Homework 5

Due Wednesday, June 24

1. Chapter 11, Exercise A2 (p. 115)
2. Chapter 11, Exercise B1 (p. 116)
3. Chapter 11, Exercise B3 (p. 116)
4. Chapter 11, Exercise D1 (p. 117)
5. Chapter 11, Exercise D2 (p. 117)
6. Chapter 12, Exercise B1 (p. 124) (Write out the elements of $[x]$ and describe the set $[x]$)
7. Chapter 12, Exercise B5 (p. 124) (Describe the set $[x]$, in particular the set $[0]$; no need to describe the partition)
8. Chapter 12, Exercise D3 (p. 125)
9. List all cyclic subgroups of $\mathbb{Z}_{12}$, and conclude that the order of all subgroups must divide the order of $\mathbb{Z}_{12}$ (this is an illustration of the statement/result in Chapter 11, Exercise B4).
10. Let $f(x) = x + 1$ be an element of $S_{\mathbb{R}}$. Write the elements of $\langle f \rangle$ and show that $\mathbb{Z} \cong \langle f \rangle$.
11. Let $f(x) = x + 1$ be an element of $\mathcal{F}(\mathbb{R})$ (see pp. 45-46 to recall the definition of $\mathcal{F}(\mathbb{R})$). Write the elements of $\langle f \rangle$ and show that $\mathbb{Z} \cong \langle f \rangle$.
12. Let $A = \begin{bmatrix} a & (b-a) \\ 0 & b \end{bmatrix}$, $a \neq 0$, $b \neq 0$, be an element of $GL_2(\mathbb{R})$.
    (a) Compute the elements $A^2, A^3, A^{-1}, A^{-2}, A^{-3}$, and check that $AA^{-1} = I$.
    (b) Find the general form of A^n, $n \in \mathbb{Z}$, to describe $\langle A \rangle$.
13. Let $f, g \in \mathcal{F}(\mathbb{R})$. Show that the following describes an equivalence relation:
$$f \sim g \iff \text{for every } x \in \mathbb{R},\ f(x) - g(x) = c \text{ for some } c \in \mathbb{R}$$
14. Let X be any set, and let $f : X \to X$ be a bijective function. Show that, for $x, y \in X$, the following describes an equivalence relation: $x \sim y$ iff there exists $n \in \mathbb{Z}$ such that $f^n(x) = y$.