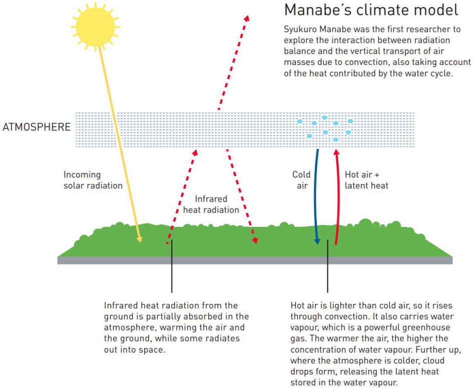
What determines the Temperature of the Earth?

- heat from the inner part of the planet? small contribution
- incoming radiation from the sun!
- But!! The radiation balance give a much too cold earth as noted by Fourier in the 1820s
- 1850s Tyndall measures the ability of water vapour (and some atmospheric gases) to absorb infra-red radiation.
- 1879 Based on Tyndall's (other) experiments Josef Stefan suggest the T^4 law.
- It is the atmosphere that makes the difference! the first one to make quantitative calculations was Svante Arrhenius. This work was published 1896.

Nobel prize 2021 to Syukuro Manabe

- physical models to incorporate vertical transport of air masses (convection), latent heat of water vapour.



https://www.nobelprize.org/prizes/physics/2021/popular-information/

Solar Radiation

Temperature on the surface of the sun: 5750 K
Stefan-Boltzmann's law for the radiation flux.

$$F = \sigma T^4, \sigma = 5.67 \times 10^{-8} W m^{-2} K^{-4} \tag{1}$$

gives a energy flux from the sun of $\sim 6.2 \times 10^7 W/m^2$. Of this $1376 W/m^2$ reaches the earth which averaged over the full area gives $344 W/m^2$

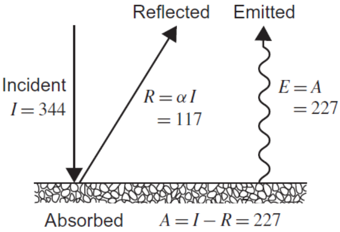


FIGURE 19.1 Simplest possible model of the earth's budget. Straight lines indicate short-wave radiation, whereas the wavy line represents long-wave radiation. (Fluxes are in watts per square meter.) Under this scenario, which does not account for the atmosphere, the earth's average temperature would be a freezing −21°C.

Stefan-Boltzmann's law for the radiation flux.

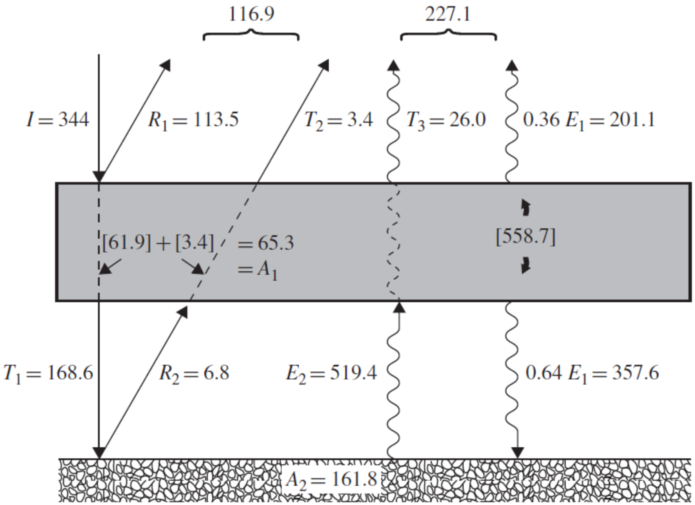
$$F = \sigma T^4, \sigma = 5.67 \times 10^{-8} Wm^{-2} K^{-4} \tag{2}$$

Emission equals Absorption if T=constant

$$\rightarrow E_2 \approx 227 W/m^2 \rightarrow T = -21^\circ C$$

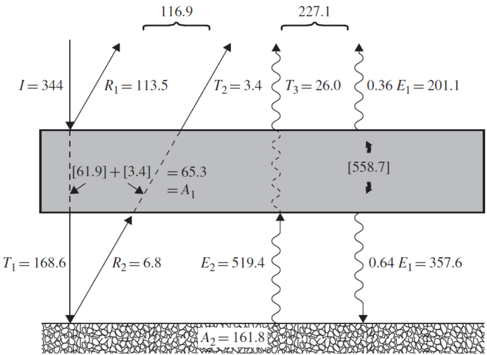
From: *Introduction to geophysical fluid dynamics : physical and numerical aspects*, Benoit Cushman-Roisin and Jean-Marie Beckers

The effect of the atmosphere



Model with one atmospheric layer.

