Third Problem Assignment

EE603 - DSP and its applications

Assigned on: August 24, 2018 Due on: August 31, 2018

Notes:

(1) Copying will be dealt with strictly. Institute disciplinary procedures will be invoked if any form of cheating is detected.

(2) The submissions must include the comments, plots AND the code in a SINGLE PDF. Without the code, the submission will not be evaluated. If you submit a zip file containing the code, plots etc., this PDF must STILL be included in the zip file.

(3) The computer assignments should be solved using GNU Octave or any other free/open source software kit approved by the instructor. Solutions that work only on Matlab will not be accepted.

Problem 1

(10 points)

Let $h[n]$ and $H(z)$ denote the impulse response and system function of stable all-pass LTI system. Let $h_i[n]$ denote the impulse response of the (stable) LTI inverse system. Assume that $h[n]$ is real. Show that $h_i[n] = h[-n]$.

Problem 2

(10 points) System S_1 has real impulse response $h_1[n]$ and real-valued frequency response $H_1(e^{j\omega})$.

(a) Does the impulse response $h_1[n]$ have any symmetry? Explain.

(b) System S_2 is a linear phase system with the same magnitude reponse as system S_1. What is the relationship between $h_2[n]$, the impulse response of the system S_2, and $h_1[n]$?

(c) Can a casual IIR filter have linear phase? Explain. If your answer is yes, provide an example. If not, prove that it is not possible for an IIR filter to have linear phase.

Problem 3

(10 points) An LTI system has generalized linear phase and system function $H(z) = a + bz^{-1} + cz^{-2}$. The impulse response has unit energy, $a \geqslant 0$, and $H(e^{j\pi}) = H(e^{j0}) = 0$.

(a) Determine the impulse response $h[n]$.

(b) Plot $|H(e^{j\omega})|$.

Problem 4

(10 points) A causal LTI has system has the system function

$$H(z) = \frac{(1 - 1.5z^{-1} - z^{-2})(1 + 0.9z^{-1})}{(1 - z^{-1})(1 + 0.7jz^{-1})(1 - 0.7jz^{-1})}$$

(a) Write down the difference equation that is satisfied by the input and output of the system.

(b) Plot the pole-zero diagram and indicate the ROC for the system function.

(c) Sketch $|H(e^{j\omega})|$.

(d) State whether the following are true or false about the system:

- (i) The system is stable.
- (ii) The impulse response approaches a constant for large n.
- (iii) The magnitude of the frequency response has a peak at approximately $\omega = \pm\pi$.
- (iv) The system has a stable and causal inverse.

Problem 5

(10 points) Consider a discrete-time LTI filter whose impulse response $h[n]$ is nonzero only over five consecutive time samples; the filter's frequency response is $H(e^{j\omega})$. Let signals $x[n]$ and $y[n]$ denote the filter's input and output, respectively.

Moreover, you are given the following information about the filter.

(a)

$$\int_{-\pi}^{\pi} H(e^{j\omega})d\omega = 4\pi.$$

(b) There exists a signal $a[n]$ that has a real and even DTFT $A(e^{j\omega})$ given by

$$A(e^{j\omega}) = H(e^{j\omega})e^{j2\omega}.$$

(c)

$$A(e^{j0}) = 8, \quad \text{and} \quad A(e^{j\pi}) = 12.$$

Completely specify the impulse response $h[n]$. That is, specify the value of $h[n]$ for all n for which it is non-zero. Plot $h[n]$ carefully, labelling its salient features.

Problem 6

(10 points) This problem concerns a discrete-time filter with a real-valued impulse response $h[n]$. Determine whether the following statement is true or false:

Statement: If the group delay of the filter is constant for $0 < \omega < \pi$, then the impulse response must have the property that either

$$h[n] = h[M - n]$$

or

$$h[n] = -h[M - n],$$

where M is an integer.
If the statement is true, show why it is true. If it is false, provide a counter example.

Problem 7

(30 points) For this problem, you will use the `rhino.wav` file provided with the homework. You may need to add the command `pkg load signal` on the top of your script for some commands to work.

(a) Load the wave file into Octave. Plot the samples. What is the sampling frequency? Play the sound using the `soundsc` command.

(b) Plot the frequency content of the waveform versus the frequency (in Hertz) for any one channel and comment on the frequency content.

(c) Add Gaussian noise of variance 0.01 to each channel. You can generate Gaussian noise with variance 0.01 by using `0.01 * randn(N, 2)`, where `N` is the sequence length.

(d) Use the `soundsc` function to play the noisy waveform. What do you observe?

(e) Now, let's try filtering the noisy signal with a Butterworth filter. You can create a Butterworth filter using the `butter` function. Create a sixth order Butterworth low-pass filter with cut-off corresponding to 1 kHz.

(f) Filter the noisy waveform with the Butterworth filter using the `filter` command. Play the sound with `soundsc`. What do you observe? Why?