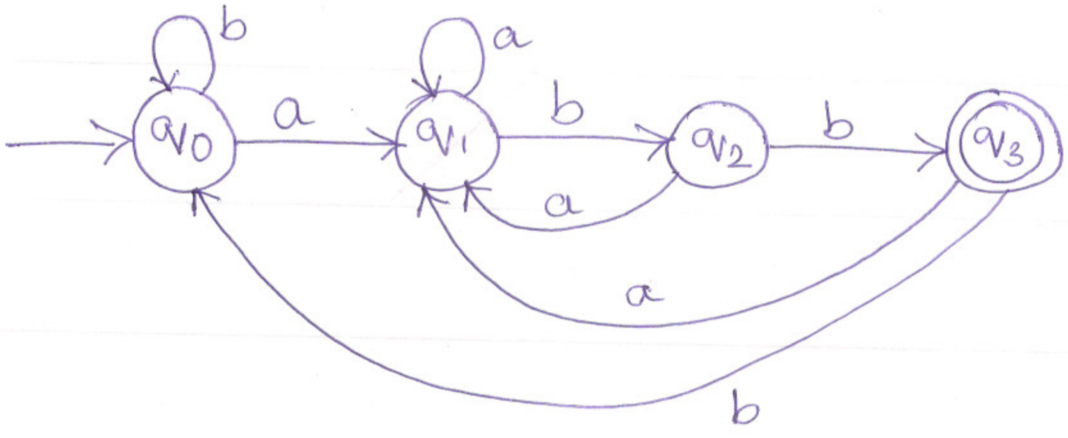
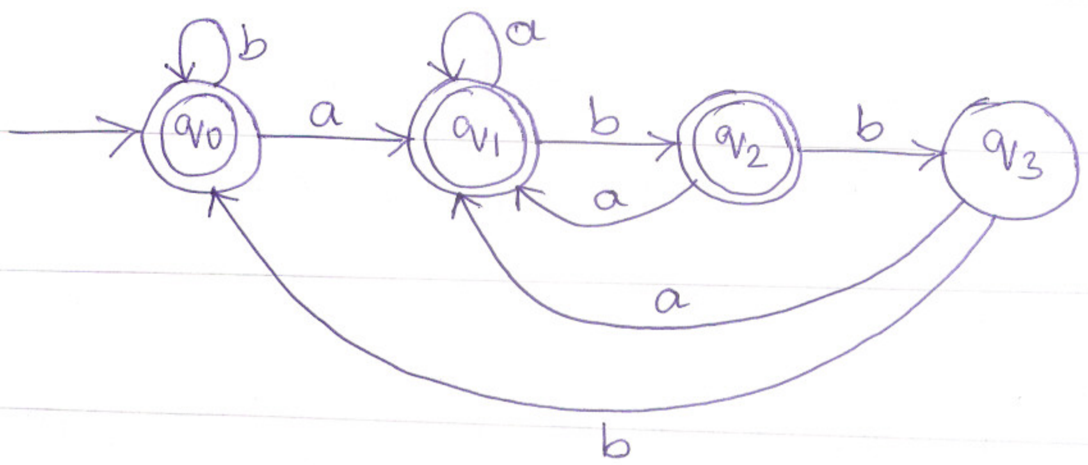
Eg: Consider dfa accepting string ending with abb



The complement of this machine is



The above DFA accepts string of a's & b's that do not end with abb.

Closure under Intersection.

Theorem: If L and M are regular languages then regular language is closed under intersection.

Proof:- Let us consider $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ which accepts L_1 and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ which accepts L_2. $L_1 \cap L_2$ is machine M, the states of the machine are $Q = Q_1 \times Q_2$. The final state

$$F = \{(p, q),\ p \in F_1 \text{ and } q \in F_2\}$$

The string ω is accepted if and only if

$\delta((q_1, q_2)\, w)$ is in F

Intial state — M_1 — M_2

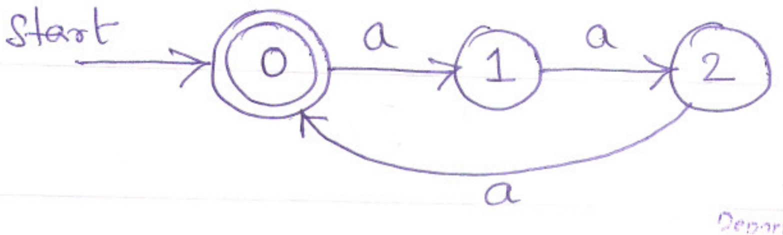
This happens if and only if

$\delta(q_1, w) \in F_1$ and $\delta(q_2, w) \in F_2$

if and only if $w \in L_1 \cap L_2$, the regular languages are closed under intersection.

