Astronomy 112: The Physics of Stars

Class 18 Notes: Neutron Stars and Black Holes

In the last class we discussed the violent deaths of massive stars via supernovae. We now turn our attention to the compact remnants left by such explosions, with a goal of understanding the structure and properties. We are at the forefront of astrophysical knowledge here, so much of what we say will necessarily be uncertain, and may change. As we discussed last time, the natural object left behind by collapse of the iron core of a massive star is a neutron star, a neutron-dominated object at huge density. We will begin with a discussion of neutron stars, and then discuss under what circumstances we may wind up with a black hole instead. Finally, we will discuss how these objects radiate so that we can detect them.

I. Neutron stars

A. Structure

We begin our discussion by considering the structure of a neutron star. We can make a rough estimate for the characteristic radius and density of such an object by considering that it is held up by neutron degeneracy pressure. The neutrons in the star are (marginally) non-relativistic, so the pressure is given by the general formula we derived for a non-relativistic degenerate gas:

$$P = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20m} n^{5/3},$$

where m is the mass per particle and n is the number density of particles. If we take the star to be composed of pure neutrons, then $m = m_n = 1.67 \times 10^{-24}$ g, and $n = \rho/m_n$. Plugging this in, we have

$$P = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20m_n^{8/3}} \rho^{5/3}.$$

This pressure must be sufficient to hold up the star. To see what this implies, we approximate the structure of the star as a polytrope, which is a reasonably good approximation since the pressure is most of the star is dominated by non-relativistic degeneracy pressure, which corresponds to an $n = 3/2$ polytrope. Using the relationship between central pressure and density appropriate to polytropes:

$$P_c = (4\pi)^{1/3} B_n G M^{2/3} \rho_c^{4/3},$$

where B_n is a constant that depends (weakly) on the polytropic index n. Combining this with the pressure-density relation for a degenerate neutron gas, we have

$$\left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20m_n^{8/3}} \rho_c^{5/3} = (4\pi)^{1/3} B_n G M^{2/3} \rho_c^{4/3}$$

$$\rho_c = \frac{4}{9}(20\pi B_n)^3 \frac{G^3 M^2 m_n^8}{h^6}.$$

We can also make use of the relationship between central density and mean density for polytropes:

$$D_n = \frac{\rho_c}{\overline{\rho}} = \rho_c \frac{4\pi R^3}{3M},$$

where $D_n = -[(3/\xi_1)(d\Theta/d\xi)_{\xi_1}]^{-1}$ is another constant that depends on the polytropic index. Plugging this in for ρ_c, we have

$$\begin{aligned}
\frac{3M}{4\pi R^3} D_n &= \frac{4}{9}(20\pi B_n)^3 \frac{G^3 M^2 m_n^8}{h^6} \\
R &= \frac{3 D_n^{1/3}}{20(2\pi)^{4/3} B_n} \left(\frac{h^2}{G m_n^{8/3}}\right) \frac{1}{M^{1/3}}
\end{aligned}$$

Plugging in the values appropriate for an $n = 3/2$ polytrope ($D_n = 5.99$ and $B_n = 0.206$) gives

$$R = 14 \left(\frac{M}{1.4\ M_\odot}\right)^{-1/3} \text{ km}.$$

The choice of 1.4 $M_\odot$ is a typical neutron star mass. This R is only slightly higher than what more sophisticated models get (10 km) for neutron stars that have had a chance to cool off from their initial formation and become fully degenerate.

The slight discrepancy has several causes. First, the fluid isn't pure neutrons; there are some protons too, which do not contribute to the neutron degeneracy pressure. Second, as we'll see shortly, the neutrons are not too far from being relativistic, and this reduces their pressure compared to the fully non-relativsitic pressure we've used. Third, the neutron matter also has a considerably more complex structure than a simple degenerate electron gas, due to nuclear forces between the neutrons. Nonetheless, our calculation establishes that neutron stars have an incredibly high density. The mean density of a star with a mass of 1.4 $M_\odot$ and a radius of 10 km is about 10^{15} g cm^{-3}, comparable to or greater than the density of an atomic nucleus.

Neutron star matter has a number of interesting and bizarre quantum mechanical properties that we only understand in general terms. First, the free neutrons spontaneously pair up with one another. This pairing of two fermions (half-integer spin particles) creates a boson (integer spin). Bosons are not subject to the Pauli exclusion principle, and this allows the pairs of neutrons to settle into the ground quantum state. Since they are already in the ground state, they cannot lose energy, which means that they are completely frictionless. The gas is therefore a superfluid, meaning a fluid with zero viscosity. Similar fluids can be made in laboratories on Earth by cooling bosons, usually helium-4.

