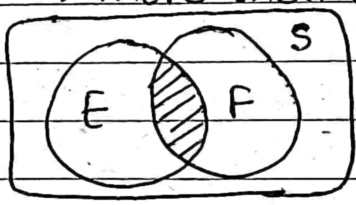
- In probability, events are the outcomes of an experiment. The probability of an event is the measure of chance that the event will occur as a result of an experiment.

- A sample space is a collection or a set of possible outcomes of a random experiment while an event is the subset of sample space

- If a die is rolled the sample space is {1,2,3,4,5,6} and event of getting an even numbo will be {2,4,6}

$$P(E) = \frac{n(E)}{n(S)}$$
↑
of a single event

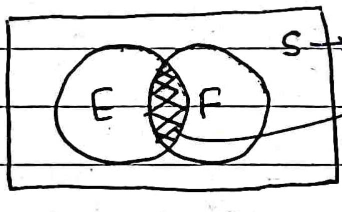
Compound event:
→ min two events
→ more than two events



something is common b/w event E & F

Compound Events:

Case I:



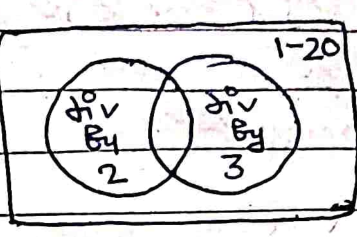
S → sample space
→ E and F, $E \cap F$ {both occur} or common region

$E = \{2, 4, 6\}$ $F \{4, 8, 9\}$
$E \cap F = \{4\}$

$$P(E \cap F) \Rightarrow \frac{\text{no of favourable region}}{\text{total region}} \Rightarrow \frac{n(E \wedge S)}{n(S)}$$

eg 1-20 digits

div by 2 (E) ; div by 3 (F)



or P(div. by 2 and 3)?

div by 2	2	4	6	8	10	12	14	16	18	20
div by 3	3	6	9	12	15	18				

$n(2 \wedge 3) = n$(nos of common elements) $= 3$

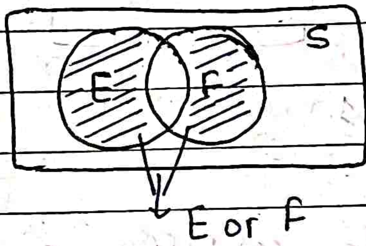
$P(2 \wedge 3) = 3/20.$

Case II. Something is common

E or F

Either E or F

$E \cup F$



$P(E \cup F) = \dfrac{n(E \cup F)}{n(S)}$; $n[E \cup F] = n[E] + n[F] - n[E \wedge F]$.

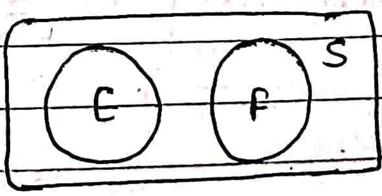
$P(E \cup F) = \dfrac{n(E) + n(F) - n(E \wedge S)}{n(S)} \Rightarrow \dfrac{n(E)}{n(S)} + \dfrac{n(F)}{n(S)} - \dfrac{n(E \wedge F)}{n(S)}$

or $\boxed{P(E) + P(F) - P(E \wedge F)}$

If nothing is common:

$\boxed{E \wedge F = 0}$

$P(E \vee F) = P(E) + P(F)$



E & F are "Mutually Exclusive" or Disjoint events.

$\therefore P(2 \text{ or } 3) = P(2) + P(3) - P(2 \text{ and } 3)$

$$\frac{10}{20} + \frac{6}{20} - \frac{3}{20} \Rightarrow \frac{13}{20}$$

Case III. Dependent / Independent events:

(A) without replacement. / dependent events-

3 yellow
2 white
2 blue
4 red

P(4 red ball drawn at random one at a time)

$\longrightarrow R_1 : 4/11$

***

$\frac{R_2}{R_1}$ → R_1 is occured & R_2 is occuring

3y
2w
2b
3r

$\longrightarrow R_2 : P\left(\frac{R_2}{R_1}\right) = \frac{3}{10}$

$\longrightarrow R_3 : P\left(\frac{R_3}{R_1, R_2}\right)$ or

$P\left(\frac{R_3}{R_1 \cap R_2}\right) = \frac{2}{9}$

$\longrightarrow R_4 : P\left(\frac{R_4}{R_1 \cap R_2 \cap R_3}\right) = \frac{1}{8}$

~~All together~~

$$\underbrace{P(R_1) \cdot P\left(\frac{R_2}{R_1}\right) \cdot P\left(\frac{R_3}{R_1 \cap R_2}\right) \cdot P\left(\frac{R_4}{R_1 \cap R_2 \cap R_3}\right)}_{\text{dependent events}} = P(R_1 \wedge R_2 \wedge R_3 \wedge R_4)$$

general form : $P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right)$

when A and B are dependent events.

