



Contents

- Project plan
- Key requirements
- Trade-off method
- Subsystem Trade-offs
- Orbit design
- Software tool



1.

Project plan

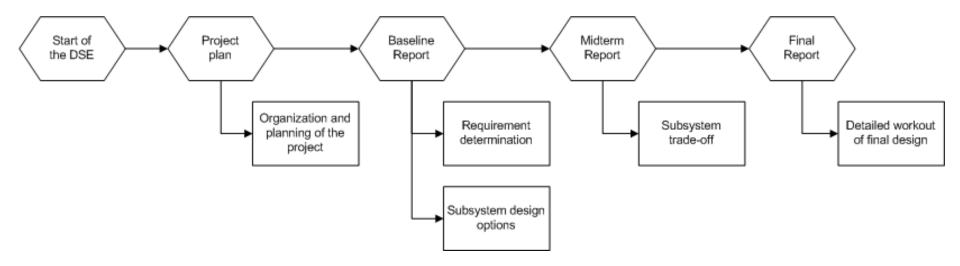


Mission need statement

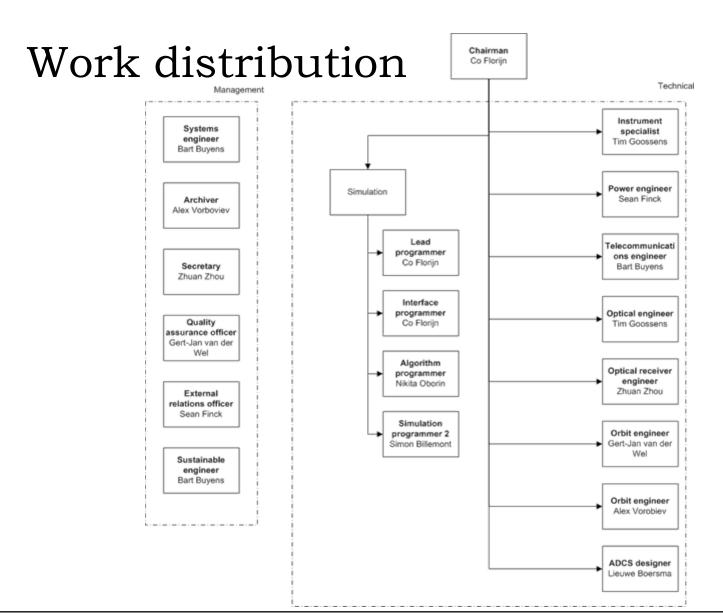
Demonstrate that a satellite constellation, consisting of a single emitter and several receivers, will perform superior (in terms of cost, lifetime and performance) to existing spaceborne laser altimetry systems.



Project Organization









2.

Key requirements



Key Requirements

Low cost

• Lifetime of ~ 5 yrs

Performance equivalent to ICESat



Additional Requirements

- Mass ≤ existing spaceborne laser altimetry systems
- No scanner may be used
- Recreation of the DEM
- Extraction of the BRDF



3.

Tradeoff method



Trade-off Method

- A set of criteria are defined
- Each criterium is assigned weight w.r.t. importance
- Varies for each subsystem
- Each subsystem is graded
- Highest score wins

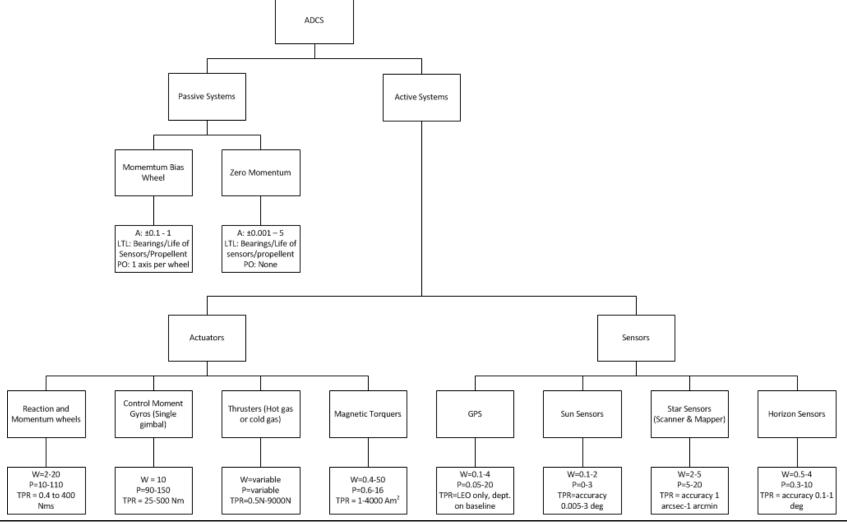


4.

Subsystem trade off



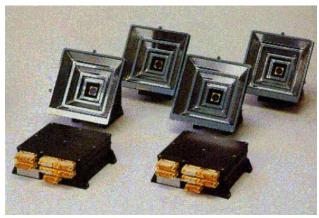
Pruned Design Option Tree ADCS

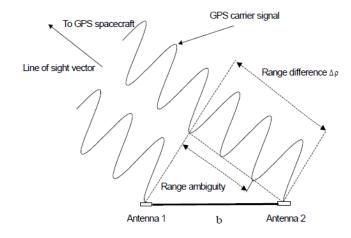




Selected ADS concepts







- 1. Maryland Aerospace Inc. IMI-100 ADACS
- 2. Sun Sensors and a Star Tracker
- GPS based attitude control

Sources: http://www.cubesatkit.com/docs/datasheet/DS CSK ADACS 634-00412-A.pdf Dr. Q.P. Chu. Spacecraft attitude dynamics and control, course notes

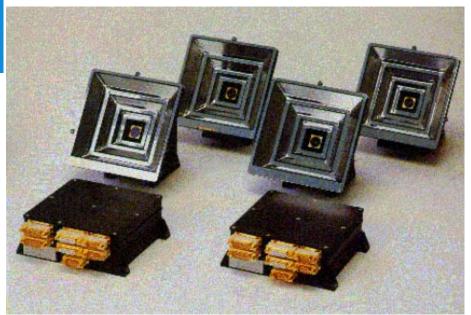


Trade-off ADS

Criteria	Weight Factor	Concept 1	Concept 2	Concept 3
Accuracy	9	4	8	4
Size	7	2	6	4
Power	7	6	5	7
Price	3	3	5	4
Development	5	8	4	5
Weighed total		141	184	150



Winner ADS

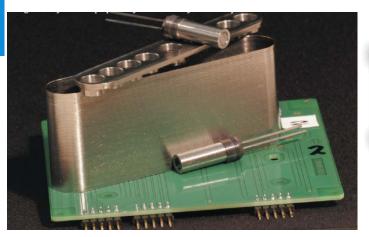


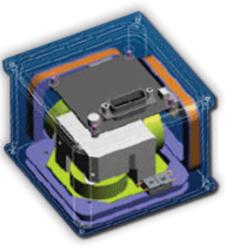


Sources: Dr. Q.P. Chu. Spacecraft attitude dynamics and control, course notes



Selected ACS concepts







- Thrusters
- 2. Reaction wheels and magnetic torquers
- 3. Maryland Aerospace Inc. IMI-100 ADACS

Sources:

http://www.tno.nl

http://www.cubesatshop.com

http://www.cubesatkit.com/docs/datasheet/DS_CSK_ADACS_634-00412-A.pdf

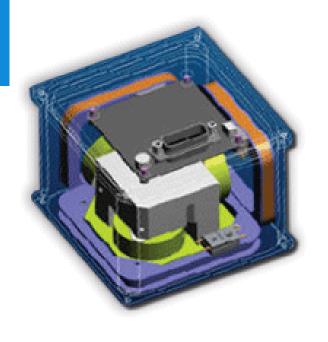


Trade-off ADS

Criteria	Weight Factor	Concept 1	Concept 2	Concept 3
Rate	5	8	6	6
Accuracy	8	4	8	7
Size	7	2	6	5
Power	7	3	6	6
Price	3	2	8	7
Development	5	4	6	8
Weighted total		133	232	224