One aspect of superfluidity relevant to neutrons stars is the way they rotate. Rotations of superfluids do not occur as macroscopic rotation like for a normal fluids, but rotate as quantized vortices. If a superfluid is held in a vessel and the vessel is rotated, at first the fluid remains perfectly stationary. Once the rotation speed of the fluid exceeds a critical value, the fluid will rotate in a vortex at that critical speed. If the speed of the vessel is increased further, the fluid will not speed up any more until the next critical speed is reached, at which point the fluid will jump to that rotation speed, and so on. Similar things happen in neutron stars – rotation of the star induces the appearance of quantized vortices in the star, and this may affect its structure.

Another property of neutron star matter is that the residual protons present also form pairs, and these pairs make the fluid a superconductor, with zero electrical or thermal resistance. This makes the star isothermal, and the superconductivity has important implications for the magnetic properties of the star, which we'll discuss in a moment.

B. Maximum mass

Neutron stars are subject to a maximum mass, just like white dwarfs, and for the same reason. The mass radius relation we derived earlier is $R \propto M^{-1/3}$, as it is for all degeneracy pressure-supported stars, so as the mass increases, the radius shrinks and the density rises.

If the star becomes too massive, the density rises to the point where the neutrons become relativistic, and a relativistic gas is a $\gamma = 4/3$ polytrope, which has a maximum mass. We can roughly estimate when the relativistic transition must set in using the same method we did for white dwarfs. The non-relativistic degeneracy pressure is

$$P = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_n} n^{5/3},$$

and the relativistic equivalent is

$$P = \left(\frac{3}{\pi}\right)^{1/3} \frac{hc}{8} n^{4/3}.$$

Equating these two, we see that the gas transitions to being relativistic at a number density

$$n = \frac{125\pi c^3 m_n^3}{24 h^3} \qquad \Longrightarrow \qquad \rho = \frac{125\pi c^3 m_n^4}{24 h^3} = 1.2 \times 10^{16} \text{ g cm}^{-3},$$

which is only slightly higher than the mean density we have already computed. Thus the gas must be close to relativistic in a typical neutron star. This is not surprising, since the escape velocity from the surface is

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = 0.64c,$$

for $M = 1.4 M_\odot$ and $R = 10$ km, i.e. the surface escape velocity is more than 60% of light speed. Thus the neutrons must be moving around at an appreciable fraction of the speed of light even at the typical neutron star mass.

For the Chandrasekhar mass of a neutron star, note that the relativistic degenerate pressure is

$$P = \left(\frac{3}{\pi}\right)^{1/3} \frac{hc}{8} n^{4/3} = \left(\frac{3}{\pi}\right)^{1/3} \frac{hc}{8m_n^{4/3}} \rho^{4/3}.$$

This is exactly the same formula as for a degenerate electron gas with $\mu_e = 1$, so we can compute the Chandrasekhar mass just by plugging $\mu_e = 1$ into the Chandrasekhar mass formula we derived earlier in the class:

$$M_{\text{Ch}} = \frac{5.83}{\mu_e^2} M_\odot = 5.83 \ M_\odot.$$

Unfortunately this turns out to a be a pretty serious overestimate of the maximum neutron star mass, for two reasons. First, this estimate is based on Newtonian physics, and we just convinced ourselves that the escape velocity is approaching the speed of light, which means that we must use general relativity. Second, our calculation of the pressure neglects the attractive nuclear forces between neutrons; electrons lack any such attractive force. The existence of an attractive force reduces the pressure compared to the electron case, which in turn means that only a smaller mass can be supported. How small depends on the attractive force, which is not completely understood. Models that do these two steps correctly suggest a maximum mass of a bit over 2 $M_\odot$, albeit with considerable uncertainty because our understanding of the equation of state of neutronized matter at nuclear densities is far from perfect – this is not an area where we can really do laboratory experiments!

C. Magnetic fields

As we discussed a moment ago, one important property of neutron stars is that they are superconductors, i.e., they have nearly infinite electrical conductivity. Therefore, electric currents flow with essentially no resistance and magnetic fields difuse very little in superconductors; fields don't diffuse in or out of them. Therefore the magnetic field within them is said to be "frozen into the fluid"; meaning that any field line that passes through a given fluid element is trapped in that fluid element and moves and deforms with it. A magnetic field that is deformed (stretched or compressed) responds by applying a restoring "Lorentz force" on the fluid.