(B) independent events & with replacement

with replacement — 3y, 3r

P(3 red ball is drawn one at a time) $= ?$

3y, 3r $\xrightarrow{\text{Ist draw}}$ $P(R_1) = \frac{3}{6}$

3y, 3r $\xrightarrow{\text{IInd draw}}$ $P\left(\frac{R_2}{R_1}\right) = \frac{3}{6}$

3y, 3r $\xrightarrow{\text{IIIrd draw}}$ $P\left(\frac{R_3}{R_1 \cap R_2}\right) = \frac{3}{6}$

$P(R_1 \cap R_2 \cap R_3) = P(R_1)\, P(R_2/R_1)\, P(R_3/R_1 \cap R_2)$

$P\left(\frac{R_2}{R_1}\right) = P(R_2)$; R_1 is not influenced

$P\left(\frac{R_3}{R_1 \cap R_2}\right) = P(R_3)$: R_3 is not influenced by $R_1 \cap R_2$

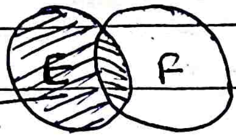
$P(R_1 \cap R_2 \cap R_3) = P(R_1) \cdot P(R_2) \cdot P(R_3)$.

general form : $\underbrace{P(A \cap B) = P(A) \cdot P(B)}$

when A & B are independent events.

(c) P(only E), P(only F), P(exactly one)

E and F are independent events



only E

$P(\text{only } E) = P(F) - P(E \cap F)$

or

$P(\text{only } E) = P(E \cap \bar{F})$

$P(A) + P(\bar{A}) = 1$

Examples:

① A integer is chosen random from 200 positive integers. find the probability that the integer chosen is divided by 6 or 8?

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\left\{ \frac{200}{6} \to 33 \quad \frac{200}{8} \to 25 \right\}$

P(div by 6) $\Rightarrow \frac{33}{200}$

P(div by 8) $\Rightarrow \frac{25}{200}$

P(6 and 8) $\Rightarrow \frac{8}{200}$

$\Rightarrow \frac{50}{200} \to 1/4$.

$$\begin{array}{r|l} 2 & 6, 8 \\ 2 & 3, 4 \\ & 3, 2 \end{array}$$

$\Rightarrow$ 24 also divisible by 24

$\frac{200}{24} \Rightarrow 8$

② Two cards are drawn from a pack of 52 cards. what is the probability that either of the cards, both are red or both are kings?

52 cards → Red (26), black (26)

Red (26) → Heart (13), Diamond (13)

black (26) → Spade (13), Club (13)

Heart (13)	Diamond (13)	Spade (13)	Club (13)
♡	◇	♣	♠

13 cards → Face Card, Non-face Card

Face Card:
→ King
→ Queen
→ Jack

$\underbrace{\text{King, Queen, Jack}}_{3\times4 \Rightarrow 12}$

Non-face Card:
A, 2, 3, 4
5, 6, 7, 8
9, 10

$\underbrace{A,2,\dots,10}_{10\times4 \Rightarrow 40}$

P(red or king) → $P(R \cup K) \rightarrow P(R) + P(K) - P(R \cap K)$

without replacement

$\boxed{\frac{26R}{52C}} \rightarrow 2R \quad \frac{26}{52} \times \frac{25}{51}$

$\boxed{\frac{4K}{52C}} \rightarrow 2K \quad \frac{4}{52} \times \frac{3}{51}$

red & king
↓
4 red, 2

$\boxed{\frac{2}{52C}} \rightarrow \frac{2}{52} \times \frac{1}{51}$

$P(R) = \frac{^{26}C_2}{^{52}C_2} \quad P(K) = \frac{^{4}C_2}{^{52}C_2} \quad P(R \cap K) = \frac{^{2}C_2}{^{52}C_2}$

(3) 3 person A, B, C independently try to hit a target, if prob. of hitting a target by A, B, C are 3/4, 1/2, 5/8 then the prob. of target hit by A or B but not C–

P(target hit by $\underbrace{\text{A or B}}$ but $\underbrace{\text{not C}}$)

$A \cup B \cap \bar{C}$

A B C

$\underbrace{\quad\quad}$

Independent / with replacement.

↓

$P(A \cup B \cap \bar{C}) = P(A \cup B) \cdot P(\bar{C})$

$1 - \frac{5}{8}$

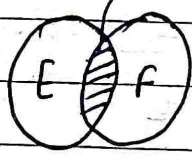
$\Rightarrow 3/8$

$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) \rightarrow P(A) \cdot P(B)$

$\Rightarrow \frac{3}{4} + \frac{1}{2} - \left(\frac{3}{4} \times \frac{1}{2}\right)$

(4) Let E and F be two independent events. The probability of both E and F happens is 1/12 and niether E or F happens is 1/2 then the value $\frac{P(E)}{P(F)} = ?$



both happen = 1/12

Independent event

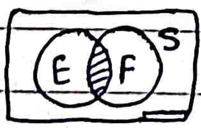
P(both happen) = 1/12

P(niether E nor F) = 1/2

$P(\bar{E} \wedge \bar{F}) = 1/2$

$P(E) \cdot P(F) = 1/12$ —— ①

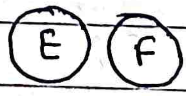
Note: Something is common



Let $P(E) = x$

$P(F) = y$

Something isn't common



$P(E) = x$

$P(F) = 1 - x$

$P(\bar{E})\,P(\bar{F}) = 1/2$

$(1-x)(1-y) = 1/2$ — ②

$1 - y - x + xy = 1/2$

from ① $x \cdot y = 1/12$

$1 - y - x + \frac{1}{12} = \frac{1}{2} \Rightarrow x + y = 1 - \frac{1}{2} + \frac{1}{12} \Rightarrow x + y = \frac{14}{24}$ or $\frac{7}{12}$

