

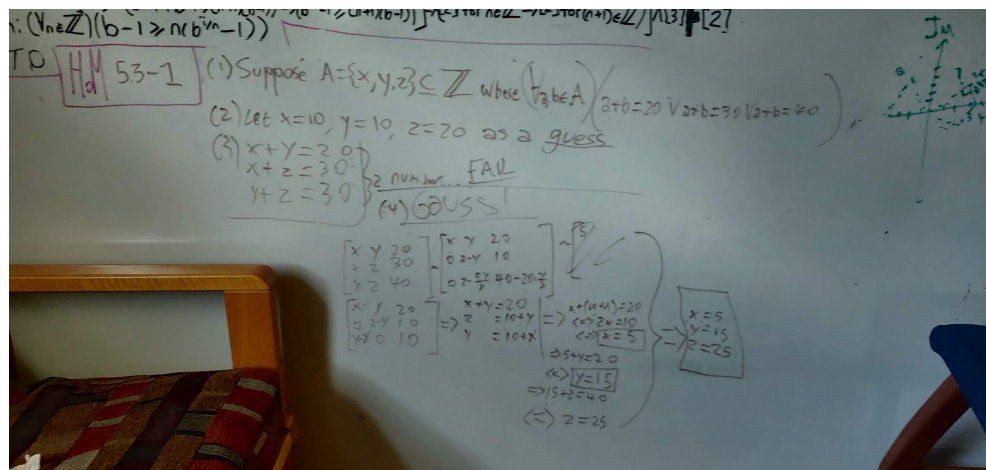
MATH4350

Homework Assignment

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Wednesday 29 July 2020

5.3-1. Book I, Problem 16



5.3-2. Book I, Problem 18

TO HM 5.3-2

(1) Suppose $A = \{x, y, z\} \subseteq \mathbb{Z} \Rightarrow \forall a, b, c \in A, (a+b)-c \in \{20, 30, 40\}$

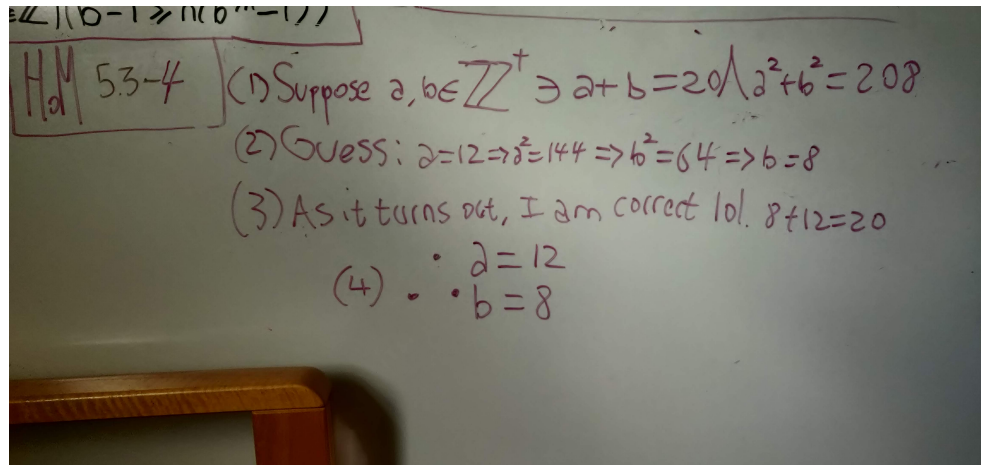
(2) ANS $\begin{bmatrix} x & y & z & 20 \\ x & z & -y & 30 \\ y & z & -x & 40 \end{bmatrix} \begin{bmatrix} x & y & z & 20 \\ 0 & 2y & -4z & 10 \\ y-x & -xy & 10 & \end{bmatrix} \Rightarrow \begin{cases} 2(x-y) = 10 \Leftrightarrow 2-x-y = 5 \Leftrightarrow z = y+5 \\ 2(y-x) = 10 \Leftrightarrow y-x = 5 \Leftrightarrow y = x+5 \\ \Leftrightarrow z = (x+5)+5 = x+10 \end{cases}$

(3) $(x+y)-z = 20 \Rightarrow x + (x+5) - (x+10) = 20 \Leftrightarrow 2x+5-x-10 = 20 \Leftrightarrow x = 25$

(4) $y = x+5 = 25+5 = 30, z = y+5 = 30+5 = 35$

(5) $\begin{aligned} (25+30)-35 &= 55-35 = 20 \\ (25+35)-30 &= 60-30 = 30 \\ (30+35)-25 &= 65-25 = 40 \end{aligned} \quad \checkmark$

5.3-4. Book I, Problem 28



5.3-8. Book III, Problem 12

Claim: $(\forall n \in \mathbb{Z}) (b-1 \geq n(b^n-1))$

// TO **HM** 5.3-8

(1) Suppose $A = \{x, y, z\} \subseteq \mathbb{Z}^*$ $\exists (a, b, c \in A) (\exists d \in \mathbb{Z} \ni ab+ac=d^2)$

(2) Given by hint: Consider in A to $A = \{x, x+6, 9\}$

(3) Then, $x(x+6)+9 = x^2+6x+9 = (x+3)^2 = d^2 \Leftrightarrow d = x+3$

(4) $x \cdot 9 + x+6 = 9x+6 = d^2 \wedge (x+3)^2 = d^2 \Rightarrow 9x+6 = (x+3)^2 \Rightarrow 9x+6 = x^2+6x+9 \Rightarrow x^2-3x+3=0$

(5) $10x+6 = d^2 \wedge 10x+54 = f^2 \Leftrightarrow f^2 - d^2 = 48 \Rightarrow (f-d)(f+d) = 48$

(6) Guess: $f=8, d=4$

(7) $(10x+6=16 \wedge 10x+54=64) \Leftrightarrow x=1$
Note LMAO

(8) $A = \{1, 7, 9\}$
 $1 \cdot 7 + 9 = 16$
 $1 \cdot 9 + 7 = 16$
 $7 \cdot 9 + 1 = 64$ ✓

(9) $\therefore A = \{1, 7, 9\} \subset$