EE 603- Assignment 2

Digital Signal Processing and it's applications

Assignment - 2

Q1) a) $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

$$x[n] = -b^n u[-n-1] = \begin{cases} b^n, & n \le -1 \\ 0 & n \ge 0 \end{cases}$$

$$X(e^{j\omega}) = \sum_{-\infty}^{-1} -b^n e^{-j\omega n}$$

restrict^n on b. $|b| \ge 1$

$$= -\left(\frac{b e^{-j\omega}}{1 - b e^{-j\omega}} \right)$$

$|b| \ge 1$

(b) $Y(e^{j\omega}) = \dfrac{2e^{-j\omega}}{1+2e^{-j\omega}}$ (b = −2 from obs.)

$x[n]$ (from prev. q^n) $= -(-2)^n u[-n-1]$

Q2) $x[n] = \cos\left(\frac{15\pi n}{4} - \frac{\pi}{3}\right)$

$$= \cos\left(\frac{15\pi n}{4}\right)\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\sin\left(\frac{15\pi n}{4}\right).$$

$$= \frac{1}{2j}\left(e^{j\frac{15\pi}{4}n} - e^{-j\frac{15\pi}{4}n}\right) + \frac{\sqrt{3}}{4}\left(e^{j\frac{15\pi n}{4}} + e^{-j\frac{15\pi n}{4}}\right)$$

Since complex exponentials are eigen f^{n}s of LTI systems

$$y[n] = \frac{1}{4}\left(H\left(e^{j\frac{15\pi}{4}}\right)e^{j\frac{15\pi n}{4}} - H\left(e^{-j\frac{15\pi}{4}}\right)e^{-j\frac{15\pi n}{4}}\right) + \frac{\sqrt{3}}{4}\left(H\left(e^{j\frac{15\pi}{4}}\right)e^{j\frac{15\pi n}{4}} + e^{-j\frac{15\pi n}{4}}H\left(e^{-j\frac{15\pi}{4}}\right)\right)$$

where,

$$H\left(e^{j\frac{15\pi}{4}}\right) = e^{-j\left[\frac{15\pi}{8} + \frac{\pi}{4}\right]} = e^{-j\left[\frac{17\pi}{8}\right]}$$

$$\& \; H\left(e^{-j\frac{15\pi}{4}}\right) = e^{-j\left(\frac{\pi}{4} - \frac{15\pi}{8}\right)} = e^{+j\left[\frac{13\pi}{8}\right]}$$

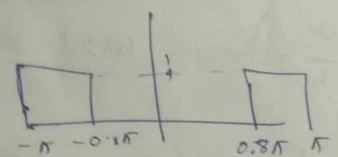
$$\Rightarrow y[n] = e^{-j\frac{\pi}{4}}\cos\left(\frac{\pi n}{4} + \frac{5\pi}{24}\right)$$

Q3)

(a) $H_1(e^{j\omega})$ is a frequency shifted form.

$= H(e^{j(\omega-\pi)})$

$$\Rightarrow H_1(e^{j\omega}) = \begin{cases} 0, & |\omega| < 0.8\pi \\ 1, & 0.8\pi \leq |\omega| \leq \pi \end{cases}$$

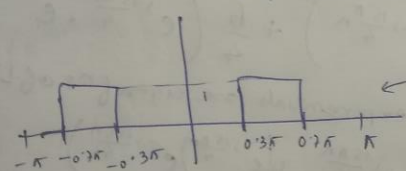


This is a high pass filter

(b) $H_2(e^{j\omega})$ is a frequency modulated form.

$$H_2(e^{j\omega}) = H(e^{j\omega}) * (\delta(\omega-0.5\pi) + \delta(\omega+0.5\pi))$$

$$= \begin{cases} 0, & |\omega| < 0.3\pi \\ 1, & 0.3\pi \leq |\omega| \leq 0.7\pi \\ 0, & 0.7\pi \leq |\omega| \leq \pi \end{cases}$$