$xy = 1/12$ — ①

$x + y = 7/12$ — ②

$x - y = \sqrt{(x+y)^2 - 4xy}$

$\sqrt{\left(\frac{7}{12}\right)^2 - 4\left(\frac{1}{12}\right)} \Rightarrow \sqrt{\frac{1}{144}}$

$\Rightarrow 1/12$ — ③

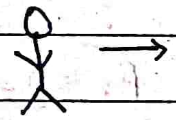
using ② & ③ $\rightarrow x = 1/3,\ y = 1/4$

$$\frac{P(E)}{P(F)} = \frac{x}{y} = \frac{\frac{1}{3}}{\frac{1}{4}} \Rightarrow \frac{4}{3}$$

★ **Mutually Exclusive Events:** nothing is common.

→ E and F are disjoint event

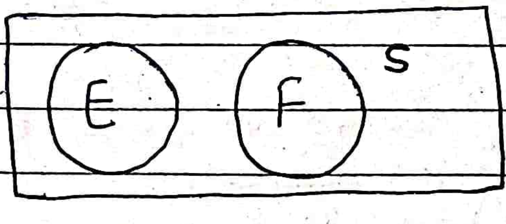
→ E and F don't occur simultaneously



win/Loose

person

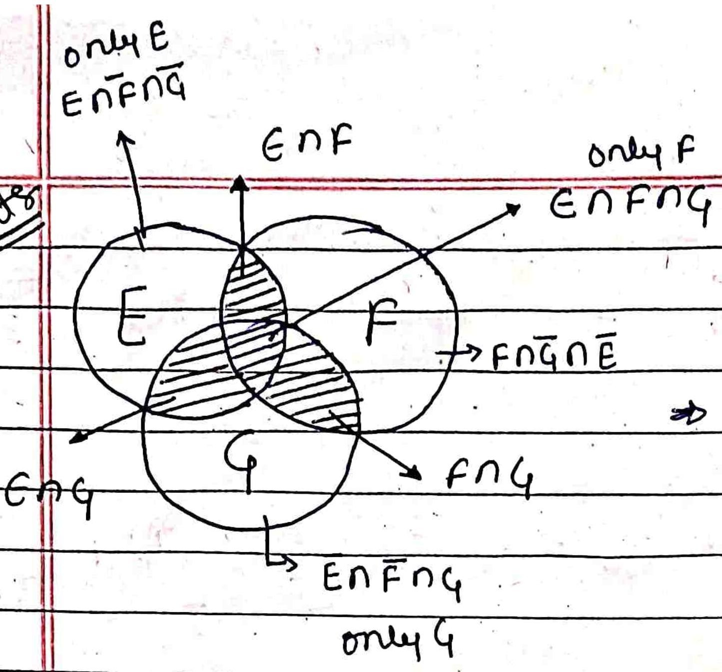
[both at same time - not possible].



• $n(E \cap F) = 0$ or $n(E \cap F) = \phi$ (null-set)

• $P(E \text{ or } F) \Rightarrow P(E \cup F) \Rightarrow P(E) + P(F) - \cancel{P(E \cap F)}^{0}$

for 3 events



P(E or F or G)
$= P(E \cup F \cup G)$

$\Rightarrow P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G) - P(E \cap G) + P(E \cap F \cap G)$

E → F → G $= E \cap G \cap F$ all E, G, F
E → F → $\bar{G}$ $= E \cap F \cap \bar{G}$ E and F, not G
E → $\bar{F}$ → G $= E \cap \bar{F} \cap G$
E → $\bar{F}$ → $\bar{G}$ $= E \cap \bar{F} \cap \bar{G}$
$\bar{E}$ → F → G $= \bar{E} \cap F \cap G$
$\bar{E}$ → F → $\bar{G}$ $= \bar{E} \cap F \cap \bar{G}$
$\bar{E}$ → $\bar{F}$ → G $= \bar{E} \cap \bar{F} \cap G$
$\bar{E}$ → $\bar{F}$ → $\bar{G}$ $= \bar{E} \cap \bar{F} \cap \bar{G}$

(5) Let S be the sample space with two mutually exclusive events A and B and $A \cup B = S$. If P denotes prob. of events then the maximum value of $P(A) \cdot P(B) = ?$

P(S) or P(sure event) = 1

$P(A \cup B) = P(S)$
$P(A) + P(B) = 1$
let, $P(A) = x$, $P(B) = 1 - x$

$\delta = P(A) \cdot P(B) \Rightarrow x(1-x)$ ⟶ $\frac{1}{2}\left(1 - \frac{1}{2}\right) \Rightarrow \frac{1}{2} \times \frac{1}{2}$
$\Rightarrow \frac{1}{4}$

$\frac{\delta \delta}{\delta x} \Rightarrow$ ~~...~~ ~~0~~ $1 - 2x = 0$, $x = 1/2$, $\frac{\delta^2 \delta}{\delta x^2} \Rightarrow$ ~~...~~ -2 neg. (max value) at $x = 1/2$