Winner ADS

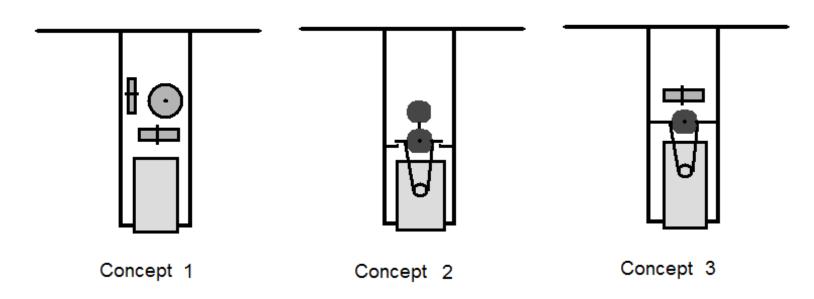




Sources: Dr. Q.P. Chu. Spacecraft attitude dynamics and control, course notes



Selected Pointing Mechanism Concepts



- Using the ADCS
- 2. Using two stepper motors
- 3. Using one axis reaction wheel and one stepper motor

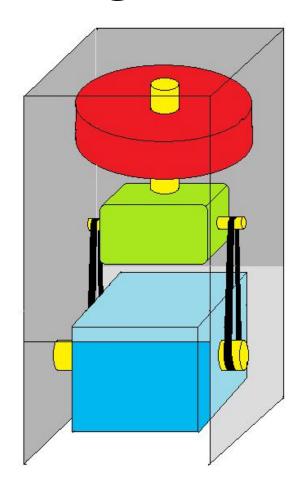


Trade-off pointing mechanism

Criteria	Weight Factor	Concept 1	Concept 2	Concept 3
Pointing accuracy	10	2	8	6
Pointing rate	10	2	8	6
Added weight	4	8	2	5
Power	4	7	2	4
Influence	6	2	3	7
Complexity	6	8	2	6
Weighted total		208	221	228



Winner Pointing Mechanism





Aspects considered

- Communications architecture
- Frequency bands
 - Ground-space link
 - Intersatellite link
- Antenna configuration
- Tracking



Aspects considered

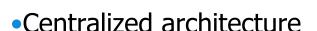
- Communications architecture
- Frequency bands
 - Ground-space link
 - Intersatellite link
- Antenna configuration
- Tracking



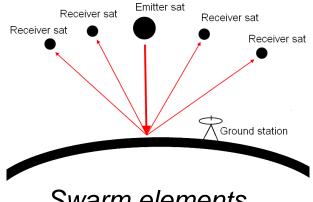
Communications architecture

•Swarm elements:

- Emitter satellite (1)
- Receiver satellites (multiple)
- Ground station



- 1 ground-space link for emitter sat.
- Intersatellite links between receiver sats & emitter sat
- Decentralized architecture
 - •Ground-space link for emitter sat & each receiver sat
 - Intersatellite links between receiver sats & emitter sat
- Extremely decentralized architecture
 - •Ground-space link for emitter sat & each receiver sat
 - No intersatellite links



Swarm elements



Communications architecture

- •Swarm elements:
 - •Emitter satellite (1)
 - Receiver satellites (multiple)
 - Ground station

Receiver sat Receiver sat Intersatellite links Ground-space link Ground station

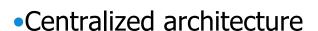
Centralized architecture

- 1 ground-space link for emitter sat.
- Intersatellite links between receiver sats & emitter sat
- Decentralized architecture
 - •Ground-space link for emitter sat & each receiver sat
 - Intersatellite links between receiver sats & emitter sat
- Extremely decentralized architecture
 - •Ground-space link for emitter sat & each receiver sat
 - No intersatellite links



Communications architecture

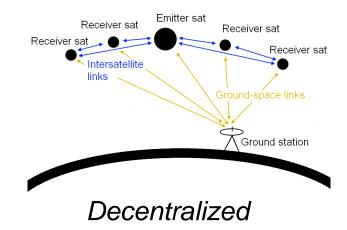
- •Swarm elements:
 - •Emitter satellite (1)
 - Receiver satellites (multiple)
 - Ground station



- 1 ground-space link for emitter sat.
- Intersatellite links between receiver sats & emitter sat

Decentralized architecture

- •Ground-space link for emitter sat & each receiver sat
- Intersatellite links between receiver sats & emitter sat
- Extremely decentralized architecture
 - •Ground-space link for emitter sat & each receiver sat
 - No intersatellite links





Communications architecture

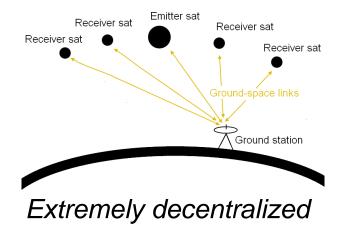
- •Swarm elements:
 - •Emitter satellite (1)
 - Receiver satellites (multiple)
 - Ground station



- 1 ground-space link for emitter sat.
- Intersatellite links between receiver sats & emitter sat
- Decentralized architecture
 - •Ground-space link for emitter sat & each receiver sat
 - Intersatellite links between receiver sats & emitter sat

Extremely decentralized architecture

- •Ground-space link for emitter sat & each receiver sat
- No intersatellite links

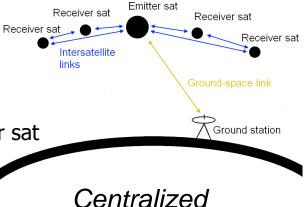




Communications architecture

Centralized architecture:

- Advantages
 - •Low mass, power consumption & volume receiver sat
 - Scientific data compressed before transmitting to the ground station
- Disadvantages
 - Less robust
 - High mass, power consumption & volume emitter sat
 - High data rate ground-space link

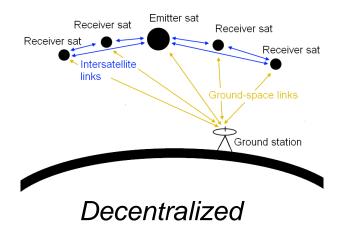




Communications architecture

Decentralized architecture:

- Advantages
 - Low data rate ground space link
 - More robust
- Disadvantages
 - Higher mass, power consumption & volume receiver sat

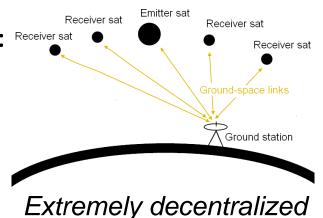




Communications architecture

Extremely decentralized architecture: Receiver sat

- Advantages
 - Low data rate ground space link
 - No frequency allocation required for intersatellite links
- Disadvantages
 - Higher mass, power consumption & volume receiver sat

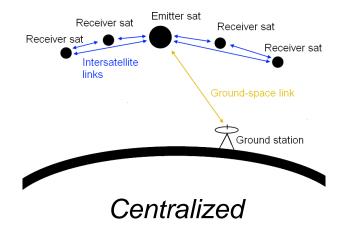


Desynchronization

Communications architecture

Winning architecture:

- Centralized architecture
 - No danger for synchronization
 - Lower total mass
 - Maximum use of allocated frequency





Aspects considered

- Communications architecture
- Frequency bands
 - Ground-space link
 - Intersatellite link
- Antenna configuration
- Tracking



Frequency allocation

Ground space link:

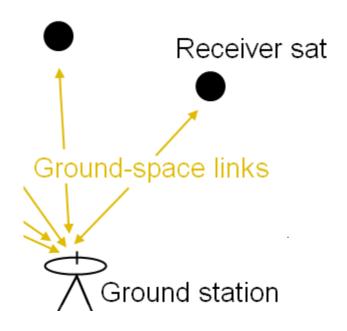
- Possible frequency bands
 - C-band
 - S-band
 - X-band
 - Ku-band
 - •Ka-band
 - •SHF/EHF-band



Frequency allocation

Ground space link:

- Possible frequency bands
 - C-band
 - S-band
 - X-band
 - High data rate possible
 - Most common for large Earth observation sats
 - Ku-band
 - Ka-band
 - •SHF/EHF-band

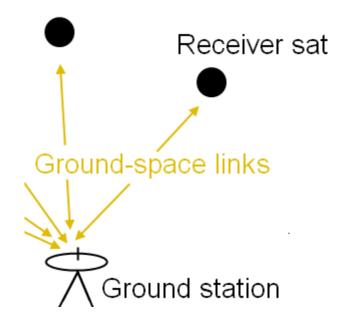




Frequency allocation

Ground space link:

