Homework 4

Due Wednesday, June 17

1. Chapter 9, Exercise A3 (p. 97)
2. Chapter 9, Exercise B3 (p. 98) (You may use the result of parts B1 and B2)
3. Chapter 9, Exercise E1 (p. 99)
4. Chapter 9, Exercise F2 (p. 100) (You may use the results from previous homeworks)
5. Chapter 9, Exercise I3 (p. 101)
6. Chapter 10, Exercise B2 (p. 108) (It will be helpful to write out the elements of $\langle 6 \rangle$)
7. Chapter 10, Exercise B3 (p. 108)
8. Chapter 10, Exercise C4 (p. 108)
9. Chapter 10, Exercise C5 (p. 108)
10. Chapter 10, Exercise D5 (p. 109) (n is a factor of m means $m = nk$ for $k \in \mathbb{Z}$)
11. Let G consists of elements (matrices)

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}, \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}, \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}, \begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}, \begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}, \begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$$

The operation is the usual matrix multiplication, but for the purpose of creating a table of G, define an operation $\star$ by: $A \star B = BA$, where $A, B \in G$. Create a table of $\langle G, \star \rangle$, and conclude that $S_3 \cong G$. (It suffices to show the isomorphism by matching the tables).

12. Find the order of element 1 in: (a) $\langle \mathbb{R}^*, \cdot \rangle$ and (b) $\langle \mathbb{R}, + \rangle$.
13. Let $A = \begin{bmatrix}1&1\\0&1\end{bmatrix}$ be an element of $GL_2(\mathbb{R})$ (to recall the definition of $GL_2(\mathbb{R})$, you may see Homework 1, Problem 7, and Homework 2 Solution, Problem 16). Find the order of A.
14. Find the order of $f(x) = 1/(1-x)$ in S_A, where $A = \mathbb{R} - \{0, 1\}$ (see Chapter 7, Exercise C, and related lecture notes for more information on S_A).
15. (a) Define $G_1 = \{f \in S_{\mathbb{R}} : f(x) = ax + b,\ a \neq 0,\ a, b \in \mathbb{R}\}$. Show that G_1 is a subgroup of $S_{\mathbb{R}}$ (under the composition operation $\circ$).

    (b) Let $G_2 = \left\{ \begin{bmatrix}a&b\\0&1\end{bmatrix} \in GL_2(\mathbb{R}) : a \neq 0 \right\}$. Show that G_2 is a subgroup of $GL_2(\mathbb{R})$.

    (c) Now, define $T : G_1 \to G_2$ as $T(f) = \begin{bmatrix}a&b\\0&1\end{bmatrix}$ (where $f(x) = ax + b$, $a \neq 0$). Show that the mapping T is an isomorphism from G_1 to G_2, and hence $G_1 \cong G_2$.