Vehicle Motion Control with Jerk Constraint by Nonlinear Model Predictive Control

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***Abstract*—In this paper, a nonlinear model predictive control considering vehicle jerk dynamics is proposed for improving ride comfort of passengers. Since the vehicle model in prediction phase requires high accuracy dynamics in order to handle the jerk motion, the approximated wheel load transfer dynamics is introduced. Also, to obtain the control capability for not only jerk but also acceleration, velocity and position, the expanded state space model including these dimensions into the one has been developed. It improves the utility as autonomous vehicle controller. By numerical simulation in assuming cornering driving scene, the effectiveness that jerk and other vehicle states enable to constraint simultaneously by individual torque distribution by electric power train is validated. Further, the principle of the optimized torque distribution by proposed method is analyzed.**

Keywords—model predictive control, jerk control, torque distribution, control allocation, autonomous vehicle, ride comfort

I. Introduction

In recent years, automobile manufacturers and IT enterprises have rapidly advanced technological development for next-generation mobility services. It is known as a MaaS (Mobility as a Service). In this trend, a mobility requires the service space to increase the value of entire travel time, not just as purpose of the transportation. On a technology perspective, existing mobility has been designed mainly as focusing on the driver comfort, as ease of operation is pursued. However, with advancing of autonomous vehicle, since the interior space will be suit for MaaS in the future, the focus of comfort will be changed from driver-centric to passenger-centric. To be concentrated on passenger tasks assuming such as the meal, sleep, entertainment and work, the importance of the posture stable control technology which does not disturb these tasks will be increased.

Some research exemplified the results that the increasing of jerk which is differential of acceleration affects the aggravation of ride comfort [1] [2]. In other words, the reduction of the jerk could be an important index to improve the ride comfort for passenger, conventionally related research for jerk control has been studied for brake control system [3]. Whilst regarding the advanced vehicle control system, recently model predictive control (MPC) has drawn attention to improve the control performance. The MPC is one of the optimization control methodologies, some researchers have applied it to the vehicle control system and reported their good performance [4] [5]. The reference [5] utilizes nonlinear MPC which can control each tire slips to realize fastest speed control.

This paper describes the consideration results of the nonlinear MPC control with jerk constraint to improve the ride-comfort of passenger.

II. Construction of Nonlinear Model Predictive Control with Jerk Constraint

A. Research Subject and Challenges

This research is for the vehicle integrated control of the autonomous driving vehicle, especially it aims to construct of the control algorithm which manages multiple actuators to acquire the desired 6DOF vehicle motion. Here, 6DOF means the triaxial translational motion and the rotational motion of each axis. Fig. 1 shows an outline of the assumed autonomous vehicle control system to clarify the research subject. This system has a path and motion planning which determines the target path based on the external recognition sensor information and map information and desired motion command. There is also a vehicle integrated control system which distributes each actuator commands of such as powertrain, brake, steering and suspension to follow the target path and desired motion. The vehicle integrated control system is this research focus. As a kind of electric powertrain, in-wheel motor (IWM) is assumed. Since the torque of the IWM can be controlled independently on each tires, it has potential to increase the controllability of vehicle motion, and IWM takes an alternative partially on not only power train but also brake system by power regeneration.

MPC is one of the optimized control methods, has been studied since around 1980s [6]. The remarkable features of MPC are able to treat complex dynamics with multi-input and multi-output system easily, and to consider the constraint explicitly. To apply MPC for 6DOF vehicle motion control considering jerk, there are some challenges in this research that

- The prediction model needs to be enough precise because the jerk is minute motion. Concretely, nonlinear prediction model including couple vehicle dynamics of 6DOF shall be constructed.

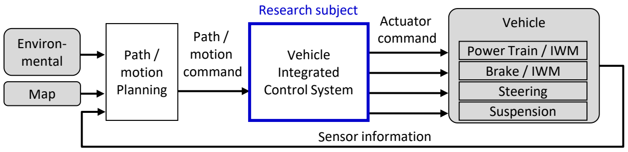


Fig. 1. Autonomous vehicle control system.

