Braking System Optimization

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Summary

Problem statement

Sub-systems

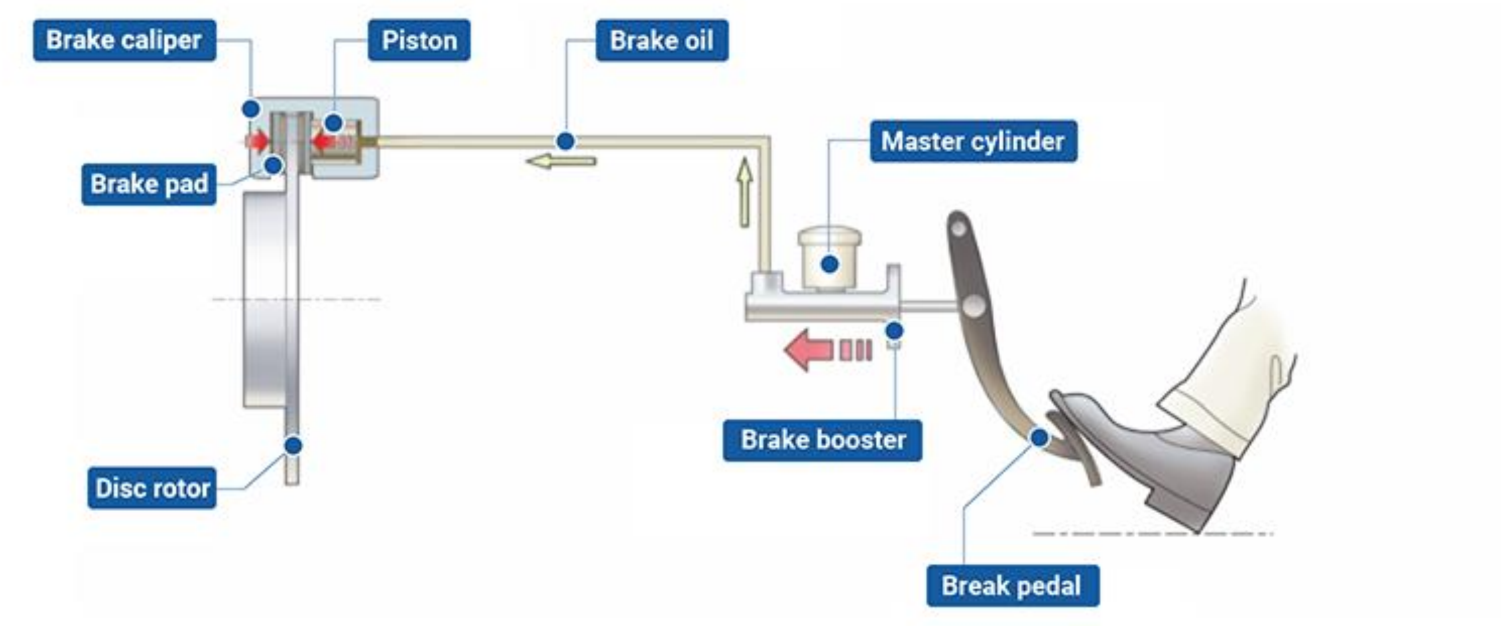
- Vibration
- Machine Element
- Thermal

Results

INTRODUCTION

- Brakes are one of the major components of any system.
- They are used to stop the moving vehicle.
- Brake system analysis has become a major area of study due to the increase in the demand for high safety.

