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Differential Equations

Quiz 1

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1) Find $y' = x^2\cos(2x)$, $y(0) = 2$

$\int y'(x)dx = \int x^2\cos(2x)dx$

$= \frac{1}{2}x^2\sin(2x) - \int x\sin(2x)dx = \frac{1}{2}x^2\sin(2x) + \frac{1}{2}x\cos(2x) - \frac{1}{4}\sin(2x) + C$

Let $u' = \cos(2x)$ $\quad v = x^2$
$u = \sin(2x)(\frac{1}{2})$ $\quad v' = 2x$

$\int (2x)(\sin(2x))dx = -x\cos(2x) \int (2\sin 2x)dx$

$u' = \sin(2x)$ $\quad v = (2x)$ $\quad = -x\cos(2x)$

$u = -\cos(2x)(\frac{1}{2})$ $\quad v' = 1$

$\hookrightarrow = -\frac{1}{2}x\cos(2x) + \int \frac{1}{2}\cos(2x)dx$

$= -x\cos(2x) - \frac{1}{4}\sin(2x) + C$

$y(x) = \frac{1}{2}x^2\sin(2x) + \frac{1}{2}x\cos(2x) - \frac{1}{4}\sin(2x) + C$

$y(0) = \frac{1}{2}(0) \cdots\cdots \frac{1}{2}0 \cdots\cdots - \frac{1}{4}\sin(2\cdot 0) + C = 2$

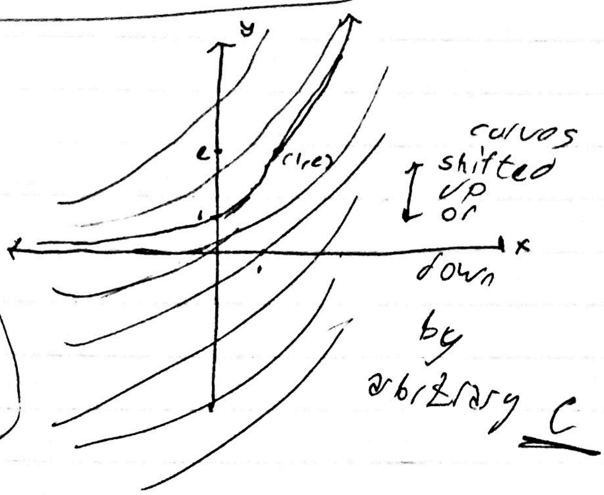
$0 + C = 2$

$\therefore y(x) = \frac{1}{2}x^2\sin(2x) + \frac{1}{2}x\cos(2x) - \frac{1}{4}\sin(2x) + 2$

2 suppose $\frac{dy}{dx} = e^x$

then, $y(x) = e^x + C$

The curve $y(x) = e^x$
passes through $(1, e)$
and has $y' = e^x$.



3 Find $\frac{dy}{dx} = x\sec^2(x^2)$

$\int \frac{dy}{dx}dx = y(x) = \int x\sec^2(x^2)\,dx$

$u = x^2$ $\quad y(u(x)) = \frac{1}{2}\int \sec^2(u)\,du$

$= \frac{1}{2}[\tan u + C]$

$= \frac{1}{2}[\tan(x^2) + C]$

OR, let $D = \frac{1}{2}C$

$\therefore y(x) = \tan(x^2) + D$

4 Show $y(x) = xe^x$ solves

$y'' - 2y' + y = 0$

we have:

i) $y = xe^x$

ii) $y' = e^x + xe^x$

iii) $y'' = e^x + e^x + xe^x = 2e^x + xe^x$

so, $y'' - 2y' + y = (2e^x + xe^x) - 2(e^x + xe^x) + xe^x = 0$

$= 2e^x + xe^x - 2e^x - 2xe^x + xe^x = 0$

$= (2e^x - 2e^x) + (xe^x + xe^x - 2xe^x) = 0$

holds. so $y(x) = xe^x$ does indeed solve $y'' - 2y' + y = 0$.