

THE CONCEPT OF POPULATION GENETICS

Hardy- Weinberg Law

&The Hardy-Weinberg Law is a basic concept in the population genetics developed independently in 1908 by the English mathematician, G H Hardy and German physician, W Weinberg.

& The Hardy-Weinberg law states that the gene frequencies and genotypic ratios in a natural breeding population remain constant from generation to generation if:

- (a) The population size is large.
- (b) There is no mutation.
- (c) There is no immigration or emigration.
- (d) Mating is random.
- (e) There is random reproductive success.

Predictions of the Hardy-Weinberg Law

- ¶ 1. If the conditions are met, the population will be in genetic equilibrium, with two expected results:
 - ¶ a. Allele frequencies do not change over generations, so the gene pool is not evolving at the locus under study.
 - ¶ b. After one generation of random mating, genotypic frequencies will be p^2 , $2pq$, and q^2 , and will stay constant in these proportions as long as the conditions above are met. This is Hardy-Weinberg equilibrium, which allows ⁴ predictions to be made about genotypic frequencies.

& The relationship between gene frequency and genotype frequency can be described by The equation $p^2 + 2pq + q^2 = 1$, where p^2 represents the frequency of dominant gene, $2pq$ is the frequency of heterozygote genotype and q^2 is the frequency of the homozygous recessive genotype.

- ¶ The Hardy-Weinberg law forms a theoretical base line for measuring evolutionary change as it predicts that there is no evolution.
- ¶ Whenever evolution occurs, the Hardy-Weinberg equilibrium is disturbed as a change in gene frequencies and indicates that evolution is in progress.

- ¶ The Hardy-Weinberg law is used to determine whether the number of harmful mutations in a population is increasing.
- ¶ In nature, the Hardy-Weinberg law is impossible.

Derivation of the Hardy-Weinberg Law

- ¶ 1. Zygotes are formed by random combinations of alleles, in proportion to the abundance of that allele in the population (Figure 24.3).
- ¶ 2. When a population is in equilibrium, genotypic frequencies will be in the proportions p^2 , $2pq$, and q^2 . This results from the expansion of the square of the allelic frequencies: $(p+q)^2=p^2+2pq+q^2$.
- ¶ 3. Mendelian principles acting on a population in equilibrium will work to ⁸maintain that equilibrium. Albinism is an example.

Mathematical proof that Gene Frequencies are Maintained from generation to generation

Mathematical proof -1

& In a population the relative frequencies of different alleles tend to be maintained constant from one generation to the next. This can be demonstrated mathematically and helps to explain why dominant traits do not automatically increase at the expense of recessives.

Mathematical proof - 2

& consider one locus with two alleles A and a . If the frequency of the allele ' A ' is p and the frequency of the allele ' a ' is q then, since each individual must have one or other allele, the sum of these allele frequencies must be one or 100%. Therefore

$$\& p + q = 1$$

Mathematical proof – 3 Table 1

		Paternal	gametes
		A	a
		(p)	(q)
	A	AA	Aa
	(p)	(p ²)	(pq)
Maternal gametes	a	Aa	aa
	(q)	(pq)	(q ²)
12			
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Mathematical proof - 4

& In production of the next generation of the three types of paternal genotype may mate with each of the three types of maternal genotype (Table 2). Table 3 indicates the genotypes of the offspring for each mating type, and as can be seen the relative frequencies of each is unchanged and the population is said to be in genetic equilibrium. Although the actual numbers of individuals with each genotype may have increased, the relative proportions of each genotype (and allele) have remained constant (AA at p^2 , Aa at $2pq$ and aa at q^2). This principle is called the Hardy Weinberg law.

