First Problem Assignment

EE603 - DSP and its applications

Assigned on: January 5, 2017 Due on: January 12, 2017

Note: Copying will be dealt with strictly. Institute disciplinary procedures will be invoked if any form of cheating is detected.

Problem 1

(10 points) Consider a discrete-time linear time-invariant system with impulse response $h[n]$. If the input $x[n]$ is a periodic sequence with period N, i.e. $x[n+N] = x[n]$, show that the output $y[n]$ is also periodic with period N.

Problem 2

(10 points) Suppose that $h[n]$ is the impulse response of a discrete-time LTI system.

(a) What are the necessary and sufficient conditions on $h[n]$ for the system to be stable?

(b) Consider the system $h[n] = (-1)^n u[n]$, where $u[n]$ is the discrete-time step sequence. Prove that this system is stable, or, alternately, provide a counter-example to show that this system is not stable.

Problem 3

(10 points) Consider a system described by the LCCDE

$$y[n] = x[n] + ay[n-1]$$

where a is a real number.

(a) Suppose that the system has an initial condition $y[-1] = b$. What is the output for $x[n] = 0$?

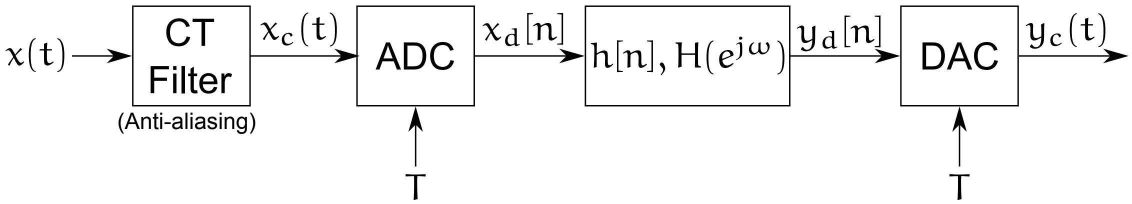
(b) What is the impulse response of this system?

(c) What is the solution for the input $x[n] = c\delta[n]$?

(d) What are the conditions on a, b and c for the system to be a stable LTI system?

Problem 4

(10 points) Consider the system shown in the figure below:



The frequency response of the discrete-time LTI system between the ADC and DAC is given by

$$H_d(e^{j\omega}) = e^{-j\omega/3}, \quad |\omega| < \pi/3$$

(a) What is the effective continuous-time frequency response $H(f)$ of the overall system?

(b) Which of the following is the most accurate statement?

(i) $y_c(t) = \frac{d}{dt} x_c(t)$.

(ii) $y_c(t) = x_c(t - \frac{T}{3})$.

(iii) $y_c(t) = \frac{d}{dt} x_c(t - 3T)$.

(iv) $y_c(t) = x_c(t - \frac{1}{3})$.

(c) Express $y_d[n]$ in terms of $y_c(t)$.

(d) Determine the impulse response $h[n]$ of the discrete-time LTI system.

Problem 5

(10 points) Consider the z-transform given by

$$H(z) = \frac{1}{1 - az^{-1}}$$

(a) Suppose that the region of convergence for this filter is $|z| > |a|$. What is the impulse response of this filter, denoted by $h_1[n]$? What is the condition for stability of this filter?

(b) Suppose that the region of convergence for another filter having the same z-transform is $|z| < |a|$. What is the impulse response of this filter, denoted by $h_2[n]$? What is the condition for stability of this filter?

(c) Comment about the stability of the filter $h_1[n] + h_2[n]$.

Problem 6

(10 points) **Computer assignment**: In this exercise, you will approximate a low-pass filter and perform some operations on it.

(a) Construct an approximate low-pass filter $h[n]$ with cut-off frequency 0.2π by approximating an ideal low-pass filter using 101 coefficients. On Octave, you can do it using: `n = -50:50; myfilter = sinc(0.2 * n);`

(b) What is the gain of this filter at dc in dB? Hint: you can use `freqz` to figure this out.

(c) How can you fix the gain for dc to be 0 dB? Provide the code for it here.

(d) Construct the filter $(-1)^n h[n]$. Provide the magnitude and phase response plot of this filter. What type of filter is this?

(e) Now, construct the filter $\cos(0.5\pi n)h[n]$. Provide the magnitude and phase response plot of this filter. What type of filter is this?

(f) Finally, construct the filter $\sin(0.1\pi n)h[n]/\pi n$. Provide the magnitude and phase response plot of this filter. What type of filter is this?

(g) Explain why you can expect the above filters after transformations.