ANT COLONY OPTIMIZATION (ACO)

Ant Colony Optimization (ACO) is a metaheuristic optimization algorithm inspired by the foraging behavior of ants. It was proposed by Marco Dorigo in the early 1990s and has since been widely used to solve combinatorial optimization problems.

The basic idea behind ACO is to simulate the foraging behavior of ants searching for food. Ants communicate with each other by depositing pheromone trails, which they use to guide their movements. Similarly, in ACO, artificial ants construct solutions to optimization problems by probabilistically selecting edges or components of a solution based on pheromone concentrations and heuristic information.

Here's a general overview of how ACO works:

1. **Initialization**: Initialize the pheromone trails and optionally the heuristic information (e.g., distance between nodes in a graph).
2. **Construction of Solutions**: Artificial ants construct solutions by iteratively adding components to a solution based on probabilistic rules. The probability of choosing an edge or component is influenced by the amount of pheromone deposited on it and possibly by heuristic information.
3. **Pheromone Update**: After all ants have constructed solutions, the pheromone trails are updated based on the quality of the solutions found. Typically, stronger solutions result in more pheromone being deposited on the edges or components used in those solutions.
4. **Evaporation**: To avoid stagnation and to allow exploration of the solution space, pheromone trails are subjected to evaporation, which reduces their strength over time.
5. **Termination Criterion**: The algorithm continues iterating through the construction, update, and evaporation steps until a termination criterion is met, such as a maximum number of iterations or convergence to a satisfactory solution.

ACO has been successfully applied to a wide range of combinatorial optimization problems, including the traveling salesman problem, the quadratic assignment problem, the vehicle routing problem, and many others. It is particularly effective for problems where the search space is large and discrete. Additionally, ACO can be adapted and extended in various ways to improve its performance, such as introducing local search procedures or incorporating parallelism.

HK Lam

Ant colony optimization

Ants live in complex social colonies, with the queen being leader and the workers foraging for food and protecting their home.

Ant colony describes.

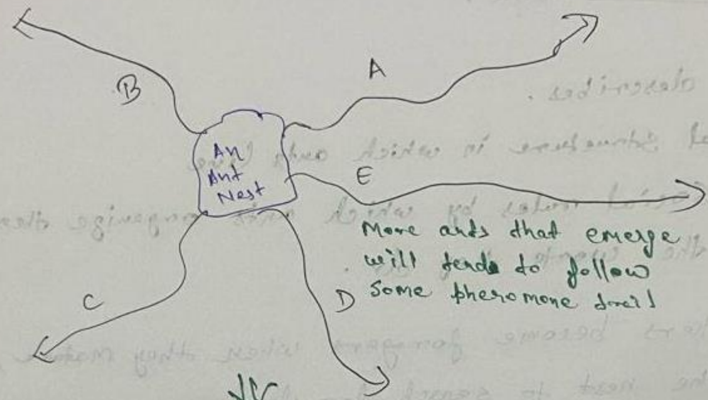
- Physical structure in which ants live
- The social rules by which ants organize themselves and the work they do.

Some workers become forgers when they mature, leaving the nest to search for food.

This cooperation and division of labour, combined with their well-developed communication systems, has allowed ants to utilise their environment in ways approached by few other animals

Ants foraging for food

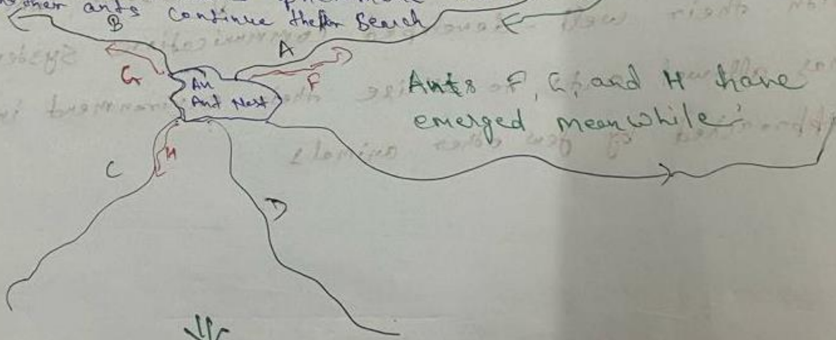
- let 5 ants A, B, C, D, and E go out in search for food.
- Each ant lays a trail of pheromone where it goes.