- Possible frequency bands
 - C-band
 - S-band
 - Low data rate
 - Good for house keeping data
 - X-band
 - Ku-band
 - Ka-band
 - •SHF/EHF-band

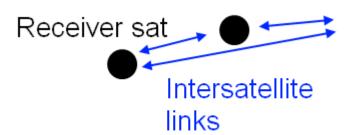




Frequency allocation

Intersatellite link:

- Possible frequency bands
 - C-band
 - S-band
 - X-band
 - Ku-band
 - Ka-band
 - •SHF/EHF-band

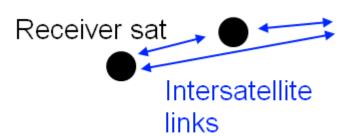




Frequency allocation

Intersatellite link:

- Possible frequency bands
 - C-band
 - S-band
 - X-band
 - Ku-band
 - •Lots of existing systems for reference during design
 - Ka-band
 - SHF/EHF-band
 - V-band





Aspects considered

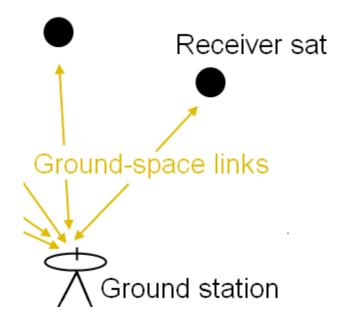
- Communications architecture
- Frequency bands
 - Ground-space link
 - Intersatellite link
- Antenna configuration
- Tracking



Antenna configuration

Ground space link:

- Possible high gain antennas
 - Parabolic reflector
 - High volume
 - Low mass
 - Phased array
 - Low volume
 - High mass

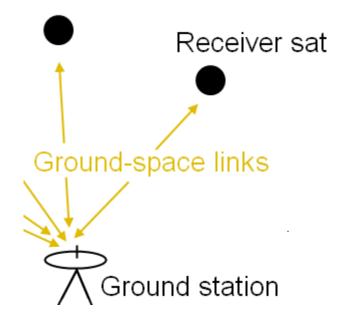




Antenna configuration

Ground space link:

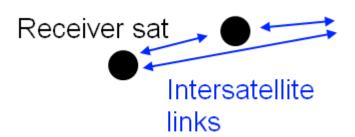
- Possible high gain antennas
 - Parabolic reflector
 - High volume
 - Low mass
 - Phased array
 - Low volume
 - •High mass





Antenna configuration

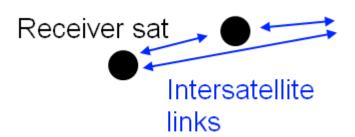
- Intersatellite links
 - Horn antenna
 - Low gain
 - •>4 Ghz
 - Helix antenna
 - Low gain
 - •<2 Ghz





Antenna configuration

- Intersatellite links
 - Horn antenna
 - Low gain
 - •>4 Ghz
 - Helix antenna
 - Low gain
 - •<2 Ghz





Aspects considered

- Communications architecture
- Frequency bands
 - Ground-space link
 - Intersatellite link
- Antenna configuration
- Tracking



Tracking method

- GPS
 - High precision
 - Provides time signal
- TDRS
 - High accuracy
 - Requires TDRS tracking antenna
- Satellite crosslinks
 - Reuses communication hardware
 - Only gives relative position
- Ground tracking
 - Well established
 - Operations intensive

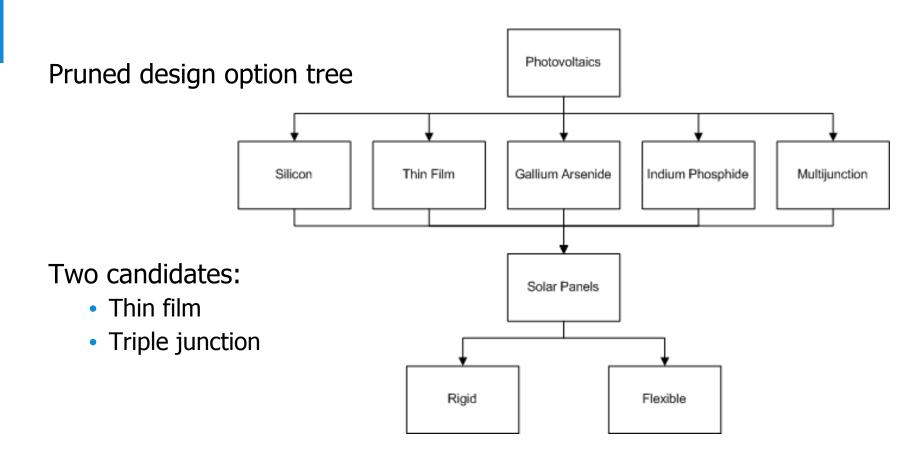


Tracking method

- GPS
 - High precision
 - Provides time signal
- TDRS
 - High accuracy
 - Requires TDRS tracking antenna
- Satellite crosslinks
 - Reuses communication hardware
 - Only gives relative position
- Ground tracking
 - Well established
 - Operations intensive



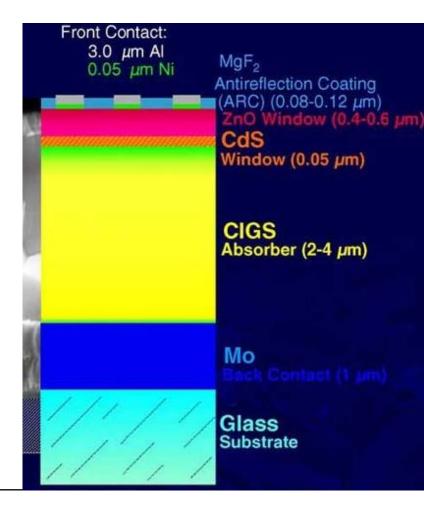
Electrical Power System





EPS – Thin Film CIGS

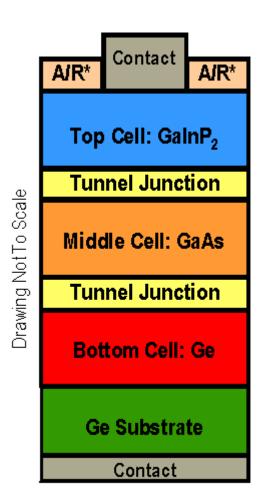
- Multiple layers of thin photovoltaic material
- Copper-Indium-Gallium-Selenium absorber
- Low efficiency
- Low production cost
- High absorptance coefficient





EPS – Triple Junction

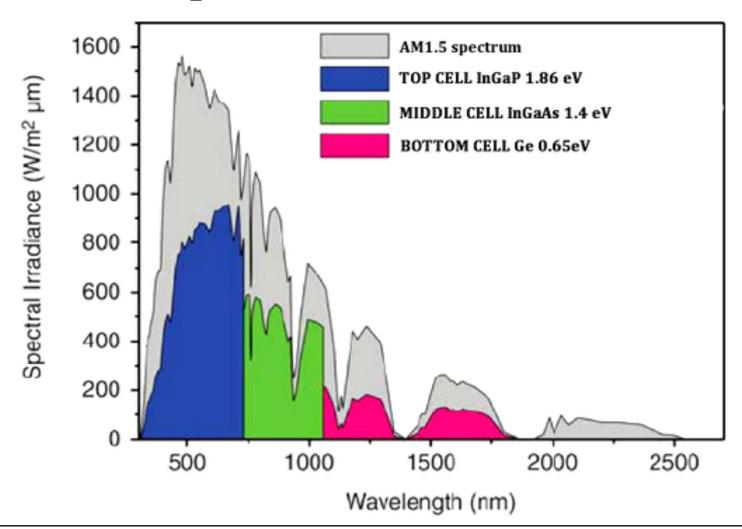
- Multiple pn-junctions
- High efficiency
- High production cost
- Larger covering of the solar spectrum



