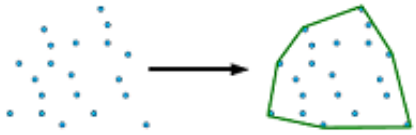
Convex Hull | Set 2 (Graham Scan) - GeeksforGeeks

Given a set of points in the plane. the convex hull of the set is the smallest convex polygon that contains all the points of it.



We strongly recommend to see the following post first.
How to check if two given line segments intersect?

We have discussed Jarvis's Algorithm for Convex Hull. Worst case time complexity of Jarvis's Algorithm is O(n^2). Using Graham's scan algorithm, we can find Convex Hull in O(nLogn) time. Following is Graham's algorithm

Let points[0..n-1] be the input array.

1) Find the bottom-most point by comparing y coordinate of all points. If there are two points with same y value, then the point with smaller x coordinate value is considered. Let the bottom-most point be P0. Put P0 at first position in output hull.

2) Consider the remaining n-1 points and sort them by polor angle in counterclockwise order around points[0]. If polor angle of two points is same, then put the nearest point first.

3 After sorting, check if two or more points have same angle. If two more points have same angle, then remove all same angle points except the point farthest from P0. Let the size of new array be m.

4) If m is less than 3, return (Convex Hull not possible)

5) Create an empty stack 'S' and push points[0], points[1] and points[2] to S.

6) Process remaining m-3 points one by one. Do following for every point 'points[i]'

4.1) Keep removing points from stack while orientation of following 3 points is not counterclockwise (or they don't make a left turn).

a) Point next to top in stack

b) Point at the top of stack

c) points[i]

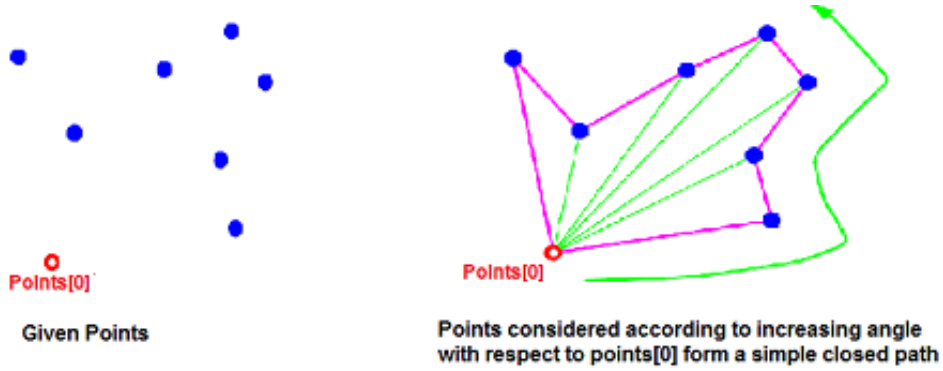
4.2) Push points[i] to S

5) Print contents of S

The above algorithm can be divided in two phases.

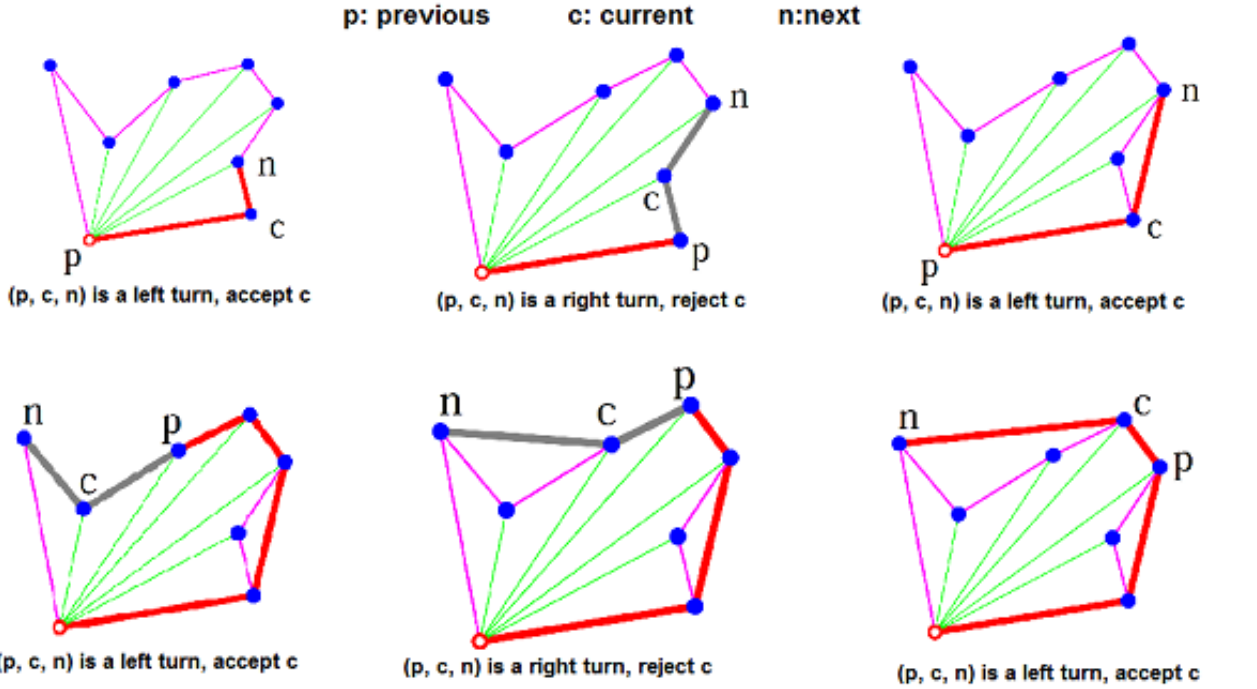
Phase 1 (Sort points): We first find the bottom-most point. The idea is to pre-process points be sorting

them with respect to the bottom-most point. Once the points are sorted, they form a simple closed path (See following diagram).