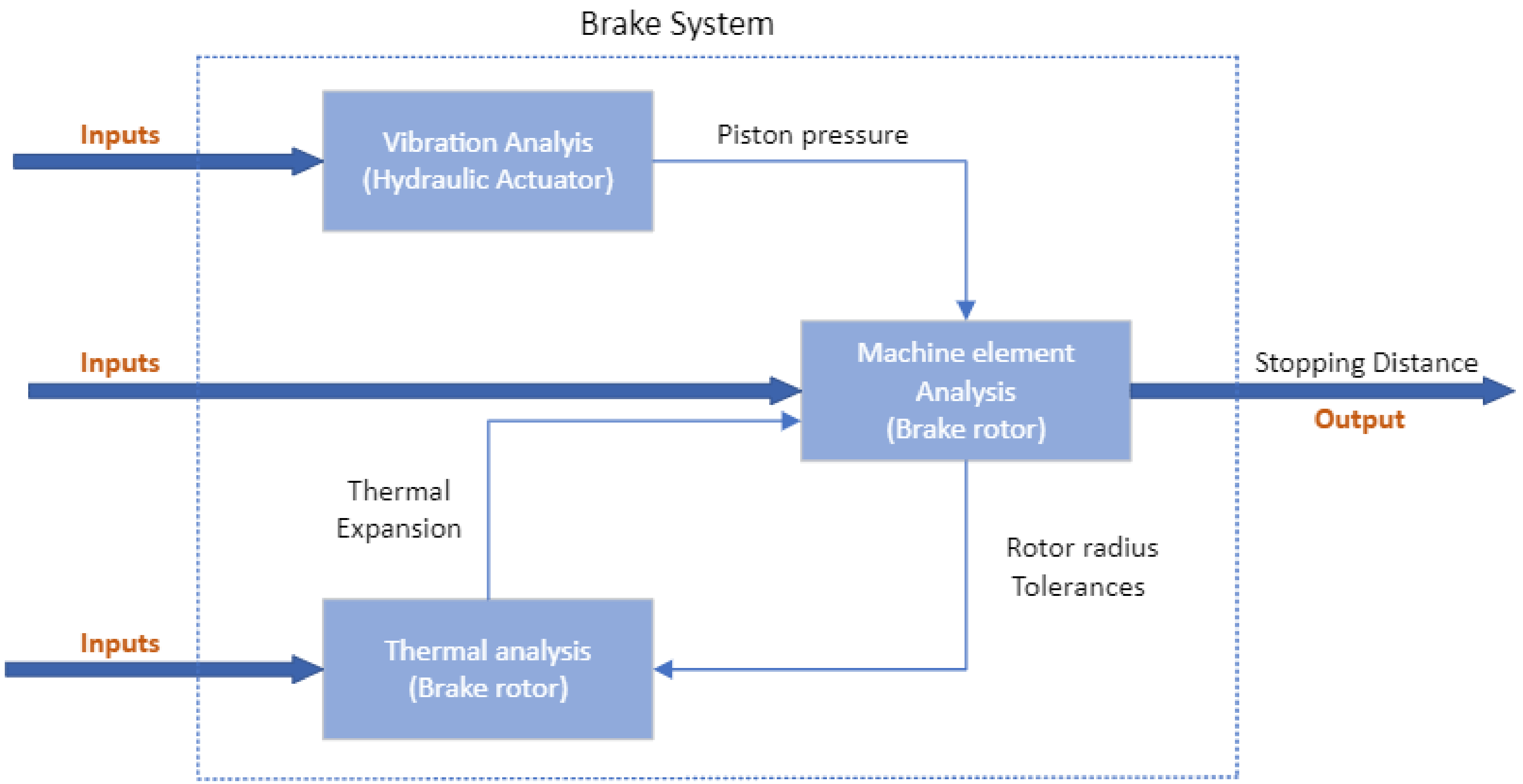
WORKING OF A DISC BRAKE.

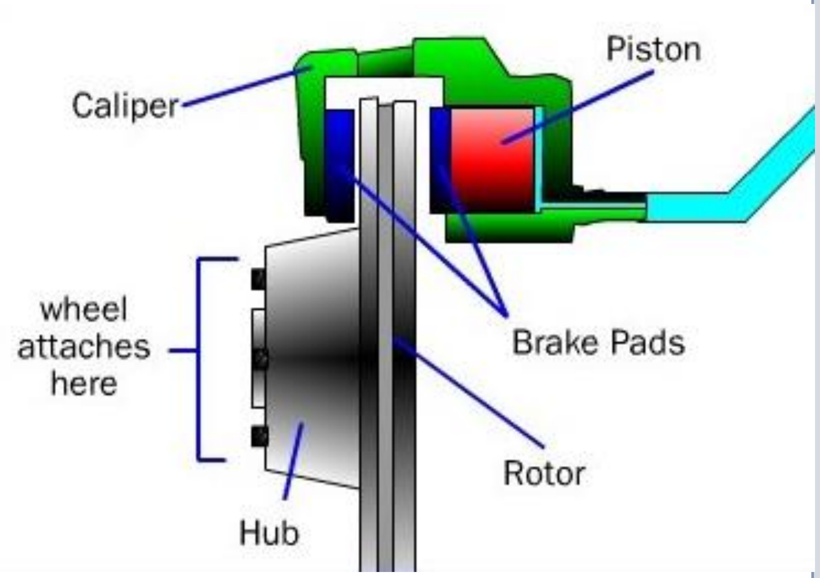


1. Problem Statement

The main aim of this project is to minimize the braking distance of the vehicle in response to certain design and manufacturing constraints.