*A/R: Anti-Reflective Coating



EPS – Triple Junction





EPS – Trade-off

	Weight factors	Candidates		
		Thin sheet (CIGS)	Triple-junction	
Efficiency	10	4	10	
Mass	10	10	3	
Cost	10	10	4	
Degradation	8	10	9	
Packing factor	7	8	8,5	
Resistance to vibrations	5	8	6	
Height	7	10	2	
Total	570	486	345,5	



Optical Receiving Payload

- Single-Photon Detection
 - Photonmultiplier tube
 - Single Photon Avalanche Diode (SPAD)
- Wavelength Estimation
 - Atmospheric transmittance
 - Wavelength ratio



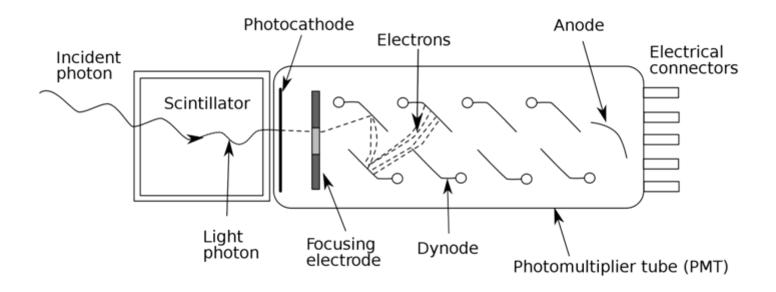
Optical Receiving Payload

Single-Photon Detection

- Convert light (photons) to measurable quantity (Voltage or current)
- Multiple ways
 - Photomultiplier tube
 - SPAD
 - Quantum dot (underdeveloped)



Photomultiplier tube



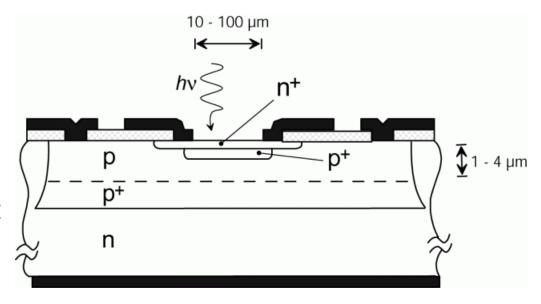
Typically 1000 to 2000 V is used



SPAD

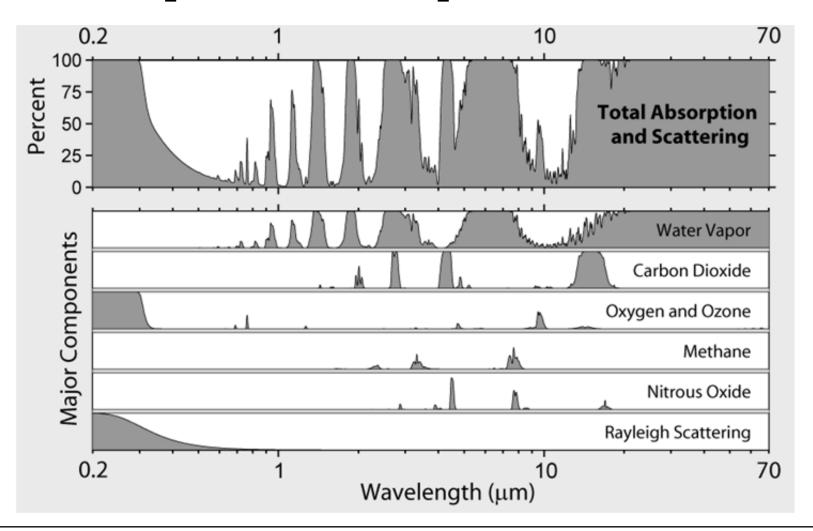
Single Photon Avalanche Diode

- Based on p-n junction
- Reversed biased voltage
- Sensing avalanche current
- Small size, less power





Atmospheric Absorption Bands





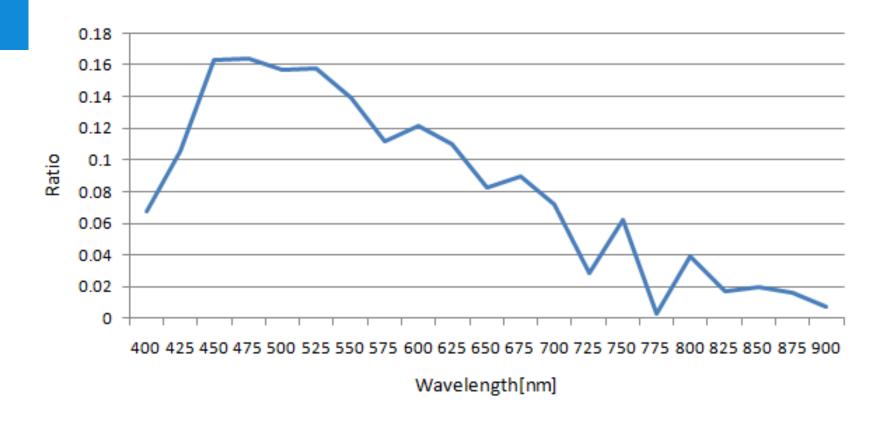
Wavelength estimation

- General sufficient wavelength range 400nm to 900nm
- Atmospheric transmittance Vs. Photon detection efficiency
- Wavelength ratio
 P = transmittance \(\Delta \rightarrow \(\Delta \rightarrow \rightarrow \Delta \rightarrow \Delta \rightarrow \(\Delta \rightarrow \Delta \rightarrow \Delta \rightarrow \Delta \rightarrow \Delta \rightarrow \(\Delta \rightarrow \Del

R = transmittance^2*efficiency



Wavelength estimation





Laser Optics

- To get the desired footprint.
- Three options:
 - No optics
 - Two lenses
 - Mirrors



Laser Optics – No Optics

Advantages:

- Really simple
- No optics to get out of focus
- Dirt-cheap

- Footprint directly depends on:
 - Laser beam divergence
 - Orbit altitude
- These two dependencies severely limit design options
- Characteristics might not be optimal





Laser Optics – Two lenses

Advantages:

Technology is well-understood

- Very heavy (even with Fresnel lenses)
- Focal length of > 4 m, so:
- Need mirrors to add light path length
- Still limits the footprint a lot
- Limits the orbit altitude a little



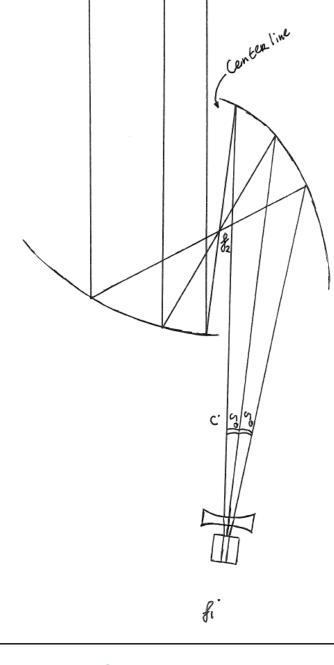


Laser Optics – Mirrors

Advantages:

- Much lighter than lenses
 - Herschel: <4 mm thick mirror
- Any footprint, any orbit altitude
- Potentially tunable in flight
- Small (~20 cm)
- Lense optional for some lasers

- Most complicated system
- Assembly must remain rigidly fixed



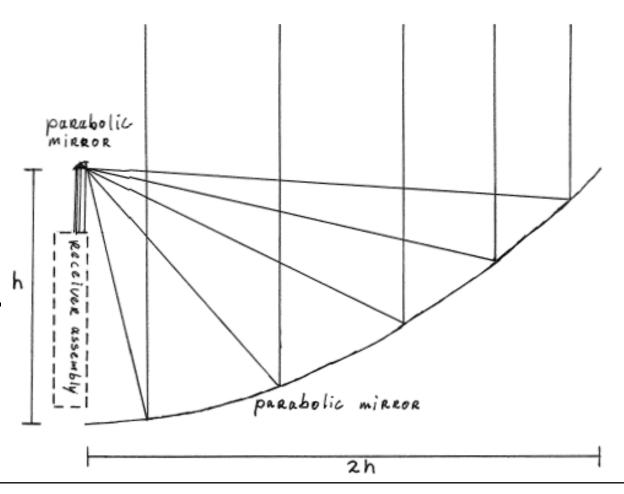


Receiver Optics – Common Part

Basically the reverse of the laser optics.