$\frac{a+b}{2}$ $\sqrt{ab}$

inequality, $AM \geq GM$

$$\frac{a+b}{2} \geq \sqrt{ab} \Rightarrow \frac{P(A)+P(B)}{2} \geq \sqrt{P(A)\cdot P(B)}$$

$$\frac{1}{2} \geq \sqrt{P(A)\cdot P(B)}$$

↓ sq. both side

$$\Rightarrow \frac{1}{4} \geq P(A)\cdot P(B)$$

Conditional Probability :

3 red, 2 white, 1 blue

$\xrightarrow[\text{1st Red}]{}$ $P(R_1) = \frac{3}{6}$ — R_1 is already occured

$\xrightarrow[\text{2nd red}]{}$ $P\left(\frac{R_2}{R_1}\right) = \frac{2}{5}$ — $\frac{R_2}{R_1}$ is occuring

both happen,

$$P(R_1 \cap R_2) = P(R_1)\cdot P\left(\frac{R_2}{R_1}\right)$$

conditional probability $\left\{ \boxed{P\left(\frac{R_2}{R_1}\right) = \frac{P(R_1 \cap R_2)}{P(R_1)}} \right.$

for dependent → $P(A \cap B) = P(A)\cdot P\left(\frac{B}{A}\right)$

for independent → $P(A \cap B) = P(A)\cdot P(B)$

Q) One ticket is selected at random from 100 tickets 00, 01, 02, ... 99. Suppose A and B are the sum & product of digits found on ticket. Then $P\left[\frac{A=7}{B=0}\right]$ is

$$P\left[\frac{A=7}{B=0}\right] = \frac{P(A=7 \cap B=0) \to 2}{P(B=0)}$$

$\downarrow$

$P(\text{product}=0) \to 19$

00, 01, 02, 03, 04, 05, 06, 07, 08, 09, 10

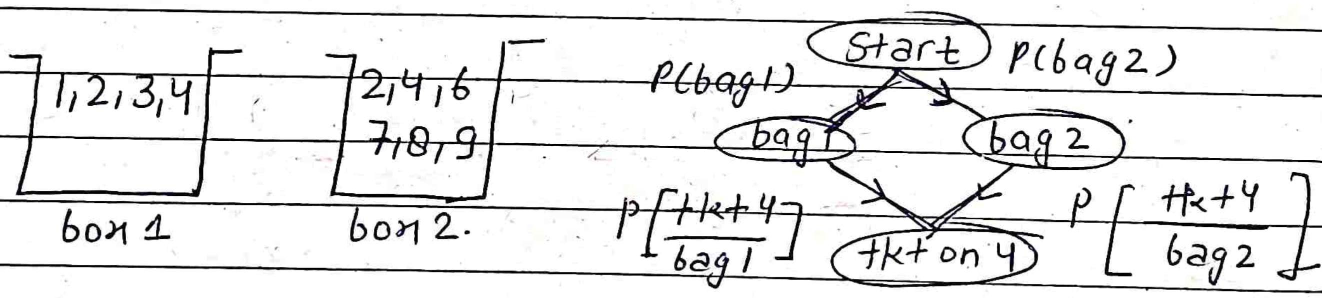
20, 30, 40, 50, 60, 70, 80, 90

$\Rightarrow 19$

Sum = 7 And prod = 0 $\Rightarrow \frac{2}{19}$.

07, 70 $\longleftrightarrow$ 19 items

Q). A bag contains 4 tickets numbers (1,2,3,4) and another bag contains 6 ticket numbers (2,4,6,7,8,9) One bag is chosen & ticket is drawn. The probability that the ticket bears the number 4 is ____

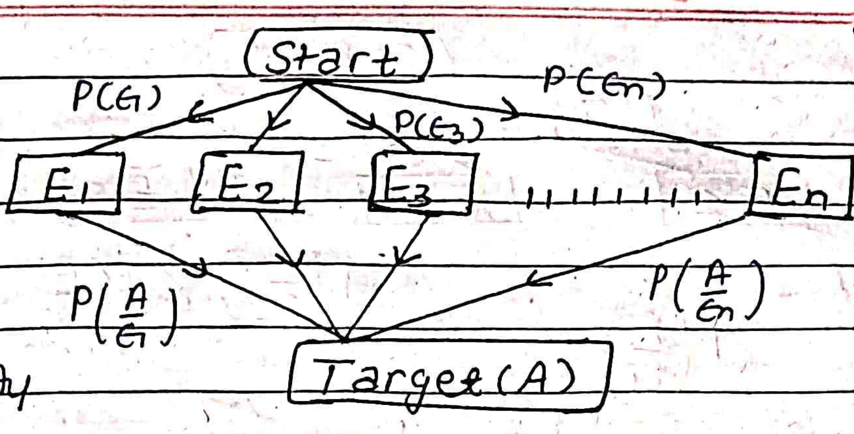


$$P[\text{Tkt } 4] = P(\text{bag}1) \cdot P\left(\frac{\text{Tkt } 4}{\text{bag}1}\right) + P(\text{bag}2) \cdot P\left(\frac{\text{Tkt } 4}{\text{bag } 2}\right)$$

$$\Rightarrow \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{6}$$

$$\Rightarrow \frac{1}{8} + \frac{1}{12} \Rightarrow \frac{5}{24}$$

★ general form :