- The algorithm needs to be able to handle both the jerk and the other physical dimensions such as position and velocity to feasible as an autonomous vehicle control.

B. Prediction Model of 6 DOF Nonlinear Vehicle Dynamics

First, the definition of 6DOF is shown in Fig. 2. The x, y and z are the local coordinate system. Roll, pitch and yaw are the rotational of each axis. The requirements of the prediction model are to express the nonlinear coupling dynamics of 6DOF, and to express anti-dive/lift force and jack-up/down force by IWM torque distribution. The first requirement is for predicting the vehicle motion accurately, and the second is one of the most advantage of the integrated control system by using IWM. The torque distribution of IWM can control vehicle posture directly without active suspension. Here, 6DOF nonlinear vehicle predictive dynamics are defined in (1) to (10). These equations consider the cornering force of each wheels individually and jack-up/down force caused by traction of IWM.

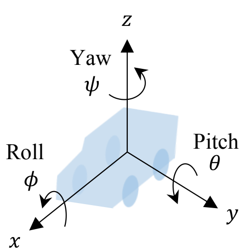


Fig. 2. Vehicle coordinate system.

$$m\ddot{x} + m_s h_p \ddot{\theta} = (m_u - m)\dot{z}\dot{\theta} + \sum_{i\in\{\mathrm{FL,FR,RL,RR}\}} (T_{x_i} - CD_i) \quad (1)$$

$$\begin{aligned} m\ddot{y} - m_s h_r \ddot{\phi} &= -m\dot{x}\dot{\psi} + (m - m_u)\dot{z}\dot{\phi} - m_s h_r \dot{\theta}\dot{\psi} \\ &+ \sum_{i\in\{\mathrm{FL,FR,RL,RR}\}} (T_{y_i} + CF_i) \end{aligned} \quad (2)$$

$$\begin{aligned} m_s\ddot{z} &= m_s h_p \dot{\theta}^2 + \rho_{z1}\dot{z} + \rho_{z2}\dot{\theta} + \rho_{z3} z + \rho_{z4}\theta - \sum_{i\in\{\mathrm{FL,FR}\}} J_i \\ &+ \sum_{i\in\{\mathrm{RL,RR}\}} J_i \end{aligned} \quad (3)$$

$$\begin{aligned} -m_s h_r \ddot{y} + I_x \ddot{\phi} &= m_s h_r(\dot{x}\dot{\psi} - \dot{z}\dot{\phi}) + (I_y - I_z)\dot{\theta}\dot{\psi} + \rho_{\phi1}\phi \\ &+ \rho_{\phi2}\dot{\phi} - \sum_{i\in\{\mathrm{FL,RR}\}} \frac{d_i}{2} J_i + \sum_{i\in\{\mathrm{FR,RL}\}} \frac{d_i}{2} J_i \end{aligned} \quad (4)$$

$$\begin{aligned} m_s h_p \ddot{x} + I_y \ddot{\theta} &= m_s h_p \dot{y}\dot{\psi} - m_s h_p \dot{z}\dot{\theta} + (I_z - I_x)\dot{\phi}\dot{\psi} + \rho_{\theta1} z \\ &+ \rho_{\theta2}\theta + \rho_{\theta3}\dot{z} + \rho_{\theta4}\dot{\theta} + \sum_{i\in\{\mathrm{FL,FR,RL,RR}\}} l_i J_i \end{aligned} \quad (5)$$

$$\begin{aligned} I_z\ddot{\psi} &= (I_x - I_y)\dot{\theta}\dot{\phi} + \sum_{i\in\{\mathrm{FL,FR}\}} l_i(CF_i + T_y) + \sum_{i\in\{\mathrm{RL,RR}\}} l_i(CF_i - T_y) \\ &- \sum_{i\in\{\mathrm{FL,RL}\}} \frac{d_i}{2} T_x + \sum_{i\in\{\mathrm{FR,RR}\}} \frac{d_i}{2} T_x \end{aligned} \quad (6)$$