Band pass filter

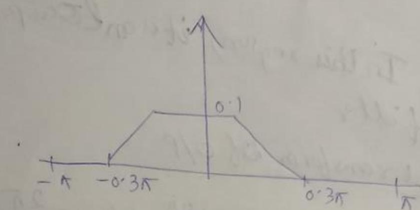
(c) $H_3(e^{j\omega})$ corresponds to a periodic convolution of $H_{lp}(e^{j\omega})$ with another low pass filter, which is -

$$H_3(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_x(e^{j\eta}) H_{lp}(e^{j\omega-\eta}) d\eta$$

where $H_x(e^{j\omega}) = \begin{cases} 1, & |\omega| < 0.1\pi \\ 0, & 0.1\pi \leq \omega \leq \pi \end{cases}$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-0.1\pi} H_{LP}(e^{j\omega-\eta})\,d\eta + \int_{0.1\pi}^{\pi} H_{LP}(e^{(j\omega-\eta)})\,d\eta\right]$$

$$H_3(e^{j\omega}) = \begin{cases} 0.1, & \omega \leq 0.1\pi \\ \frac{-|\omega|}{2\pi} + 0.15, & 0.1\pi \leq |\omega| \leq 0.3\pi \\ 0, & 0.3\pi < |\omega| < \pi \end{cases}$$



Q4) $y[n] = \frac{1}{2} y[n-1] + x[n]$

Finding solution by recursion.

$x[n] = \delta[n]$

$y[0] = 1$ $\quad y[2] = \frac{1}{4}$

$y[1] = \frac{1}{2}$ $\quad y[3] = \frac{1}{8}$ ...

$y[n] = \left(\frac{1}{2}\right)^n u[n]$ (as $n \geq 0$)

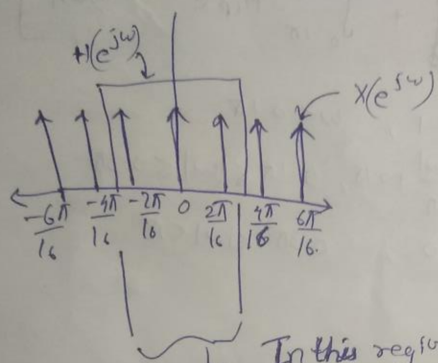
The system is stable, as $\left(\frac{1}{2}\right)^n u[n]$ is infinitely summable, ie.

$$\sum_{-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n] = \frac{1}{1-\frac{1}{2}} = 2.$$

Q5). $$X(e^{j\omega}) = \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}_{(n,k)} \delta(n+16k)\, e^{-j\omega n}$$

$$= \frac{1}{16} \sum_{k=-\infty}^{\infty} \delta\left(\omega + \frac{2\pi k}{16}\right)$$

$X(e^{j\omega})$ is also a periodic impulse train. There are 16 impulses in the range $-\pi \leq \omega \leq \pi$



In this region, it is an low pass filter.

~~EXP~~ fourier transform of O/P

$$Y(e^{j\omega}) = \frac{1}{16}\delta(\omega) + \frac{1}{16}e^{-j\frac{4\pi}{16}}\delta\left(\omega - \frac{2\pi}{16}\right) + \frac{1}{16}e^{j\frac{4\pi}{16}}\delta\left(\omega + \frac{2\pi}{16}\right)$$

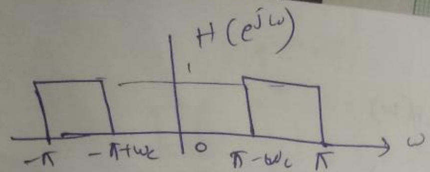
$$\Rightarrow y[n] = \frac{1}{16} + \frac{1}{8}\cos\left(\frac{2\pi n}{16} + \frac{3\pi}{8}\right)$$

Q6) $V(e^{j\omega}) = X(e^{j(\omega+\pi)})$

$$W(e^{j\omega}) = H_1 e^{(j\omega)}(V(e^{j\omega}))$$
$$= H_1(e^{j\omega})X(e^{j(\omega+\pi)})$$

$$Y(e^{j\omega}) = W(e^{j(\omega-\pi)})$$
$$= H_1(e^{j(\omega-\pi)})X(e^{j\omega})$$

$$\Rightarrow H(e^{j\omega}) = H_1(e^{j(\omega-\pi)})$$
$$H(e^{j\omega}) = H_1(e^{j(\omega-\pi)})$$
$$H(e^{j\omega}) = \begin{cases} 0, & |\omega| < \pi - \omega_c \\ 1, & \pi - \omega_c < |\omega| \leq \pi \end{cases}$$