2. Sub-systems





ASSUMPTIONS

1) MATERIAL OF THE ROTOR – CAST IRON

2) VELOCITY – 180 KM/H.

3) 4 STUD BOLT HOLES.

4) BRAKE PAD MATERIAL – CERAMIC

5) NO SLIP CONDITION

6) ROAD GRADIENT IS NOT TAKEN INTO CONSIDERATION.

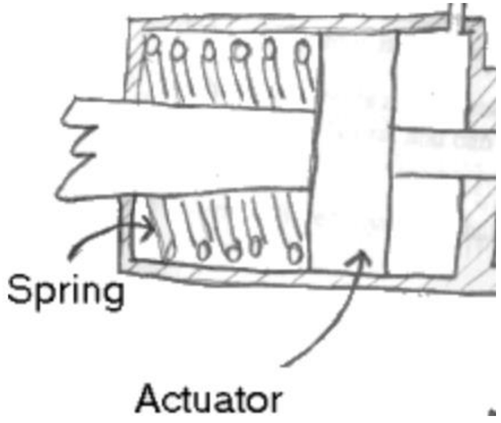
2.a Vibration Analysis

- Following a displacement, minimize settling time of actuator

Constraints:

- Under ABS braking, limit resonant amplitude
- System is slightly underdamped
- Limit power dissipation through damping

$m_a, \delta, F_0, x_{max}, P_{max}$ → Hydraulic Actuator (vibration analysis) → k, c → p



Parameters:

m - actuator mass (0.5 kg)

δ - percentage of initial displacement at which actuator is considered settled (2%)

F_0 - amplitude of ABS force (1500 N)

x_{max} - maximum permissible actuator displacement

P_{max} - maximum power dissipated through damping (3000 W)

Optimization Variables:

k - spring stiffness (N/m)

c - effective damping coefficient(N*s/m)

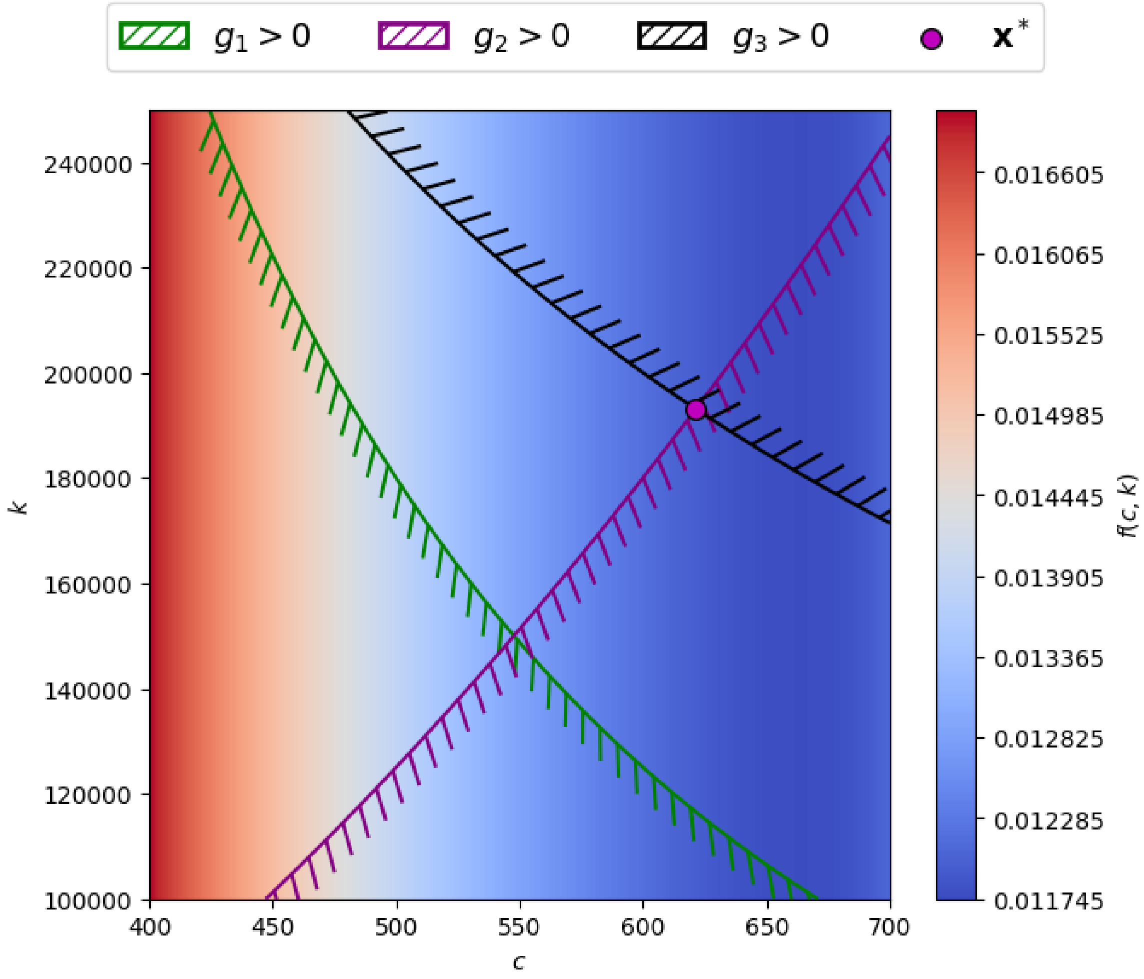
Formulation:

$$\min f(c,k;\delta,m) = -\ln\left(\delta\sqrt{1-\frac{c^2}{4mk}}\right)\frac{2m}{c}$$

$$g_1(c,k;m,F_0,x_{max}) = \frac{F_0\sqrt{\frac{m}{k}}}{c} - x_{max} \leq 0$$

$$g_2(c,k;m) = \frac{c}{2\sqrt{mk}} - 1 \leq 0$$

$$g_2(c,k;m,x_{max},P_{max}) = \frac{x_{max}^2 ck}{2m} - P_{max} \leq 0$$



Methods:
- KKT points (nonlinear optimization problem)

Results :
- c = 621.4 Ns/m
- k = 193 kN/m

Output:
- p = 1194 kN/m^2 (brake line pressure)

2.b Machine Element Analysis

Minimize the braking distance of the vehicle subject to a maximum braking force (Fb) and maximum stress.

Calculating braking distance:

- Piston pressure given after the Vibration analysis of 1st sub-system (brake caliper) is used to calculate the normal force exerted by brake pads on the rotor for a given rotor radius -

$$Fn = Pp*Ab$$

- Braking force generated due to friction between the rotor and brake pads -

$$Fb = 2\mu Fn$$

- Stopping distance -

Work done against friction = Kinetic energy of the vehicle;

$$X = (\tfrac{1}{2}(m)v^2)/Fb$$

Min X $(R_o, R_{i,}\ R; r, P_p, \theta)$

$= (½ * m * v^2) / (2 * \mu * P_p * \theta * (R_0{}^2 - R_i{}^2))$

$= \rho(\Pi * t)(R^2 - r_c{}^2 - r^2)v^2 / (\mu * Pp * \theta(R_0{}^2 - R_i{}^2))$

Such that $\alpha r < \sigma_y$

$\sigma r_{\_max} = (3 + \nu)(\rho * \omega^2 / \nu)(R - r_c)^2 < \sigma_y$

$\sigma_{t\_max} = (\rho * \omega^2 / 4)[(1 - \nu)r_c{}^2 + (3 + \nu)R^2] < \sigma_y$

