→ Generalized Likelihood ratio test

$H_0 : \quad \theta \in \Theta_0$

$H_A : \quad \theta \notin \Theta_0$

$X_1 \ldots X_n$ r.s.

the likelihood ratio is defined by

$$\Lambda = \frac{\sup_{\theta \in \Theta_0} L(\theta)}{\sup_{\theta \in \Theta_0} L(\theta)}$$

For $n \to \infty$, Under H_0

$$-2 \log \Lambda \sim \chi^2_{\dim \Theta - \dim \Theta_0}$$

→ $N_p(\mu, \Sigma)$

$$\begin{array}{ccc} \mu & , & \Sigma \\ \downarrow & & \downarrow \\ p & & \frac{p(p+1)}{2} \end{array}$$

→ Central Limit Theorem

→ Theory of Multivariate Statistics (6.3)

Asymptotic distributions

The central limit theorem states that for any sample

$x_1 \ldots x_n$

→ Lemma 6.3 (Multivariate Slutsky)

$$X_n \xrightarrow{d} c \quad \Rightarrow \quad X_n \xrightarrow{p} c$$

Eg: $\begin{pmatrix} X_n \\ Y_n \end{pmatrix} \sim N_2\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix} \right)$

$$X_n \xrightarrow{d} N(0,1)$$

$$Y_n \xrightarrow{d} N(0,1)$$

$\Rightarrow \begin{pmatrix} X_n \\ Y_n \end{pmatrix} \xrightarrow{d} N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right)$ (contradiction)

→ Prop^n 6.2 (Delta method)