Q7) $y[n] = \nabla(x[n]) = x[n] - x[n-1]$

(a) linear & time invariant

$$y[ax_1[n] + bx_2[n]] = ax_1[n] + bx_2[n] - ax_1[n-1] - bx_2[n-1]$$
$$= a(x_1[n] - x_1[n-1]) + b(x_2[n] - x_2[n-1])$$
$$= y(ax_1[n]) + y(ax_2[n])$$

$\therefore$ linear

$$y(x[n-1]) = x[n-1] - x[n-2]$$
$$= y[n-1]$$

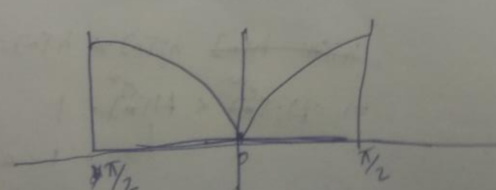
$\therefore$ Time invariant

(b) $x[n] = \delta[n]$

$y[n] = \delta[n] - \delta[n-1]$

(c) $H(e^{j\omega}) = 1 - e^{-j\omega}$

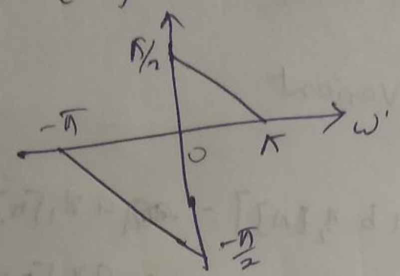
$$|H(e^{j\omega})| = \sqrt{(1-e^{-j\omega})(1-e^{+j\omega})}$$
$$= \sqrt{(1+1-e^{-j\omega}-e^{j\omega})}$$
$$= \sqrt{(2-2\cos\omega)}$$



phase

$$H(e^{jw}) = (1-\cos w) + j\sin(w)$$

$$\theta_{H(e^{jw})} = \tan^{-1}\left(\frac{\sin(w)}{1-\cos w}\right)$$



(d) $y[n] = f[n] * g[n]$

$\downarrow$

$x[n] - x[n-1] \equiv \underbrace{x[n]}_{f[n]*g[n]\ (\text{let})} * \overbrace{(\delta[n] - \delta[n-1])}^{t[n]}$

$\Rightarrow y[n] = f[n] * g[n] * t[n]$

$= f[n] * g[n] * (\delta[n] - \delta[n-1])$

$= [f[n] * (\delta[n] - \delta[n-1])] * g[n]$ $\quad = f[n] * (g[n] * (\delta[n] - \delta[n-1]))$

