KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY
COLLEGE OF SCIENCE

FACULTY OF PHYSICAL AND COMPUTATIONAL SCIENCES
DEPARTMENT OF MATHEMATICS

CALCULUS WITH ANALYSIS

Differentiation 2

1. Determine the absolute maximum and minimum points for each given function $f(x)$, or state that they do not exist.

   a) $f(x) = x^2 - 6x - 1, 2 \leq x \leq 5$          b) $f(x) = x^2 - 6x - 1, 0 \leq x \leq 2$

   c) $f(x) = \dfrac{2x^2 - 2x + 5}{x^2 + 2}, -3 \leq x \leq 4$          d) $f(x) = \dfrac{x^2 - x + 2}{x^2 + 3}, -2 \leq x \leq 3$

   e) $f(x) = \sin x + \cos^2 x, 0 \leq x \leq \pi$          f) $f(x) = 2\sec x - \tan x, 0 \leq x \leq \dfrac{\pi}{3}$

2. If the total cost of producing $x$ units of a commodity is given by,

$$C(x) = x^3 - 100x + 1500, x \geq 1,$$

   find the value of $x$ that minimizes this cost, and determine the minimal cost.

3. If the velocity $v$ of air through a bronchial tube of radius $r$ under pressure is,

$$v = kr^2(a - r),$$

   Where $a$ is the radius when no pressure is applied, and $k$ is a constant, find the bronchial tube radius that will provide the maximum air velocity.

4. The maximum height $h$ attained by a projectile fired with initial velocity $v_0$ at an angle of $\theta$ to the horizontal is,

$$h = \frac{v_0^2 \sin^2 \theta}{2g},$$

where $g$ is the acceleration due to gravity. For what angle is $h$ a maximum?

5. Find an equation of the tangent to the graph of,

a) $y^6 + 2xy^2 + 3 = 0 \, at \, (-2, 1)$ 　　　　b) $y = x \sin x \, at \, x = \dfrac{7\pi}{6}$

6. Find an equation of the line perpendicular to the tangent to the graph of the following,

a) $x^3 + y^3 = 8 \, at \, (2, 0)$ 　　　　b) $(x + y)^{\frac{4}{3}} + xy = 1 \, at \, (0, 1)$

7. What is the slope of the graph of $\sqrt{2} \cos(x + y) = 1 \, at \, the \, point \left( \dfrac{\pi}{6}, \dfrac{\pi}{12} \right)$?

8. An open trough is to be constructed in the form of a half-cylinder. If the trough is to contain 300 liters of water, what dimensions minimize the total surface area?

9. Express $dy$ in terms of $x$ and $dx$,

a) $y = 7x^2 - 3x$ 　　　　　　　　　　b) $y = (x^3 - 5x^2)^3$

c) $y = x^{\frac{4}{3}} + x^{\frac{1}{2}}$ 　　　　　　　　　d) $y = \sin^2(x^3 - 2x)$

10. Use differentials to approximate the indicated functional value,

a) $g(x) = \sqrt{x^2 - 9}, g(5, 1)$ 　　　　b) $g(x) = x^{\frac{5}{2}}, g(101)$

c) $g(x) = \sin x \cos x, g(.02)$

11. Use differentials to approximate the change in the volume of a cube as it's side length decreases from 5.02cm to 5cm.

12. Find $y^I$ using implicit differentiation.

   a) $y=\left(x^2 y^3 - 2xy\right)^2$

   b) $\tan\left(x^2 y\right)+y^2 \cos\left(xy\right)=xy \sin y$

   c) $x^2 \sec y+\sin x \cos y= y^2 \tan x$