Greedy Algorithms | Set 3 (Huffman Coding) - GeeksforGeeks

Huffman coding is a lossless data compression algorithm. The idea is to assign variable-legth codes to input characters, lengths of the assigned codes are based on the frequencies of corresponding characters. The most frequent character gets the smallest code and the least frequent character gets the largest code.
The variable-length codes assigned to input characters are Prefix Codes, means the codes (bit sequences) are assigned in such a way that the code assigned to one character is not prefix of code assigned to any other character. This is how Huffman Coding makes sure that there is no ambiguity when decoding the generated bit stream.
Let us understand prefix codes with a counter example. Let there be four characters a, b, c and d, and their corresponding variable length codes be 00, 01, 0 and 1. This coding leads to ambiguity because code assigned to c is prefix of codes assigned to a and b. If the compressed bit stream is 0001, the de-compressed output may be “cccd” or “ccb” or “acd” or “ab”.

See this for applications of Huffman Coding.

There are mainly two major parts in Huffman Coding
1) Build a Huffman Tree from input characters.
2) Traverse the Huffman Tree and assign codes to characters.

Steps to build Huffman Tree
Input is array of unique characters along with their frequency of occurrences and output is Huffman Tree.

1. Create a leaf node for each unique character and build a min heap of all leaf nodes (Min Heap is used as a priority queue. The value of frequency field is used to compare two nodes in min heap. Initially, the least frequent character is at root)

2. Extract two nodes with the minimum frequency from the min heap.

3. Create a new internal node with frequency equal to the sum of the two nodes frequencies. Make the first extracted node as its left child and the other extracted node as its right child. Add this node to the min heap.

4. Repeat steps#2 and #3 until the heap contains only one node. The remaining node is the root node and the tree is complete.

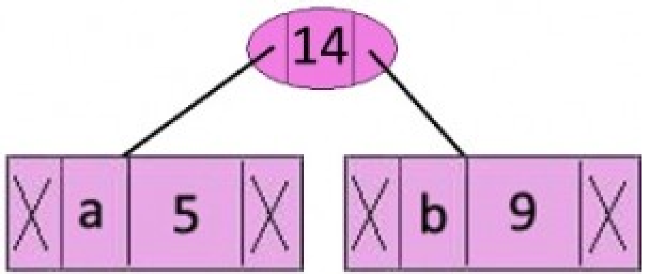
Let us understand the algorithm with an example:

```
character   Frequency
    a           5
```

```
     b            9
     c            12
     d            13
     e            16
     f            45
```

Step 1. Build a min heap that contains 6 nodes where each node represents root of a tree with single node.

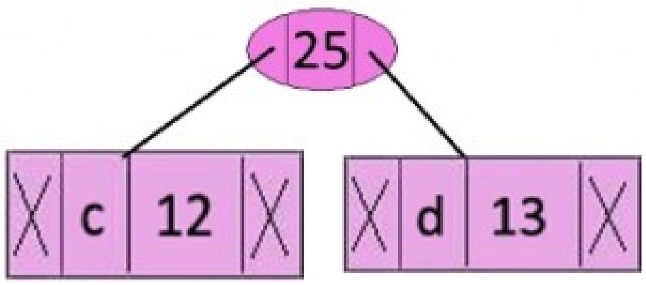
Step 2 Extract two minimum frequency nodes from min heap. Add a new internal node with frequency 5 + 9 = 14.



Now min heap contains 5 nodes where 4 nodes are roots of trees with single element each, and one heap node is root of tree with 3 elements

```
character           Frequency
       c               12
       d               13
 Internal Node         14
       e               16
       f                45
```

Step 3: Extract two minimum frequency nodes from heap. Add a new internal node with frequency 12 + 13 = 25

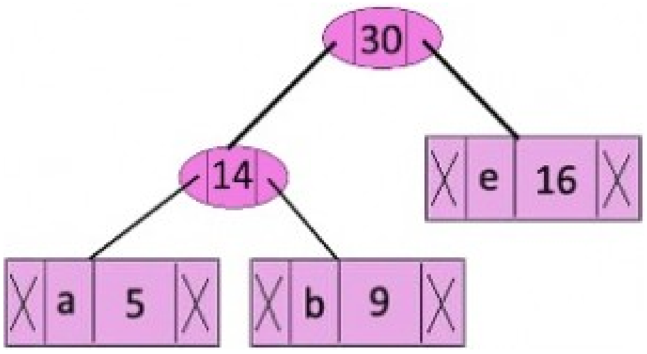


Now min heap contains 4 nodes where 2 nodes are roots of trees with single element each, and two heap nodes are root of tree with more than one nodes.

```
character           Frequency
Internal Node          14
```

```
       e                 16
Internal Node            25
       f                 45
```