Mathematical proof - 5

Table 2		Paternal genotypes		
Maternal genotypes		AA	Aa	aa
		(p ²)	(2pq)	(q ²)
	AA	AA x AA	AA x Aa	AA x aa
	(p ²)	(p ⁴)	(2p ³ q)	(p ² q ²)
	Aa	Aa x AA	Aa x Aa	Aa x aa
	(2pq)	(2p ³ q)	(4p ² q ²)	(2pq ³)
		aa	aa x AA	aa x aa
		(q ²)	(p ² q ²)	(q ⁴)

Mathematical proof - 6

Table 3						
Mating type	RESULTS	Frequency (from Table 2)	Offspring			
			AA	Aa	aa	
AA x AA	AA,AA,AA,AA	p^4	p^4	-	-	
AA x Aa	AA,AA, Aa, Aa	$4p^3q$	$2p^3q$	$2p^3q$	-	
AA x aa	Aa, Aa, Aa, Aa	$2p^2q^2$	-	$2p^2q^2$	-	
Aa x Aa	AA,Aa, Aa, aa	$4p^2q^2$	p^2q^2	$2p^2q^2$	p^2q^2	
Aa x aa	Aa,Aa,aa,aa	$4pq^3$	-	$2pq^3$	$2pq^3$	
aa x aa	aa, aa,aa,aa	q^4	-	-	q^4	2/22/2017

Mathematical proof - 7

$$\& AA \text{ offspring} = p^4 + 2p^3q + p^2q^2$$

$$= p^2(p^2 + 2pq + q^2)$$

$$= p^2(p + q)^2$$

$$= p^2(1)^2 = p^2$$

$$\& Aa \text{ offspring} = 2p^3q + 4p^2q^2 + 2pq^3$$

$$= 2pq(p^2 + 2pq + q^2)$$

$$= 2pq$$

$$\& aa \text{ offspring} = p^2q^2 + 2pq^3 + q^4$$

$$= q^2(p^2 + 2pq + q^2)$$

$$= \frac{1}{16}q^2$$

Extensions of the Hardy-Weinberg Law to Loci with More than Two Alleles

- ¶ 1. Often more than two alleles are possible at a given locus, and the frequencies of possible genotypes are still given by the square of the allelic frequencies.
- ¶ 2. If three alleles are present (e.g., alleles A , B , and O) with frequencies p , q , and r , the frequencies of the genotypes at equilibrium will be:
$$(p+q+r)^2 = p^2(AA) + 2pq(AB) + q^2(BB) + 2pr(AO) + 2qr(BO) + r^2(OO)$$
- ¶ 3. Human blood groups is an example.

Extensions of the Hardy-Weinberg Law to Sex-Linked Alleles

- ¶ 1. In species where sex is chromosomally determined, humans or *Drosophila* for example, females have two X chromosomes while males have only one. In females, Hardy-Weinberg frequencies are the same as for any other locus. In males, frequencies of the genotypes are the same as frequencies of the alleles in the population.
- ¶ 2. Because males receive their X chromosome from their mothers, the frequency of an X-linked allele will be the same as the frequency of that allele in their mothers. For females the frequency will be the average of both parents.

Extensions of the Hardy-Weinberg Law to Sex-Linked Alleles

¶3. With random mating, the difference in allelic frequency between the sexes will be reduced by half in each generation . One generation after allelic frequencies become equal in males and females, the genotypes will be in Hardy-Weinberg proportions.

Testing for Hardy-Weinberg Proportions

- ¶ 1. Data from real populations rarely match Hardy-Weinberg proportions. Use a chi-square test to check whether deviation is larger than expected by chance.
- ¶ 2. If the deviation is larger than expected, researchers begin to study which of the Hardy-Weinberg assumptions is being violated.

¶ If we know the frequency of each allele in a population, we can predict the genotypes and phenotypes we should see in that population. If the phenotypic frequencies in a population are not those predicted from the allele frequencies, the population is not in Hardy-Weinberg equilibrium, because an assumption has been violated.

- ¶ Either non-random mating or evolution is occurring.
- ¶ But as long as the Hardy-Weinberg assumptions are not violated, recessive alleles and dominant alleles do not change their frequencies over time.

Humans can inherit various alleles from the liver enzyme ADH (alcohol dehydrogenase), which breaks down ingested alcohol. People of Italian and Jewish descent commonly have a form of ADH that detoxifies very rapidly. People of northern European descent have forms of ADH that are moderately effective in alcohol breakdown, while people of Asian descent typically have ADH that is less effective at processing alcohol. Explain why researchers have been able to use this information to help trace the origin of Human use of alcoholic beverages.