Recursion Problem Set

For each of these problems, identify the base case, and the recurrence relation.

Coins:

In Byteland they have a very strange monetary system.
Each Bytelandian gold coin has an integer number written on it. A coin n can be exchanged in a bank into three coins: n/2, n/3 and n/4. But these numbers are all rounded down (the banks have to make a profit).
You can also sell Bytelandian coins for American dollars. The exchange rate is 1:1. But you cannot buy Bytelandian coins.
You have n gold coins. What is the maximum amount of American dollars you can get for it?

Lottery Ticket

Nick likes to play the lottery. The cost of a single lottery ticket is price. Nick has exactly four banknotes with values b1, b2, b3 and b4 (some of the values may be equal). He wants to know if it's possible to buy a single lottery ticket without getting any change back. In other words, he wants to pay the exact price of a ticket using any subset of his banknotes. Return true if it is possible and false otherwise.

Truckloads

We have a pile of crates at our warehouse that we want to load onto trucks. Our plan is to divide the pile in half forming two smaller piles, then continuing dividing each of the small piles in half until we get piles that will fit on a truck. (Of course, when we divide an odd number of crates in "half", one of the resulting piles will have one more crate than the other.) Our problem is to determine how many trucks we will need to ship the crates. Given N number of crates and capacity of each truck c.

Lines in a plane:

Given a plane, and N lines, you have to find the maximum number of partitions of the plane by the given lines. The position/direction of the lines is chosen such that the number of partitions is maximized.

Sequencing differences:

Given a sequence of K elements, we can calculate its difference sequence by taking the difference between each pair of adjacent elements. For instance, the difference sequence of {5, 6, 3, 9,-1} is {6-5, 3-6, 9-3,-1-9} = {1,-3, 6,-10}. Formally, the difference sequence of the sequence a1, a2, ..., ak is b1, b2, ... , bk-1, where bi = ai+1 - ai.

The derivative sequence of order N of a sequence A is the result of iteratively applying the above process N times. For example, if A = {5,6,3,9,-1}, the derivative sequence of order 2 is: {5,6,3,9,-1} -> {1,-3,6,-10} -> {-3-1,6-(-3),-10-6} = {-4,9,-16}.

You will be given a sequence a as an int array and the order n. Return an int array representing the derivative sequence of order n of a.

Mining a pyramid:

You are given a pyramid where every level has i number of blocks, number of levels is n.

Example

n=5

```
    _
   _ _
  _ _ _
 _ _ _ _
_ _ _ _ _
```

Each block has a bit of gold in it, and the amount of gold is given.

Example:

```
    6
   6 3
  6 2 5
 1 4 8 7
2 9 2 5 6
```

You can safely assume that stealing the top block will not make the structure unstable. For each level i below that, as long as you remove one of the 2 blocks, that was directly under the block you removed in level i-1.

Example, if you remove 2 in level 3, you can remove either 4 or 8 from level 4.

Find the maximum amount of gold you can remove, while keeping the structure stable.

Save Thy Toys

Leonard is very fond of buying rare and expensive science fiction toys. He keeps his collection in a sequential order of the date on which the toy was bought in a special closet so that his roomie Sheldon never gets hold of his toys. But because of his bad luck Leonard once loses a bet to Sheldon and Sheldon demands a share Leonard's toys. Since Leonard doesn't want to lose much money, he decides upon a strategy to reduce his loss to minimum.

Leonard, beginning from the first toy in his closet will pick some toys, say "x" toys in sequence. Sheldon will then pick the next "x" toys (Note that Sheldon picks equal no. of toys as picked by Leonard in his move unless the remaining toys are less than "x". In that case he picks all of the remaining). This will keep going on till no more toys are left in the closet for Leonard to pick. You are given the sequence of toys with their price. Help Leonard in maximizing the total price of all toys he picks.

Leonard in his each turn can either pick only 1 or 2 or 3 toys ("x" described above can take value either 1, 2 or 3).

Tower of Hanoi:

The **Tower of Hanoi** consists of three rods, and a number of disks of different sizes which can slide onto any rod. The puzzle starts with the disks in a neat stack in ascending order of size on one rod, the smallest at the top.

a)

Number of Disks: n

Here n=3

```
   |      |        |
   -      |        |
  _ _     |        |
 _ _ _    |        |
   A      B        C
```

Number of moves to move disks from A to C given:

1. Only one disk can be moved at a time.
2. Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack i.e. a disk can only be moved if it is the uppermost disk on a stack.
3. No disk may be placed on top of a smaller disk.

b) Added Condition:

III) You can only move a disk to the adjacent tower. i.e., A->B, B->A, B->C, and C -> B are the only legal moves.

Josephus Problem:

There are N people standing in a circle waiting to be executed. The counting out begins at some point in the circle and proceeds around the circle in a fixed direction. In each step, K people are skipped and the next person is executed. The elimination proceeds around the circle (which is becoming smaller and smaller as the executed people are removed), until only the last person remains, who is given freedom. Where should you stand to gain freedom?

a) K= input variable

b) K=2

c) K=i+2, where i = number of people killed. Initially, K=2 (as number of people killed = 0)

d) Instead of killing K^{th} position, the kth person is saved. Solve part (a) with this change.

Escape from Prison:

a)

You have been imprisoned. You are placed in a cell that is N-1 cells away from the prison door (i.e., the N^{th} cell). You want to escape from the prison. At any time, you can move to the cell right next to your current location, or any one after that up to the K^{th} cell from your location. After that you check for guards and make another move until you get to the prison door, which is open and unguarded. You can safely assume that you won't be caught as long as you are in a cell.

Given N and K, you have to find the number of ways you can escape the prison.
Example: N = 12, K=3
Initial Position:
x _ _ _ _ _ _ _ _ _ _ _ |gate|
Legal Moves:
I)
_ x _ _ _ _ _ _ _ _ _ _ |gate|
II)
_ _ x _ _ _ _ _ _ _ _ _ |gate|
III)
_ _ _ x _ _ _ _ _ _ _ _ |gate|

b) Instead of having cells stacked in a single row, the cells are present in rows and columns. If you take the origin as the prison door, you are in the square shaped unit cell where the left lower coordinate is (n, m) and you can take k1, k2 moves in x, y directions respectively. How many ways are there for you to escape?

c) Consider the same case as before, with the added constraint k1, k2=0, 1 or 2, but not 0 at the same time. However this time, when you get to the door, you see it is locked. You have to get back to your own cell before the guards check all cells at midnight. You have enough time to return. However, you can only return along the cells you came. How many total paths are there from your cell, to the door, and back?

Hint: When you skip 2 cells along x from position (p, q) to (p-2, q), you move through the intermediate cell (p-1, q) and similarly for y. So, when you're returning, you can stop at (p-1, q) and check instead of directly going to (p, q) from (p-2, q).

That’s All Folks