Fourth Problem Assignment

EE603 - DSP and its applications

Assigned on: September 26, 2018 Due on: October 5, 2018

Notes:

(1) Copying will be dealt with strictly. Institute disciplinary procedures will be invoked if any form of cheating is detected.

(2) The submissions must include the comments, plots AND the code in a SINGLE PDF. Without the code, the submission will not be evaluated. If you submit a zip file containing the code, plots etc., this PDF must STILL be included in the zip file.

(3) The computer assignments should be solved using GNU Octave or any other free/open source software kit approved by the instructor. Solutions that work only on Matlab will not be accepted.

Problem 1

(5 points) The commonly used windows can all be expressed in terms of rectangular windows. This fact can be used to obtain expression for the Fourier transforms of the Bartlett window and the raised cosine family of windows, which includes the Henning, Hamming and Blackman windows.

(a) Show that the $(M+1)$ -point Barlett window can be expressed as the convolution of two smaller rectangular windows. Use this fact to show that the Fourier transform of the $(M+1)$-point Barlett window is

$$W_B(e^{j\omega}) = e^{-j\omega M/2}(2/M)\left(\frac{\sin(\omega M/4)}{\sin(\omega/2)}\right)^2 \quad \text{for } M \text{ even},$$

or

$$W_B(e^{j\omega}) = e^{-j\omega M/2}(2/M)\left(\frac{\sin(\omega(M+1)/4)}{\sin(\omega/2)}\right)\left(\frac{\sin(\omega(M-1)/4)}{\sin(\omega/2)}\right) \quad \text{for } M \text{ odd},$$

(b) It can be seen that the $(M+1)$ -point raised-cosine windows can all be expressed in the form

$$w[n] = [A + B\cos(2\pi n/M) + C\cos(4\pi n/M)]w_R[n]$$

where $w_R[n]$ is an $(M+1)$ -point rectangular window. Use this relation to find the Fourier transform of the general raised-cosine window.

(c) Using appropriate choices for A, B, and C and the result determined in part (b), sketch the magnitude of the Fourier transform of the Hamming window.

Problem 2

(5 points)

Let $h_d[n]$ denote the impulse response of ideal desired system with corresponding frequency response $H_d(e^{j\omega})$ and $h[n]$ and$H(e^{j\omega})$ denote the impulse response and frequency response, respectively, of an FIR approximation to the ideal system. Assume that $h[n] = 0$ for $n < 0$ and $n > M$. We wish to choose the $(M + 1)$ samples of the impulse response so as to minimum the mean-square error of the frequency response defined as

$$\epsilon^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} | H_d(e^{j\omega}) - H(e^{j\omega}) |^2 \, d\omega$$

(a) Use Parseval's relation to express the function in terms of the sequences $h_d[n]$ and $h[n]$.

(b) Using the result of part (a), determine the values of $h[n]$ for $0 \leqslant n \leqslant M$ that minimize ϵ^2.

(c) The FIR filter determined in part (b) could have been odtained by a windowing operation. That is, $h[n]$ could have been obtained by multiplying the desired infinite-length sequence $h_d[n]$ by a certain finite-length sequence $\omega[n]$. Determine the necessary window $\omega[n]$ such that the optimal impulse response is $h[n] = \omega[n]h_d[n]$.

Problem 3

(5 points) Consider the following ideal frequency response for a multiband filter

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega M/2}, & 0 \leqslant |\omega| \leqslant 0.3\pi \\ 0, & 0.3\pi \leqslant |\omega| \leqslant 0.6\pi \\ 0.5e^{-j\omega M/2}, & 0.6\pi \leqslant |\omega| \leqslant \pi \end{cases}$$

The impulse response $h_d[n]$ is multiplied by a Kaiser window with $M = 48$ and $\beta = 3.68$ resulting in a linear-phase FIR system with impluse response $h[n]$.

(a) What is the delay of the filter?

(b) Determine the ideal desired impulse response $h_d[n]$.