$$T_x = \frac{1}{r}\tau\cos\delta, T_y = \frac{1}{r}\tau\sin\delta \quad (7)$$

$$J = T_x \tan\xi_i \quad (8)$$

$$CD = K \cdot \beta \quad (9)$$

$$\begin{aligned} CF|_{i\in\{\mathrm{FL,FR}\}} &= K_i\left(\delta_i - \beta_i - \frac{l_i}{v}\dot{\psi}\right) \\ CF|_{i\in\{\mathrm{RL,RR}\}} &= K_i\left(\delta_i - \beta_i + \frac{l_i}{v}\dot{\psi}\right) \end{aligned} \quad (10)$$

Where, m is a vehicle mass, I is a moment of inertia, T is a traction force, CD is a cornering drag, CF is a cornering force, K is a cornering power, β is a side slip angle, h_p is a pitch center height, h_r is a roll center height, l is a length from C.G. to axle, J is a jack-up force, d is a tread width, r is a tire radius, ξ is an anti-dive and lift angle, ρ is a spring and damping coefficient, τ is a IWM torque, δ is a steering angle. Further the suffix s and u mean sprung and unsprung mass of the vehicle; the first characters of suffix F and R which correspond to i are front and rear wheel; the first characters of suffix L and R which correspond to i are the left and right wheel.

Above dynamics are tailored to take advantage of IWM by introducing the jack-up/down force. Though the dynamics is complicated, MPC can obtain theoretically the control system. However, one difficulty exists to apply their dynamics for MPC. K : cornering power has a high nonlinearity by load transfer of each wheel so that usually it is treated as the table data in the simulator or a constant value in the vehicle controller. Whilst this research is important to consider the changing of cornering power in order to distribute each IWM torque accurately. Therefore the vary K is introduced and approximated in (11), and the modeling concept is also shown in Fig. 3. Where K_0 represents a constant value of cornering power, which is set by balanced wheel load; ε represents a coefficient of changing cornering power depends on the wheel load; F represents a displacement from initial balanced load. On the basic MPC algorithm, the state space (SS) model is required to ease calculation, the proposed model makes easy to be applied to the SS model in MPC. However the approximated model shall be assumed in using small side slip angle by linearization.

$$K_i = K_0 + \varepsilon F_i \quad (11)$$

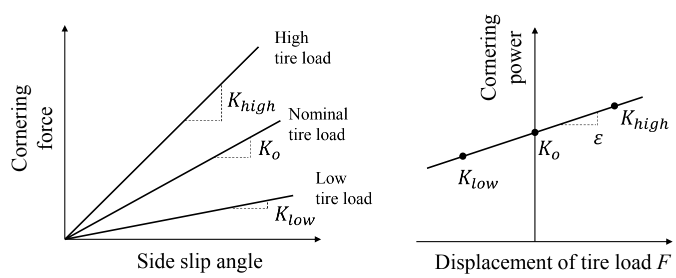


Fig. 3. Concept of load transfer model.

C. Algorithm Design of Nonlinear MPC with Jerk Dynamics

To obtain the SS model for constructed predictive model, the state vector $\boldsymbol{X}$ and input vector $\boldsymbol{U}$ are defined in (12) and (13). Then SS model is constructed by (14) which can treat nonlinear dynamics. In this case, the equation (12) doesn't include the jerk states so that it is not suite for jerk control. Hence, second derivative of dynamics is introduced. The derived states, inputs and SS denotes in (15), (16) and (17). As a result, the state vector can include jerk states, e.g., longitudinal jerk $\dddot{x}$. The input vector is identically represented by the second derivative of actuation commands of IWM torque and front tire steering angle. If this model is used for vehicle controller, the jerk and accelerate states can be controlled but other physical dimensions such as position and velocity are ignored because the state vector doesn't include these parameters. Moreover the actuator commands $\boldsymbol{U}$ aren't included as well so that the limitation of actuator, e.g. the maximum torque of IWM cannot be considered as the constraints condition in MPC algorithm. It might be difficult to be used for autonomous vehicle control system.