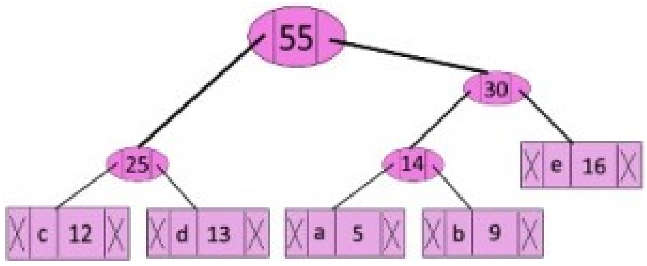
Step 4: Extract two minimum frequency nodes. Add a new internal node with frequency 14 + 16 = 30



Now min heap contains 3 nodes.

```
character          Frequency
Internal Node         25
Internal Node         30
      f               45
```

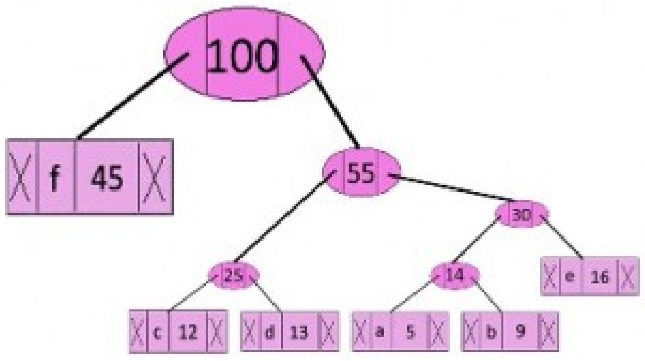
Step 5: Extract two minimum frequency nodes. Add a new internal node with frequency 25 + 30 = 55



Now min heap contains 2 nodes.

```
character     Frequency
       f         45
Internal Node    55
```

Step 6: Extract two minimum frequency nodes. Add a new internal node with frequency 45 + 55 = 100

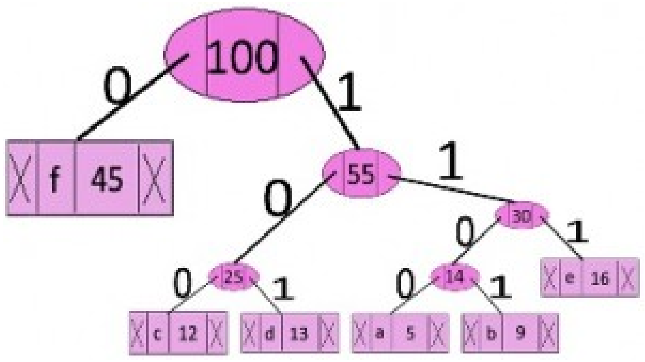


Now min heap contains only one node.

```
character      Frequency
Internal Node    100
```

Since the heap contains only one node, the algorithm stops here.

Steps to print codes from Huffman Tree:
Traverse the tree formed starting from the root. Maintain an auxiliary array. While moving to the left child, write 0 to the array. While moving to the right child, write 1 to the array. Print the array when a leaf node is encountered.