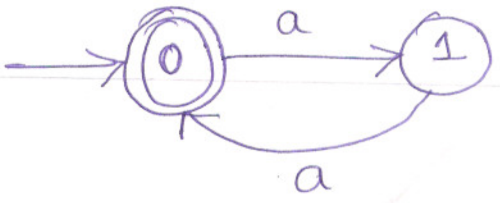
Example: DFA to accept $|w| \bmod 3 = |w| \bmod 2$

Ans: DFA for $|w| \bmod 3$ is



$Q_1 = \{0, 1, 2\}$

DFA for $|w| \bmod 2$ is



$Q_2 = \{0, 1\}$

$Q_1 \times Q_2 = \{(0,0)\,(0,1)\,(1,0)\,(1,1)\,(2,0)\,(2,1)\}$

State of first M/C — State of second m/c

$\delta((0,0)a) = (\delta(0,a), \delta(0,a)) = (1,1)$

$\delta((1,1)a) = (\delta(1,a), \delta(1,a)) = (2,0)$

$\delta((2,0)a) = (\delta(2,a), \delta(0,a)) = (0,1)$

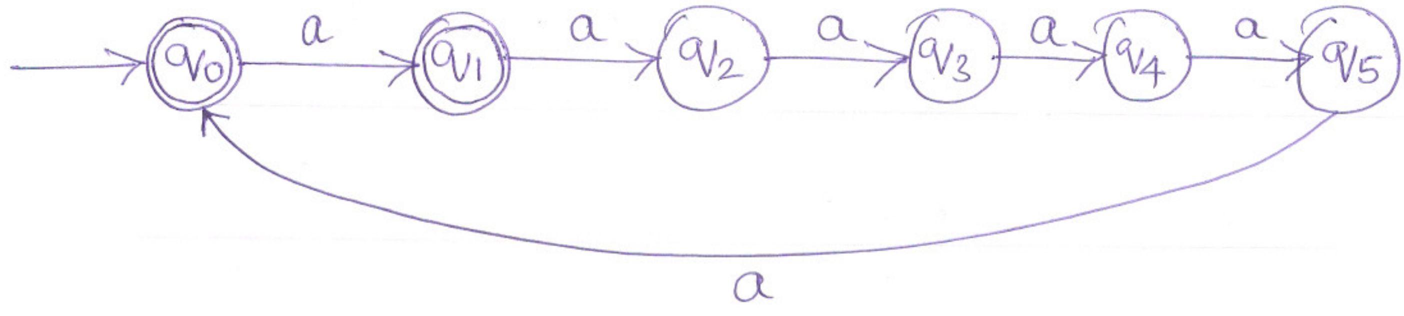
$\delta((0,1)a) = (\delta(0,a), \delta(1,a)) = (1,0)$

$\delta((1,0)a) = (\delta(1,a), \delta(0,a)) = (2,1)$

$\delta((2,1)a) = (\delta(2,a), \delta(1,a)) = (0,0)$

To accept string $|w| \bmod 3 = |w| \bmod 2$, the pair (x,y) such that $x=y$ are final states.

$F = \{(0,0)(1,1)\}$



Closure under Difference

If L_1 and L_2 are regular languages, then regular languages is closed under difference. i.e if L_1 and L_2 are regular languages, the $L_1 - L_2$ is also regular.

Proof: Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F)$ which accepts L_1 and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$. $M = (Q, \Sigma, \delta, q, F)$ recognizing $L_1 - L_2$ is

$$Q = Q_1 \times Q_2$$

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$$\Sigma = \text{is same}$$

$$\delta = \delta(Q, \text{i/p Symbol})$$

$$F = \{(p, q) \mid p \in F_1 \text{ and } q \notin F_2)$$

The string w is accepted if and only if

$$\delta((q_1 q_2) w) \text{ is in } F$$

$$(\delta((q_1, w)\ \delta(q_2, w))) \text{ is in } F$$

This will happen if and only if

$\delta(q_1, w) \in F_1$ and $\delta(q_2, w) \notin F_2$ i.e $L_1 - L_2$

Closure under Reversal

If L is regular then L^R is also regular.