(c) Determine the set of approximation error specifications that is satisfied by the FIR filter; i.e., determine the parameters $\delta_1, \delta_2, \delta_3, B, C, \omega_{p1}, \omega_{s1}, \omega_{s2}$ and ω_{p2} in

$$B - \delta_1 \leqslant |H(e^{j\omega})| \leqslant B + \delta_1, 0 \leqslant \omega \leqslant \omega_{p1}$$
$$|H(e^{j\omega})| \leqslant \delta_2, \omega_{s1} \leqslant \omega \leqslant \omega_{s2}$$
$$C - \delta_3 \leqslant |H(e^{j\omega})| \leqslant C + \delta_3, \omega_{p2} \leqslant \omega \leqslant \pi.$$

Problem 4

(5 points)

(a) Suppose that we are given an ideal low pass discrete-time filter with frequency response

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \pi/4 \\ 0, & \pi/4 < |\omega| \leqslant \pi \end{cases}$$

We wish to derive new filters from this prototype by manipulations of the impulse response $h[n]$.

(i) Plot the frequency response $H_1(e^{j\omega})$ for th system whose impulse response is $h_1[n] = h[2n]$.

(ii) Plot the frequency response $H_2(e^{j\omega})$ for th system whose impulse response is

$$h_2[n] = \begin{cases} h[n/2], & n = 0, \pm 2, \pm 4, ... \\ 0, & \text{otherwise} \end{cases}$$

(iii) Plot the frequency response $H_3(e^{j\omega})$ for the system whose impulse response is $h_3[n] = e^{j\pi n}h[n] = (-1)^n h[n]$.

Problem 5

(20 points) You have to design a filter with these specifications:

(1) Passband cutoff frequency: $\omega_p = 0.65\pi$

(2) Stopband cutoff frequency: $\omega_s = 0.7\pi$

(3) Maximum passband ripple: $1 - \delta_1 \geqslant -1\text{dB}$

(4) Minimum stopband attenuation: $\delta_2 \leqslant -75\text{dB}$

(a) Find the order of the filter (i.e. the number of delays) needed to realize these specifications using the following filter design approaches. (You do **not** need to develop the whole filter design.)

- (i) FIR design using a carefully-selected Kaiser window.
- (ii) FIR design using the Parks-McClellan algorithm. Use the remezord function attached.
- (iii) IIR design using the bilinear transform in conjunction with an elliptical prototype filter. (You can do this using the appropriate frequency warping functions and the Octave routine ellipord. See also the help file for ellip.

(b) Design each of the above filters. For each design, specify the numerator and denominator coefficients of the z-transform and provide a neat plot of the magnitude and phase responses. The x-ticks on the axes must be $0, \pi/4, \pi/2, 3\pi/4, \pi$.

(c) We would now like to compare the total computational efficiency of "efficient" implementations of each of the designs. Assume that you have realized the filters using the following implementations:

- (1) The FIR design using windowing techniques is realized using the "linear-phase" implementation (with half the number of delays and multiplications compared to the direct form).
- (2) The FIR design using the Parks McClellan algorithm is also realized using the standard linear-phase implementation.
- (3) The IIR design using the bilinear transform and an elliptical prototype filter is realized using the LCCDE implementation.

Using each of the above implementations, what is the total number of multiplications required when the filter is used to process an arbitrarily long input sequence directly in the time domain? Express your answers in terms of multiplications per sample of input.

Problem 6

(10 points) In this problem, you will listen to different frequency components of a music file to see what components of the music you are able to discern.

(1) First, fetch Johann Bach's music piece "Brandenburg Concerto No. 3 – 1. Allegro" from here: `https://en.wikipedia.org/wiki/File:Bach_-_Brandenburg_Concerto_No._3_-_1._Allegro.ogg`

(2) Convert the first 20 seconds to a WAV file.

(3) Use `audioread` in Octave to load the file.

Now, do the following:

(a) Develop four linear phase filters using `remez`, each of length 200. The first should be a low-pass filter with cut-off corresponding to 4 kHz. The second should be a band-pass filter with pass-band between 4 to 8 kHz. The third is a band-pass filter with pass-band from 8 to 12 kHz. The final filter is a band-pass filter with pass-band from 12 kHz to 16 kHz.

(b) Filter both the left and right channels with each of the above filters. Listen to the output. What do you observe in each case?

(c) Repeat with Butterworth filters for each of the above cases. Do you observe any difference from the previous case?