The codes are as follows:

```
character   code-word
    f          0
    c          100
    d          101
    a          1100
    b          1101
    e          111
```

```
// C program for Huffman Coding#include <stdio.h>#include <stdlib.h>// This
constant can be avoided by explicitly calculating height of Huffman
Tree#define MAX_TREE_HT 100// A Huffman tree node
struct MinHeapNode
```

```
{
      char data;  // One of the input characters
      unsigned freq;  // Frequency of the character
      struct MinHeapNode *left, *right; // Left and right child of this node
};// A Min Heap:  Collection of min heap (or Hufmman tree) nodes
struct MinHeap
{
      unsigned size;    // Current size of min heap
      unsigned capacity;   // capacity of min heap
      struct MinHeapNode **array;  // Attay of minheap node pointers
};// A utility function allocate a new min heap node with given character//
and frequency of the character
struct MinHeapNode* newNode(char data, unsigned freq)
{
      struct MinHeapNode* temp =
            (struct MinHeapNode*) malloc(sizeof(struct MinHeapNode));
      temp->left = temp->right = NULL;
      temp->data = data;
      temp->freq = freq;
      return temp;
}// A utility function to create a min heap of given capacity
struct MinHeap* createMinHeap(unsigned capacity)
{
      struct MinHeap* minHeap =
           (struct MinHeap*) malloc(sizeof(struct MinHeap));
      minHeap->size = 0;  // current size is 0
      minHeap->capacity = capacity;
      minHeap->array =
       (struct MinHeapNode**)malloc(minHeap->capacity * sizeof(struct
MinHeapNode*));
      return minHeap;
}// A utility function to swap two min heap nodes
void swapMinHeapNode(struct MinHeapNode** a, struct MinHeapNode** b)
{
      struct MinHeapNode* t = *a;
      *a = *b;
      *b = t;
}// The standard minHeapify function.
void minHeapify(struct MinHeap* minHeap, int idx)
{
      int smallest = idx;
      int left = 2 * idx + 1;
```

```
    int right = 2 * idx + 2;
    if (left < minHeap->size &&
        minHeap->array[left]->freq < minHeap->array[smallest]->freq)
      smallest = left;
    if (right < minHeap->size &&
        minHeap->array[right]->freq < minHeap->array[smallest]->freq)
      smallest = right;
    if (smallest != idx)
    {
        swapMinHeapNode(&minHeap->array[smallest], &minHeap->array[idx]);
        minHeapify(minHeap, smallest);
    }
}// A utility function to check if size of heap is 1 or not
int isSizeOne(struct MinHeap* minHeap)
{
    return (minHeap->size == 1);
}// A standard function to extract minimum value node from heap
struct MinHeapNode* extractMin(struct MinHeap* minHeap)
{
    struct MinHeapNode* temp = minHeap->array[0];
    minHeap->array[0] = minHeap->array[minHeap->size - 1];
    --minHeap->size;
    minHeapify(minHeap, 0);
    return temp;
}// A utility function to insert a new node to Min Heap
void insertMinHeap(struct MinHeap* minHeap, struct MinHeapNode* minHeapNode)
{
    ++minHeap->size;
    int i = minHeap->size - 1;
    while (i && minHeapNode->freq < minHeap->array[(i - 1)/2]->freq)
    {
        minHeap->array[i] = minHeap->array[(i - 1)/2];
        i = (i - 1)/2;
    }
    minHeap->array[i] = minHeapNode;
}// A standard funvtion to build min heap
void buildMinHeap(struct MinHeap* minHeap)
{
    int n = minHeap->size - 1;
    int i;
    for (i = (n - 1) / 2; i >= 0; --i)
        minHeapify(minHeap, i);
```

```
}// A utility function to print an array of size n
void printArr(int arr[], int n)
{
    int i;
    for (i = 0; i < n; ++i)
        printf("%d", arr[i]);
    printf("\n");
}// Utility function to check if this node is leaf
int isLeaf(struct MinHeapNode* root)
{
    return !(root->left) && !(root->right) ;
}// Creates a min heap of capacity equal to size and inserts all character
of // data[] in min heap. Initially size of min heap is equal to capacity
struct MinHeap* createAndBuildMinHeap(char data[], int freq[], int size)
{
    struct MinHeap* minHeap = createMinHeap(size);
    for (int i = 0; i < size; ++i)
        minHeap->array[i] = newNode(data[i], freq[i]);
    minHeap->size = size;
    buildMinHeap(minHeap);
    return minHeap;
}// The main function that builds Huffman tree
struct MinHeapNode* buildHuffmanTree(char data[], int freq[], int size)
{
    struct MinHeapNode *left, *right, *top;
    // Step 1: Create a min heap of capacity equal to size.  Initially,
there are
    // modes equal to size.
    struct MinHeap* minHeap = createAndBuildMinHeap(data, freq, size);
    // Iterate while size of heap doesn't become 1
    while (!isSizeOne(minHeap))
    {
        // Step 2: Extract the two minimum freq items from min heap
        left = extractMin(minHeap);
        right = extractMin(minHeap);
        // Step 3:  Create a new internal node with frequency equal to the
        // sum of the two nodes frequencies. Make the two extracted node as
        // left and right children of this new node. Add this node to the
min heap
        // '$' is a special value for internal nodes, not used
        top = newNode('$', left->freq + right->freq);
        top->left = left;
        top->right = right;
        insertMinHeap(minHeap, top);
    }
    // Step 4: The remaining node is the root node and the tree is
complete.
    return extractMin(minHeap);
}// Prints huffman codes from the root of Huffman Tree.  It uses arr[] to//
store codes
void printCodes(struct MinHeapNode* root, int arr[], int top)
{
    // Assign 0 to left edge and recur
    if (root->left)
    {
        arr[top] = 0;
        printCodes(root->left, arr, top + 1);
    }
    // Assign 1 to right edge and recur
    if (root->right)
    {
        arr[top] = 1;
        printCodes(root->right, arr, top + 1);
    }
    // If this is a leaf node, then it contains one of the input
    // characters, print the character and its code from arr[]
    if (isLeaf(root))
    {
        printf("%c: ", root->data);
        printArr(arr, top);
    }
}// The main function that builds a Huffman Tree and print codes by
traversing// the built Huffman Tree
void HuffmanCodes(char data[], int freq[], int size)
{
   //  Construct Huffman Tree
   struct MinHeapNode* root = buildHuffmanTree(data, freq, size);
   // Print Huffman codes using the Huffman tree built above
   int arr[MAX_TREE_HT], top = 0;
   printCodes(root, arr, top);
}// Driver program to test above functions
int main()
{
    char arr[] = {'a', 'b', 'c', 'd', 'e', 'f'};
    int freq[] = {5, 9, 12, 13, 16, 45};
    int size = sizeof(arr)/sizeof(arr[0]);
    HuffmanCodes(arr, freq, size);
    return 0;
}
```

```
f: 0
c: 100
d: 101
a: 1100
b: 1101
e: 111
```

Time complexity: O(nlogn) where n is the number of unique characters. If there are n nodes, extractMin() is called 2*(n – 1) times. extractMin() takes O(logn) time as it calles minHeapify(). So, overall complexity is O(nlogn).

If the input array is sorted, there exists a linear time algorithm. We will soon be discussing in our next post.

Reference:
http://en.wikipedia.org/wiki/Huffman_coding

This article is compiled by Aashish Barnwal and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.