The effect of the atmosphere



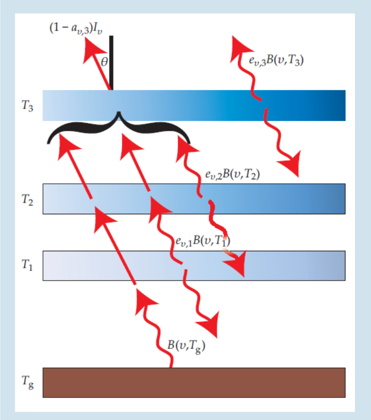
$$A_1 = I - R_1 - T_1 + R_2 - T_2 = 65.3 W/m^2$$

$$E_2 = A_2 + \chi E_1$$

$$E_1 = A_1 + E_2 - T_3$$

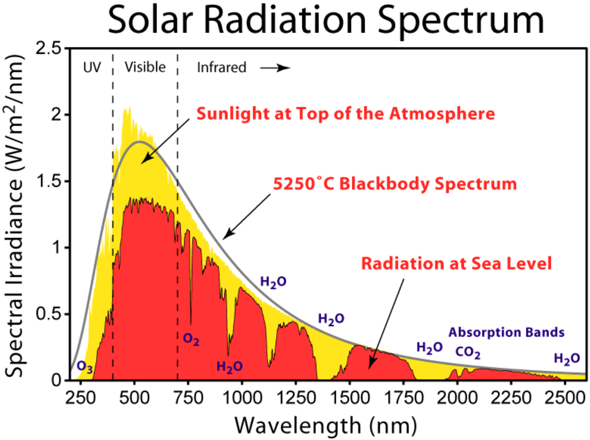
Several good guesses give:

$$\rightarrow E_2 \approx 520 W/m^2 \rightarrow T = 36^\circ C$$



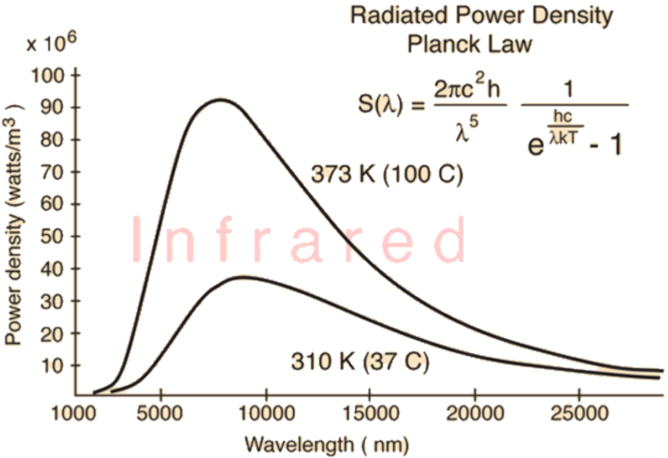
Many layers, each with absorption, transmission and emission.
This is the model you will build.

Solar Radiation Spectrum

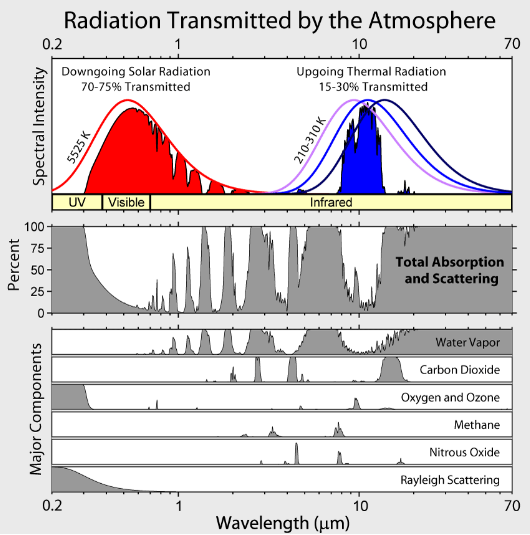


Of the energy flux from the sun ($\sim 6.2 \times 10^7 W/m^2$) $1376 W/m^2$ reaches the earth which averaged over the full area gives $344 W/m^2$

Emitted Radiation Spectrum



The radiation emitted from the earth is in the form of long wave length radiation.



