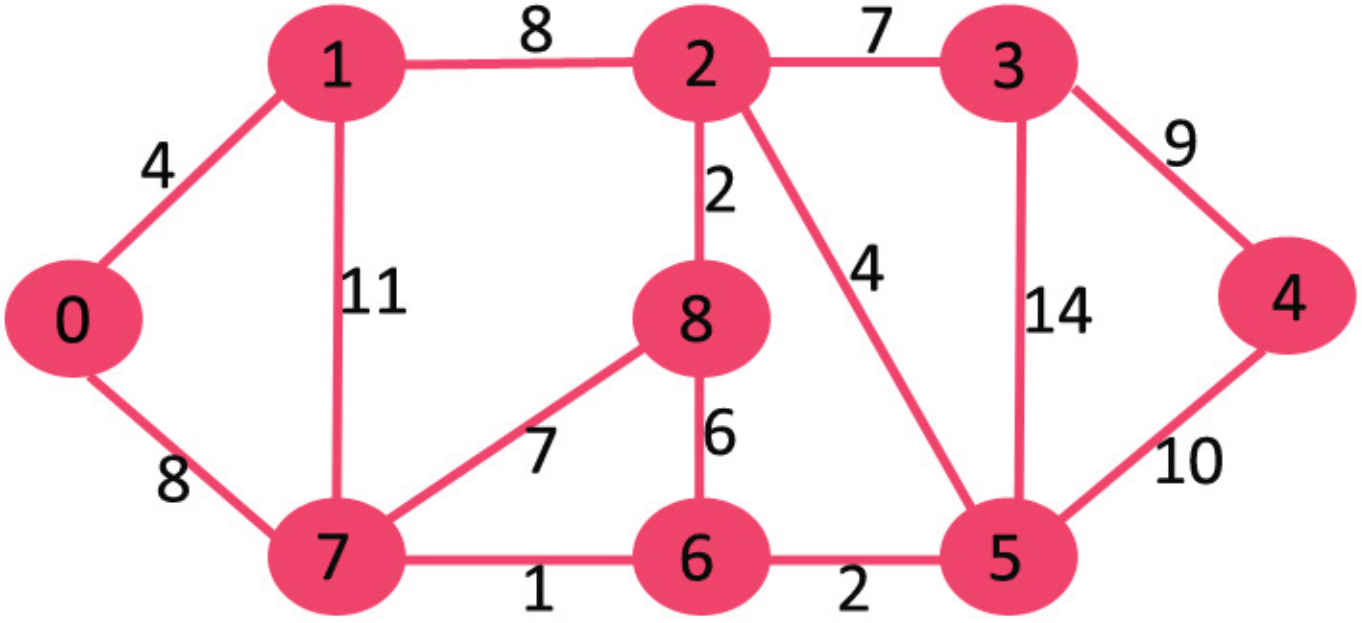
Greedy Algorithms | Set 6 (Prim's MST for Adjacency List Representation) - GeeksforGeeks

We recommend to read following two posts as a prerequisite of this post.

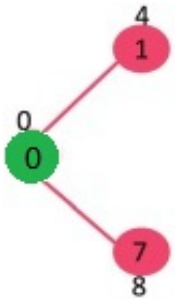
We have discussed Prim's algorithm and its implementation for adjacency matrix representation of graphs. The time complexity for the matrix representation is O(V^2). In this post, O(ELogV) algorithm for adjacency list representation is discussed.
As discussed in the previous post, in Prim's algorithm, two sets are maintained, one set contains list of vertices already included in MST, other set contains vertices not yet included. With adjacency list representation, all vertices of a graph can be traversed in O(V+E) time using BFS. The idea is to traverse all vertices of graph using BFS and use a Min Heap to store the vertices not yet included in MST. Min Heap is used as a priority queue to get the minimum weight edge from the cut. Min Heap is used as time complexity of operations like extracting minimum element and decreasing key value is O(LogV) in Min Heap.

Following are the detailed steps.
1) Create a Min Heap of size V where V is the number of vertices in the given graph. Every node of min heap contains vertex number and key value of the vertex.
2) Initialize Min Heap with first vertex as root (the key value assigned to first vertex is 0). The key value assigned to all other vertices is INF (infinite).
3) While Min Heap is not empty, do following
…..**a)** Extract the min value node from Min Heap. Let the extracted vertex be u.
…..**b)** For every adjacent vertex v of u, check if v is in Min Heap (not yet included in MST). If v is in Min Heap and its key value is more than weight of u-v, then update the key value of v as weight of u-v.

