Homework 8

Due Wednesday, July 15

1. Chapter 15, Exercise A1 (p. 153)
2. Chapter 15, Exercise A4 (p. 153)
3. Chapter 15, Exercise C1 (p. 153)
4. Chapter 15, Exercise D1 (p. 154)
5. Chapter 15, Exercise E2 (p. 154)
6. Chapter 15, Exercise E5 (p. 154)
7. Chapter 15, Exercise E6 (p. 154)
8. Refer to Problem 7. Show that each element in $\mathbb{Q}/\mathbb{Z}$ is the same as that in the equivalence class described in Chapter 12, Exercise B2 (i.e., for a fixed $r \in \mathbb{Q}$, show that $[r] = r + \mathbb{Z}$; see also June 18 notes).
9. Let $G = A_4 = \{e, (12)(34), (13)(24), (14)(23), (123), (132), (124), (142), (134), (143), (234), (243)\}$ and $H = \{e, (12)(34), (13)(24), (14)(23)\}$ (see Homework 6, Problem 10). We have already computed the cosets of H and established that H is a normal subgroup, so that $G/H = \{H, H(123), H(132)\}$. Perform the coset multiplications of the elements of G/H to (a) write the table of G/H and (b) confirm that $\mathrm{ord}(Ha) = 3$ if $Ha \in G/H$ for $a \notin H$.
10. Refer to Homework 7, Problem 2. Let $G = \mathcal{D}(\mathbb{R})$ and $H = \ker(\phi)$. Show that for any $g \in G$, $[g] = H + g$, where $[g]$ is an equivalence class obtained from the equivalence relation described in Homework 5, Problem 13. Conclude that each element $H + g \in G/H$ contains all the functions that differ from g by a constant (or equivalently, shifted vertically from g).
11. Let $G = \mathcal{C}(\mathbb{R})$ (the set of all continuous functions, see page 46). Define $\psi : G \to \mathbb{R}$ by $\psi(f) = \int_0^1 f(x)\,dx$. Show that ψ is a homomorphism, find $H = \ker(\psi)$, and describe G/H. (See also Homework 2, Problem 9).
12. Let $G = GL_2(\mathbb{R})$, and let $H = \{X \in G : \det(X) = 1\} = SL_2(\mathbb{R})$ be a subgroup of G. Show that H is a normal subgroup of G, and describe G/H.
13. Let $G = GL_2(\mathbb{R})$, and let $H = \{X \in G : \det(X) > 0\}$.
    (a) Define $\psi : G \to P$, by $\psi(A) = 1$ iff $\det(A) > 0$, and $\psi(A) = -1$ iff $\det(A) < 0$, where $P = \{-1, 1\}$ is a parity group with multiplication (write the table of P, with 1 as the identity). Show that ψ is a homomorphism, find $\ker(\psi)$, and conclude that a H is a normal subgroup of G.
    (b) Show that $G/H = \{H, HA\}$ where $A \notin H$, and verify that $(HA)^2 = H$.
14. Chapter 15, Exercise F (p. 154) (Do all parts; the definition of a center of a group G is given by $C = Z(G) = \{a \in G : ax = xa,\ \forall x \in G\}$)