The secondary mirror is really small (mm range).

The difference is in the receiver assembly.





Receiver Optics – Fill Factor

Fill factor = \sim 2%. Then fraction of light detected is: QDE x FF = 37% x 2% = 0.74%

This is clearly unacceptable. Therefore, we need focusing optics after the main collector.



Receiver Optics – Noise

As the Sun bombards the Earth with photons, we need to filter the light, to prevent an unacceptable SNR.

Optical filters degrade fast and also filter put some of the wanted photons.

Therefore, we will use a prism to filter out unwanted noise.

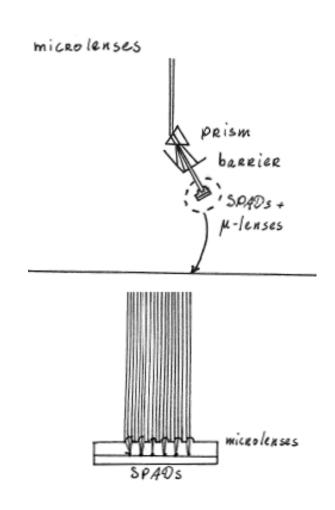


Receiver Optics – Microlenses

Advantages:

- Lightweight
- Conventional

- Only improves the fill factor to 10%
- This means: QDE x FF = 37% x 10% = 3.7%
- Needs to be rigidly fixed
- Still unacceptible



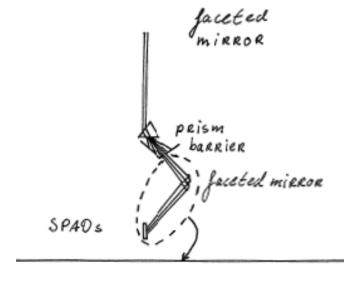


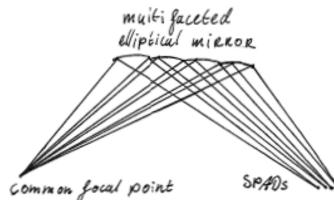
Receiver Optics - Faceted Mirror

Advantages:

- Improves the fill factor to over 80-95%
- This means: QDE x FF = 37% x 80-95% = 30-35%
- Is acceptable

- Manufacturing is complicated
- Needs to be rigidly fixed







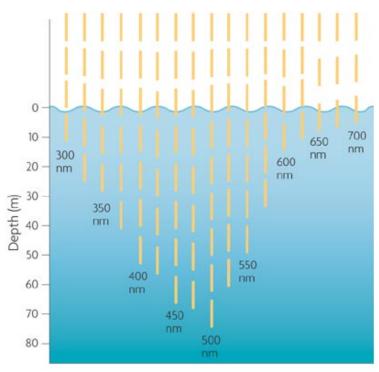
Receiver Trade-off

Criteria	Weight Factor	MPD	SILAT	SPAD + microlenses	SPAD + mirrors
Power	7	6	6	9	9
Mass	8	5	5	9	8
Volume	8	4	3	8	8
Reliability	7	8	8	6	5
Efficiency	10	7	6	1	5
Cost	5	4	3	7	5
Availability	3	10	8	5	3
Lifetime	10	8	8	6	6
Resolution	8	7	7	10	10
FOV	6	6	6	9	9
Weighed total		462	433	495	504



Ocean Reflectance

Large part of the Earth is covered by water



However, the fractional reflectance is highest

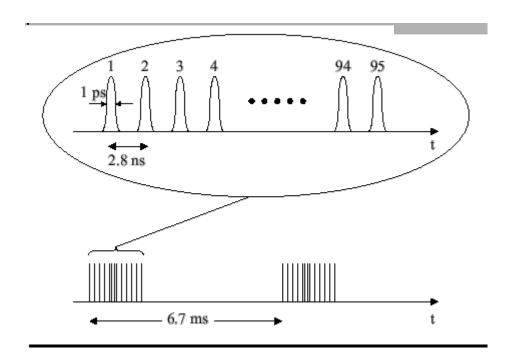
'blue' has the highest absorption depth





Continuous Versus Pulsed Waves

- By default: continuous
- By altering the laser: pulsed ~ nano- or picoseconds



Analysis of individual pulses

Increased spatial resolution



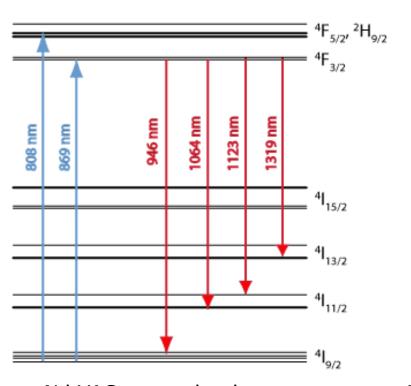
Blue types of laser

- Optimum wavelength according to analysis ~ 425 500 [nm]
- Possible 'blue' lasers
 - Gas lasers
 - Wavelength: 441.6 [nm] (Helium-Cadmium)
 - Wavelength: 488 [nm] (Argon)
 - Solid-State laser (Nd-YAG: Neodymium-doped Yttrium Aluminium Garnet)
 - Wavelength: 946 [nm]
 - Diode laser
 - Difficult to produce for lifetimes > 1 year



Nd-YAG wavelength correction

Second Harmonic Generation (non-linear optics)



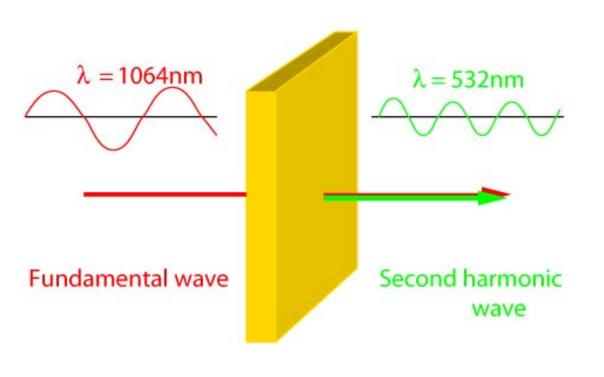


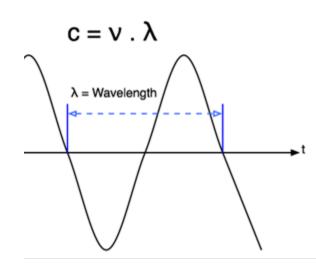
Nd-YAG energy levels

Non-linear (Lithium-Boron) frequency doubling crystal



From 946 [nm] to 473 [nm]

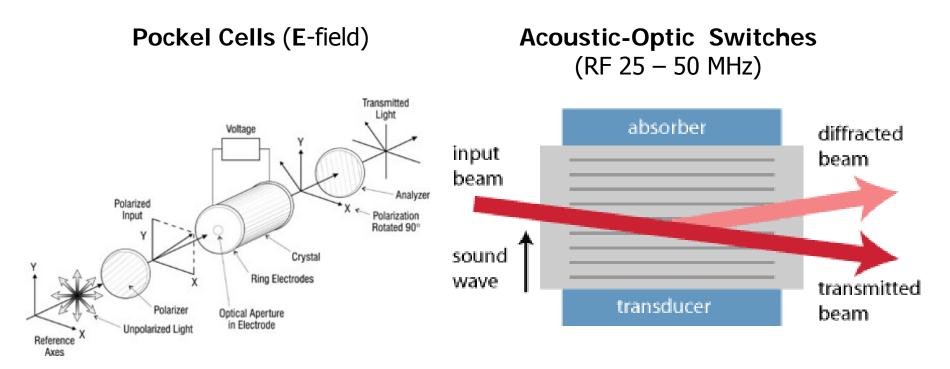






Pulse Duration Deviation

 Change pulse length (and pulse energy) over specific time intervals





5.

Orbit design



- Polar orbit
- Repeat orbit
- Sun Synchronous orbit
- Frozen orbit



Repeat orbit

Allows an area to be viewed more than once.