Let L is language corresponding to regular expression E

Then $L(E^R) = (L(E))^R$

By definition

i) ϕ is a Regular expression

ii) ε —"—"—

iii) a —"—"—

Reversal of E^R is

i) $\{\phi\}^R = \phi$

ii) $\{\varepsilon\}^R = \varepsilon$

iii) $\{a\}^R = \{a\}$

Again by definition

i) $E_1 + E_2$ is Regular expression

ii) $E_1 \cdot E_2$ —"—"—

iii) E_1^* is Regular Expression

Case 1: If $E_1 + E_2$ is regular expression, then

$E^R = E_1^R + E_2^R$ is regular expression, denoting

language $L(E^R) = L(E_1^R) \cup L(E_2^R)$

Case 2: If $E_1 \cdot E_2$ is regular expression then

$E^R = E_1^R \cdot E_2^R$ is regular expression denoting

language $L(E^R) = L(E_1^R) \cdot L(E_2^R)$

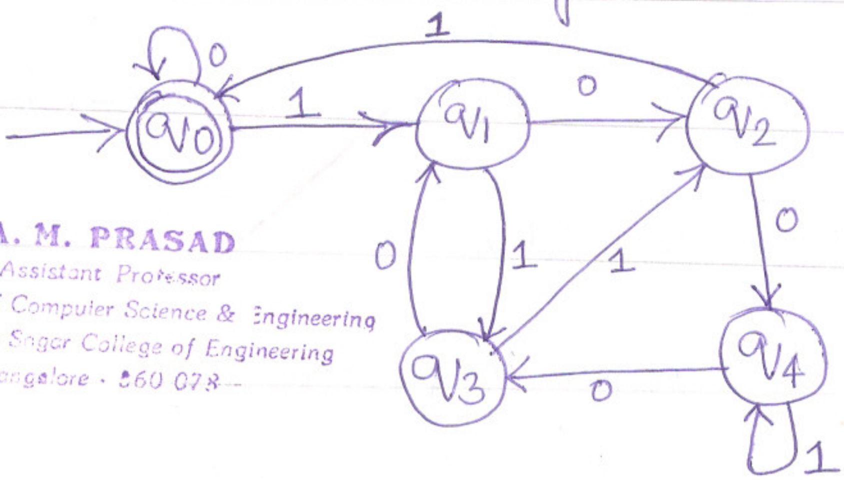
Case 3: If $E_1^* = E_1$ is regular expression, then

$E^R = E_1^R$ is regular expression denoting

Language $L(E^R) = L(E_1^R)$

Note: Reversal of FA is obtained by reversing all arcs in transition diagram.

Example: The DFA for binary number divisible by 5



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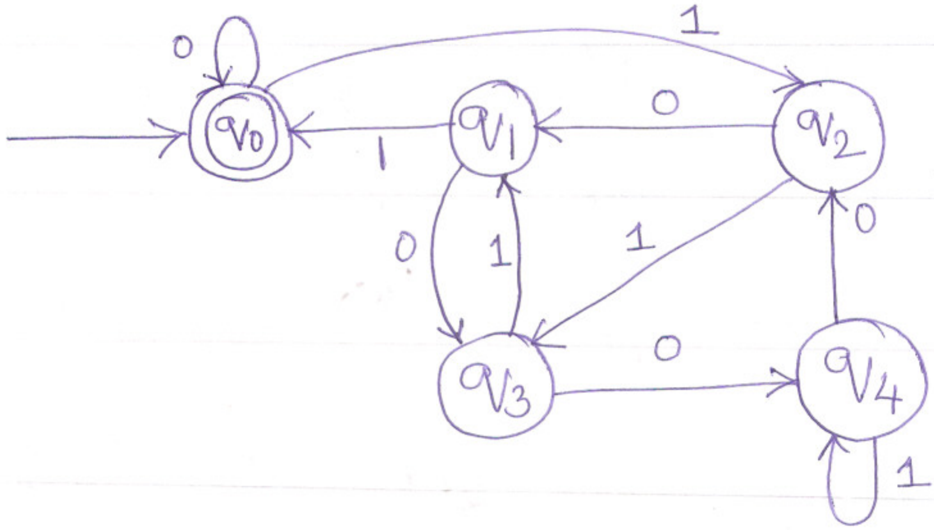
The DFA accepts $|w| \bmod 5 = 0$. By changing the direction of the arcs the automata accepts strings in reverse divisible by 5



Closure under Homomorphism

If L is regular and h is homomorphism, then homomorphic image h(L) is regular.

Let R be regular expression and L(R) be the corresponding regular language. We can find h(R) by substituting h(a) for each a in Σ. By definition of regular expression, h(R) is a regular expression and so h(L) is regular languages

Example: Let Σ and Γ are set of alphabets. The homomorphism function $h: \Sigma \to \Gamma^*$ is called homomorphism. If $w = a_1 a_2 \cdots a_n$

then $h(w) = h(a_1) h(a_2) \cdots h(a_n)$

If L is made of alphabets from Σ then $h(L) = \{ h(w) \mid w \in L \}$ is called homomorphic image.

Let $\Sigma = \{0, 1\}$ $\Gamma = \{0, 1, 2\}$ and $h(0) = 01$, $h(1) = 112$

then $h(010) = h(0) h(1) h(0)$

$= 0111201$

$L(00, 010) = L(0101, 0111201)$

is homomorphic image.

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Decision Property of Regular Languages

By this property, if two automata define same language, then states of automata can be minimized to few states as possible. The minimization is important design in switching circuits.

Equivalence of two states

The reduction of states of DFA can be achieved by finding the distinguishable and indistinguishable states.

Two states p and q of a DFA are equivalent

⟨i⟩ $\delta(p, w) \in F$

$\delta(q, w) \in F$

then states p and q are indistinguishable.

