The Real and Complex Number Systems

by David Moser

What is a real number? Are real numbers really real? This seems like it should be an easy question to answer: the answer is contained in the question. Real numbers should, be, well, *real*. They're not. Numbers are all *abstractions*. This means that they are used to represent something else. They do not really exist like the things they are used to represent. (I would expect that mathematicians might take issue with the idea that numbers do not exist as independent objects. This is natural, as I am infringing on their ability to earn a living by implying that their stock in trade is not real, and we know how hard it is to impress one's spouse without gainful employment. I mean no harm, and am myself in similar straits, as words are abstractions, too.) We possess the ability to think and talk about them, but their existence is entirely in the mind as truths which cannot be apprehended by simple observation, but only through abstract thought.

If real numbers are not real, what are they?

Well, numbers are named for the kind of things they are used for: What do they describe? Looking at some simpler cases, this becomes more clear. First, the "counting" numbers. What are they used for? Counting, of course. The smallest counting number is one (1). It is the smallest number of things we can have and still have something. Then there are the oddly named "whole" numbers. These are comprised of the counting numbers, plus the number zero (0). I suppose that when one considers counting with numbers larger than nine (9), it becomes important to have zero as a placeholder, and it must be included to complete the set, or make it "whole". What does it mean? Nothing, of course.

Beyond this, we come to the somewhat intimidating "integers." These are named using the same Latin word that became our English word "integral," or "whole." Coincidental? I can say no more. The integers include all the whole numbers plus all the "negative" whole numbers. Negative numbers are used to count what we do not have (and by implication, want). It seems odd to me that anyone would want to get involved with something as difficult to quantify as desire, but all of us seem to know exactly what it is that we want and do not have, and have prevailed upon the mathematical-financial types to give us a way to keep track of it. The banker and the tax collector seem to be more adept than anyone at the constructive use of these numbers.

The next step is a big one. Up to this point in their history, numbers were used as an abstract tool for counting things; keeping track of how much or how little of something one did or did not have. In the early days, this was enough. It was a sort of number Eden, and the living was easy. Then came the mathematicians. They started *thinking* about numbers. They tried to figure out what could be done with them, and to them, and they invented the idea of "rational" numbers. These are numbers that could be expressed as a ratio of integers, or one integer divided by another. Any integer could be expressed as a ratio of one integer to another, and that was okay. Other numbers were discovered, though, that could be expressed this way, that were **not** integers. These came to be called fractions, from a Latin word that means "broken."