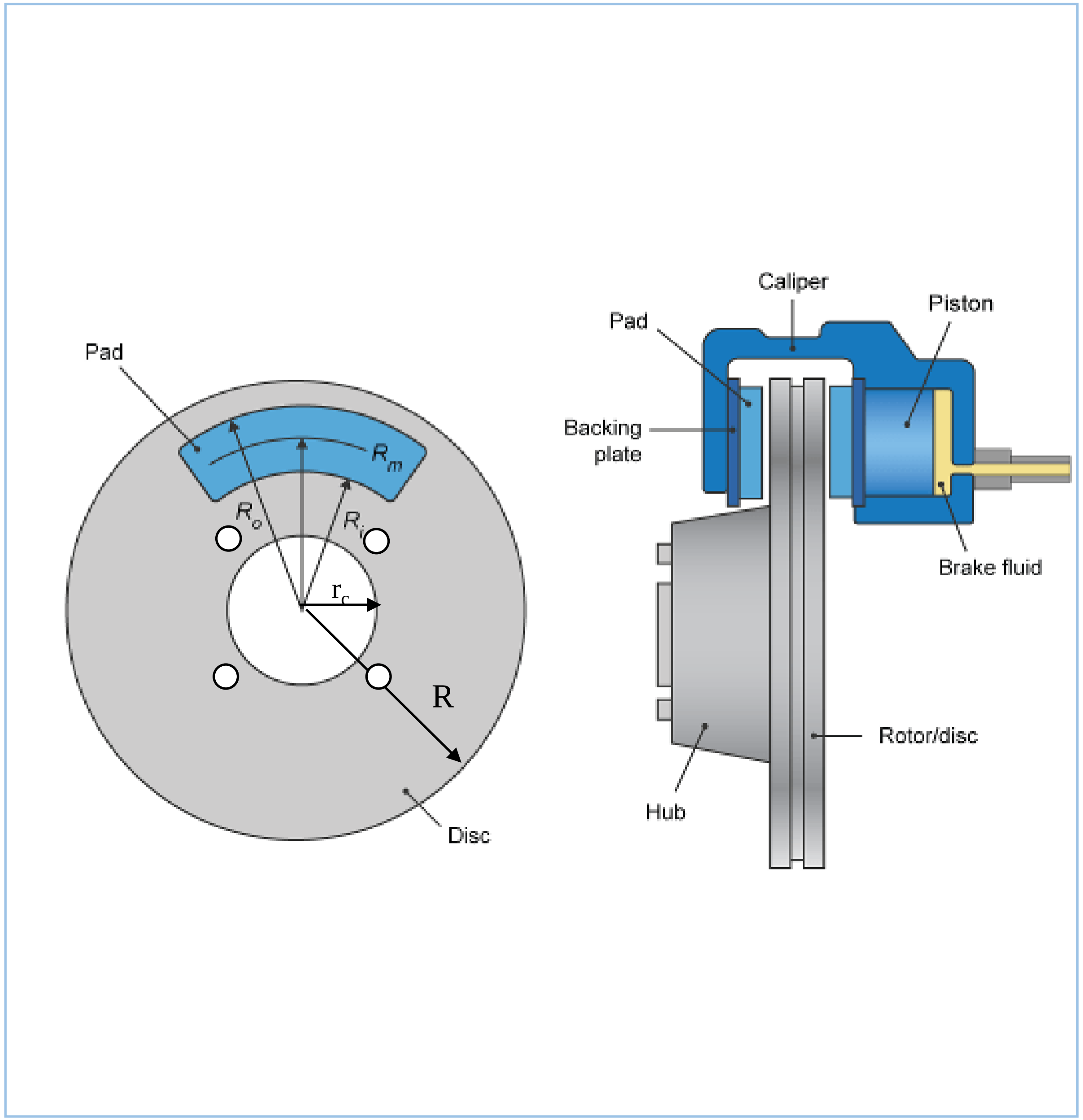
$V = - \Pi (R^2 - r_c{}^2 - r^2)t < 0$

$R_o - R <= 0$

$r_{i\ -}\ Ro <= 0$

$X, w, \sigma_{r\ \_max}, \sigma_{t\_max} > 0$

R	Radius of the rotor	ω	Angular velocity of the disc
R_o	Outer radius of brake pad	R_i	Inner radius of brake pad
P_p	Piston pressure	σ_y	Yielding Stress
r_c	Inner radius of rotor	t	Thickness of rotor
r	Bolt radius		



Manufacturing Constraints	
Hub bolt tolerances	11.85 mm < 2r < 12.0 mm -> 11.85mm - 2r < 0 2r - 12.0mm < 0
Rotor thickness tolerance	$t_{min} < t < t_{max}$ 16.95 < t < 17.05 -> 16.95 - t < 0 t -17.05 < 0
Brake pad inner radius	Ri > = 0.0728
Rotor radius	R ≤ 0.12

Results	
Initial guesses	Ro = 0.99, Ri = 0.07, R = 0.12
Results	Ro = 0.1106, Ri = 0.0816, R = 0.1732 !