⟨ii⟩ $\delta(p, w) \notin F$

$\delta(q, w) \notin F$

then states p and q are also indistinguishable.

Two states p and q are distinguishable then

$\delta(p, w) \in F$ and $\delta(q, w) \notin F$ & Viceversa.

Procedure to Minimize DFA.

Step 1 :- Using table filling algorithm find distinguish -ble and indistinguishable pairs.

Step 2 :- Write the transition table for indistinguishable pairs and individual distinguishable states.

Step 3 :- The start state of reduced DFA is the pair in which start state of given problem is present.

Step 4 :- The final state of reduced DFA is the pairs in which final state of given problem is present.

Problems : 1. Minimize the following DFA

δ	a	b
A	B	F
B	G	C
* C	A	C
D	C	G
E	H	F
F	C	G
G	G	E
H	G	C

Ans :- **Step 1** :- The Various states of given DFA are A, B, C, D, E, F, G, H. The pairs of these states are

B							
C							
D							
E							
F							
G							
H							
	A	B	C	D	E	F	G

Step 2 :- Mark the pair, in which one is final and other is nonfinal state.

B							
C	X	X					
D			X				
E			X				
F			X				
G			X				
H			X				
	A	B	C	D	E	F	G

Step 3: Write the transition for unmarked pair (p, q)

$\delta((p, q)\, a) = r, \quad \delta((p, q)\, b) = s.$

Mark the pair (p, q), if r is markable or s is markable or both are markable.

δ	a (r)	b (s)
(A,B)	(B,G)	(F,C) ✓
(A,D)	(B,C) ✓	(F,G)
(A,E)	(B,H)	(F,F)
(A,F)	(B,C) ✓	(F,G)
(A,G)	(B,G)	(F,E)
(A,H)	(B,G)	(F,C) ✓
(B,D)	(G,C) ✓	(C,G) ✓
(B,E)	(G,H)	(C,F) ✓
(B,F)	(G,C) ✓	(C,G) ✓
(B,G)	(G,G)	(C,E) ✓
(B,H)	(G,G)	(C,C)
(D,E)	(C,H) ✓	(G,F)
(D,F)	(C,C)	(G,G)
(D,G)	(C,G) ✓	(G,E)
(D,H)	(C,G) ✓	(G,C) ✓
(E,F)	(H,C) ✓	(F,G)

	pair r	pair s
δ	a	b
(E,G)	(H,G)	(F,E)
(E,H)	(H,G)	(F,C) ✓
(F,G)	(C,G) ✓	(G,E)
(F,H)	(C,G) ✓	(G,C) ✓
(G,H)	(G,G)	(E,C) ✓

Step 4: The unmarked pairs are

(A, E), (A, G), (B, H), (D, F) and (E, G).

B	X						
C	X	X					
D	X	X	X				
E		X	X	X			
F	X	X	X		X		
G		X	X	X		X	
H	X		X	X	X	X	X
	A	B	C	D	E	F	G

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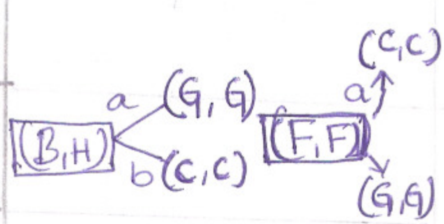
Write the transition for unmarked pairs, if transition of unmarked pairs r or s can marked then mark the unmarked pair.

δ p	a (r)	b (s)
(A, E)	(B, H)	(F, F)
(A, G) ✓	(B, G)	(F, E)
(B, H)	(G, G)	(C, C)
(D, F)	(C, C)	(G, G)
(E, G) ✓	(H, G)	(F, E)



if any of the four pairs can be marked then mark pair p. Similarly for all pairs.

The pairs (A, G) and (H, G) can be marked.