More ants that emerge will tends to follow some pheromone trail

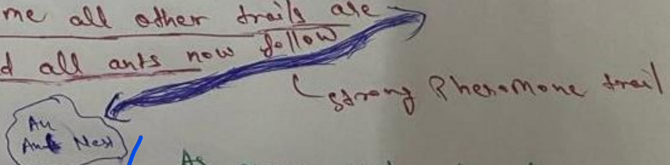
Ants A finds some food.

- Let ant A find some food. It grabs it and dutifully head backs
- It follows its own pheromone trail back
- And other ants continue their search



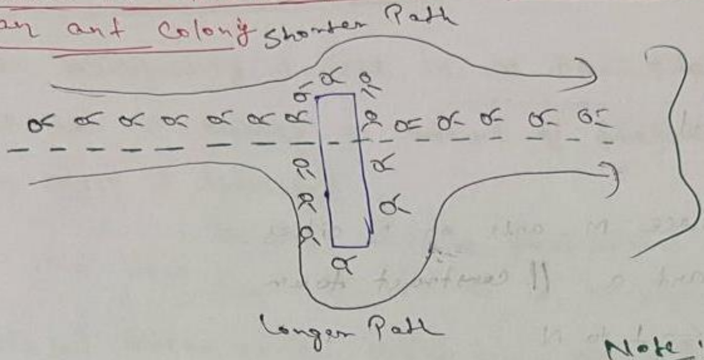
Ants F, G, and H have emerged meanwhile.

After some time all other trails are abandoned and all ants now follow the trail formed by Ant A.



As more ants travel on the trail they deposit more pheromone and the trail gets stronger and stronger, eventually becoming the caravan you might have see raiding your food.

when an obstacle is placed on the pheromone trail of an ant colony



They quickly finds the shortest diversion around it.

⇓

Note:-

Shorter Path Ants foraging for food will done faster

⇓

They will return fastly to their home depositing pheromones and increasing pheromones intensity.

⇓

Longer Path will disappear after few time

Ant colony optimization (ACO)

- It is an optimization method that takes inspiration by the bio-semiotic communication between ants.
- Each ant constructs a solution using a stochastic greedy method using a combination of a heuristic function and pheromone trail following.
- Related to the class of algorithm known as swarm optimization
- Applicable to problems that ~~redue~~ can be reduced to graph search
- Presented by Macro Dorigo in 1992

ACO Algorithm

```
TSP_ACO()
best Tour <- Nil
repeat
    randomly place M ants on N cities.
    for each ant a            // construct tour
        for n <- 1 to N
            ant a selects an edge from distribution P_n^a
    update best tour
    for each ant a            // update pheromone
        for each edge (u, v) in the ant's tour
            deposit pheromone ∝ 1/tour-length on edge (u,v)
until some termination criteria
return best Tour
```

Ant Colony optimization for TSP.

On a city i the k^{th} ant moves to city j with a probability given by:

$$P_{ij}^{K}(t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^{\alpha} * [\eta_{ij}]^{\beta}}{\sum_{h \in allowed_K(t)} \left([\tau_{ih}(t)]^{\alpha} * [\eta_{ih}]^{\beta} \right)} & \text{if } j \in allowed_K(t) \\ 0 & \text{otherwise} \end{cases}$$

valid tours for ant_K to move at time t. the cities ant_K is allowed to move to.

where, $\tau_{ij}(t)$ is pheromone on edge ij

η_{ij} is called visibility which is inversely proportional to the distance between cities i and j.

α :- ~~It is~~ component / Parameter used to control the effect of pheromone. ~~on edge~~ (How much importance we want to give Pheromone)

β :- It is Parameter used to control the effect of distance between cities i and j (How much importance we want to give distance)

ACO : updating Pheromone

- After constructing a tour in n time steps
- Each ant K deposits an amount of pheromone Q/L_K on the edges it traversed
- which is ~~directly~~ inversely proportional to the cost of the tour L_K it found. Q is an appropriate constant.
- Total Pheromone deposited on edge$_{ij}$ is :

$$\Delta \tau_{ij}(t, t+n)$$

Note:

$$\Delta \tau_{ij}^{K}(t) = \begin{cases} \frac{Q}{L_K(t)}, & \text{if } (i,j) \in T_K(t) \\ & \text{if } (i,j) \notin T_K(t) \end{cases}$$

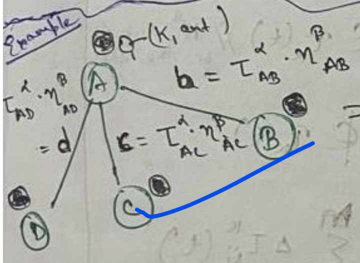
- The total pheromone on edge$_{ij}$ is updated as

$$\tau_{ij}(t+n) = (1-\rho) * \tau_{ij}(t) + \Delta \tau_{ij}(t, t+n)$$

where ρ is the rate of evaporation of pheromone.

