

## ASSIGNMENT 5

*Homeworks are due at the beginning of class or in my mail box by 2pm on the due date. The point value for the 6000 level is indicated in small font.*

Consider the following options on a stock  $S$  (in all cases  $S_0$  is the initial stock price):

1.  $C(S_0, K, T)$ : the *European Call* option with strike  $K$  and exercise time  $T$ .
2.  $B(S_0, K, B, T)$ : the *Barrier option* with payoff  $K$ , barrier  $B$  and horizon  $T$ . If and when the price hits the barrier  $B$  the holder may buy at  $B$  and sell at  $K$ .
3.  $A(S_0, T)$ : the *average strike Asian call option* with expiry  $T$  – a European call option with strike at the average value of the stock price over  $[0, T]$ .
4.  $M(S_0, T)$ : the *minimum strike call option* – a European call option with strike at the minimum price over  $[0, T]$ .

### 1 (25 (25) points) Simulating Stock Paths

Assume the initial stock price is  $S_0$  and it follows real and risk neutral dynamics given by

$$\Delta S = \mu S \Delta t + \sigma S \Delta W \quad \Delta \tilde{S} = r \tilde{S} \Delta t + \sigma \tilde{S} \Delta \tilde{W}.$$

Write a program that takes as input  $\mu, r, \sigma, S_0, T, \Delta T$  and simulates the stock price from time 0 to  $T$  in time steps of  $\Delta t$  for the risk neutral world, using each of the following modes:

- (a) Binomial mode I: compute  $\lambda_{\pm}$  from  $\mu, \sigma$  assuming that  $p = \frac{1}{2}$ , and then computing  $\tilde{p}$ .
- (b) Binomial mode II: compute  $\lambda_{\pm}$  from  $\mu, \sigma$  assuming that  $p = \frac{2}{3}$ , and then computing  $\tilde{p}$ .
- (c) Continuous mode: using the continuous risk neutral dynamics  $r, \sigma$  generate at time step  $\Delta t$  as if the discrete model were taken to the limit  $dt \rightarrow 0$ .

For each of the three methods, give plots of representative price paths for  $S_0 = 1, \mu = 0.07, r = 0.03, \sigma = 0.2, T = 2$  using  $\Delta t = 0.1, 0.01, 0.0001$ .

## 2 (25 (25) points) Pricing Options Using Monte Carlo

Use Monte Carlo simulation to price the 4 options. Assume that  $S_0 = 1$ ,  $\mu = 0.07$ ,  $r = 0.03$ ,  $\sigma = 0.2$ ,  $T = 2$ . For each case, use each of the three modes above, and compute the price using each of the three time discretizations,  $\Delta t = 0.1, 0.01, 0.0001$ .

In all cases, make some intelligent choice for the number of Monte Carlo samples that you need to take to get an accurate price. [Hint: first take a few samples to get an estimate of the variance of the Monte Carlo sample values.]

- (a) Compute  $C(1, 1, 2)$  as efficiently as you can and compare with the analytic formula.
- (b) Compute  $B(1, 1, 0.95, 2)$
- (c) Compute  $A(1, 2)$ , for three possible definitions of “average”: the harmonic, arithmetic, and geometric means. Explain the relative ordering of these prices.
- (d) Compute  $M(1, 2)$ .

## 3 Bonus - Unspecified Number of Points

The barrier and the minimum option values can be computed analytically. Can you compute them, and compare with your numerical estimates?