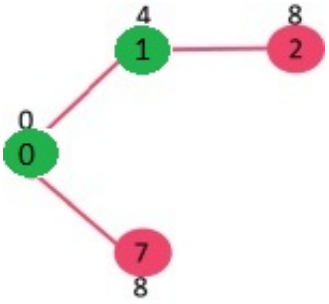
Let us understand the above algorithm with the following example:



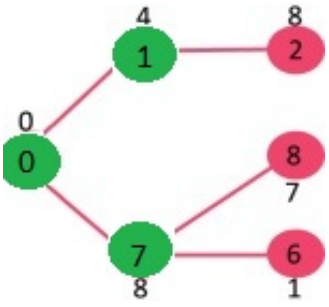
Initially, key value of first vertex is 0 and INF (infinite) for all other vertices. So vertex 0 is extracted from Min Heap and key values of vertices adjacent to 0 (1 and 7) are updated. Min Heap contains all vertices except vertex 0.
The vertices in green color are the vertices included in MST.



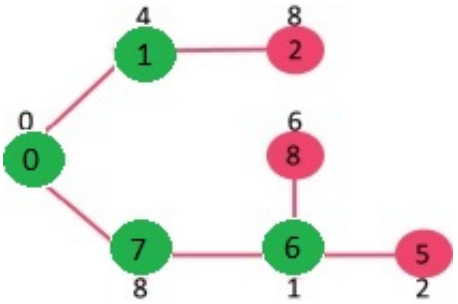
Since key value of vertex 1 is minimum among all nodes in Min Heap, it is extracted from Min Heap and key values of vertices adjacent to 1 are updated (Key is updated if the a vertex is not in Min Heap and previous key value is greater than the weight of edge from 1 to the adjacent). Min Heap contains all vertices except vertex 0 and 1.



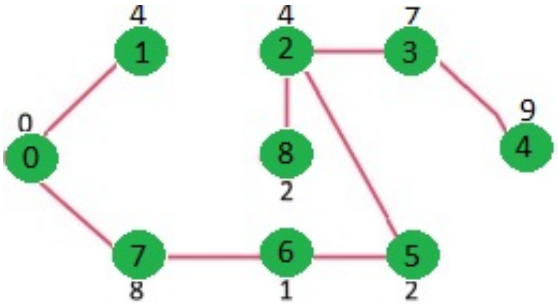
Since key value of vertex 7 is minimum among all nodes in Min Heap, it is extracted from Min Heap and key values of vertices adjacent to 7 are updated (Key is updated if the a vertex is not in Min Heap and previous key value is greater than the weight of edge from 7 to the adjacent). Min Heap contains all vertices except vertex 0, 1 and 7.



Since key value of vertex 6 is minimum among all nodes in Min Heap, it is extracted from Min Heap and key values of vertices adjacent to 6 are updated (Key is updated if the a vertex is not in Min Heap and previous key value is greater than the weight of edge from 6 to the adjacent). Min Heap contains all vertices except vertex 0, 1, 7 and 6.



