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COLLEGE OF SCIENCE
FACULTY OF PHYSICAL AND COMPUTATIONAL SCIENCES
DEPARTMENT OF MATHEMATICS

CALCULUS WITH ANALYSIS : FUNCTIONS

TUTORIAL **JUNE 2021**

1. Find the domain and the range of each of the following.

1) $y = |x| - 2$ 2) $y = \sqrt{16 - x^2}$ 3) $y = 2e^{-x} - 3$

4) $y = 5 - \sqrt{x^2 - 2x - 3}$ 5) $y = 4sin\left(\frac{1}{x}\right)$ 6) $y = 2 + \frac{3x^2}{x^2 + 4}$

7) $f(x) = 1 + x^2$ 8) $f(x) = 1 - \sqrt{x}$ 9) $F(x) = \sqrt{5x + 10}$

10) $g(x) = \sqrt{x^2 - 3x}$ 11) $f(t) = \frac{4}{3 - t}$ 12) $G(t) = \frac{2}{t^2 - 16}$

2. Determine whether the function is even, odd, or neither.

1) $g(x) = x^3 + 1$ 2) $h(t) = \frac{1}{t - 1}$ 3) $g(x) = \frac{x}{x^2 - 1}$

4)$h(t) = 2|t| + 1$ 5) $y = 1 + \cos x$ 6) $y = \frac{x^4 + 1}{x^3 - 2x}$

7) $y = \sec x \tan x$ 8) $y = x \cos x$ 9) $y = \ln(x^2 - |x|) + \frac{\sin x}{1 - \sec x}$

3. Find the domain of (a), the range of (b) and both domain and range in (c) and (d).

$$a)\ y = \frac{x+3}{4-\sqrt{x^2-9}} \qquad b)\ y = 2+\sqrt{9+x^2}$$

$$c)\ y = \begin{cases} \sqrt{-x}, & -4 \le x \le 0 \\ \sqrt{x}, & 0 < x \le 4 \end{cases} \qquad d)\ y = \begin{cases} -x-2, & -2 \le x \le -1 \\ x, & -1 < x \le 1 \\ -x+2, & 1 < x \le 2 \end{cases}$$

4. Find the largest interval on which the given function is increasing

$a)\ g(x) = |x-2|+1 \quad b)\ h(x) = (x+1)^4 \quad c)\ h(x) = (x+1)^{\frac{1}{3}} \quad d)\ g(x) = \sqrt{2x-1}$

5. Use the addition formulas to derive the identities

$$a)\ \cos\left(x - \frac{\pi}{2}\right) = \sin x \qquad b)\ \cos\left(x + \frac{\pi}{2}\right) = -\sin x$$

$$c)\ \sin\left(x + \frac{\pi}{2}\right) = \cos x \qquad d)\ \sin\left(x - \frac{\pi}{2}\right) = -\cos x$$

6. Consider the point (x, y) lying on the graph of the line $2x+4y = 5$. Let L be the distance from the point (x, y) to the origin $(0, 0)$. Write L as a function of x.

7. Consider the point (x, y) lying on the graph of $y = \sqrt{x-3}$. Let L be the distance between the points (x, y) and $(4, 0)$. Write L as a function of y.

8. Specify the intervals over which the function is increasing and the interval where it is decreasing.

$$1)\ y = -x^3 \qquad 2)\ y = -\frac{1}{x^2} \qquad 3)\ y = \sqrt{|x|} \qquad 4)\ y = \frac{1}{|x|}$$

9. Find the largest interval on which the given function is increasing.

$1)\ f(x) = |x-2|+1 \quad 2) f(x) = (x+1)^4 \quad 3)\ g(x) = (3-1)^{\frac{1}{3}} \quad 4)\ h(x) = \sqrt{2x-1}$

10. Find the limits in the following

$$1)\ \lim_{x\to -2^-} (x+3)\frac{|x+2|}{x+2} \qquad 2)\ \lim_{x\to 1^-} \frac{\sqrt{2x}(x-1)}{|x-1|} \qquad 3)\ \lim_{t\to 0} \frac{\sin kt}{t},\ k \in \mathbb{R}.$$

$$4)\ \lim_{x\to 0} \frac{x\csc 2x}{\cos x} \qquad 5)\ \lim_{x\to 0} \frac{x + x\cos x}{\sin x \cos x} \qquad 6)\ \lim_{x\to 0} \frac{1-\cos 3x}{2x}$$

$$7)\ \lim_{x\to \infty} \frac{\sqrt{x^2+1}}{x+1} \qquad 8)\ \lim_{x\to \infty} \frac{2+\sqrt{x}}{2-\sqrt{x}} \qquad 9)\ \lim_{x\to \infty} \frac{x^{-1}+x^{-4}}{x^{-2}-x^{-3}}$$

$$10)\ \lim_{x\to -\infty} \frac{4-3x^3}{\sqrt{x^6+9}} \qquad 11)\ \lim_{x\to \infty} \left(\sqrt{x+9}-\sqrt{x+4}\right) \qquad 12)\ \lim_{x\to \pm\infty} \ln(x^2-|x|) + \frac{\sin x}{1-\sec x}$$

11. At what points are the functions continuous ?

$$1)\ y = \frac{1}{x-2} - 3x \quad 2)\ y = \frac{1}{(x-2)^2} + 4 \qquad 3)\ y = \frac{1}{|x|+1} - \frac{x^2}{2}$$

$$4)\ y = \frac{x+2}{\cos x} \qquad 5)\ g(x) = \begin{cases} \frac{x^2-x-6}{x-3}, & x \neq 3 \\ 5, & x = 3 \end{cases} \quad 5)\ f(x) = \begin{cases} \frac{x^3-8}{x^2-4}, & x \neq 2, x \neq -2 \\ 3, & x = 2 \\ 4, & x = -2 \end{cases}$$

12. Are the functions continuous at the points being approached ?

$$1)\ \lim_{x \to \pi} \sin(x - \sin x) \qquad 2)\ \lim_{x \to \pi} \sin\left(\frac{\pi}{2}\cos(\tan t)\right)$$

13. At what value of a and b are the following functions continuous at every x?

$$1)\ f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases} \qquad 2)\ g(x) = \begin{cases} a^2x - 2a, & x \geq 2 \\ 12, & x < 2 \end{cases}$$

$$3)\ g(x) = \begin{cases} \frac{x-a}{a+1}, & x < 0 \\ x^2 + a, & x > 2 \end{cases} \qquad 4)\ g(x) = \begin{cases} ax - 2b, & x \leq 0 \\ x^2 + 3a - b, & 0 < x \leq 2 \\ 3x - 5, & x > 2 \end{cases}$$

14. Define $f(1)$ in a way that extends $f(s) = \dfrac{s^3-1}{s^2-1}$ to be continuous at $s = 1$.

15. Define $g(4)$ in a way that extends $g(x) = \dfrac{s^2-16}{s^2-3x-4}$ to be continuous at $s = 4$.

16. Explain why the equation $\cos x = x$ has at least one solution.

17. Show that the equation $x^3 - 15x + 1 = 0$ has three solutions in the interval $[-4, 4]$.

18. Show that the function $F(x) = (x-a)^2(x-b)^2 + x$ takes on the value $\frac{1}{2}(a+b)$ for some value of x.

19. 1) What is a horizontal, vertical and an oblique asymptote of the graph of a function?

    2) Find the asymptotes of the graph of the following functions

$$a)\ f(x) = -2x + 1\frac{1}{x} \qquad b)\ f(x) = \frac{x^2+3}{x-1} \qquad c)\ f(x) = \sqrt{x^2 - x}.$$

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