$$\boldsymbol{X} = \left[\dot{x}\ \dot{y}\ \dot{z}\ \dot{\phi}\ \dot{\theta}\ \dot{\psi}\ x\ y\ z\ \phi\ \theta\ \psi\right]^{\mathrm{T}} \tag{12}$$

$$\boldsymbol{U} = [\tau_i\ \delta_F\ ]^{\mathrm{T}} \qquad i \in \{FL, FR, RL, RR\} \tag{13}$$

$$\dot{\boldsymbol{X}} = \boldsymbol{f}(\boldsymbol{X}, \boldsymbol{U}) \tag{14}$$

$$\ddot{\boldsymbol{X}} = \left[\dddot{x}\ \dddot{y}\ \dddot{z}\ \dddot{\phi}\ \dddot{\theta}\ \dddot{\psi}\ \ddot{x}\ \ddot{y}\ \ddot{z}\ \ddot{\phi}\ \ddot{\theta}\ \ddot{\psi}\right]^{\mathrm{T}} \tag{15}$$

$$\ddot{\boldsymbol{U}} = \left[\ddot{\tau}_i\ \ddot{\delta}_F\right]^{\mathrm{T}} \qquad i \in \{FL, FR, RL, RR\} \tag{16}$$

$$\dddot{\boldsymbol{X}} = \ddot{\boldsymbol{f}}(\ddot{\boldsymbol{X}}, \ddot{\boldsymbol{U}}) \tag{17}$$

To solve this problem, the state vector is expanded to include additional states and inputs, the expanded state vector and SS model are redefined by (18) and (19). Additionally the output vector is also denotes by (20). Where $\mathbf{C}$ is defined as an unit matrix. The improved SS model includes 48 dimensions in state vector and 5 dimensions in input vector. Thus proposed SS model can treat various physical dimension such as jerk, acceleration, velocity and position within one prediction model. Furthermore, the method can also handle the limitation of actuators simultaneously by introducing expanded state vector. Consequently, the proposed method can realize the optimized actuators allocation to follow the desired vehicle motion under the jerk constraint and actuator limitation in the nonlinear MPC (NMPC) algorithm. NMPC executes optimized calculation on the cost function defined in (21) and (22) under the constraint status of (23). Where V, H_p, $\boldsymbol{E}$ and $\boldsymbol{Q}$ are a cost, a predictive horizon, a state error vector, and a weight matrix respectively. Also $\mathbf{G_X}$ and $\mathbf{G_U}$ represents coefficient matrix that determines each constraint conditions on predicted states and inputs respectively. The optimized input is generated in time series through the predicted future, but only the first input actually can be used as control command and a new optimization calculation is carried out again in the next control cycle in real-time.

$$\tilde{\boldsymbol{X}} = \left[\ddot{\boldsymbol{X}}\ \boldsymbol{X}\ \dot{\boldsymbol{U}}\ \boldsymbol{U}\right]^{\mathrm{T}} \tag{18}$$

$$\dot{\tilde{\boldsymbol{X}}} = \boldsymbol{f}(\tilde{\boldsymbol{X}}, \tilde{\boldsymbol{U}}) \qquad \because \tilde{\boldsymbol{U}} = \ddot{\boldsymbol{U}} \tag{19}$$

$$\tilde{\boldsymbol{Y}} = \mathbf{C}\tilde{\boldsymbol{X}} \tag{20}$$

$$V = \int_{t=0}^{H_p} \boldsymbol{E}^{\mathrm{T}}(t)\mathbf{Q}(t)\boldsymbol{E}\,(t)\,dt \tag{21}$$

$$\boldsymbol{E}(t) = \boldsymbol{X}(t) - \boldsymbol{X_r}(t) \tag{22}$$

$$\begin{aligned} &\mathbf{G}_X\left(\boldsymbol{X}(t),\ \cdot\ \cdot\ \cdot \boldsymbol{X}(t+H_p)\right)\cdot \boldsymbol{X} \leq \mathbf{0} \\ &\mathbf{G}_U\left(\boldsymbol{U}(t),\ \cdot\ \cdot\ \cdot \boldsymbol{U}(t+H_P)\right)\cdot \boldsymbol{U} \leq \mathbf{0} \end{aligned} \tag{23}$$

