Second Problem Assignment

EE603 - DSP and its applications

Assigned on: August 17, 2018 Due on: August 24, 2018

Notes:

(1) Copying will be dealt with strictly. Institute disciplinary procedures will be invoked if any form of cheating is detected.

(2) The submissions must include the comments, plots AND the code in a SINGLE PDF. Without the code, the submission will not be evaluated. If you submit a zip file containing the code, plots etc., this PDF must STILL be included in the zip file.

(3) The computer assignments should be solved using GNU Octave or any other free/open source software kit approved by the instructor. Solutions that work only on Matlab will not be accepted.

Problem 1

(10 points) The DTFT pair

$$a^n u[n] \Leftrightarrow \frac{1}{1 - ae^{-j\omega}}, \quad |a| < 1$$

is given.

(a) Using the above equation, determine the DTFT, $X(e^{j\omega})$, of the sequence

$$x[n] = -b^n u[-n-1] = \begin{cases} -b^n, & n \leqslant -1 \\ 0, & \geqslant 0. \end{cases}$$

What restriction on b is necessary for the DTFT of $x[n]$ to exist?

(b) Determine the sequence $y[n]$ whose DTFT is

$$Y(e^{j\omega}) = \frac{2e^{-j\omega}}{1 + 2e^{-j\omega}}.$$

Problem 2

(10 points) Consider the LTI system with frequency response

$$H(e^{j\omega}) = e^{-j[(\omega/2)+(\pi/4)]}, \qquad -\pi \leqslant \omega < \pi$$

Determine $y[n]$, the output of the system, if the input to the system is

$$x[n] = \cos\left(\frac{15\pi n}{4} - \frac{\pi}{3}\right)$$

Problem 3

(10 points) This is much like your previous homework problem. Consider an ideal low-pass filter with impulse response $h_{lp}[n]$ and frequency response

$$H_{lp}(e^{j\omega}) = \begin{cases} 1, & |\omega| < 0.2\pi \\ 0, & 0.2\pi \leqslant |\omega| \leqslant \pi \end{cases}$$

(a) A new filter is designed by the equation $h_1[n] = (-1)^n h_{lp}[n] = e^{j\pi n} h_{lp}[n]$. Determine an equation for the frequency response of $H_1(e^{j\omega})$, and plot the equation for $|\omega| < \pi$. What kind of filter is this?

(b) A new filter is designed by the equation $h_2[n] = 2h_{lp}[n]\cos(0.5\pi n)$. Determine an equation for the frequency response of $H_2(e^{j\omega})$, and plot the equation for $|\omega| < \pi$. What kind of filter is this?

(c) A new filter is designed by the equation

$$h_3[n] = \frac{\sin(0.1\pi n)}{\pi n} h_{lp}[n]$$

Determine an equation for the frequency response of $H_3(e^{j\omega})$, and plot the equation for $|\omega| < \pi$. What kind of filter is this?

Problem 4

(10 points) Consider the following system for which the input $x[n]$ and output $y[n]$ satisfy the difference equation

$$y[n] - \frac{1}{2}y[n-1] = x[n]$$

and for which $y[-1]$ is constrained to be zero for every point. Determine whether or not the system is stable. If you conclude that system is stable, show your reasoning. If you conclude that the system is not stable, give an example of a bounded input that results in an unbounded output.

Problem 5

(10 points) A linear time-invariant system has frequency response

$$H(e^{j\omega}) = \begin{cases} e^{-j\omega 3}, |\omega| < \frac{2\pi}{16}\left(\frac{3}{2}\right) \\ 0, \frac{2\pi}{16}\left(\frac{3}{2}\right) \leqslant |\omega| \leqslant \pi \end{cases}$$

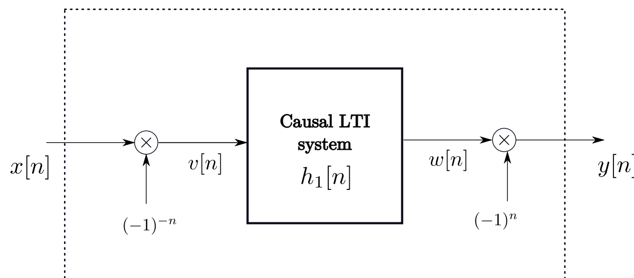
The input to the system is a periodic unit-impulse train with period $N = 16$ i.e

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n + 16k]$$

Find the output of the system.

Problem 6

(10 points) The overall system in the dotted box in the figure below can be shown to be linear and time invariant.



(a) Determine an expression for $H(e^{j\omega})$, the frequency response of the overall system from the input $x[n]$ to the output $y[n]$, in terms of $H_1(e^{j\omega})$, the frequency response of the internal LTI system. Remember that $(-1)^n = e^{j\omega n}$.

(b) Plot $H(e^{j\omega})$ for the case when the frequency response of the internal LTI system is

$$H_1(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & \omega_c \leqslant |\omega| \leqslant \pi \end{cases}$$

Problem 7

(10 points) A commonly used numerical operation called the first backward difference is defined as

$$y[n] = \nabla(x[n]) = x[n] - x[n-1]$$

where $x[n]$ is the input and $y[n]$ is the output of the first-backward-difference system.

(a) Show that this system is linear and time invariant.
(b) Find the impulse response of the system.
(c) Find and sketch the frequency response (magnitude and phase)
(d) Show that if

$$x[n] = f[n] * g[n]$$

$$\nabla(x[n]) = \nabla(f[n]) * g[n] = f[n] * \nabla(g[n])$$

where $*$ denotes discrete convolution.

(e) Find the impulse response of a system that could be cascaded with the first-difference system to recover the input; i.e find $h_1[n]$ where

$$h_1[n] * \nabla(x[n]) = x[n]$$

Problem 8

(30 points) **Use Octave!**
Consider the following continuous-time signal:

$$x(t) = \cos(2\pi t) + 2\sin(12\pi t)$$

The above signal is sampled with $f_s = 20$ Hz. You will work with this signal for values `n = -100:100` for all the questions below.

(a) Plot $x[n]$.
(b) Plot the frequency and phase response of $x[n]$ (just use `freqz`).
(c) Create a discrete-time low-pass filter with unity gain and cut-off frequency corresponding to 2 Hz by approximating an ideal low-pass filter, just like in HW1. Plot the frequency and phase response.
(d) Create a discrete-time band-pass filter with centre pass-band between 5 Hz and 7 Hz by modulating the filter from the previous section. (Hint: multiply by $\cos \omega_0 n$ for appropriate ω_0. Plot the frequency response using `freqz`).
(e) Filter the $x[n]$ with the band-pass filter and plot the response vs. n, as well as the `freqz` output. What do you observe? Why?
(f) Finally, use the `interp` function to interpolate the resulting sequence and check the frequency of the output signal. Use any appropriate factor as a parameter to `interp`. Does it match your intuition?