$= (\nabla(f[n])) * g[n] \quad = \quad = f[n] * \nabla(g[n])$

(e) to find $h_i[n]$ where

$h_i[n] * \nabla(x[n]) = x[n]$

if $x[n] = \delta[n]$

~~$h_i[n]\ h[n]$~~ $h_i[n] * h[n] = \delta[n]$

$\Rightarrow H_i[e^{jw}] \times H[e^{jw}] = 1$

$\hookrightarrow = 1 - e^{-jw}$

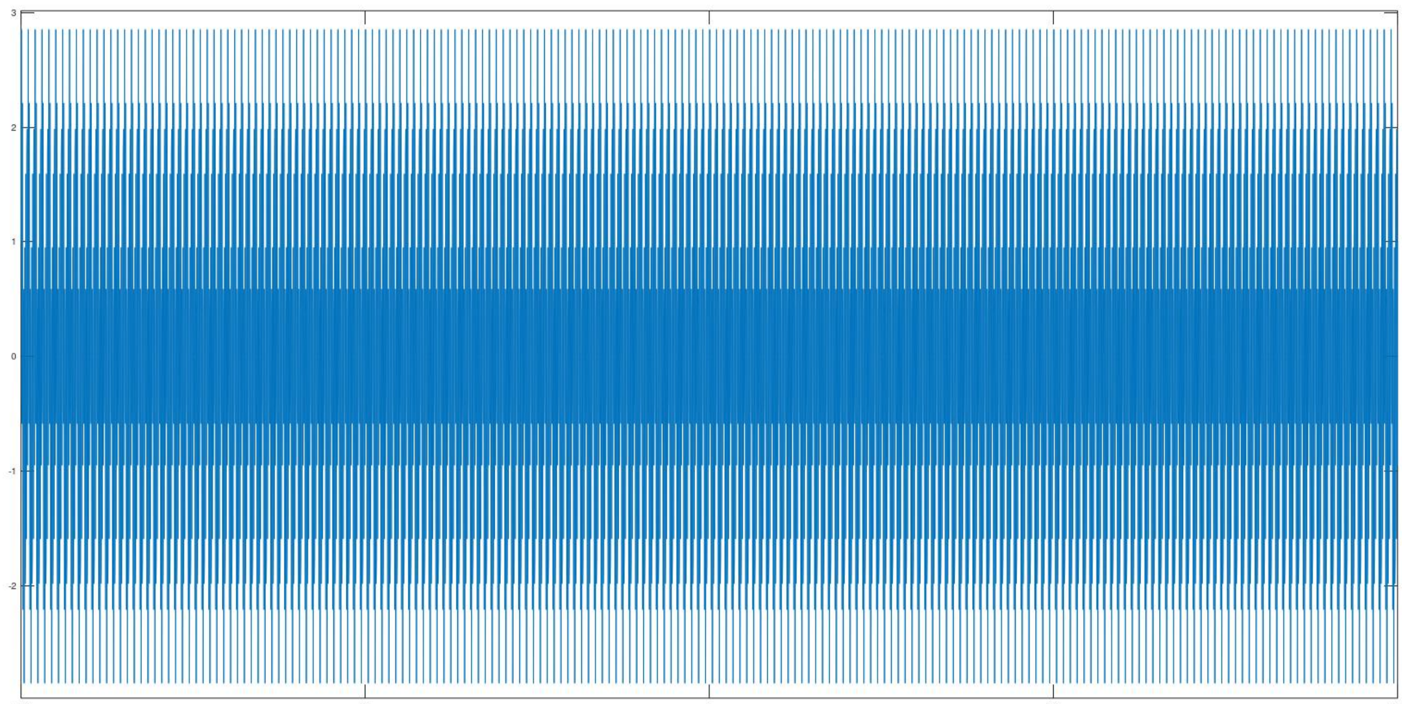
$\Rightarrow H_i(e^{jw}) = \frac{1}{1-e^{-jw}} \Rightarrow \underline{h_i[n] = U[n]}$

Question 8

a. Code:

```
n =-100:100;
Fs= 20;
t=-100:1/Fs:100;
x=cos(2*pi*t)+2*sin(12*pi*t);
plot(t,x);
```