III. ACCURACY VERIFICATOIN OF THE PREDICTION MODEL

A. *Verification Environment*

Three models are prepared to verify the accuracy of the predictive model; a predictive model constructed in above discussion, a predictive model without load transfer dynamics, a high accuracy model developed for vehicle simulator as an ideal model. The simulator model realizes high accuracy by includes mechanics friction, road friction, actuator response and so on. In below discussion, the principal vehicle specifications accordance with Table I.

B. *Verification Results*

Open-loop simulation without controller of these models are conducted to compare the results. The initial velocity is set on two patterns; 40 [km/h] and 80 [km/h], and the other vehicle states are zero. For the simulation IWM torque and steering angle are varied as shown in Fig. 4. From 1 to 2 second, IWM torque is increased by minus 300 [Nm] on each wheel, and the steering angle is also increased by 3 [deg] simultaneously. This case simulates deceleration and left turn. Though such a simulation case of both braking and steering is not typical driving scene, but the rapid deceleration and cornering cause large load transfer of each tire so that it is better scenario for the purpose to verify the accuracy of developed model.

TABLE I. VEHICLE SPECIFICATIONS

Vehicle mass	[kg]	1800	Pitch center	[m]	0.4
Wheelbase	[m]	3	Roll center	[m]	0.3
C.G. height	[m]	0.5	Inertia of x	[kgm^2]	700
Anti-dive, lift angle	[deg]	10	Inertia of y	[kgm^2]	2100
Tire radius	[m]	0.3	Inertia of z	[kgm^2]	2100

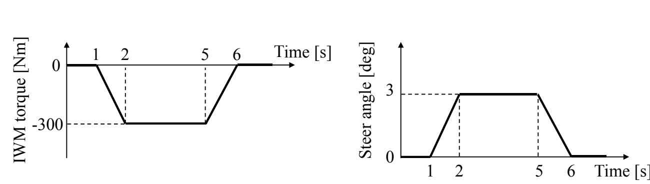
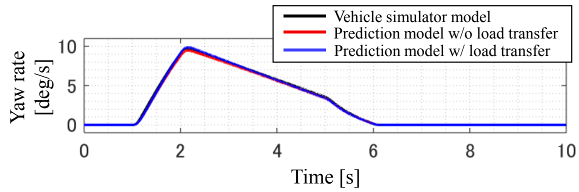
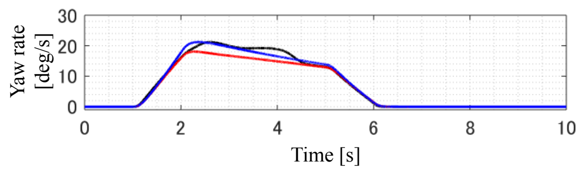


Fig. 4. Actuator commands for the open-loop simulation.



(i) Initial velocity 40 km/h.



(ii) Initial velocity 80 km/h.

Fig. 5. Simulation result of yaw rate.

As a representative example within the simulation results, yaw rate on the initial velocity on 40 [km/h] and 80 [km/h] are shown in Fig. 5. The result of the vehicle simulator model is represented in black line, the predict model without load transfer dynamics is in red line and proposed predictive model is in blue line. According to the results in (i), the yaw rate of both prediction models align with the vehicle simulator model. Thus these predictive models are verified to be accurate whether they have load transfer dynamics or not. On the other hand, we can recognize the difference in (ii). The prediction model with load transfer is closer to the vehicle simulator model than the prediction model without load transfer. The result means that the approximated load transfer dynamics can improve the model accuracy.