Assuming a footprint size of 100 meters:

40.000.000/(2*100)=200.000 revolutions 200.000*90 (minutes) = 34 years



Sun synchronous orbit

- Orbital plane fixed w.r.t. the sun vector
- Most useful orbit is the dawn/dusk orbit
 - Solar panels are in the sunlight continuously
 - Allows pointing to the night side of the Earth



Frozen orbit

- Reduces the need for orbit station keeping.
- A constellation in formation flight has strict constraints.
 - > A frozen orbit helps meet these constraints



Frozen orbit design equations

$$\dot{e} = \frac{3}{2} \frac{J_3 r_{eq}^3}{p^3} (1 - e^2) n \sin i \cdot \cos \omega \left(\frac{5}{4} \sin^2 i - 1 \right)$$

$$\frac{di}{dt} = \frac{3}{2} \frac{J_3 n}{\left(1 - e^2\right)^3} \left(\frac{R_e}{a}\right)^3 e \cos i \cdot \cos \omega \left(\frac{5}{4} \sin^2 i - 1\right)$$

$$\dot{\omega} = \frac{3J_2n}{\left(1 - e^2\right)^2} \left(\frac{R_e}{a}\right)^2 \left(1 - \frac{5}{4}\sin^2 i\right) F$$

$$F = 1 + \frac{J_3}{2J_2(1 - e^2)} \left(\frac{R_e}{a}\right) \left(\frac{\sin^2 i - e^2 \cos^2 i}{\sin i}\right) \frac{\sin \omega}{e}$$



Frozen orbit design equations

$$\frac{di}{dt} = \frac{3}{2} \frac{J_3 n}{\left(1 - e^2\right)^3} \left(\frac{R_e}{a}\right)^3 e \cos i \cdot \cos \omega \left(\frac{5}{4} \sin^2 i - 1\right) = 0$$

Circular orbit, so e = 0

$$\frac{di}{dt} = 0$$
 for any a, i or ω



Frozen orbit design equations

$$\dot{e} = \frac{3}{2} \frac{J_3 r_{eq}^3}{p^3} (1 - e^2) n \sin i \cdot \cos \omega \left(\frac{5}{4} \sin^2 i - 1 \right) = 0$$

With e = 0 this becomes

$$\dot{e} = \frac{3}{2} \frac{J_3 r_{eq}^3}{a^3} n \sin i \cdot \cos \omega \left(\frac{5}{4} \sin^2 i - 1 \right) = 0$$

Equation is satisfied for any a and i if ω = 90 degrees



Frozen orbit design equations

$$\dot{\omega} = \frac{3J_{2}n}{\left(1 - e^{2}\right)^{2}} \left(\frac{R_{e}}{a}\right)^{2} \left(1 - \frac{5}{4}\sin^{2}i\right) F$$

$$F = 1 + \frac{J_{3}}{2J_{2}\left(1 - e^{2}\right)} \left(\frac{R_{e}}{a}\right) \left(\frac{\sin^{2}i - e^{2}\cos^{2}i}{\sin i}\right) \frac{\sin \omega}{e}$$

With e = 0 these equations reduce to

$$\dot{\omega} = 3J_2 n \left(\frac{R_e}{a}\right)^2 \left(1 - \frac{5}{4}\sin^2 i\right) F$$

$$F = 1$$



Frozen orbit

$$\dot{\omega} = 3J_2 n \left(\frac{R_e}{a}\right)^2 \left(1 - \frac{5}{4}\sin^2 i\right)$$

- Is equal to zero if i = 63.4 OR i = 116.6 degrees
- However a polar orbit is an orbit of 90 degrees inclination
 - ➤ Definition: An orbit is a polar orbit if $80 \le i \le 100$ degrees



Frozen orbit

$$\dot{\omega} = 3J_2 n \left(\frac{R_e}{a}\right)^2 \left(1 - \frac{5}{4}\sin^2 i\right)$$

- The orbit is circular
 - \triangleright It does not matter if ω rotates in the orbit plane
- Taking collision avoidance into collision avoidance
 - \rightarrow i = 85 degrees



Summary

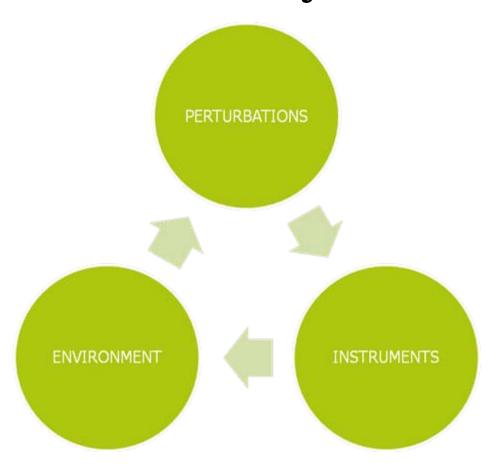
- Sun synchronous is not required
- Repeat orbit is unfeasible
- The end result is a

Frozen, polar orbit with

- > e = 0 degrees
- > i = 90 degrees
- $\triangleright \omega = 90 \text{ degrees}$

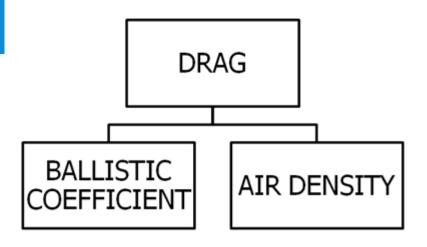


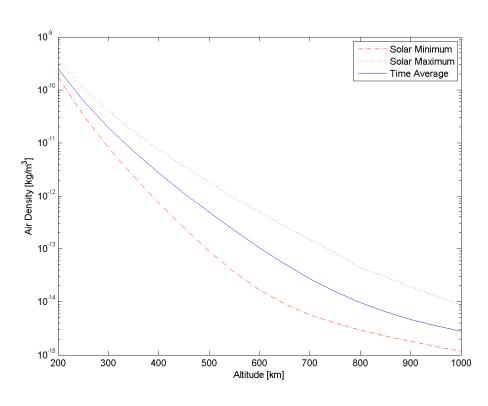
Orbit Altitude Analysis





Orbit Altitude Analysis Perturbations





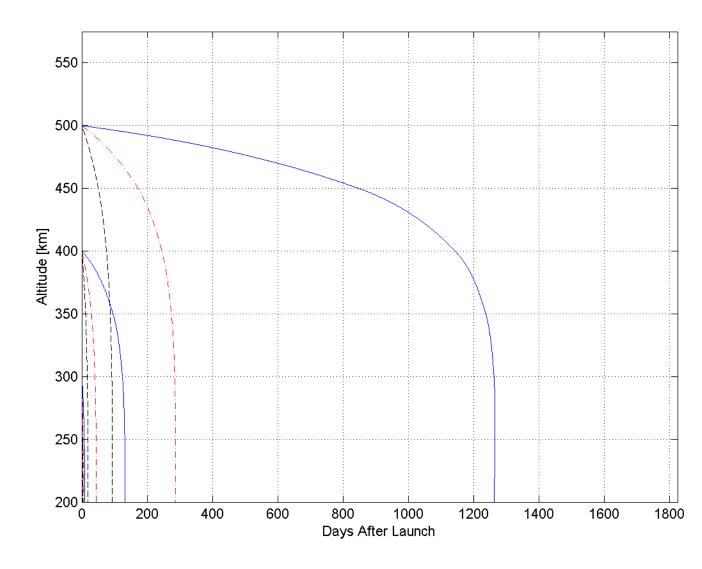


Drag

$$\Delta a = -2\pi \left(C_D \frac{A}{m} \right) \rho a^2$$

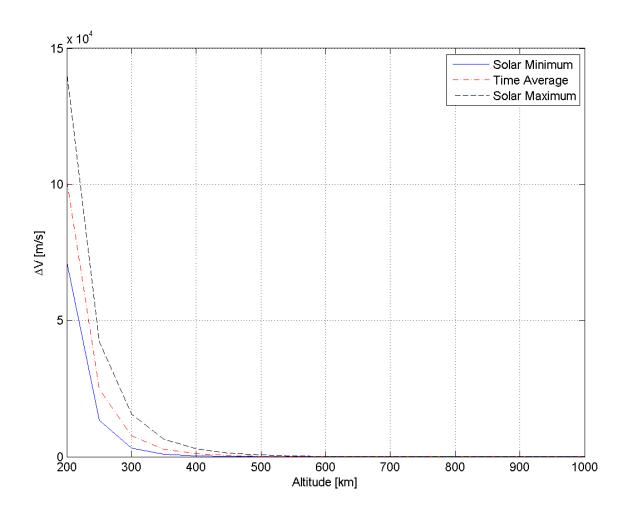
$$\Delta P = -6\pi \left(C_D \frac{A}{m} \right) \rho \frac{a^2}{V}$$