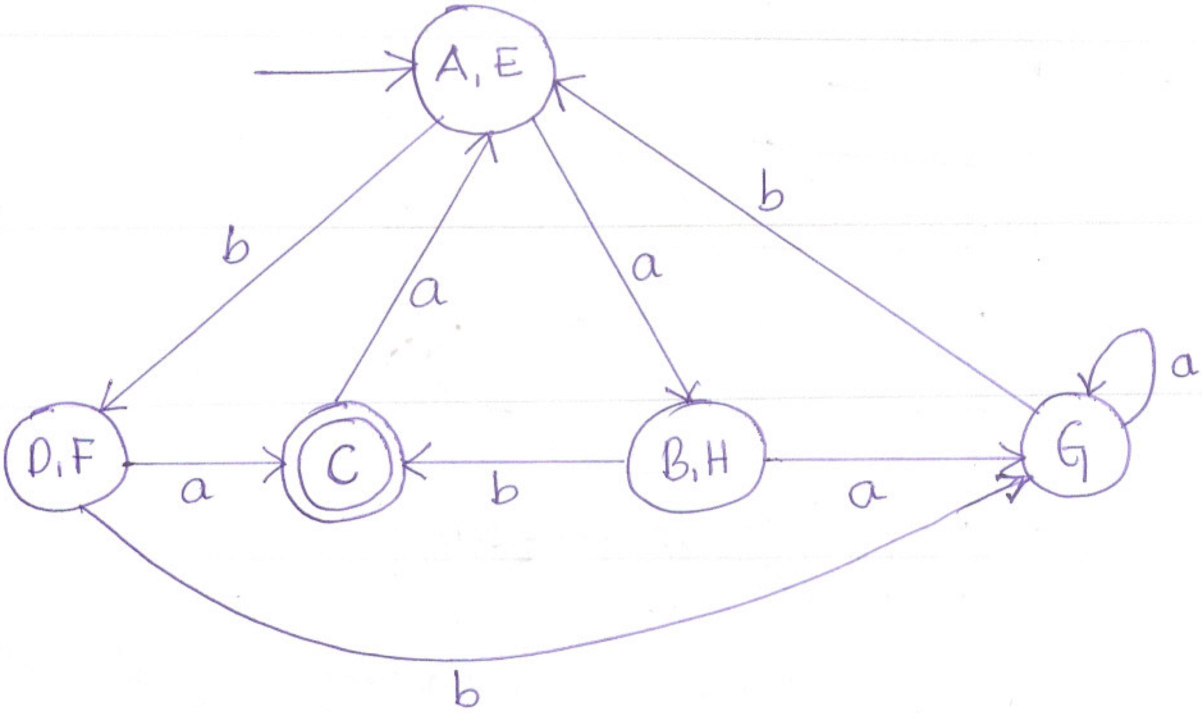
step 5:- The pair unmarked are
(A, E), (B, H), (D, F)

	A	B	C	D	E	F	G
B	X						
C	X	X					
D	X	X	X				
E		X	X	X			
F	X	X	X		X		
G	X	X	X	X	X	X	
H	X		X	X	X	X	X

Write the transition for unmarked pairs

	δ	a	b
→	(A, E)	(B, H)	(D, F)
	(B, H)	G	C
*	C	(A, E)	C
	(D, F)	C	G
	G	G	(A, E)

Step 6 :- Write transition diagram for states in transition table.



2. Find the minimized DFA for the following

	δ	0	1
→	A	B	A
	B	A	C
	C	D	B
*	D	D	A
	E	D	F
	F	G	E
	G	F	G
	H	G	D

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Step 1: The states of DFA are A, B, C, D, E, F, G, H.

The pairs of these states are

B							
C							
* D							
E							
F							
G							
H							
	A	B	C	D *	E	F	G

Step 2: Mark the pair in which one is final state and other is nonfinal state.

B							
C							
D	X	X	X				
E				X			
F				X			
G				X			
H				X			
	A	B	C	D	E	F	G

Step 3: Write the transition for unmarked pairs. On transition any of the or both obtained pairs can be marked, then corresponding pair can be marked.

δ	a	b
(A,B)	(A,B)	(A,C)
(A,C)	(B,D)	(A,B)
(A,E)	(B,D)	(A,F)
(A,F)	(B,G)	(A,E)
(A,G)	(B,F)	(A,G)
(A,H)	(B,G)	(A,D)
(B,C)	(A,D)	(C,B)
(B,E)	(A,D)	(C,F)
(B,F)	(A,G)	(C,E)
(B,G)	(A,F)	(C,G)
(B,H)	(A,G)	(C,D)
(C,E)	(D,D)	(B,F)
(C,F)	(D,G)	(B,E)
(C,G)	(D,F)	(B,G)
(C,H)	(D,G)	(B,D)
(E,F)	(D,G)	(F,E)
(E,G)	(D,F)	(F,G)
(E,H)	(D,G)	(F,D)

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(F, G)	(G, F)	(E, G)
(F, H)	(G, G)	(E, D) ✓
(G, H)	(F, G)	(D, G) ✓

Step 4: The unmarked pairs are

(A, B) (A, F) (A, G) (B, F) (B, G) (C, E) (F, G)

B							
C	X	X					
D	X	X	X				
E	X	X		X			
F			X	X	X		
G			X	X	X		
H	X	X	X	X	X	X	X
	A	B	C	D	E	F	G

Write the transition for unmarked pairs.