IV. Control Performance Analysis

A. Simulation Condition

The control performance of proposed NMPC is described by simulation results in this chapter. In this simulation, proposed NMPC is used for controller and the prediction model used in NMPC is also used as the plant model (Fig. 6). The intention of using the same model between prediction and the plant is to make sure the algorithm performance by getting rid of the model errors. In other words, we would verify if the jerk and other vehicle states are able to constraint properly. The vehicle specification is the same as mentioned in chapter III, and the only IWM is used for the actuator in this simulation. It means the steering angle don't use as actuator and IWM makes the vehicle turning by torque distribution. Here, the simulation constitution and simulation cases are shown in Fig. 7 and Table 2. These simulation cases are difference on the constraint conditions by using proposed NMPC. The case (a) sets only maximum IWM torque constraint. This is as hardware constraint.

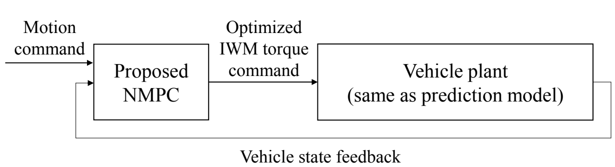


Fig. 6. Simulation constitution.

The case (b) sets additionally jerk constraint of lateral direction. The case (c) sets additionally roll rate constraint. The lateral jerk and roll rate are assumed as the index of ride comfort. Moreover, control parameters of proposed NMPC are described in Table 3. This simulation assume the cornering scenario with constant velocity by 60 [km/h]. Where, the prediction time is 0.1 [s], the prediction horizon is 10 steps, the control cycle is 0.01 [s] and the control step is 2 steps within the prediction horizon. Then the weight for steering angle is enough larger than IWM torque in order to deny the command generation as control input.

TABLE II. SIMULATION CASE

Case	Constraint condition		
	Max. IWM torque [Nm/wheel]	y jerk [m/s^3]	Roll rate [deg/s]
(a)	within ±1000	n/a	n/a
(b)	within ±1000	within ±4	n/a
(c)	within ±1000	within ±4	within ±3

TABLE III. CONTROL PARAMETERS OF PROPOSED NMPC

Prediction time	[s]	0.1
Prediction horizon	[-]	10
Control cycle	[s]	0.01
Control step	[-]	2
Weight of each vehicle states	[-]	10^4 : Longitudinal acceleration 10^5 : Velocity 10^5 : Yaw angular acceleration 10^6 : Yaw rate 10 : IWM torque 10^5 : Steering angle

B. Simulation Results and Considerations

The cornering scenario outputs the motion command of yaw rate by 7 [deg/s] at 0.1 [s], the proposed NMPC tries to align with the commands of yaw rate and velocity.

The simulation results are shown in Fig. 7. According to the results, all results are convergent to yaw rate command without steady state error by proposed method. Moreover, (b) keeps the constraint of y jerk and (c) is also able to realize the y jerk and roll rate constraint. Incidentally, the bottom figure shows the IWM torque differential between left wheel and right wheel. This value effects to the yaw moment, its positive torque difference causes positive yaw rate and left turn. A notable feature of proposed method is that (b) stops the increasing of torque difference at around 0.3 [s] to keep the y jerk constraint. Further (c) stops it earlier than (b) at 0.2 [s] in order to prevent the excess of not only y jerk constraint but also roll rate constraint simultaneously. Additionally, the IWM torque distribution of each wheels is shown in Fig. 8 to analyze the optimized control strategy by proposed NMPC. In the case with (a), only the torque between left and right is distributed because this control purpose is only for the yaw rate. On the other hand, in the case with (b), the distribution becomes complicate. To reduce y jerk, proposed method generates the positive roll rate on purpose by jack-up and down force. The principle diagram regarding generating of jack-up and down force is shown in Fig. 9. The jack-up force upon the suspension can be generated when the traction force directs pinch by front and rear, and the jack-down force is its opposite direction. To understand the phenomenon on mathematical perspective, the equation of lateral jerk (Γ_y) is described in (24). The equation is introduced from (2) under the constant velocity and yaw rate conditions. This values is the same as a differential of centrifugal force as imitating passenger feel and is in the reverse sign to vehicle y-coordinate. According to the equation, increasing of differential of roll angle acceleration $\dddot{\phi}$ can decrease the y jerk Γ_y. Thus the proposed method distributes IWM torque to increase the positive $\dddot{\phi}$ and realizes to decrease Γ_y. These complicated commands shows the benefit of optimized torque distribution by prediction considering coupling dynamics.