The above steps are repeated for rest of the nodes in Min Heap till Min Heap becomes empty




```
// C / C++ program for Prim's MST for adjacency list representation of graph#include <stdio.h>#include <stdlib.h>#include <limits.h>// A structure to represent a node in adjacency list
struct AdjListNode
{
    int dest;
    int weight;
    struct AdjListNode* next;
};// A structure to represent an adjacency liat
struct AdjList
{
    struct AdjListNode *head;  // pointer to head node of list
};// A structure to represent a graph. A graph is an array of adjacency lists.// Size of array will be V (number of vertices in graph)
struct Graph
{
    int V;
    struct AdjList* array;
};// A utility function to create a new adjacency list node
struct AdjListNode* newAdjListNode(int dest, int weight)
{
    struct AdjListNode* newNode =
            (struct AdjListNode*) malloc(sizeof(struct AdjListNode));
    newNode->dest = dest;
    newNode->weight = weight;
    newNode->next = NULL;
    return newNode;
```

```
}// A utility function that creates a graph of V vertices
struct Graph* createGraph(int V)
{
    struct Graph* graph = (struct Graph*) malloc(sizeof(struct Graph));
    graph->V = V;
    // Create an array of adjacency lists.  Size of array will be V
    graph->array = (struct AdjList*) malloc(V * sizeof(struct AdjList));
     // Initialize each adjacency list as empty by making head as NULL
    for (int i = 0; i < V; ++i)
        graph->array[i].head = NULL;
    return graph;
}// Adds an edge to an undirected graph
void addEdge(struct Graph* graph, int src, int dest, int weight)
{
    // Add an edge from src to dest.  A new node is added to the adjacency
    // list of src.  The node is added at the begining
    struct AdjListNode* newNode = newAdjListNode(dest, weight);
    newNode->next = graph->array[src].head;
    graph->array[src].head = newNode;
    // Since graph is undirected, add an edge from dest to src also
    newNode = newAdjListNode(src, weight);
    newNode->next = graph->array[dest].head;
    graph->array[dest].head = newNode;
}// Structure to represent a min heap node
struct MinHeapNode
{
    int  v;
    int key;
};// Structure to represent a min heap
struct MinHeap
{
    int size;      // Number of heap nodes present currently
    int capacity;  // Capacity of min heap
    int *pos;      // This is needed for decreaseKey()
    struct MinHeapNode **array;
};// A utility function to create a new Min Heap Node
struct MinHeapNode* newMinHeapNode(int v, int key)
{
    struct MinHeapNode* minHeapNode =
           (struct MinHeapNode*) malloc(sizeof(struct MinHeapNode));
    minHeapNode->v = v;
    minHeapNode->key = key;
```

```
    return minHeapNode;
}// A utilit function to create a Min Heap
struct MinHeap* createMinHeap(int capacity)
{
    struct MinHeap* minHeap =
         (struct MinHeap*) malloc(sizeof(struct MinHeap));
    minHeap->pos = (int *)malloc(capacity * sizeof(int));
    minHeap->size = 0;
    minHeap->capacity = capacity;
    minHeap->array =
         (struct MinHeapNode**) malloc(capacity * sizeof(struct
MinHeapNode*));
    return minHeap;
}// A utility function to swap two nodes of min heap. Needed for min
heapify
void swapMinHeapNode(struct MinHeapNode** a, struct MinHeapNode** b)
{
    struct MinHeapNode* t = *a;
    *a = *b;
    *b = t;
}// A standard function to heapify at given idx// This function also
updates position of nodes when they are swapped.// Position is needed for
decreaseKey()
void minHeapify(struct MinHeap* minHeap, int idx)
{
    int smallest, left, right;
    smallest = idx;
    left = 2 * idx + 1;
    right = 2 * idx + 2;
    if (left < minHeap->size &&
        minHeap->array[left]->key < minHeap->array[smallest]->key )
      smallest = left;
    if (right < minHeap->size &&
        minHeap->array[right]->key < minHeap->array[smallest]->key )
      smallest = right;
    if (smallest != idx)
    {
        // The nodes to be swapped in min heap
        MinHeapNode *smallestNode = minHeap->array[smallest];
        MinHeapNode *idxNode = minHeap->array[idx];
        // Swap positions
        minHeap->pos[smallestNode->v] = idx;
```

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        minHeap->pos[idxNode->v] = smallest;
        // Swap nodes
        swapMinHeapNode(&minHeap->array[smallest], &minHeap->array[idx]);
        minHeapify(minHeap, smallest);
    }
}// A utility function to check if the given minHeap is ampty or not
int isEmpty(struct MinHeap* minHeap)
{
    return minHeap->size == 0;
}// Standard function to extract minimum node from heap
struct MinHeapNode* extractMin(struct MinHeap* minHeap)
{
    if (isEmpty(minHeap))
        return NULL;
    // Store the root node
    struct MinHeapNode* root = minHeap->array[0];
    // Replace root node with last node
    struct MinHeapNode* lastNode = minHeap->array[minHeap->size - 1];
    minHeap->array[0] = lastNode;
    // Update position of last node
    minHeap->pos[root->v] = minHeap->size-1;
    minHeap->pos[lastNode->v] = 0;
    // Reduce heap size and heapify root
    --minHeap->size;
    minHeapify(minHeap, 0);
    return root;
}// Function to decreasy key value of a given vertex v. This function//
uses pos[] of min heap to get the current index of node in min heap
void decreaseKey(struct MinHeap* minHeap, int v, int key)
{
    // Get the index of v in  heap array
    int i = minHeap->pos[v];
    // Get the node and update its key value
    minHeap->array[i]->key = key;
    // Travel up while the complete tree is not hepified.
    // This is a O(Logn) loop
    while (i && minHeap->array[i]->key < minHeap->array[(i - 1) / 2]->key)
    {
        // Swap this node with its parent
        minHeap->pos[minHeap->array[i]->v] = (i-1)/2;
        minHeap->pos[minHeap->array[(i-1)/2]->v] = i;
        swapMinHeapNode(&minHeap->array[i],  &minHeap->array[(i - 1) / 2]);
```

