Real Analysis Assignment 1

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1(a):
Suppose we have the following equation:

$$|x-1| + |x+2| = 0.5 \tag{1}$$

We have $x \in \mathbb{R}$ if and only if:

$$x \leq -2 \vee -2 \leq x \leq 1 \vee 1 \leq x \tag{2}$$

since $(-\infty, -2] \cup [-2, 1] \cup [1, \infty) = \mathbb{R}$.

Assume $x \leq -2$. Then, $x - 1 < 0 \implies |x-1| = -(x-1)$, and $x + 2 \leq 0 \implies |x+2| = -(x+2)$, so we can solve (1) as follows:

$$\begin{aligned}
&-(x-1) - (x+2) && = 0.5 && (3)\\
=& -x + 1 - x - 2 && = 0.5 && (4)\\
=& -2x - 1 && = 0.5 && (5)\\
\implies& -2x && = 1.5 && (6)\\
\implies& x && = \frac{1.5}{-2} = \frac{-3}{4} && (7)
\end{aligned}$$

However, we have by assumption that $x \leq -2$, and by (7) we have that $x = \frac{-3}{4} > -2$, which is a contradition. Therefore, $x \leq -2$ is false for any real solution to (1).

Assume $-2 \leq x \leq 1$. Then, $x - 1 \leq 0 \implies |x-1| = -(x-1)$, and $x + 2 \geq 0 \implies |x+2| = x + 2$, so we can solve (1) as follows:

$$\begin{aligned}
&-(x-1) + x + 2 && = 0.5 && (8)\\
=& -x + 1 + x + 2 && = 0.5 && (9)\\
=& 3 && = 0.5 && (10)
\end{aligned}$$

But of course, $3 = 0.5$ is absurd, therefore we reach a contradiction and conclude that $-2 \leq x \leq 1$ is false for any real solution to (1).

Assume $x \geq 1$: Then, $x - 1 \geq 0 \implies |x-1| = x-1$, and $x + 2 > 0 \implies |x+2| = x+2$, so we can solve (1) as follows:

$$\begin{align} x - 1 + x + 2 &= 0.5 \tag{11} \\ =2x + 1 &= 0.5 \tag{12} \\ \implies 2x &= -0.5 \tag{13} \\ \implies x &= \frac{-0.5}{2} = \frac{-1}{4} \tag{14} \end{align}$$

However, we have by assumption that $x \geq 1$, and by (14) we have that $x = \frac{-1}{4} < 1$, which is a contradition. Therefore, $x \geq 1$ is false for any real solution to (1).

Finally, since we have $x \notin (-\infty, -2] \wedge x \notin [-2, 1] \wedge x \notin [1, \infty)$, we have $x \notin \mathbb{R} = (-\infty, -2] \cup [-2, 1] \cup [1, \infty)$, therefore we have demonstrated that there are no real solutions to (1).

1(b):