KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

COLLEGE OF SCIENCE

FACULTY OF PHYSICAL AND COMPUTATIONAL SCIENCES

DEPARTMENT OF MATHEMATICS

CALCULUS WITH ANALYSIS: Differentiation

TUTORIAL PART 1 **JULY 2021**

1. Find the derivatives of each of the following function.

1) $f(x) = (x^2 + 8)(x^3 - 2)$ 2) $f(x) = x^3(x - 3)$

3) $f(x) = (x^3 - 2x)^2$ 4) $f(x) = 6x^5 + 7x^{-5}$

5) $f(x) = 14x^{-3} - 2x^{-4}$ 6) $f(x) = -4x^3 + 7x^2 - 2x^{-3}$

7) $f(x) = \dfrac{1}{2x^4 + 1}$ 8) $f(x) = \dfrac{x^2 - 3x + 5}{x^2 + 3}$

9) $f(x) = (x^2 - 3)(x + 4)(x - 5)$

2. Verify that the given function is a solution to the given differential by substituting the function into the equation and showing that the equation is satisfied

1) $\dfrac{dy}{dx} - 2x = 5,\ \ y = x^2 + 5x + 7$ 2) $\dfrac{dy}{dx} - 3x^2 = 2x,\ \ y = x^3 + x^2 + 5$

3) $y + xy^{'} = 6x,\ \ y = kx^{-1} + 3x$ 4) $xy^{'} - y = x^2 + 1,\ \ y = x^2 + kx - 1$

3. Find $\frac{dy}{dx}$ of the the following,

$a) y = 3u^2 - 5u + 2, where\ \ u = x^2 - 1$ $\quad$ $b) y = 4u^5 - 7u + u^{-2}, where\ \ u = x + x^2$

$c) y = (u^2+1)^{-5}, where\ \ u = x^{-3}$ $\quad$ $d) y = \dfrac{u^3+1}{u^2-1}, where\ \ u = (x^2+4)^3$

$e) y = \dfrac{u^3+5u}{u^5-1}, where\ \ u = x^7 + x^{-1}$ $\quad$ $f) y = \left(\dfrac{x^{-3}+1}{x^2-1}\right)^{-2}$

$g) y = \left(\dfrac{x^{-1}+3x^{-2}}{4x+5}\right)^{-6}$ $\quad$ $h) y = (4x^{-1} - 2x^{-2})^{-3}$

4. Differentiate the following functions

$a) f(x) = sin(x^5 - 3x)$ $\quad$ $b) f(x) = \sin^3 5x \cos^4 6x$

$c) h(x) = \dfrac{\sin 9x}{\cos 6x}$ $\quad$ $d) f(x) = \sin^2(x^3-2)\cos^3(x^3+7)$

$c) h(x) = \dfrac{\tan^4(x^3+1)}{\sec^3(x^2-2)}$ $\quad$ $d) f(x) = \tan^5(sin^2 4x)$

5. Determine where the graph is (a) increasing. (b) decreasing. (c) concave upward, and (d) concave downward. Also find (e) the critical points, and (f) the points of inflection, and sketch the graph of the equation.

$a) y = 12x - x^3 + 1$ $\quad$ $b) y = (x+3)(x^2+6x+6)$

$c) y = \dfrac{x-2}{x+1}$ $\quad$ $d) y = x + \dfrac{1}{x}$

$c) y = \sin x + \cos x,\ \ -\dfrac{3\pi}{4} \leq x \leq \dfrac{5\pi}{4}$ $\quad$ $d) y = \dfrac{x}{2} + \sin x,\ \ -\pi \leq x \leq 2\pi$

6. Find all values of c whose existence is assured by the Mean Value Theorem for the indicated functions and intervals

$a) f(x) = 3x^2 - 5x + 7,\ \ 1 \leq x \leq 9$ $\quad$ $b) f(x) = x^3 - 2,\ \ -4 \leq x \leq 5$

$c) f(x) = 8 - 6x^2,\ \ 3 \leq x \leq 7$ $\quad$ $d) f(x) = \dfrac{1}{x} + 3,\ \ \dfrac{1}{3} \leq x \leq 6$

7. Find an equation of the tangent to $y = (3x^{-2} - 2x^3)^5$ at the point (1, 1)

8. Find an equation of the tangent to $x = (4v-2)^2$, $y = (3v^2 - 2v)^{-1}$ when v = 1.

9. Find the values of x where the graph of $y = (x^3 - 4)^5$ has a horizontal tangent.
10. Find an equation of the tangent to $x = (t^{-3} + 4t)^2$, $y = (t^4 - t^{-4})^2$ when t = 1.
11. Find $[f \circ g]'(2)$ if $f(u) = u^2 - 1$, $g(2) = 3$, and $g'(2)$
12. Find an equation of the tangent to the graph of $y = x \sin x$ at $x = \frac{7\pi}{6}$.
13. Differentiate both sides of the identity $\sin 2x = 2 \sin x \cos x$ to get an identity fo $\cos 2x$

AKA