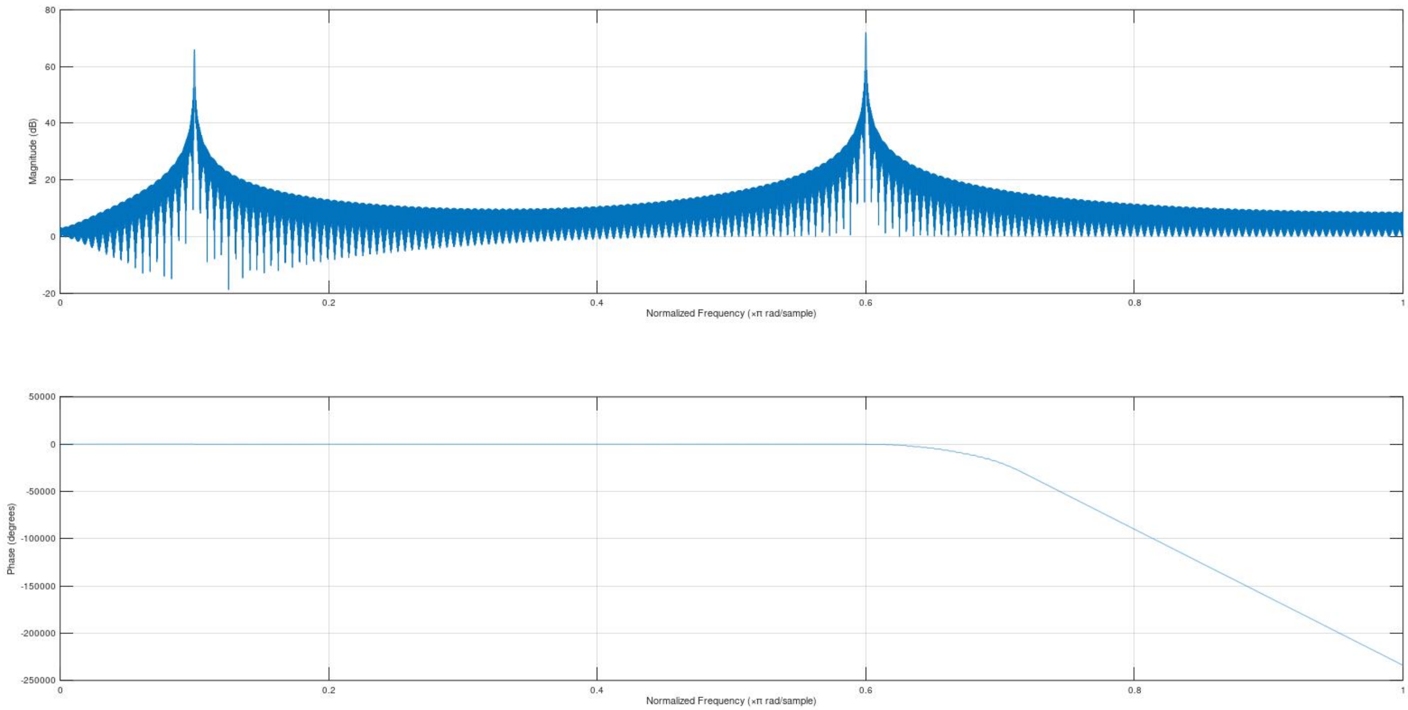
Plot:



b. Code:

```
n =-100:100;
Fs= 20;
t=-100:1/Fs:100;
x=cos(2*pi*t)+2*sin(12*pi*t);
freqz(x);
```

