Differential Equations Summer 2020 UMC Diamond Brent

Quiz 2

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1 $P(t) = Ae^{kt}$ $\frac{dP}{dt} = kP \iff P = Xe^{kt}$

$P(0) = 20$

$P(2) = 54$

$P(t_0) = 100 \Rightarrow t_0 = ?$

$\int \frac{1}{P}\frac{dP}{dt}\,dt = \ln|P| = kt + C$

$P(0) = 20 = De^{k(0)} \Rightarrow D = 20$ $\quad P = De^{kt}$ $(D = 20)$

$\therefore P(t) = 20e^{kt}$

$P(2) = 54 = 20e^{2k} \iff \frac{27}{10} = e^{2k} \iff \ln\left|\frac{27}{10}\right| = 2k$

$\iff \ln\left(\sqrt{\frac{27}{10}}\right) = k$

$\Rightarrow P(t) = 20e^{\ln\left(\frac{27}{10}\right)^{t/2}} = 20\left(\frac{27}{10}\right)^{t/2}$

$\therefore P(t) = 20\cdot\left(\frac{27}{10}\right)^{t/2}$

$P(t) = 100 = 20\cdot\left(\frac{27}{10}\right)^{t/2} \Rightarrow 5 = \left[\sqrt{\left(\frac{27}{10}\right)}\right]^{t} \iff t = \log_{\sqrt{27/10}} 5$

$\therefore \Rightarrow t \approx 3.2407$

2 $\frac{dy}{dx} = \frac{1}{y^3(1-x^2)^{1/2}}$; $y(0) = 2$

$\iff y^3\frac{dy}{dx} = (1-x^2)^{-1/2} \iff \int y^3\,dy = \int \frac{1}{\sqrt{1-x^2}}\,dx$

$\iff \frac{1}{4}y^4 = \sin^{-1}x + C$

$y(x) = \sqrt[4]{4\sin^{-1}(x) + 4C}$

$y(0) = \sqrt[4]{4\sin^{-1}(0) + 4C} = 1$

$\sqrt[4]{4C} = 1$

$4C = 1 \iff C = \frac{1}{4}$

$\therefore y(x) = \sqrt[4]{4\sin^{-1}(x) + 1}$

3 $\frac{dy}{dx} - \frac{3}{x}y = x$

let $\rho(x) = e^{-3\int \frac{1}{x}dx} = e^{-3\ln|x|+c} = e^{\ln(|x^{-3}|)+c} = Dx^{-3}$ $D = e^c$

Then $x^{-3}y = \int x^{-2}dx = -x^{-1} + C$

$y = \frac{-x^{-1}+C}{x^{-3}} = \frac{-x^{-1}}{x^{-3}} + \frac{C}{x^{-3}}$

$y(x) = -x^2 + \frac{C}{x^{-3}} = Cx^3 - x^2$

$\therefore y(x) = Cx^3 - x^2$ where $C \in \mathbb{R}$

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