

MATH4350

Homework Assignment

Joel Savitz

Wednesday 20 July 2020

3.2-1

TO

Ham 3.2-1

(1) Plutarch says: $8t_n + 1 = k^2 \Rightarrow k \in \mathbb{N}$

(2) $8(t_1) + 1 = 8(1) + 1 = 9 = 3^2 \wedge 3 \in \mathbb{N}$

(3) Assume [1] holds for some $n \in \mathbb{N}$. Then, $8t_n + 1 = k^2$ (even)

(4) $t_n = \frac{1(1+1)}{2} \Rightarrow 8t_n + 1 = 8\left(\frac{1(1+1)}{2}\right) + 1 = 4n^2 + 4n = k^2$

(5) $4n^2 + 4n = k^2 \Rightarrow 4n^2 + 4n + 1 = 4n^2 + 4n + 1 = (2n+1)^2 = k^2 + 4$

(6) honestly just look at the picture

$t_0 = 1 \Rightarrow 8t_0 + 1 = 9 = 3^2$
 $3 \Rightarrow 25$
 $5 \times 5 = 25$

Intuitively, if we let the triangle grow arbitrarily, this proof roughly holds in general (with tweaks).

3.2-4

HM 3.2-4

(a) $56 = t_{10} + t_1 = 55 + 1$

(b) $69 = t_9 + t_6 + t_2 = 45 + 21 + 3$

(c) $185 = t_8 + t_{11} + t_7 = 91 + 66 + 28$

(d) $287 = t_{21} + t_{10} + t_1 = 171 + 106 + 15$

10 1 2 3 4 5 6 7 8 9 10 11 12

3.3-1 and 4

Handwritten notes on a whiteboard:

Hom 3.3-1

(a) (1) Suppose $a, b \in \mathbb{N}$. Then, $4ab + a^2 + b^2 = 4ab + a^2 + b^2$
 (2) Therefore, $4ab + a^2 - 2ab + b^2 = a^2 + 2ab + b^2$
 (3) $4ab + (a-b)^2 = (a+b)^2$
 (4) $\therefore ab + \left(\frac{a-b}{2}\right)^2 = \left(\frac{a+b}{2}\right)^2$
 (b) (5) Let $a = 2n^2 \exists n \in \mathbb{N}$, $b = 2$. Then, $\left(n^2 + \left(\frac{2-1}{2}\right)^2\right)^2 = \left(\frac{2n^2+2}{2}\right)^2$
 (6) $\left(\frac{2n^2+2}{2}\right)^2 = (n^2+1)^2$
 (7) Let $a = (2n+1)^2$, $b = 1$. Then, $\left((n+1)^2 + \left(\frac{(2n+1)-1}{2}\right)^2\right)^2 = \left(\frac{(2n+1)+1}{2}\right)^2$
 (8) $4n^2 + 4n + 1 + \left(\frac{(2n+1)-1}{2}\right)^2 = \left(\frac{(2n+1)+1}{2}\right)^2$
 (9) $4n^2 + 4n + 1 + (n(n+1))^2 = \frac{1}{4} \text{ will be } 2 \text{ then } 1$
 (10) $(2n+1)(n+1)^2 = k^2$ *min bc*

3.3-4

$(x, x+1, x+2)$
 $x^2 + (x+1)^2 = (x+2)^2$
 $2x^2 + 2x + 1 = x^2 + 4x + 4$
 $x^2 - 2x - 1$
 $(x-3)(x-3)$
 $\rightarrow x = 3$