$$\Gamma_y = -\dddot{y} = \dot{x}\ddot{\psi} - \frac{m_s}{m} h_r \dddot{\phi} \tag{24}$$

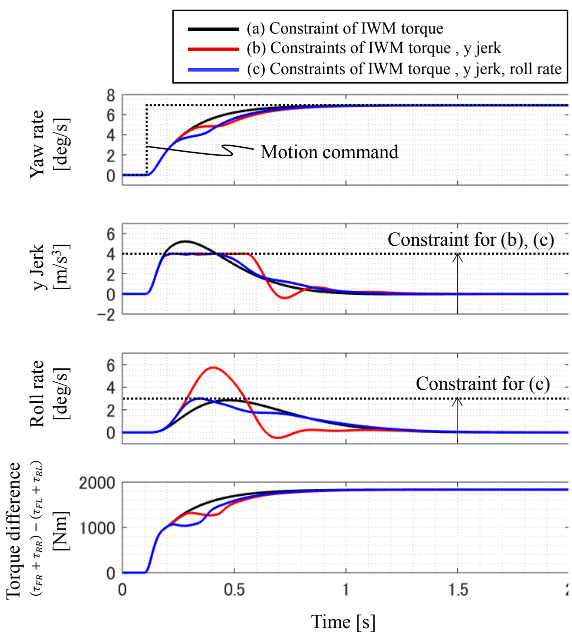
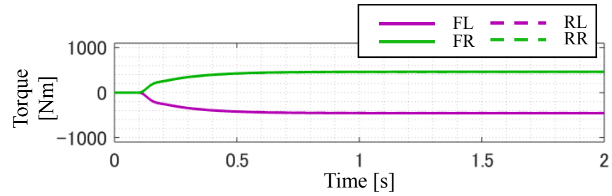


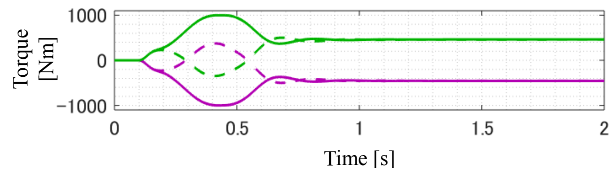
Fig. 7. Simulation results of each constraint conditions.

V. Conclusions

This paper described the consideration result of the vehicle integrated control algorithm for autonomous driving that controls the 6DOF vehicle motion in order to realize the improvement of ride comfort. The nonlinear prediction model which can be applied to MPC algorithm was derived by approximated load transfer dynamics. And jerk dynamics as important index on ride comfort was able to coexist with velocity, position in one control system. According to the simulation results, the jerk and roll rate were able to be constrained simultaneously by the proposed method. Further, it was verified that the IWM torque distribution by optimization calculation was appropriate in principle perspective.



(i) Case (a).



(ii) Case (b).

Fig. 8. Comparison of IWM torque distribution in case (a) and (b).

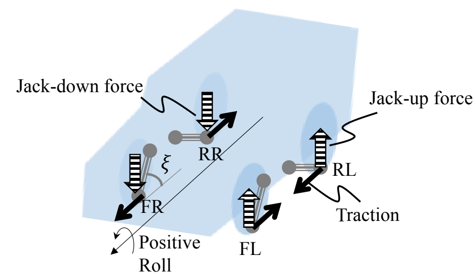


Fig. 9. Principle diagram of jack-up and down force by IWM.

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