mutually exclusive events

A is already occured

A is Occuring

$\frac{A}{E_1}$

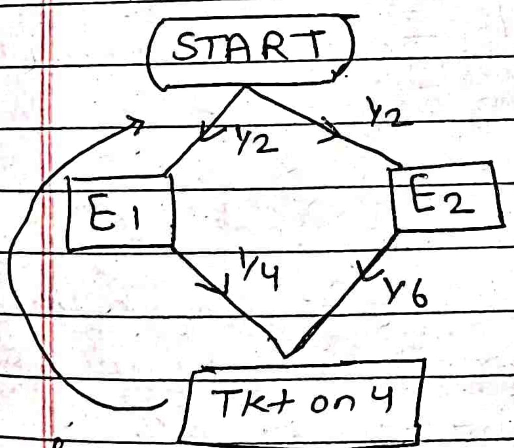
$$\boxed{P(A) = P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + \ldots + P(E_n)\,P\left(\frac{A}{E_n}\right)}$$

possible Ques

→ P(Tkt on 4) ⇒ $\frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{6} \Rightarrow 5/24$

→ P(Tkt on 4 from bag 1) ⇒ ?

→ P(Tkt on 4 from bag 2) ⇒ ?



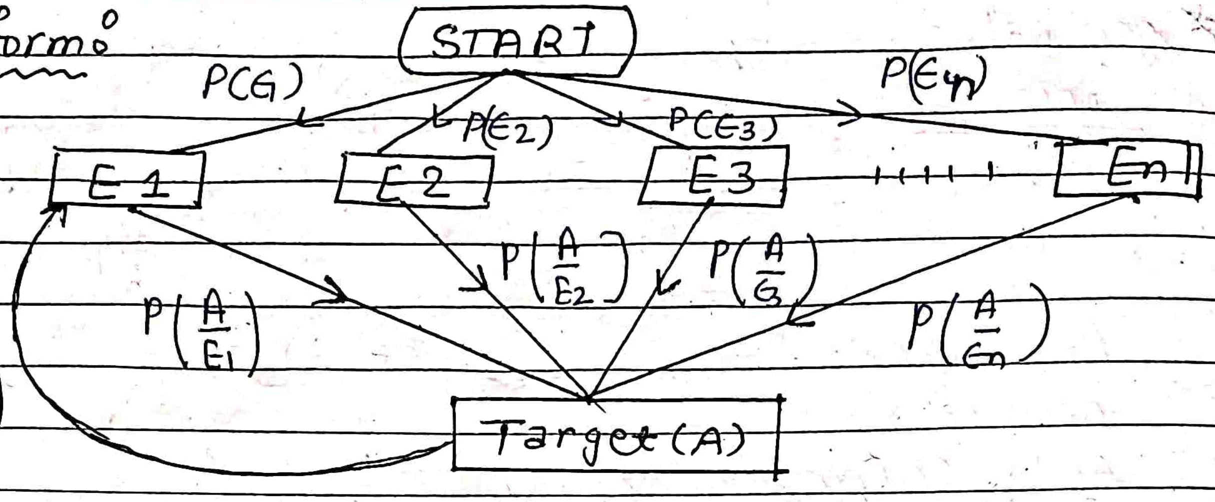
favourable part

$$P\left(\frac{\text{bag } 1}{\text{Tkt on } 4}\right) \Rightarrow \frac{\frac{1}{2} \times \frac{1}{4}}{\frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{6}}$$

$$P\left(\frac{\text{bag } 2}{\text{Tkt on } 4}\right) \Rightarrow \frac{\frac{1}{2} \times \frac{1}{6}}{\frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{6}}$$

Baye's Theorem

★ general form:



Total probability,

$$P(A) = P(\text{target}) = P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right) + \ldots + P(E_n)\cdot P\left(\frac{A}{E_n}\right)$$

$P(E_1)$ → has occured, $P\left(\frac{A}{E_1}\right)$ → A is occuring E_1 has occured, $P\left(\frac{E_1}{A}\right)$ E_1 occuring given A {inverse probability}

$$\boxed{P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{\sum_{i=1}^{n} P(E_i)\, P\left(\frac{A}{E_i}\right)}}$$

Bayes Theorem

• Conditional probability: The possibility of an event or outcome happening, based on the existence of a previous event or outcome. It's calculated by multiplying the probability of the succeding or conditional event.

• **Bayes Theorem:** Let $E_1, E_2, \ldots E_n$ a set of events associated with a sample space S, where all the events E_1, $E_2, E_3, \ldots E_n$ have non-zero probability of occurrence and they form a partition S. Let A be any event associated with S, then.