$$\Delta V = \pi \left(C_D \frac{A}{m} \right) \rho aV$$



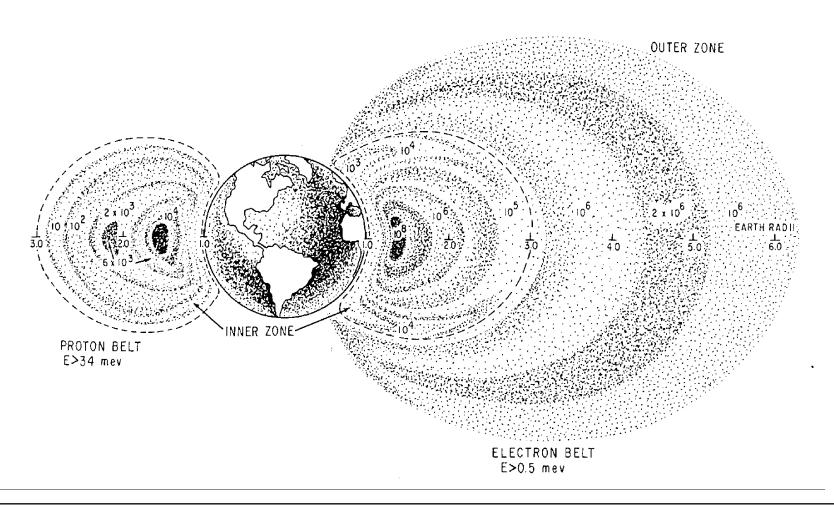


Drag - ΔV



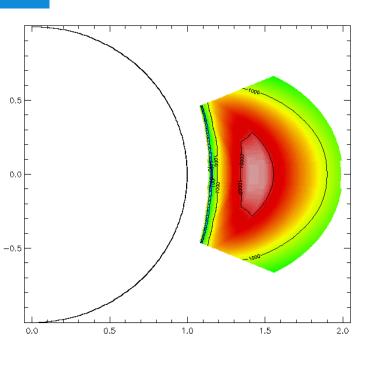


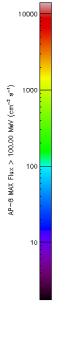
Environment Trapped particle radiation

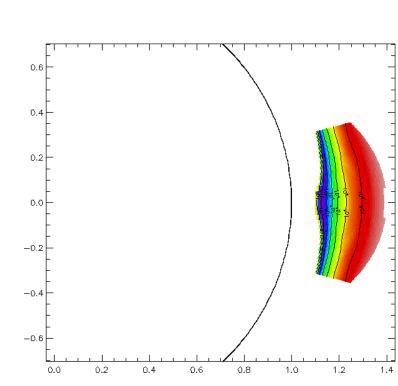


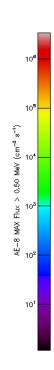


Environment Trapped particle radiation









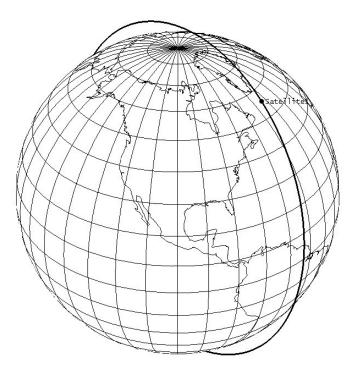


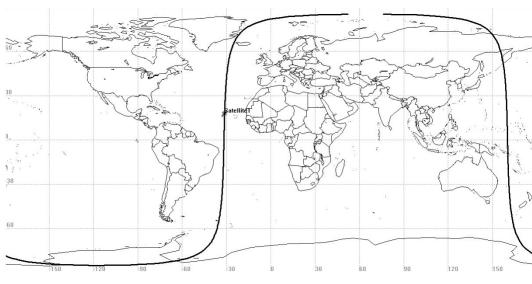
Orbit Altitude Summary

- As high as possible to reduce propellant mass
- Mission timeframe is crucial solar min/max
- Keep ballistic coefficient close



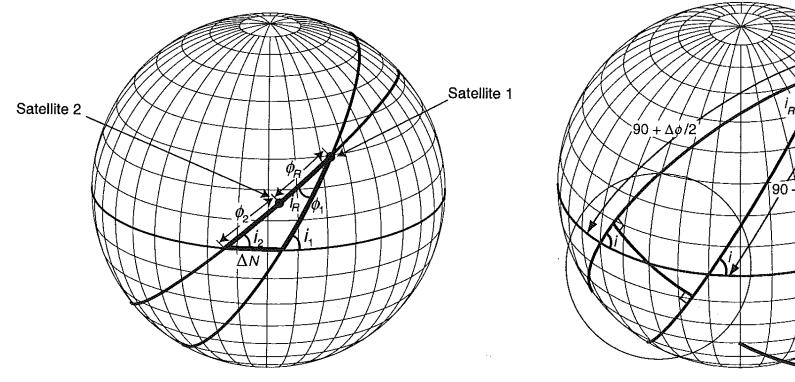
Orbit Altitude Summary

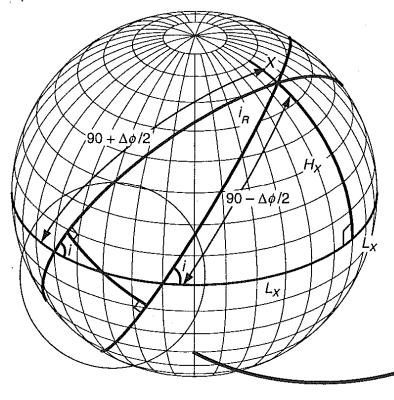






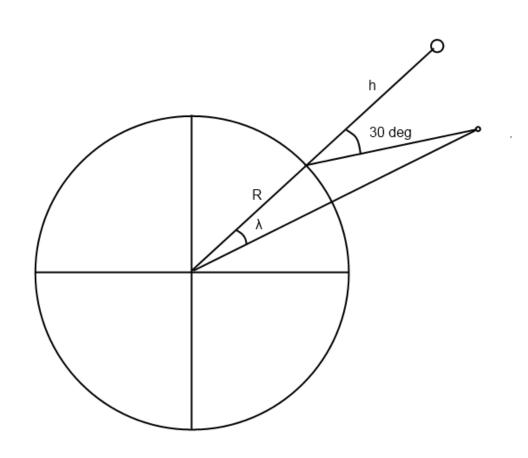
Formation Design







Formation Design



$$\lambda = 2.18^{\circ}$$

 $i_R = 2.18^{\circ}$

Formation Design Stationkeeping

Keeping the general constellation to insure better measurement data.

What affects it?

- Perturbations
- Differences in initial conditions



Formation Design Stationkeeping

What can be done?

- Nothing
- Relative Stationkeeping
- Absolute Stationkeeping



Formation Design

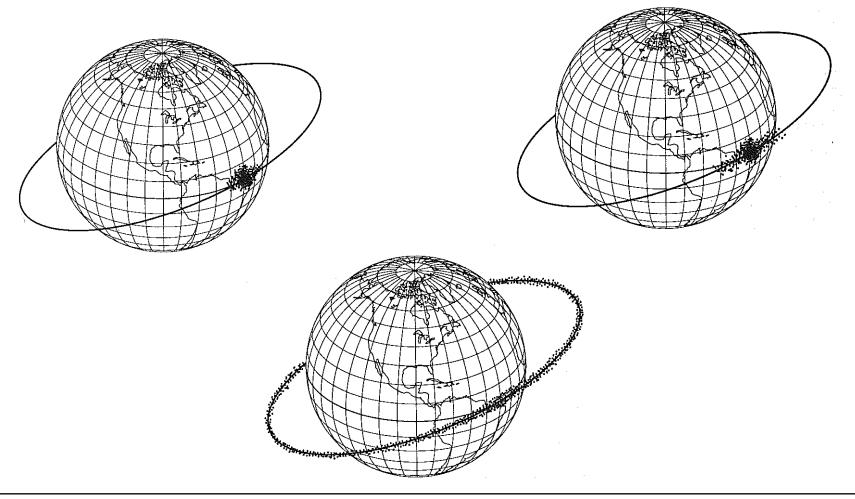
Collision Avoidance

Why is it important?

