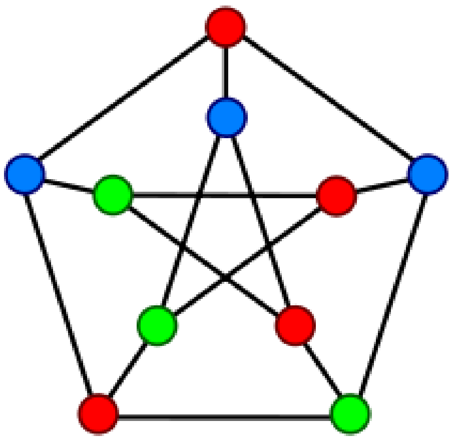
Backtracking | Set 5 (m Coloring Problem) - GeeksforGeeks

Given an undirected graph and a number m, determine if the graph can be colored with at most m colors such that no two adjacent vertices of the graph are colored with same color. Here coloring of a graph means assignment of colors to all vertices.

Input:
1) A 2D array graph[V][V] where V is the number of vertices in graph and graph[V][V] is adjacency matrix representation of the graph. A value graph[i][j] is 1 if there is a direct edge from i to j, otherwise graph[i][j] is 0.
2) An integer m which is maximum number of colors that can be used.

Output:
An array color[V] that should have numbers from 1 to m. color[i] should represent the color assigned to the ith vertex. The code should also return false if the graph cannot be colored with m colors.

Following is an example graph (from Wiki page) that can be colored with 3 colors.



Naive Algorithm
Generate all possible configurations of colors and print a configuration that satisfies the given constraints.

```
while there are untried conflagrations
{
   generate the next configuration
   if no adjacent vertices are colored with same color
   {
      print this configuration;
   }
}
```

There will be V^m configurations of colors.

Backtracking Algorithm

The idea is to assign colors one by one to different vertices, starting from the vertex 0. Before assigning a color, we check for safety by considering already assigned colors to the adjacent vertices. If we find a color assignment which is safe, we mark the color assignment as part of solution. If we do not a find color due to clashes then we backtrack and return false.

Implementation of Backtracking solution

```
#include<stdio.h>// Number of vertices in the graph#define V 4
void printSolution(int color[]);
/* A utility function to check if the current color assignment
   is safe for vertex v */
bool isSafe (int v, bool graph[V][V], int color[], int c)
{
    for (int i = 0; i < V; i++)
        if (graph[v][i] && c == color[i])
            return false;
    return true;
}/* A recursive utility function to solve m coloring problem */
bool graphColoringUtil(bool graph[V][V], int m, int color[], int v)
{
    /* base case: If all vertices are assigned a color then
       return true */
    if (v == V)
        return true;
    /* Consider this vertex v and try different colors */
    for (int c = 1; c <= m; c++)
    {
        /* Check if assignment of color c to v is fine*/
        if (isSafe(v, graph, color, c))
        {
           color[v] = c;
           /* recur to assign colors to rest of the vertices */
           if (graphColoringUtil (graph, m, color, v+1) == true)
             return true;
            /* If assigning color c doesn't lead to a solution
               then remove it */
           color[v] = 0;
        }
    }
    /* If no color can be assigned to this vertex then return false */
    return false;
}/* This function solves the m Coloring problem using Backtracking.
  It mainly uses graphColoringUtil() to solve the problem. It returns
  false if the m colors cannot be assigned, otherwise return true and
  prints assignments of colors to all vertices. Please note that there
  may be more than one solutions, this function prints one of the
  feasible solutions.*/
bool graphColoring(bool graph[V][V], int m)
{
    // Initialize all color values as 0. This initialization is needed
    // correct functioning of isSafe()
    int *color = new int[V];
    for (int i = 0; i < V; i++)
       color[i] = 0;
    // Call graphColoringUtil() for vertex 0
    if (graphColoringUtil(graph, m, color, 0) == false)
    {
      printf("Solution does not exist");
      return false;
    }
    // Print the solution
    printSolution(color);
    return true;
}/* A utility function to print solution */
void printSolution(int color[])
{
    printf("Solution Exists:"
            " Following are the assigned colors \n");
    for (int i = 0; i < V; i++)
      printf(" %d ", color[i]);
    printf("\n");
}// driver program to test above function
int main()
{
    /* Create following graph and test whether it is 3 colorable
      (3)---(2)
       |   / |
       |  /  |
       | /   |
      (0)---(1)
    */
    bool graph[V][V] = {{0, 1, 1, 1},
        {1, 0, 1, 0},
        {1, 1, 0, 1},
        {1, 0, 1, 0},
    };
    int m = 3; // Number of colors
    graphColoring (graph, m);
    return 0;
}
```