$$I(x) = I_0 e^{-\sigma_a \rho x},$$

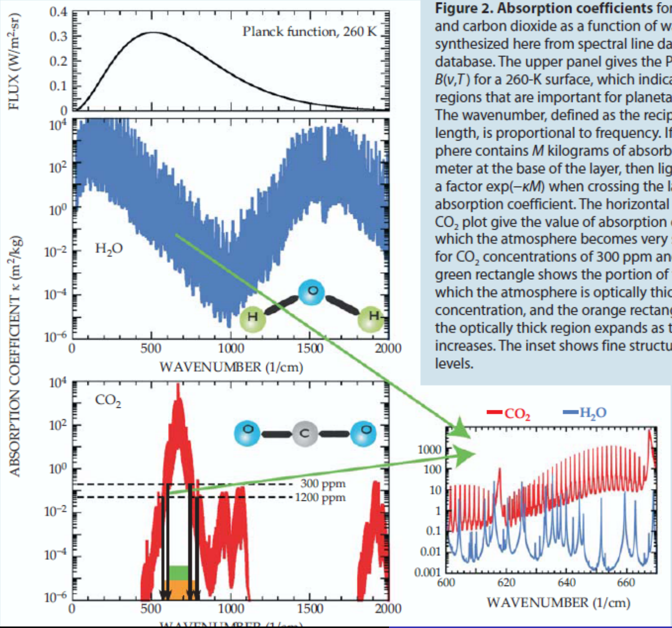


Figure 2. Absorption coefficients for water vapor and carbon dioxide as a function of wavenumber are synthesized here from spectral line data in the HITRAN database. The upper panel gives the Planck function $B(\nu,T)$ for a 260-K surface, which indicates the spectral regions that are important for planetary energy balance. The wavenumber, defined as the reciprocal of the wavelength, is proportional to frequency. If a layer of atmosphere contains M kilograms of absorber for each square meter at the base of the layer, then light is attenuated by a factor $\exp(-\kappa M)$ when crossing the layer, where κ is the absorption coefficient. The horizontal dashed lines on the CO_2 plot give the value of absorption coefficient above which the atmosphere becomes very strongly absorbing for CO_2 concentrations of 300 ppm and 1200 ppm; the green rectangle shows the portion of the spectrum in which the atmosphere is optically thick for the lower concentration, and the orange rectangle indicates how the optically thick region expands as the concentration increases. The inset shows fine structure due to rotational levels.

$$I(x) = I_0 e^{-\sigma_a \rho x},$$

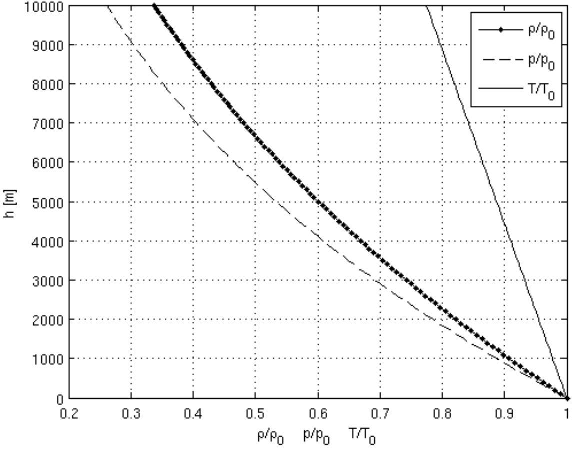
$I(x)$: intensity at x, I_0: incoming radiation at $x = 0$
σ absorption cross section (dimension L^2), different for each type of atom/molecule. ρ, density of absorbing molecule. Alternatively, σ can be expressed in $L^2/mass$ and ρ in $mass/L^3$ (e.g. on previous slide).

Often people average over the different types of molecules and express the effect of absorption as:

$$I(x) = I_0 e^{-\alpha x},$$

α attenuation coefficient (dimension L^{-1}).
Visible light $\alpha = (10^{-5} - 10^{-4})\mathrm{m}^{-1}$ at the surface of the earth.
Scale with density to find it at different heights.

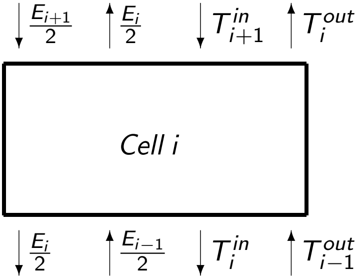
Height variations of T, ρ, and p



Knowing ρ_0, T_0, p_0, we may approximate $\rho(h)$ with $pV = nRT$ assuming a constant T. $\rho = n/V$ moles/volume

Approximations

- No water cycle
- No convection



The radiation going in and out of cell i (of height h). E_i is the emission from cell i, which is equal to the absorption in it. T_i^{inV}/T_i^{inIR} is the incoming visible/IR light that is transmitted through cell i from above, and T_i^{out} is the outgoing long wave length radiation transmitted through i from below.

$$T_i^{inV} = T_{i+1}^{inV} e^{-\sigma_{vis}\rho_i h}, \;\; T_i^{inIR} = \left(T_{i+1}^{inIR} + \frac{E_{i+1}}{2} \right) e^{-\sigma_{IR}\rho_i h}$$

$$T_i^{out} = \left(T_{i-1}^{out} + \frac{E_{i-1}}{2} \right) e^{-\sigma_{IR}\rho_i h}$$

$$E_i = A_i = \left(\frac{E_{i-1} + E_{i+1}}{2} + T_{i-1}^{out} + T_{i+1}^{inIR} \right) \left(1 - e^{-\sigma_{IR}\rho_i h} \right) + T_{i+1}^{inV} \left(1 - e^{-\sigma_{vis}\rho_i h} \right)$$

It is easiest to solve this system of equations through iterations.