2.c Thermal analysis

- Thermal expansion of the dimensions
- Thermal stress
- Assumptions

Problem formulation and constraints

Rotor under braking

Objective: $\max Q_{transfer}$

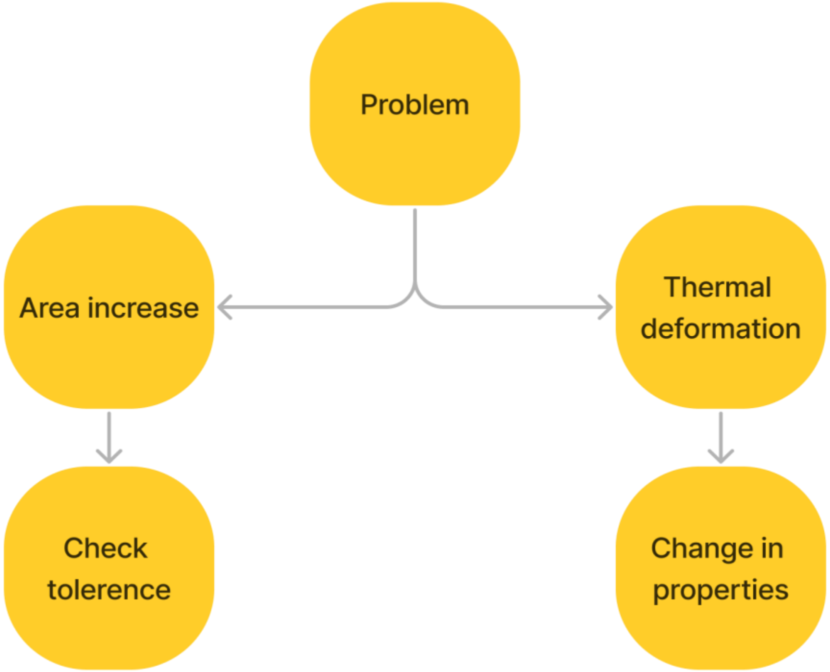
Constraints:

$$T_{max} < 343℃$$

$$LB \leq \delta_{radius} \leq UB$$

$$LB \leq \delta_{thickness} \leq UB$$

What is the problem?



