Alpha decay and quantum tunnelling

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

$$\frac{\hbar}{2m_\alpha}u''(r) + (V(r) - E)u(r) = 0 \tag{1}$$

where

$$V(r) = \begin{cases} -V_0 + V_C(R), & r < R \\ V_C(r), & r \geq R \end{cases} \tag{2}$$

and

$$V_C(r) = \frac{2(Z-2)e^2}{4\pi\epsilon_0 r_j}, \quad r_j \leq r < r_{j+1} \tag{3}$$

where $r_j = R + j\Delta r$, $j = 0, 1, \ldots N+1$ and N is the number of barriers. The value of V_0 is estimated by ... in ... to be $V_0 = 134$ MeV. The radius R can be calulated with the nuclear radius relationship (??):

$$R = R_0\left(4^{1/3} + (A-4)^{1/3}\right) \tag{4}$$

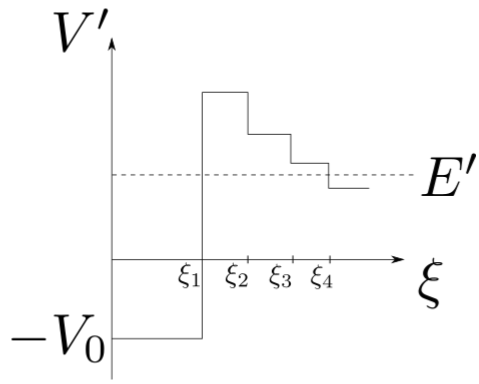
where A is the mass number of the nucleus and $R_0 \approx 1.2$ fm. The width of the barrier D is given by $V_C(D) = E$.

Numerical solution

It is convenient to write the differential equation (1) in the dimensionless form:

$$\begin{aligned} u''(\xi) + \frac{2m_\alpha D^2 U}{\hbar^2}(V'(\xi) - E')u(\xi) \\ = u''(\xi) + \alpha(V'(\xi) - E')u(\xi) = 0 \end{aligned} \tag{5}$$

where $\xi = r/D$, $V' = V/U$ and $E' = E/U$. The constants D and U are in the units of length and energy, respectively, which makes the constant α dimensionless. Suitable choices D and U for this particular problem are $D = 1$ fm and $U = 1$ MeV. Since the potential is constant on the interval $(\xi_j, \xi_{j+1}) = (r_j/D, r_{j+1}/D)$, the solution on this interval is of the form:



$$u(\xi) = \begin{cases} A_j e^{i\omega_j\xi} + B_j e^{-i\omega_j\xi}, & E' > V'(\xi_j) \\ A_j e^{\omega_j\xi} + B_j e^{-\omega_j\xi}, & E' < V'(\xi_j) \end{cases}$$

where $\omega_j = \alpha\sqrt{|V' - E'|}$. When $j = N+1$, the potential has dropped below the total energy. In that region we're assuming an outgoing wave:

$$u(\xi) = Fe^{i\omega_{N+1}\xi}, \quad \xi \geq \xi_{N+1}.$$

The unknowns A_j, B_j and F, in total $2N+3$, can be found by demanding that u and its derivative are continuous at the interfaces of the potential, ξ_j. That only gives us $2N+2$ equations but since the equation is linear, we can set $A_0 = 1$ for instance and that gives us totally $2N+3$ equations. One problem encountered when setting up the matrix for this linear system of equations was that the matrix elements, composed of exponentials of $\pm\omega_j\xi_j$, spanned several orders of magnitude because of the large value of $\omega_j\xi_j$. The best solution I found to this was to shift the system to the left so that the middle of the barrier was situated around $\xi = 0$.