What should be the sorting criteria? computation of actual angles would be inefficient since trigonometric functions are not simple to evaluate. The idea is to use the orientation to compare angles without actually computing them (See the compare() function below)

Phase 2 (Accept or Reject Points): Once we have the closed path, the next step is to traverse the path and remove concave points on this path. How to decide which point to remove and which to keep? Again, orientation helps here. The first two points in sorted array are always part of Convex Hull. For remaining points, we keep track of recent three points, and find the angle formed by them. Let the three points be prev(p), curr(c) and next(n). If orientation of these points (considering them in same order) is not counterclockwise, we discard c, otherwise we keep it. Following diagram shows step by step process of this phase (Source of these diagrams is Ref 2).



In the above algorithm and below code, a stack of points is used to store convex hull points. With reference to the code, p is next-to-top in stack, c is top of stack and n is points[i].

Following is C++ implementation of the above algorithm.

```
// A C++ program to find convex hull of a set of points. Refer// for
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```
explanation of orientation()#include <iostream>#include <stack>#include
<stdlib.h>
using namespace std;
struct Point
{
    int x, y;
};// A globle point needed for  sorting points with reference// to  the
first point Used in compare function of qsort()Point p0;// A utility
function to find next to top in a stackPoint nextToTop(stack<Point> &S){
    Point p = S.top();
    S.pop();
    Point res = S.top();
    S.push(p);
    return res;
}// A utility function to swap two points
int swap(Point &p1, Point &p2)
{
    Point temp = p1;
    p1 = p2;
    p2 = temp;
}// A utility function to return square of distance// between p1 and p2
int distSq(Point p1, Point p2)
{
    return (p1.x - p2.x)*(p1.x - p2.x) +
           (p1.y - p2.y)*(p1.y - p2.y);
}// To find orientation of ordered triplet (p, q, r).// The function
returns following values// 0 --> p, q and r are colinear// 1 -->
Clockwise// 2 --> Counterclockwise
int orientation(Point p, Point q, Point r)
{
    int val = (q.y - p.y) * (r.x - q.x) -
              (q.x - p.x) * (r.y - q.y);
    if (val == 0) return 0;  // colinear
    return (val > 0)? 1: 2; // clock or counterclock wise
}// A function used by library function qsort() to sort an array of//
points with respect to the first point
int compare(const void *vp1, const void *vp2)
{
   Point *p1 = (Point *)vp1;
   Point *p2 = (Point *)vp2;
   // Find orientation
   int o = orientation(p0, *p1, *p2);
```

```
    if (o == 0)
        return (distSq(p0, *p2) >= distSq(p0, *p1))? -1 : 1;
    return (o == 2)? -1: 1;
}// Prints convex hull of a set of n points.
void convexHull(Point points[], int n)
{
    // Find the bottommost point
    int ymin = points[0].y, min = 0;
    for (int i = 1; i < n; i++)
    {
        int y = points[i].y;
        // Pick the bottom-most or chose the left
        // most point in case of tie
        if ((y < ymin) || (ymin == y &&
            points[i].x < points[min].x))
            ymin = points[i].y, min = i;
    }
    // Place the bottom-most point at first position
    swap(points[0], points[min]);
    // Sort n-1 points with respect to the first point.
    // A point p1 comes before p2 in sorted ouput if p2
    // has larger polar angle (in counterclockwise
    // direction) than p1
    p0 = points[0];
    qsort(&points[1], n-1, sizeof(Point), compare);
    // If two or more points make same angle with p0,
    // Remove all but the one that is farthest from p0
    // Remember that, in above sorting, our criteria was
    // to keep the farthest point at the end when more than
    // one points have same angle.
    int m = 1; // Initialize size of modified array
    for (int i=1; i<n; i++)
    {
        // Keep removing i while angle of i and i+1 is same
        // with respect to p0
        while (i < n-1 && orientation(p0, points[i],
                                      points[i+1]) == 0)
            i++;
        points[m] = points[i];
        m++;   // Update size of modified array
    }
    // If modified array of points has less than 3 points,
```

```
    // convex hull is not possible
    if (m < 3) return;
    // Create an empty stack and push first three points
    // to it.
    stack<Point> S;
    S.push(points[0]);
    S.push(points[1]);
    S.push(points[2]);
    // Process remaining n-3 points
    for (int i = 3; i < m; i++)
    {
        // Keep removing top while the angle formed by
        // points next-to-top, top, and points[i] makes
        // a non-left turn
        while (orientation(nextToTop(S), S.top(), points[i]) != 2)
            S.pop();
        S.push(points[i]);
    }
    // Now stack has the output points, print contents of stack
    while (!S.empty())
    {
        Point p = S.top();
        cout << "(" << p.x << ", " << p.y <<")" << endl;
        S.pop();
    }
}// Driver program to test above functions
int main()
{
    Point points[] = {{0, 3}, {1, 1}, {2, 2}, {4, 4},
                      {0, 0}, {1, 2}, {3, 1}, {3, 3}};
    int n = sizeof(points)/sizeof(points[0]);
    convexHull(points, n);
    return 0;
}
```