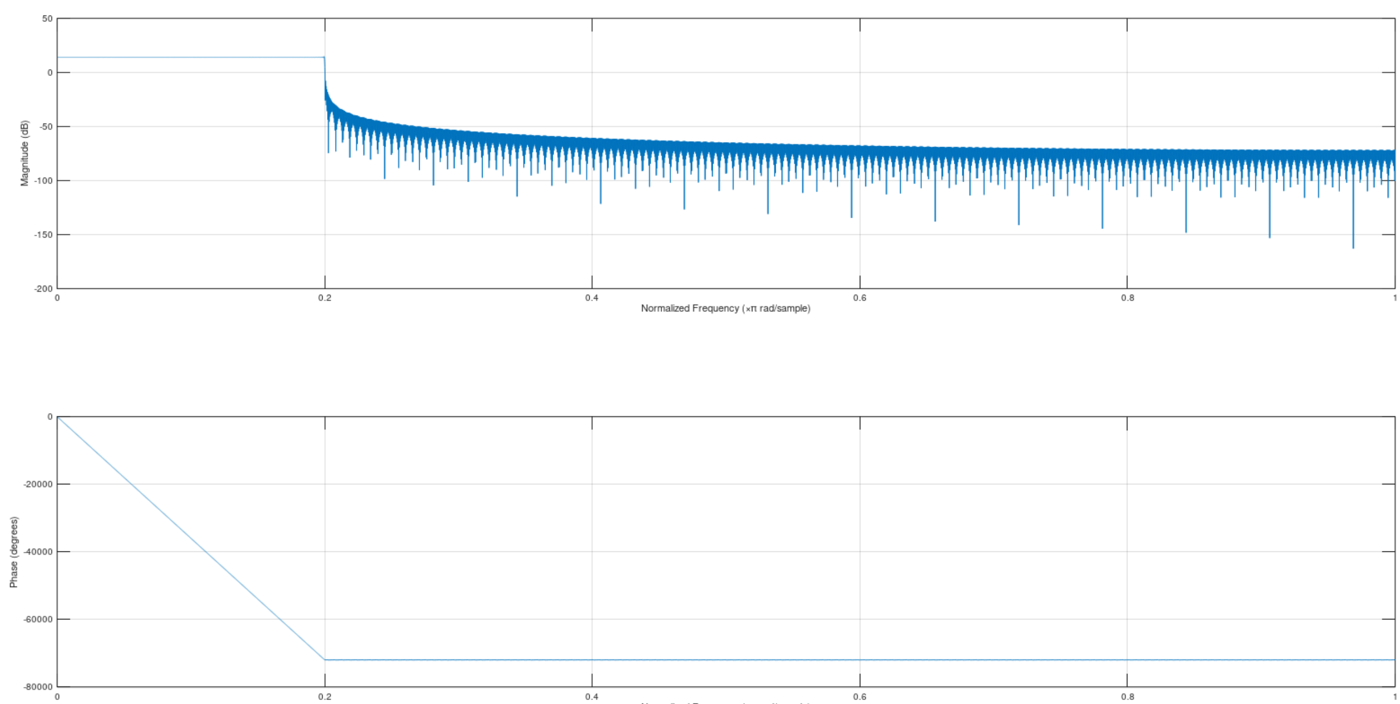
Plot:



c. Code:

```
Fs= 20;
t=-100:1/Fs:100;
myfilter=sinc(4*t);
freqz(myfilter);
```

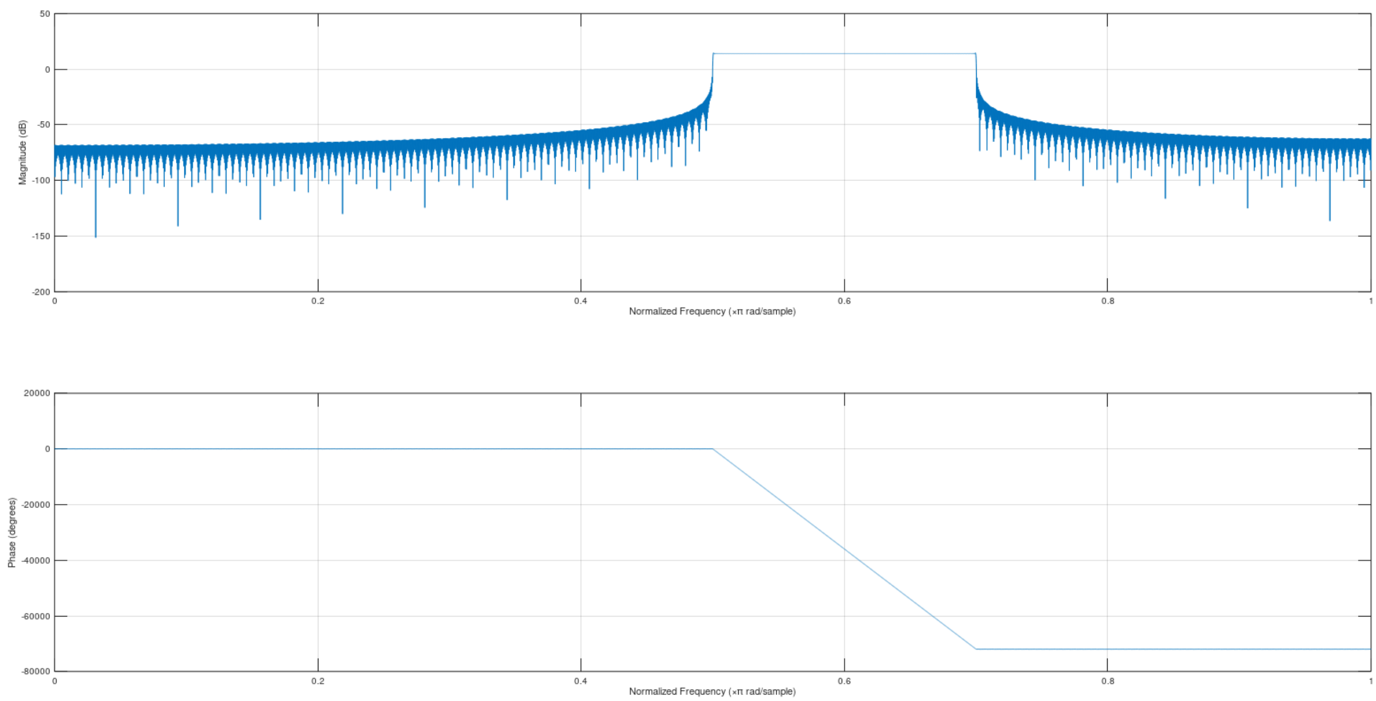
Plot:



d. Code:

```
Fs= 20;
t=-100:1/Fs:100;
myfilter=sinc(2*t);
myfilter2=myfilter.*cos(12*pi*t);
freqz(myfilter2);
```

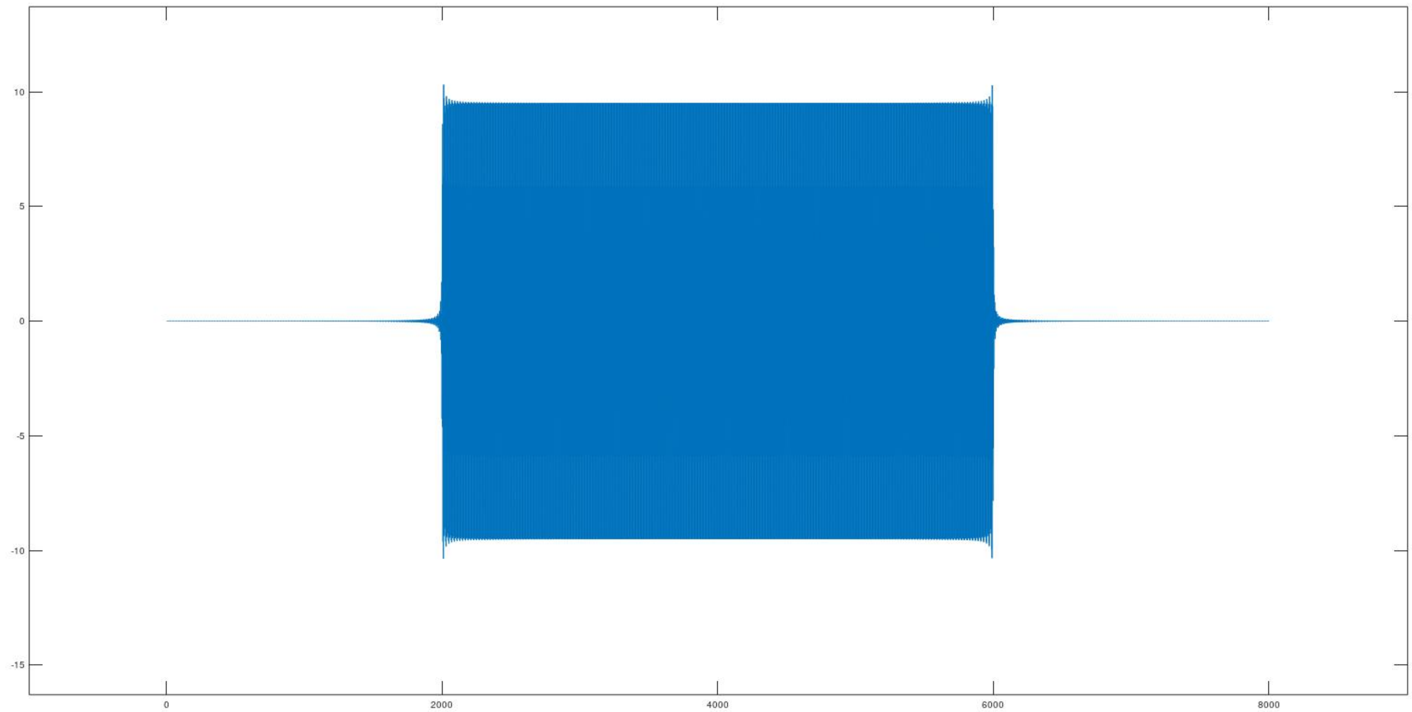
Plot:



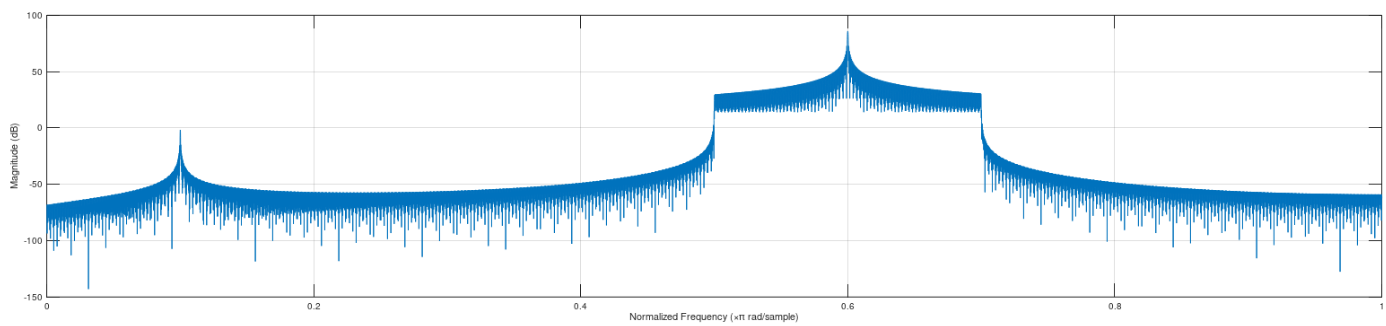
e. Code:

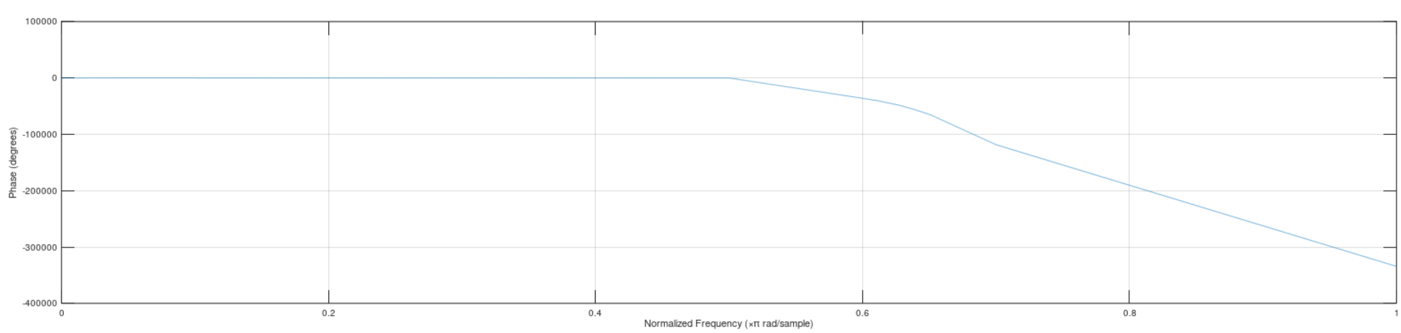
```
Fs= 20;
t=-100:1/Fs:100;
x=cos(2*pi*t)+2*sin(12*pi*t);
myfilter=sinc(2*t);
myfilter2=myfilter.*cos(12*pi*t);
plot(conv(x,myfilter2));
```

Plot:



Freqz:





We observe a high varying output between 5 Hz and 7 Hz, and on the plot we see a box like figure. This is because the filter filtered out all frequencies in the input signal x(t), except for the frequencies 5 to 7 Hz. This means that the sine wave is passed(the peak at 0.6) but the cos wave is blocked.