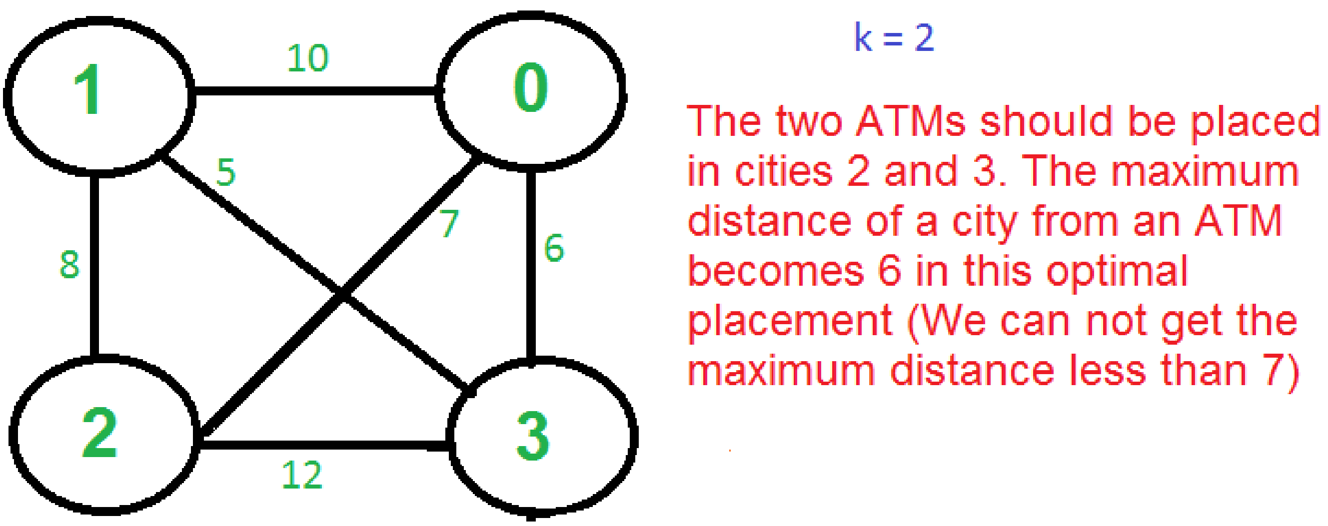
# K Centers Problem | Set 1 (Greedy Approximate Algorithm) - GeeksforGeeks

Given n cities and distances between every pair of cities, select k cities to place warehouses (or ATMs or Cloud Server) such that the maximum distance of a city to a warehouse (or ATM or Cloud Server) is minimized.

For example consider the following four cities, 0, 1, 2 and 3 and distances between them, how do place 2 ATMs among these 4 cities so that the maximum distance of a city to an ATM is minimized.

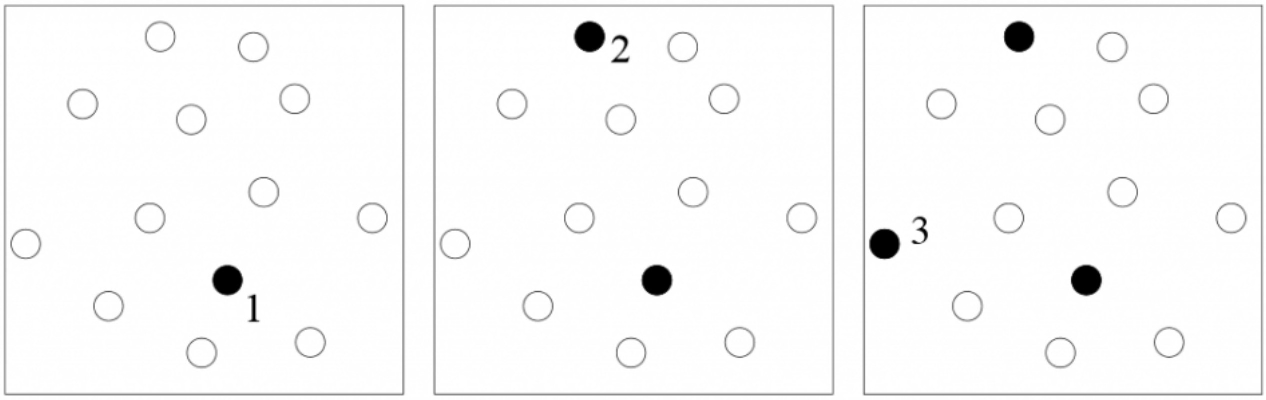



There is no polynomial time solution available for this problem as the problem is a known NP-Hard problem. There is a polynomial time Greedy approximate algorithm, the greedy algorithm provides a solution which is never worse that twice the optimal solution. The greedy solution works only if the distances between cities follow Triangular Inequality (Distance between two points is always smaller than sum of distances through a third point).

**The 2-Approximate Greedy Algorithm:**
1) Choose the first center arbitrarily.

2) Choose remaining k-1 centers using the following criteria.
    Let c1, c2, c3, … ci be the already chosen centers. Choose
    (i+1)'th center by picking the city which is farthest from already
    selected centers, i.e, the point p which has following value as maximum
        Min[dist(p, c1), dist(p, c2), dist(p, c3), …. dist(p, ci)]

The following diagram taken from here illustrates above algorithm.




**Example (k = 3 in the above shown Graph)**

a) Let the first arbitrarily picked vertex be 0.

b) The next vertex is 1 because 1 is the farthest vertex from 0.

c) Remaining cities are 2 and 3. Calculate their distances from already selected centers (0 and 1). The greedy algorithm basically calculates following values.

Minimum of all distanced from 2 to already considered centers
Min[dist(2, 0), dist(2, 1)] = Min[7, 8] = 7

Minimum of all distanced from 3 to already considered centers
Min[dist(3, 0), dist(3, 1)] = Min[6, 5] = 5

After computing the above values, the city 2 is picked as the value corresponding to 2 is maximum.

Note that the greedy algorithm doesn't give best solution for k = 2 as this is just an approximate algorithm with bound as twice of optimal.

**Proof that the above greedy algorithm is 2 approximate.**

Let OPT be the maximum distance of a city from a center in the Optimal solution. We need to show that the maximum distance obtained from Greedy algorithm is 2*OPT.

The proof can be done using contradiction.

a) Assume that the distance from the furthest point to all centers is > 2·OPT.

b) This means that distances between all centers are also > 2·OPT.

c) We have k + 1 points with distances > 2·OPT between every pair.

d) Each point has a center of the optimal solution with distance <= OPT to it.

e) There exists a pair of points with the same center X in the optimal solution (pigeonhole principle: k optimal centers, k+1 points)

f) The distance between them is at most 2·OPT (triangle inequality) which is a contradiction.

**Source:**
http://algo2.iti.kit.edu/vanstee/courses/kcenter.pdf

This article is contributed by **Harshit**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above