δ p	a (r)	b (s)
(A, B)	(B, A)	(A, C) ✓
(A, F)	(B, G)	(A, E) ✓
(A, G)	(B, F)	(A, G)
(B, F)	(A, G)	(C, E)
(B, G)	(A, F)	(C, G) ✓
(C, E)	(D, D)	(B, F)
(F, G)	(G, F)	(E, G) ✓

Any of the pair either r or s or both are marked then mark corresponding pair p

Step 5 :- The unmarked pairs are

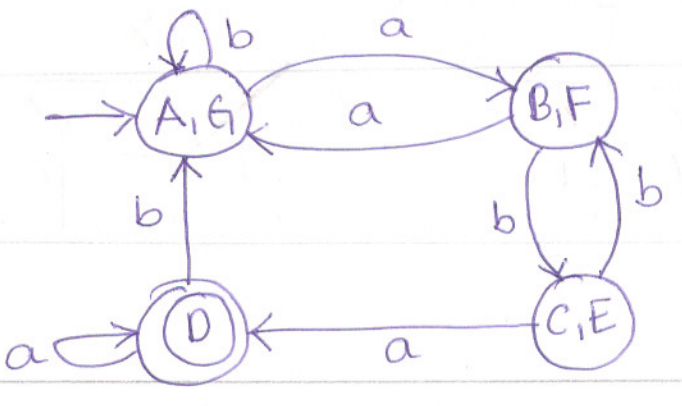
(A, G) (B, F) (C, E)

B	X						
C	X	X					
D	X	X	X				
E	X	X		X			
F	X		X	X	X		
G		X	X	X	X	X	
H	X	X	X	X	X	X	X
	A	B	C	D	E	F	G

Write the transition for unmarked pairs.

	δ	a	b
→	(A, G)	(B, F)	(A, G)
	(B, F)	(A, G)	(C, E)
	(C, E)	D	(B, F)
*	D	D	(A, G)

Step 6: Write transition diagram



3. Minimize the following DFA.

δ	0	1
A	B	E
B	C	F
* C	D	H
D	E	H
E	F	I
* F	G	B
G	H	B
H	I	C
* I	A	E

Step 1:- Table for the pairs of DFA are shown. Mark the pair in which one is final & other is non final.

B								
* C	X	X						
D			X					
E			X					
* F	X	X		X	X			
G			X			X		
H			X			X		
* I	X	X		X	X		X	X
	A	B	C	D	E	F	G	H

Step 2: Write the transition for unmarked pairs.

δ	a	b
(A, B)	(B, C) ✓	(B, F)
(A, D)	(B, E)	(B, H)
(A, E)	(B, F) ✓	(B, I)
(A, G)	(B, H)	(B, B)
(A, H)	(B, I)	(B, C) ✓
(B, D)	(C, E) ✓	(F, H)
(B, E)	(C, F)	(F, I)
(B, G)	(C, H) ✓	(F, B)
(B, H)	(C, I)	(F, C)
(D, E)	(E, F)	(H, I) ✓
(D, G)	(E, H)	(H, B)
(D, H)	(E, I) ✓	(H, C)
(E, G)	(F, H) ✓	(I, B)
(E, H)	(F, I)	(I, C)
(H, G)	(I, H)	(C, B) ✓

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The unmarked pairs are (A, D), (A, G), (D, G)
(B, E) (B, H), (E, H), (C, F), (C, I), (F, I)

B	X							
C	X	X						
D		X	X					
E	X		X	X				
F	X	X		X	X			
G		X	X		X	X		
H	X	X	X	X		X	X	
I	X	X		X	X		X	X
	A	B	C	D	E	F	G	H

Step 3:- The unmarked states can be grouped as follows

(A, D) (A, G) (D, G) can be written as (A, D, G)

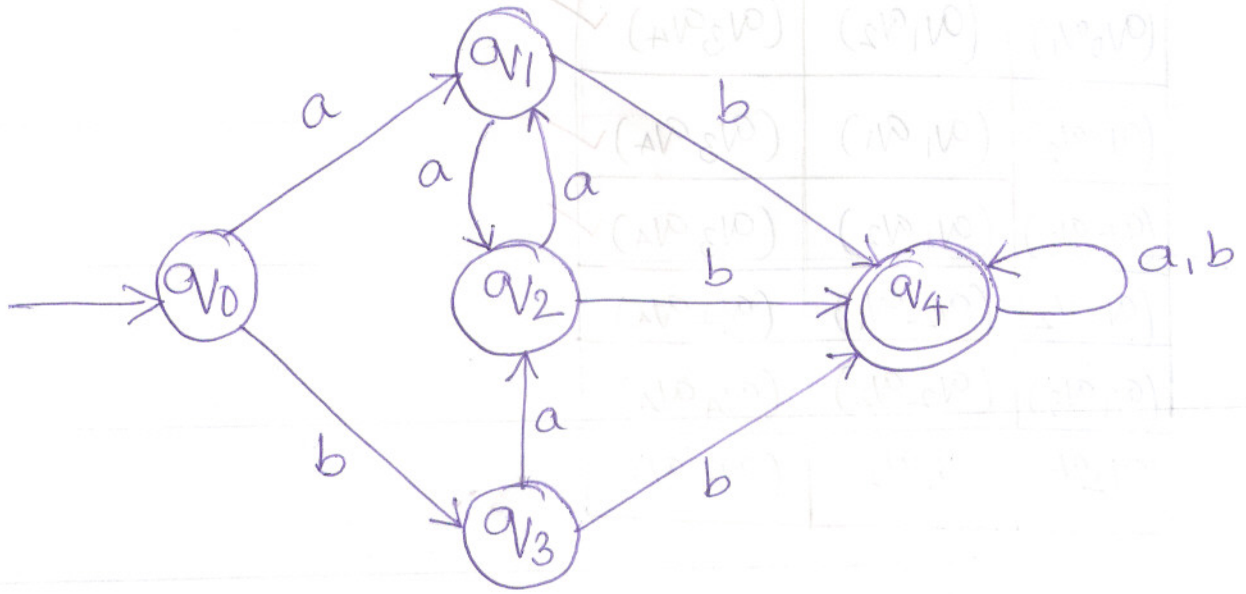
(B, E) (B, H) (E, H) —— " —— " —— (B, E, H)