```
        // move to parent index
        i = (i - 1) / 2;
    }
}// A utility function to check if a given vertex// 'v' is in min heap or
not
bool isInMinHeap(struct MinHeap *minHeap, int v)
{
    if (minHeap->pos[v] < minHeap->size)
        return true;
    return false;
}// A utility function used to print the constructed MST
void printArr(int arr[], int n)
{
    for (int i = 1; i < n; ++i)
        printf("%d - %d\n", arr[i], i);
}// The main function that constructs Minimum Spanning Tree (MST)// using
Prim's algorithm
void PrimMST(struct Graph* graph)
{
    int V = graph->V;// Get the number of vertices in graph
    int parent[V];    // Array to store constructed MST
    int key[V];       // Key values used to pick minimum weight edge in cut
    // minHeap represents set E
    struct MinHeap* minHeap = createMinHeap(V);
    // Initialize min heap with all vertices. Key value of
    // all vertices (except 0th vertex) is initially infinite
    for (int v = 1; v < V; ++v)
    {
        parent[v] = -1;
        key[v] = INT_MAX;
        minHeap->array[v] = newMinHeapNode(v, key[v]);
        minHeap->pos[v] = v;
    }
    // Make key value of 0th vertex as 0 so that it
    // is extracted first
    key[0] = 0;
    minHeap->array[0] = newMinHeapNode(0, key[0]);
    minHeap->pos[0]   = 0;
    // Initially size of min heap is equal to V
    minHeap->size = V;
    // In the followin loop, min heap contains all nodes
    // not yet added to MST.
```

```
    while (!isEmpty(minHeap))
    {
        // Extract the vertex with minimum key value
        struct MinHeapNode* minHeapNode = extractMin(minHeap);
        int u = minHeapNode->v; // Store the extracted vertex number
        // Traverse through all adjacent vertices of u (the extracted
        // vertex) and update their key values
        struct AdjListNode* pCrawl = graph->array[u].head;
        while (pCrawl != NULL)
        {
            int v = pCrawl->dest;
            // If v is not yet included in MST and weight of u-v is
            // less than key value of v, then update key value and
            // parent of v
            if (isInMinHeap(minHeap, v) && pCrawl->weight < key[v])
            {
                key[v] = pCrawl->weight;
                parent[v] = u;
                decreaseKey(minHeap, v, key[v]);
            }
            pCrawl = pCrawl->next;
        }
    }
    // print edges of MST
    printArr(parent, V);
}// Driver program to test above functions
int main()
{
    // Let us create the graph given in above fugure
    int V = 9;
    struct Graph* graph = createGraph(V);
    addEdge(graph, 0, 1, 4);
    addEdge(graph, 0, 7, 8);
    addEdge(graph, 1, 2, 8);
    addEdge(graph, 1, 7, 11);
    addEdge(graph, 2, 3, 7);
    addEdge(graph, 2, 8, 2);
    addEdge(graph, 2, 5, 4);
    addEdge(graph, 3, 4, 9);
    addEdge(graph, 3, 5, 14);
    addEdge(graph, 4, 5, 10);
    addEdge(graph, 5, 6, 2);
```

```
    addEdge(graph, 6, 7, 1);
    addEdge(graph, 6, 8, 6);
    addEdge(graph, 7, 8, 7);
    PrimMST(graph);
    return 0;
}
```

Output:

```
0 - 1
5 - 2
2 - 3
3 - 4
6 - 5
7 - 6
0 - 7
2 - 8
```

Time Complexity: The time complexity of the above code/algorithm looks O(V^2) as there are two nested while loops. If we take a closer look, we can observe that the statements in inner loop are executed O(V+E) times (similar to BFS). The inner loop has decreaseKey() operation which takes O(LogV) time. So overall time complexity is O(E+V)*O(LogV) which is O((E+V)*LogV) = O(ELogV) (For a connected graph, V = O(E))

This article is compiled by Aashish Barnwal and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.