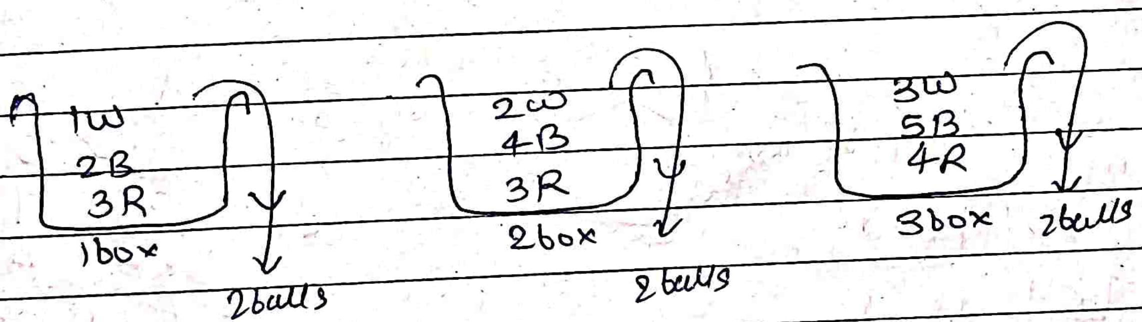
$$\boxed{P\left(\frac{E_i}{A}\right) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{K=1}^{n} P(E_K) \cdot P(A/E_K)}}$$

for $k = 1, 2, 3, \ldots, n$

Q). Three boxes B_1, B_2, B_3 contains balls

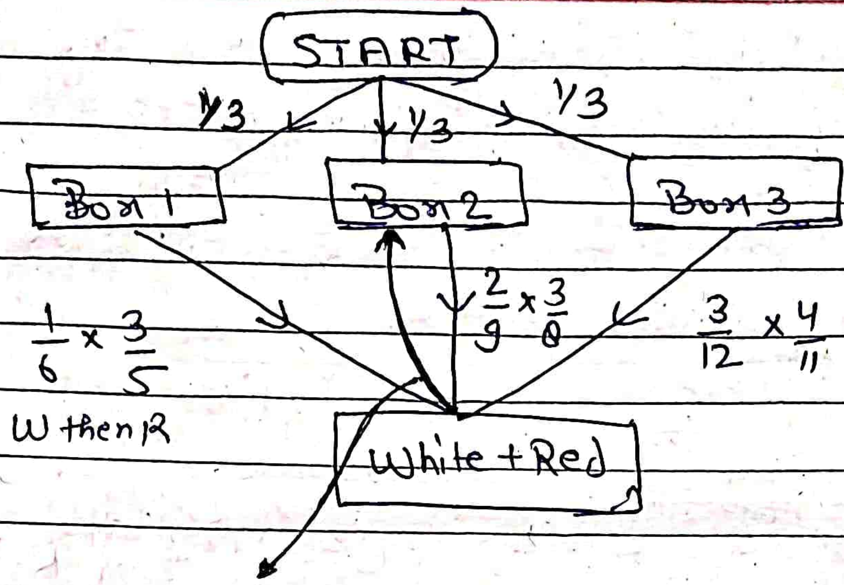
$B_1 \to 1W, 2B, 3R$
$B_2 \to 2W, 4B, 3R$
$B_3 \to 3W, 5B, 4R$

without replacement, if 2 balls are drawn from randomly selected box. Find the probability one of the ball drawn is white & other ball is red from box 2 [order is specified.



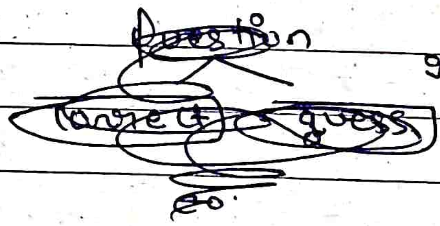
$\xrightarrow{1W}$ $P(W) = \frac{1}{6}$ $\xrightarrow{1W}$ $P(W) = 2/9$ $\xrightarrow{1W}$ $P(W) = 3/12$

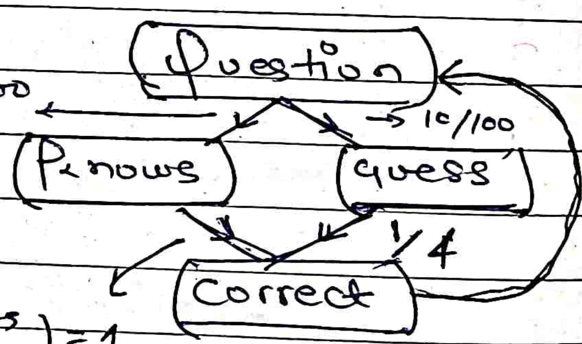
$\xrightarrow{1R}$ $P\left(\frac{R}{W}\right) = \frac{3}{5}$ $\xrightarrow{1R}$ $P\left(\frac{R}{W}\right) = 3/8$ $\xrightarrow{1R}$ $P\left(\frac{R}{W}\right) = \frac{4}{11}$



$$\frac{\text{fav}}{\text{total}} \Rightarrow \frac{\frac{1}{3} \times \frac{2}{9} \times \frac{3}{8}}{\frac{1}{3} \times \frac{1}{6} \times \frac{3}{5} + \frac{1}{3} \times \frac{2}{9} \times \frac{3}{8} + \frac{1}{3} \times \frac{3}{12} \times \frac{4}{11}}$$

Q). In the entrace test there are mcq, there are 4 possible answers to each questions of which one is correct. The probability that a student knows the answer of the question is 90%. If he gets the correct answer to the question then the probability that he was guessing is ___