(C, F) (C, I) (F, I) —— " —— " —— (C, F, I)

Write the transition for these states

δ	0	1
→ (A, D, H)	(B, E, H)	(B, E, H)
(B, E, H)	(C, F, I)	(C, F, I)
* (C, F, I)	(A, D, H)	(B, E, H)

4. Minimize the following DFA



Step 1: Write the table for all pairs of states

q_0, q_1, q_2, q_3, q_4

q_1				
q_2				
q_3				
* q_4	X	X	X	X
	q_0	q_1	q_2	q_3

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Mark the pair in which one is final and other is nonfinal state

Step 2: Write the transitions for unmarked pairs.

On transition to unmarked pair, any of the resultant pair or both are marked then mark corresponding pair.

pair	a	b
$(q_0 q_1)$	$(q_1 q_2)$	$(q_3 q_4)$
$(q_0 q_2)$	$(q_1 q_1)$	$(q_3 q_4)$
$(q_0 q_3)$	$(q_1 q_2)$	$(q_3 q_4)$
$(q_1 q_2)$	$(q_2 q_1)$	$(q_4 q_4)$
$(q_1 q_3)$	$(q_2 q_2)$	$(q_4 q_4)$
$(q_2 q_3)$	$(q_1 q_2)$	$(q_4 q_4$

q_1	X			
q_2	X			
q_3	X			
q_4	X	X	X	X
	q_0	q_1	q_2	q_3

step 3:

δ	a (r)	b (s)
$(q_1 q_2)$	$(q_2 q_1)$	$(q_4 q_4)$
$(q_1 q_3)$	$(q_2 q_2)$	$(q_4 q_4)$
$(q_2 q_3)$	$(q_1 q_2)$	$(q_4 q_4)$

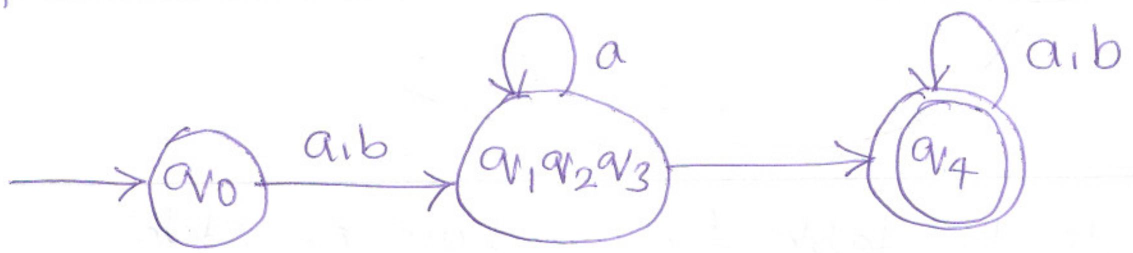
No pair can be marked either r or s, hence no corresponding pair can be marked.