$$\sum_{K=1}^{h} \Delta \tau_{ij}^{K}(t)$$

Example



$$\Rightarrow P_{AB}^{K_1}(t) = \frac{\tau_{AB}^{\alpha}(t) * \eta_{AB}^{\beta}}{\tau_{AB}^{\alpha}(t) * \eta_{AB}^{\beta} + \tau_{AC}^{\alpha}(t) * \eta_{AC}^{\beta} + \tau_{AD}^{\alpha}(t) * \eta_{AD}^{\beta}}$$

$$P_{AB}^{K_1}(t) = \frac{b}{b+c+d} \quad (\text{say } 0.2)$$

$$P_{AC}^{K_1}(t) = \frac{c}{b+c+d} \quad (\text{say } 0.3)$$

$$P_{AD}^{K_1}(t) = \frac{d}{b+c+d} \quad (\text{say } 0.5)$$

Note :

$$P_{A \text{ to All next valid cities}}^{K_1}(t) = P_{AB}^{K_1}(t) + P_{AC}^{K_1}(t) + P_{AD}^{K_1}(t) = 1$$

$$= 0.2 + 0.3 + 0.5 = 1.$$

Note :-

Ant Transition Probabilistic Selection

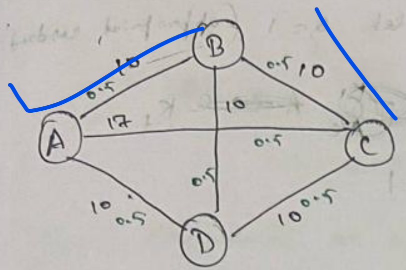
$$P_{ij}^{K}(t) = \begin{cases} \dfrac{[\tau_{ij}^{K}(t)]^{\alpha} [\eta_{ij}^{K}]^{\beta}}{\sum\limits_{m \in \text{allowed city}_K(t)} [\tau_{im}^{K}(t)]^{\alpha} \cdot [\eta_{im}^{K}]^{\beta}} & \text{if } j \in \text{allowed}_K(t) \\ 0 & \text{Otherwise} \end{cases}$$

allowed$_K(t)$ — the cities and K is allowed to move to

Pheromone updation

$$\Delta\tau_{ij}^{K}(t) = \begin{cases} \dfrac{Q}{L_K(t)} & \text{if } (i,j) \in T_K(t) \quad // \; K^{th} \text{ ant travels on the edge } i,j \\ 0, & \text{if } (i,j) \notin T_K(t) \end{cases}$$

$$\tau_{ij}(t+1) = (1-\rho) \cdot \tau_{ij}(t) + \sum_{K=1}^{m} \Delta\tau_{ij}^{K}(t)$$



$K=1$

	route	length	Phero mone
K_1	A B D C A	47	$4/47 = 0.085$
K_2	B D A C B	47	$4/47 = 0.085$
K_3	C D A B C	40	$4/40 = 0.100$

1) Initially 0.5 Pheromone for each Path.

2) 36%. Pheromone evaporated ($\rho = 0.36$).

$$\text{Remaining Pheromone} = (1-\rho) \cdot \tau_{ij}(t)$$

$$= (1-0.36)\, \tau_{ij}(t)$$

$$= 0.64 \cdot \tau_{AB}(t) \Rightarrow 0.64 * 0.5 = 0.32$$

for route [ABDCA]

$$\tau_{ij}(t+1) = (1-\rho) \cdot \tau_{ij}(t) + \sum_{K=1}$$

$K_1 = A \quad \alpha = 1, \beta = 1$

$$P^{K_1}_{AB} = \frac{\tau^{\alpha}_{AB} \cdot \eta^{\beta}_{AB}}{\tau^{\alpha}_{AB} \cdot \eta^{\beta}_{AB} + \tau^{\alpha}_{AD} \cdot \eta^{\beta}_{AD} + \tau^{\alpha}_{AC} \cdot \eta^{\beta}_{AC}} = \frac{(0.5)^1 \cdot (10)^1}{(0.5)^1 \cdot (10)^1 + (0.5)^1 \cdot (10)^1 + (0.5)^1 \cdot (17)^1}$$