A B

C D

$P\left(\frac{\text{knows}}{\text{answer}}\right) = 1$

knows → 1

guess → 1/4

$$P\left[\frac{\text{guess}}{\text{correct}}\right] \Rightarrow \frac{\frac{10}{100} \times \frac{1}{4}}{\frac{10}{100} \times \frac{1}{4} + \frac{90}{100} \times 1}$$

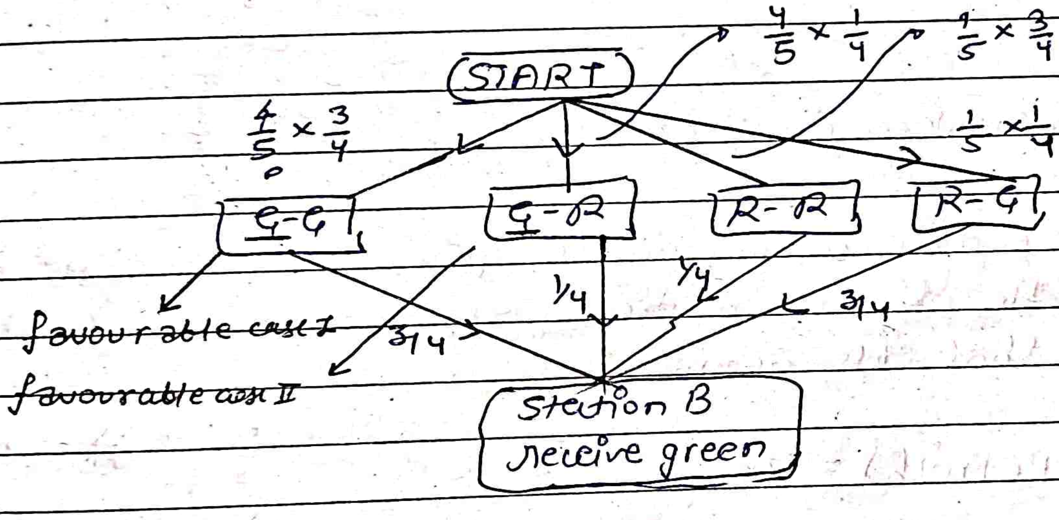
Q. A signal which can be green or red with probability 4/5 & 1/5 respectively, is received by station A and then to station B. The prob. of each ~~signal~~ station receiving the signal correctly is 3/4. If the signal received at station B is given, then prob. that the original signal was given is ___

$P(green) = \frac{4}{5}$ $\quad P(red) = \frac{1}{5}$ $\quad P(correct) = \frac{3}{4}$

Original source $\longrightarrow$ Station A (received) $\longrightarrow$ Station B (received)

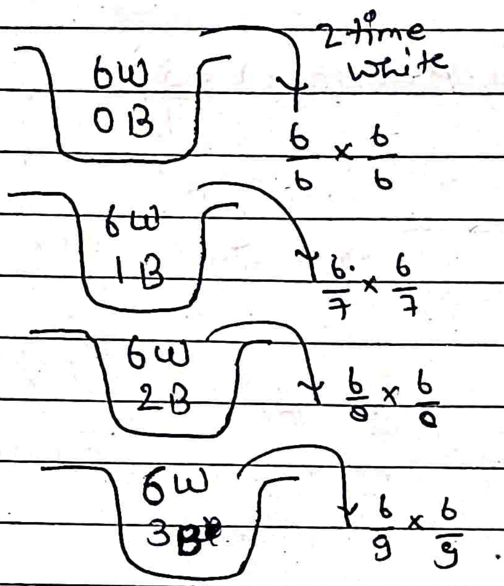
$$\left.\begin{array}{l} G \longrightarrow G \longrightarrow G \\ G \longrightarrow R \longrightarrow R \\ R \longrightarrow R \longrightarrow R \\ R \longrightarrow G \longrightarrow G \end{array}\right\} \text{4 possible cases}$$

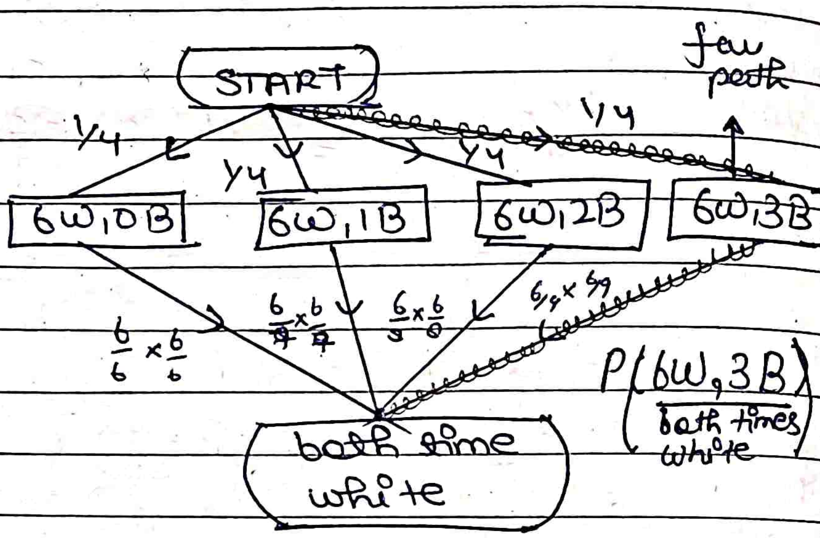
Station B green



$$\frac{\frac{4}{5} \times \frac{3}{4} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{4} \times \frac{1}{4}}{\text{Total}}$$