- Loss of 2 satellites, possibly vital
- · Increased possibility of collision due to debris spread



Formation Design Collision Avoidance





Formation Design Collision Avoidance

Parameters	5 Satellite Formation in 3 Planes	9 Satellite Formation in 5 Planes
No. of satellites	5	9
No. of orbit planes	3	5
Vertical dispersion [km]	1	1
In-track dispersion [km]	276	276
Potential impact area [km ²]	276	276
Collision opp. per orbit	20	72
Orbit period [min]	94.62	94.62
Collision opp. per year	$1.1*10^5$	$4.0*10^5$
Collision opp. in 5 years	$5.6*10^5$	$2.0*10^6$
Collision prob. per opp.	$1.0*10^{-6}$	$1.0*10^{-6}$
Mean number of collisions per	0.11	0.4
year		

Information based on Wertz, 2001



6.

Software tool



Software tool

Two parts

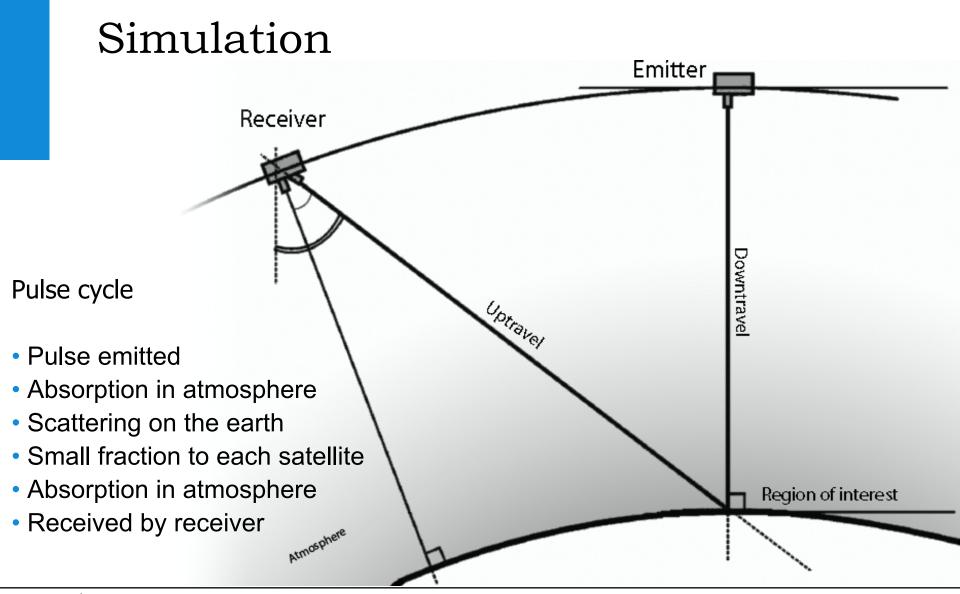
Simulation

Simulate the laser pulse photons and noise as received by the sensor.

Data analysis

Reconstructing the digital elevation model and BRDF from the received time series.







Noise

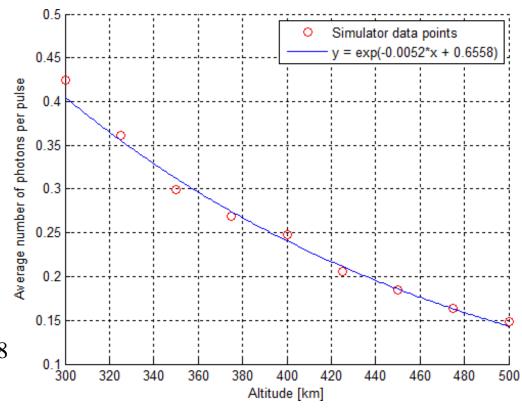
- Noise is introduced into the system
- Sources are the Earth and the Sun
- In a selective wavelength band
- Strong dependence on
 - Receiver footprint area
 - Receiver sensitivity band
 - Constellation altitude



Photon count variation with altitude

- Exponential decrease in photon count with altitude
- Lower is better
- Higher altitudes:
 - larger receiver aperture
 - higher emitter power

$$photons = e^{-0.0052 \cdot alt + 0.6558}$$





Solar noise photons fraction

Orbit at 450 km, 33W laser, 10 nm filter

Simulation results:

• Pulses sent: 24989 (about 5s)

Photons from pulses received: 12289 (86.5%)

Sun noise photons received: 1930 (13.5%)

Total photons received: 14219

Majority of the photons from the emitter laser Noise can be filtered out (constellation)



Terrain Reconstruction Algorithm

- Define time range
- Find the peaks
- Calculate altitudes
- •Find the most common altitude



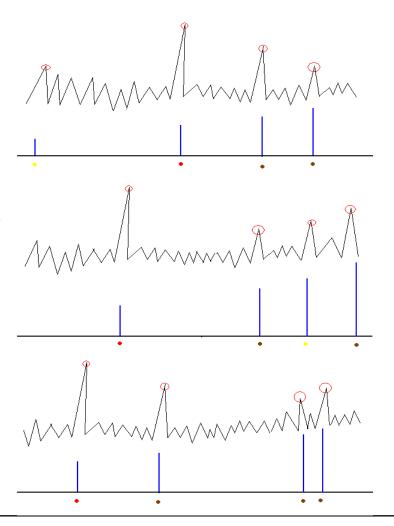
Defining Range

- Known:
 - Time pulse sent
 - Time <u>SOME</u> pulse received
- Window: 1/5000 sec = 200 micro sec
- Offset: $500 \text{km/c}^2 = 3.333 \text{ milliseconds travel time}$
- Height range: 60 km



Find the Peaks

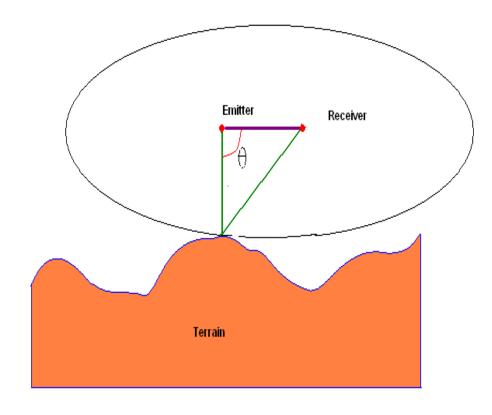
- The peaks correspond to received pulses
- Noise introduced creates "false" peaks
- N*mean threshold
- Intermediate step for BRDF determination





Calculate altitudes

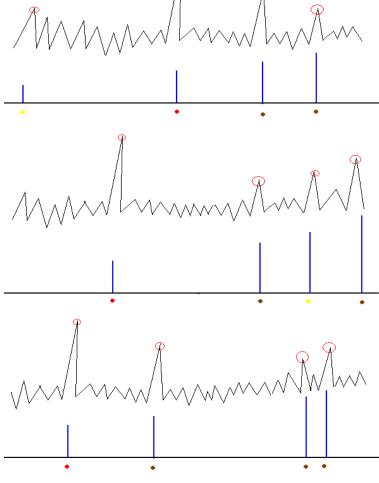
- Required:
 - Position of emitter
 - Position of receiver
 - Travel time





Finding most common altitude

- Filtering noise
- Peak = specific altitude
- Least standard deviation configuration





7.

Summary and conclusions



Summary

ADCS
 Sun sensor & star tracker

Reaction wheels and magneto torquers

COMS Centralized architecture

EPS Thin film solar cells

ORP 32x32 SPAD with faceted mirror

OEP Nd-YAG laser 473 nm

• ORBIT Frozen polar orbit, 500 km, $I = 85^{\circ}$, $\lambda_{max} = 2.18^{\circ}$