$$= \frac{5}{18.5} = 0.270.$$

$$P^{K_1}_{AD} = \frac{\tau^{\alpha}_{AD} \cdot \eta^{\beta}_{AD}}{\tau^{\alpha}_{AB} \cdot \eta^{\beta}_{AB} + \tau^{\alpha}_{AD} \cdot \eta^{\beta}_{AD} + \tau^{\alpha}_{AC} \cdot \eta^{\beta}_{AC}} = \frac{(0.5)^1 (10)^1}{18.5} = \frac{5}{18.5} = 0.270$$

$$P^{K_1}_{AC} = \frac{\tau^{\alpha}_{AC} \cdot \eta^{\beta}_{AC}}{\tau^{\alpha}_{AB} \cdot \eta^{\beta}_{AB} + \tau^{\alpha}_{AD} \cdot \eta^{\beta}_{AD} + \tau^{\alpha}_{AC} \cdot \eta^{\beta}_{AC}} = \frac{(0.5)^1 \cdot (17)^1}{18.5} = \frac{8.5}{18.5} = 0.4594$$

$\boxed{K_1 = A - C}$

$$P^{K_1}_{CB} = \frac{\tau^{\alpha}_{CB} \cdot \eta^{\beta}_{CB}}{\tau^{\alpha}_{CB} \cdot \eta^{\beta}_{CB} + \tau^{\alpha}_{CD} \cdot \eta^{\beta}_{CD}} = \frac{(0.5)^1 \cdot (10)^1}{(0.5)^1 \cdot (10)^1 + (0.5)^1 \cdot (10)^1} = \frac{5}{10} = 0.5$$

$$P^{K_1}_{CD} = \left(\frac{\tau_{CD} \cdot \eta_{CD}}{\tau^{\alpha}_{CB} \cdot \eta^{\beta}_{CB}}\right) = \frac{(0.5)^1 \cdot (10)^1}{10} = \frac{5}{10} = 0.5$$

$K_1 :: A - C - B - D - A \Rightarrow L_{K_1} = 17 + 10 + 10 + 1\ldots$

$K_2 :- C - D - A - B - C \Rightarrow L_{K_2}^{\left(\substack{\text{total} \\ \text{cost}}\right)} = 10 + 10 + 10 + 10 = 40$

<u>Note</u> Evaporation rate $\rho = 0.34$, let $Q = 1$ (appropriate cons...

Pheromone deposited on edges $A - C$ K_1

$$\Delta \tau_{AC}^{K_1}(t) = \frac{Q}{L_{K_1}} = \frac{1}{47} = 0.021$$

$$\Delta \tau_{CB}^{K_1} = \frac{Q}{L_{K_1}} = \frac{1}{47} = 0.021$$

$$\Delta \tau_{BD}^{K_1} = \frac{Q}{L_{K_1}} = \frac{1}{47} = 0.021$$

$$\Delta \tau_{DA}^{K_1} = \frac{Q}{L_{K_1}} = \frac{1}{47} = 0.021.$$

$\therefore \sum_{K=1}$

~~Total Pheromone on edges is updated as~~

$$\cancel{\tau_{AC}^{K_1}(t+1) = (1-\rho) \cdot \tau_{AC}^{K_1}(t) + \sum_{K=1}^{h} \Delta \tau}$$

Pheromone deposited on edges by K_2

$$\Delta \tau_{CD}^{K_2}(t) = \frac{Q}{L_{K_2}} = \frac{1}{40} = 0.025$$

$$\Delta \tau_{DA}^{K_2}(t) = \frac{Q}{L_{K_2}} = \frac{1}{40} = 0.025$$

$$\Delta \tau_{AB}^{K_2} = \frac{Q}{L_{K_2}} = \frac{1}{40} = 0.025$$

$$\Delta \tau_{BC}^{K_2} = \frac{Q}{L_{K_2}} = \frac{1}{40} = 0.025$$

$$\cancel{\Delta \tau_{CA}^{K_2} = \frac{Q}{L_{K_2}} = \frac{1}{40} = 0.025}$$

$$\cancel{\Delta \tau^{K_2}}$$

$\cancel{\Delta \tau^{K}_{AC}}$

$$\sum_{K=1}^{M} \Delta \tau^{K}_{BC} = \Delta \tau^{K_1}_{BC} + \Delta \tau^{K_2}_{BC}$$

$$= 0.021 + 0.025$$

$$= 0.046.$$

Total Pheromone on edges is update as.

$$\tau_{BC} = (1-\rho)\,\tau_{BC}(t) + \sum_{K=1}^{M} \Delta \tau^{K}_{BC}$$

$$= (1-0.34)$$

$$= 0.64 \times 0.5 + 0.46$$