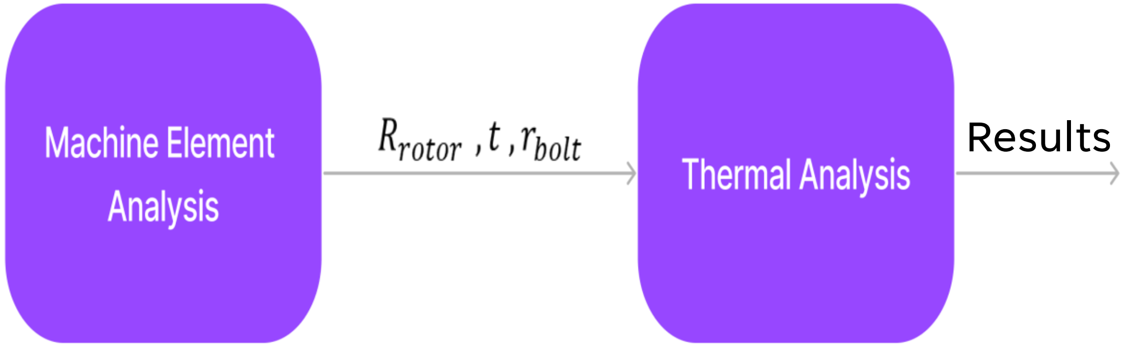
<u>Trade off</u>

- Relation between Thermal and machine element analysis

Results

Possible problems

- Inconsistent constraints
- Formulation of the problem

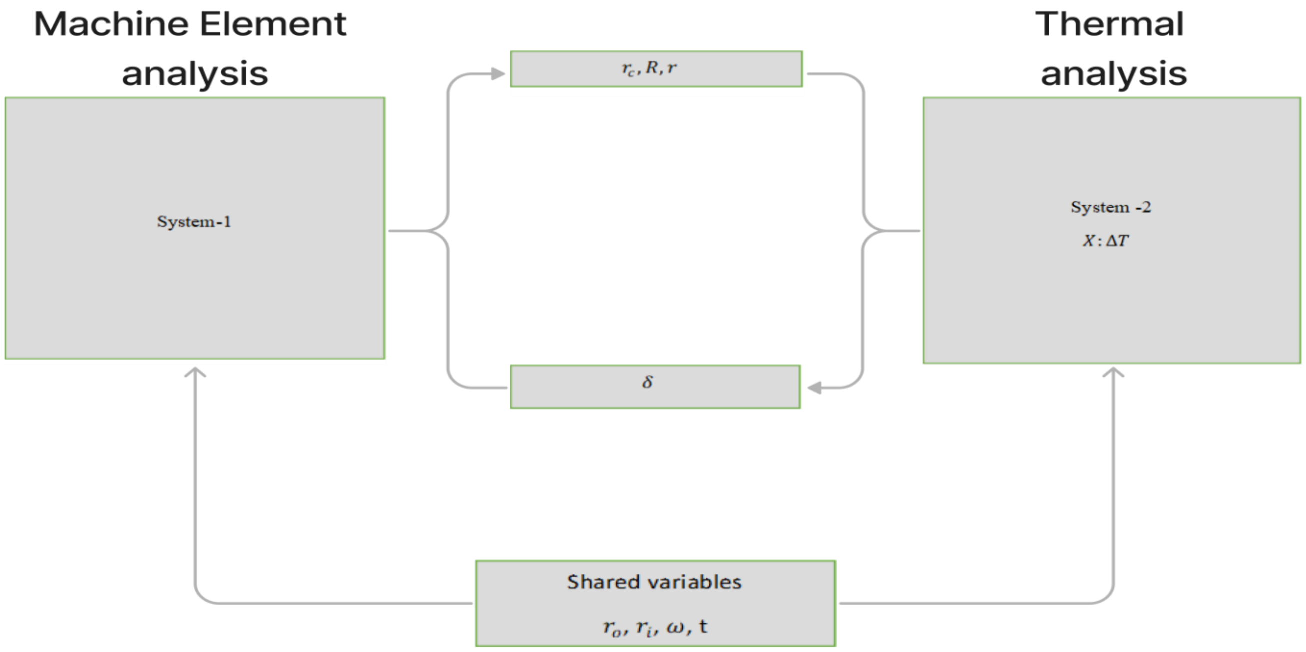


$$\delta_{radius}, \delta_t \ll 0.1$$
$$\Delta T = 0.6283 \ ℃$$

MDO

NHATC

Formulation



Book keeping of variables

Index	subproblem	name	coupling type	link	lower bound	upper bound	baseline
1	1	r0(pad)	shared	2	0.099	0.1255	1
2	1	ri(pad)	shared	2	0.07	0.099	1
3	1	omega	shared	2	3.5	6.585	1
4	1	t	shared	2	0.01695	0.01705	1
5	1	rc(centre)	feed forward	2	0.038	0.042	1
6	1	R(rotor)	feedforward	2	0.07	0.1317	1
7	1	r (Bolt)	feedforward	2	0.00722	0.00798	1
8	1	tol	feedback	2	-0.05	0.05	1
9	1	σy	uncoupled	None	1	200000000	1
10	1	rk	uncoupled	None	0.06676	0.0737	1
11	2	r0(pad)	shared	1	0.099	0.1155	1
12	2	ri(pad)	shared	1	0.07	0.099	1
13	2	omega	shared	1	3.5	6.585	1
14	2	t	shared	1	0.01695	0.01705	1
15	2	ΔT	uncoupled	None	1	316	1
16	2	rc(centre)	feedback	1	0.038	0.042	1
17	2	R(rotor)	feedback	1	0.07	0.1317	1
18	2	r (Bolt)	feedback	1	0.00722	0.000798	1
19	2	tol	feedforward	1	-0.05	0.05	1

Trouble

- Improper formulation of black box model
- Inconsistent constraints
- Invalid lower and upper bounds of variables
- Any suggestions?

Thank you

Any Questions?

Acknowledgement

- Dr. Khalil Al Handawi
- Dr. Ahmed Bayoumy

Equations for the calculation

- Heat generation due to friction between rotor and pad → $Q_{gen} = \frac{0.9\mu R\omega}{2\pi}$
- Heat convection between rotor and environment → $Q = hA(T_{rotor} - T_{air})$
- Thermal expansion of the material → $\alpha\Delta T$
- Equivalent radius → $R_{eq} = \frac{2(r_o^3 - r_i^3)}{3(r_o^2 - r_i^2)}$

Where,

$$r_o = Inner\ radius\ of\ brake\ pad$$
$$r_i = outer\ radius\ of\ brake\ pad$$