Step 4:- The unmarked pairs are

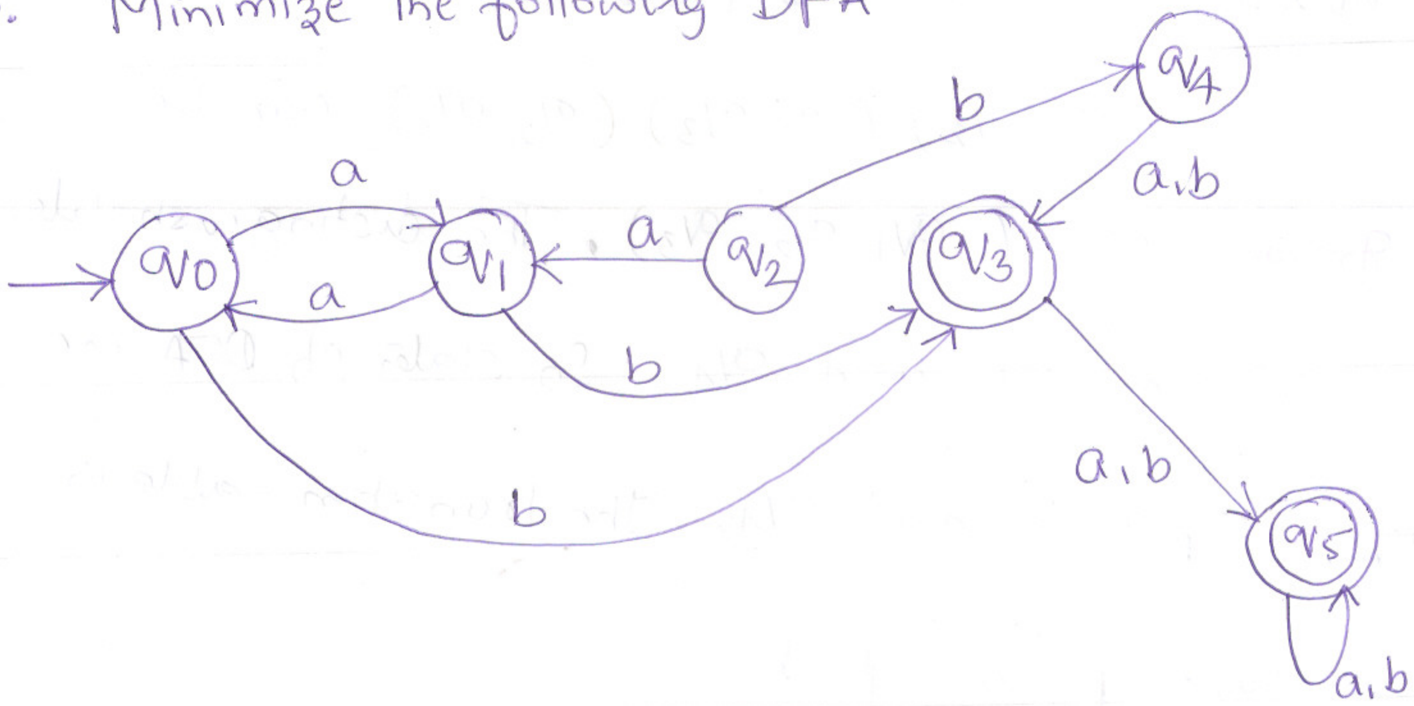
$(q_1\, q_2)\ (q_1\, q_3)\ (q_2\, q_3)$ can be grouped as $(q_1\, q_2\, q_3)$. The distinguishable states are q_0 and q_4. So states of DFA are q_0, $q_1 q_2 q_3$ and q_4. The transition table is

States	a	b
q_0	$q_1q_2q_3$	$q_1q_2q_3$
$q_1q_2q_3$	$q_1q_2q_3$	q_4
q_4	q_4	q_4

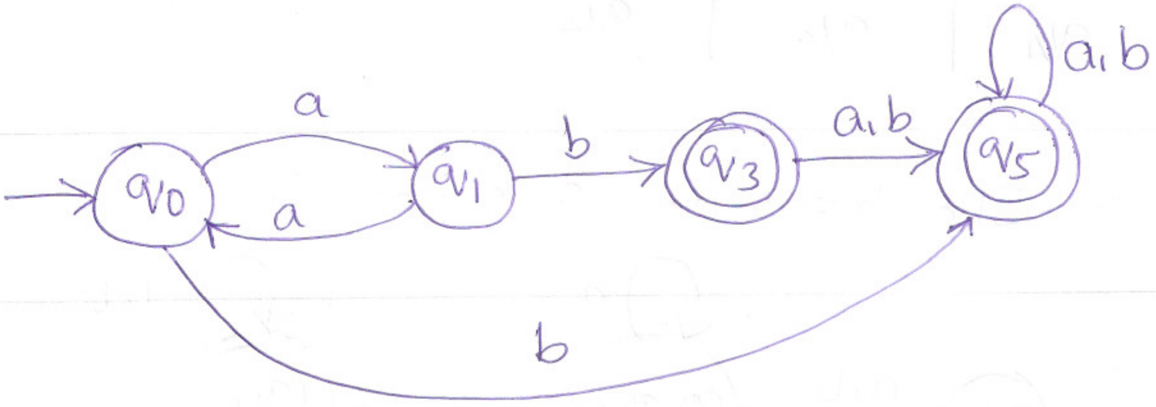
Step 5:- The transition diagram is



5. Minimize the following DFA



Step 1:- Remove the states which are not reachable from start state ie q_2 and q_4



Write the table for all pairs of states

q_1			
q_3	X	X	
q_5	X	X	
	q_0	q_1	q_3

Mark the pair in which one is final & other is nonfinal

Step 2: Write transition for unmarked pairs

δ	a	b
$(q_0 q_1)$	$(q_0 q_1)$	$(q_3 q_5)$
$(q_3 q_5)$	$(q_3 q_5)$	$(q_3 q_5)$

No pair can be marked because their resultant transition pairs are not marked. They are the states of DFA. ie $(q_0 q_1)$ $(q_3 q_5)$ and transition table is same as above.

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Step 3: Write the transition diagram