

Dyck path - GeeksforGeeks

Consider a $n \times n$ grid with indexes of top left corner as $(0, 0)$. Dyck path is a staircase walk from bottom left, i.e., $(n-1, 0)$ to top right, i.e., $(0, n-1)$ that lies above the diagonal cells (or cells on line from bottom left to top right).

The task is to count the number of Dyck Paths from $(n-1, 0)$ to $(0, n-1)$.

Examples :

Input : $n = 1$

Output : 1

Input : $n = 2$

Output : 2

Input : $n = 3$

Output : 5

Input : $n = 4$

Output : 14

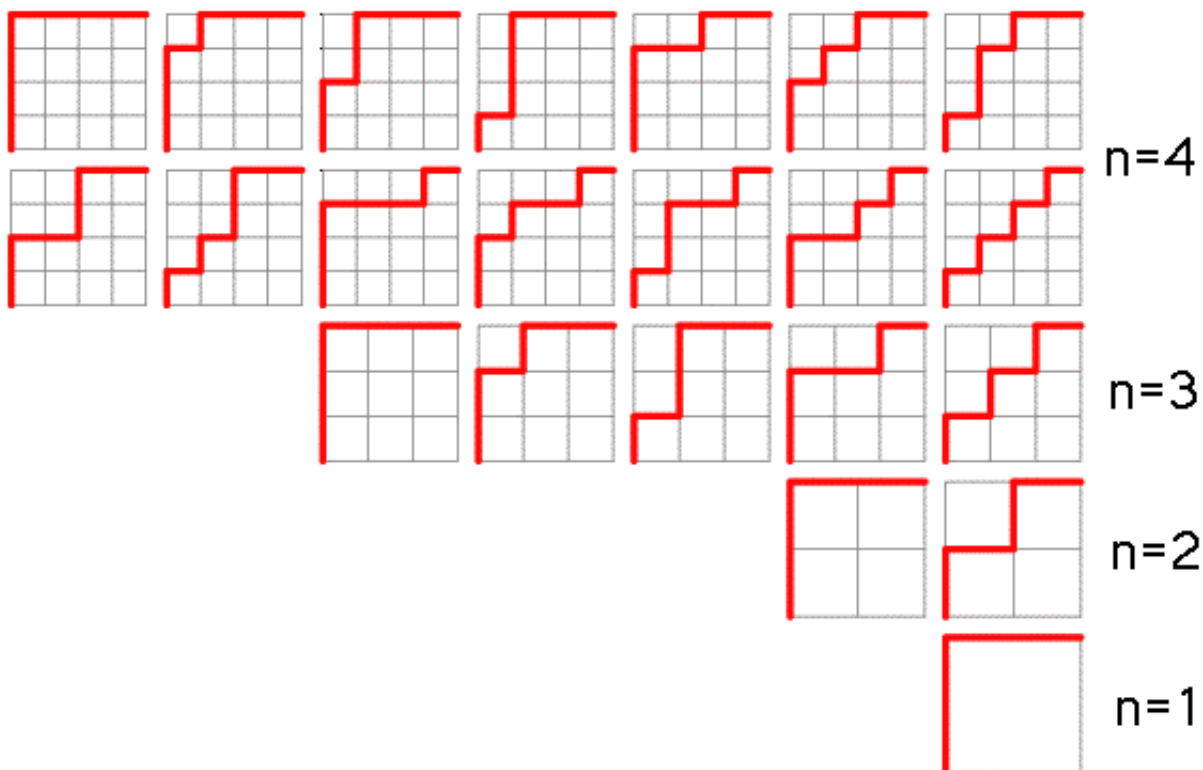


Image Source : <http://mathworld.wolfram.com/DyckPath.html>

The number of Dyck paths from $(n-1, 0)$ to $(0, n-1)$ can be given by the [Catalan number](#) $C(n)$.

$$C_n = \frac{(2n)!}{(n+1)!n!} = \prod_{k=2}^n \frac{n+k}{k} \quad \text{for } n \geq 0$$

Below are C++ and Java implementations to find count of Dyck Paths (or n'th Catalan number).

- C++
- Java

```
// C++ program to count number of Dyck Paths
#include<iostream>
using namespace std;

// Returns count Dyck paths in n x n grid
int countDyckPaths(unsigned int n)
{
    // Compute value of 2nCn
    int res = 1;
    for (int i = 0; i < n; ++i)
    {
        res *= (2*n - i);
        res /= (i + 1);
    }

    // return 2nCn/(n+1)
    return res / (n+1);
}

// Driver program to test above functions
int main()
{
    int n = 4;
    cout << "Number of Dyck Paths is " << countDyckPaths(n);
    return 0;
}
```

Output:

Number of Dyck Paths is 14

Exercise:

- Find number of sequences of 1 and -1 such that every sequence follows below constraints :
 - The length of a sequence is 2n
 - There are equal number of 1's and -1's, i.e., n 1's, n -1s
 - Sum of prefix of every sequence is greater than or equal to 0. For example, 1, -1, 1, -1 and 1, 1,

-1, -1 are valid, but -1, -1, 1, 1 is not valid.

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2. Number of paths of length $m + n$ from $(m-1, 0)$ to $(0, n-1)$ that are restricted to east and north steps.

This article is contributed by **Aditya Chatterjee**. If you like GeeksforGeeks and would like to contribute, you can also write an article and mail your article to contribute@geeksforgeeks.org. See your article appearing on the GeeksforGeeks main page and help other Geeks.

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