Q). A bag contains 6 white & an unknown number of black balls (≤ 3) Balls are drawn one by one with replacement from the bag twice & is found to be white on both occasions
find, the probability that the bag had exactly '3' black balls

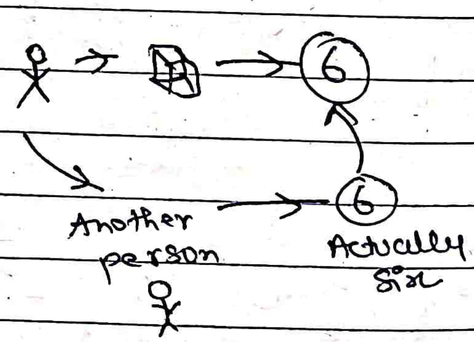




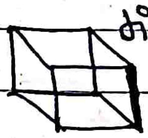
$$\frac{\frac{1}{4} \times \frac{6}{9} \times \frac{6}{9}}{\text{total}}$$

★ problem based on truth :

Q) A man is known to speak truth 3 out of 4 times He throws a die & report it is six, the prob. that it's actually six,

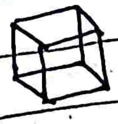


P(truth) = 3/4



1,2,3,4,5,6

$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$



$\{1,2,3,4,5,6\}$ $P(\text{correct}) = p$

$P(\text{not correct}) = 1-p$

$\{6\}$ ~~truth~~ $\downarrow$ $1/6$

$\{1,2,3,4,5,\}$ lie $\downarrow$ $1-\frac{1}{6} \Rightarrow 5/6$

$\{1,2,3,4,5\} \rightarrow$ particular wrong

w w w w wrong

$1-p$

$1-p/5$ $\quad \frac{1-p}{5}$ $\quad \frac{1-p}{5}$ $\quad \frac{1-p}{5}$ $\quad \frac{1-p}{5}$

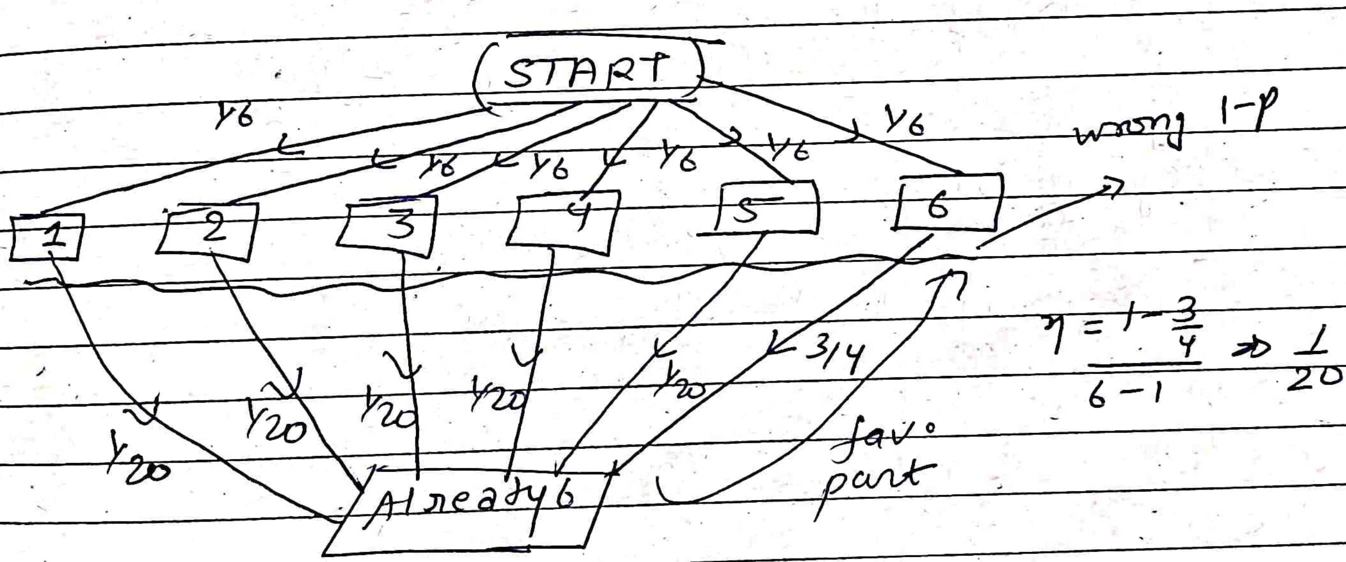
let, p(particular wrong) = x

$p(1) + p(2) + \ldots + p(6) = 1$

then ~~[illegible]~~ $x + x + x + x + x = 1-p$

$x(6-1) = 1-p$

$$x = \frac{1-p}{n-1}$$ particular wrong letter probo



$$x = \frac{1-\frac{3}{4}}{6-1} \Rightarrow \frac{1}{20}$$

$$\frac{\frac{3}{4} \times \frac{1}{6}}{\frac{3}{4} \times \frac{1}{6} + \frac{1}{20} \times 5 \times \frac{1}{6}} \Rightarrow \frac{3}{4}$$