Abstract Algebra: Homework #1

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1 Chapter 3: Excercise A1

Suppose $*$ is defined on $\mathbb{R}$ as $a*b = a+b+k$ for any $a, b \in \mathbb{R}$ for some $k \in \mathbb{R}$.

Theorem 1. *$\langle \mathbb{R}, * \rangle$ is a group.*

Proof. Let a, b, and c be some arbitrary real numbers. Because $a * b = a + b + k \in \mathbb{R}$, we have that the real numbers are closed under $*$. Then, observe that $a * (b * c) = a * (b + c + k) = a + (b + c + k) + k = (a + b + k) + c + k = (a + b + k) * c = (a * b) * c$, so $*$ is associative. We also have $a * -k = a + (-k) + k = -k + a + k = -k * a = a$, so $-k$ is the identity for the real numbers under $*$ Finally, consider the quantity $-(2k + a)$. Since we have $a * -(2k + a) = a + (-2k) + (-a) + k = -(2k + a) * a = -k$, that quantity is the inverse of any a. Since $*$ is closed under the real numbers and $*$ is associative and $-k$ is the identity of $*$ under the real numbers and any real number a has an inverse under $*$ of $-(2k - a)$, $\langle \mathbb{R}, * \rangle$ is a group. $\square$