To see what this implies, suppose that the stellar core out of which a neutron star formed was threaded by an initial magnetic field intensity B_i (also called the magnetic flux density, in units of, for example, gauss, where 1 gauss $= 10^{-4}$ Webers/m^2). The core was a superconductor because of electron degeneracy, so the same magnetic flux that passed through the core must now pass through the neutron star – think of this as the number of field lines passing through the core being the same as the number that now go through the neutron star.

The magnetic flux through a small region of surface area A on the initial core was $A\,B_i$. For the final neutron star, this surface area has shrunk in proportion to the radius squared. Therefore,

$$R_i^2 B_i = R_f^2 B_f \qquad \Longrightarrow \qquad B_f = \left(\frac{R_i}{R_f}\right)^2 B_i.$$

Thus when the core collapses to make a neutron star, the magnetic field that is trapped in the core is enhanced by a factor of $(R_i/R_f)^2$. The initial radius we said last time is about 10^4 km and the final one is 10 km, so the field intensity is boosted by a factor of 10^6 and the magnetic field energy, which is proportional to the square of the field intensity, is increased by a factor of 10^{12}!

We're not sure exactly how strong the magnetic field is before the supernova, but we can take the observed magnetic fields of white dwarfs as a rough guess, since the massive star core is basically an iron white dwarf before it collapses. These cover a very wide range, but typical values are $\sim 10^5$ gauss, which means that we expect neutron stars to have magnetic fields of order $10^{11} - 10^{12}$ gauss, with some going much higher and some much lower. Indeed the highest observed neutron star magnetic fields reach nearly 10^{15} gauss, although 10^{12} gauss is more typical.

To put this in perspective, the Earth's surface magnetic field is around 0.6 gauss, a typical refrigerator magnet is around 100 gauss, the strongest magnets we can make on Earth are well under 10^6 gauss, and the strongest magnetic field ever achieved briefly (using focused explosives) are around 10^7 gauss. Even a 10^6 gauss field cannot be created using conventional materials because the magnetic forces generated exceed the tensile strength of terrestrial materials, i.e. a 10^6 gauss electromagnet would crush itself because steel would not be strong enough to hold it up. A 10^{12} gauss magnetic field is high enough that atoms cannot have a normal structure, and instead the electron orbitals become highly distorted and flattened.

In the stars with the strongest magnetic fields, $\sim 10^{15}$ gauss, known as magnetars, sudden re-arrangements of the magnetic field can generate bursts of gamma rays. One such even on August 27, 1998 was sufficient to ionize large parts of the Earth's outer atmosphere, disrupting radio communications.

[Slide 1 – x-ray light curve of SGR 1900+14]

D. Rotation and Pulsars

The strong magnetic field is particularly important when coupled with another aspect of neutron stars: rapid rotation. Neutron stars are rapid rotators for exactly the same reason they are strongly magnetized: conservation during collapse, in this case conservation of angular momentum. Consider a massive star core rotating with an initial angular velocity ω_i and an initial moment of inertia $I_i = C_i M R_i^2$, where C_i is a constant or order unity that depends on the core's density structure. Its angular momentum is

$$L = I_i \omega_i = C_i M R_i^2 \omega_i.$$

As it collapses it must conserve angular momentum, so its angular momentum after collapse is

$$L = I_f \omega_f = C_f M R_f^2 \omega_f = C_f M R_i^2 \omega_i \qquad \Longrightarrow \qquad \omega_f \approx \omega_i \left(\frac{R_i}{R_f} \right)^2,$$

where we have dropped the constants of order unity.

Thus the angular velocity of the core is enhanced by the same factor of $\sim 10^6$ as the magnetic field. The period, which is simply $P = 2\pi/\omega$, decreases by the same factor. As with the magnetic field, we're not exactly sure what rotation rates should be for massive star cores, but we can guess based on white dwarfs. The fastest rotating of these, which are probably the youngest and closest to their original state, have periods of about an hour, or a few $\times 10^3$ s. The period of a neutron star should be roughly a million times smaller than this, which is a few milliseconds. Thus newborn neutron stars should be extremely rapidly rotating.

The combination of a strong magnetic field and rapid rotation gives rise to an interesting phenomenon: pulsation. The strong magnetic field of the pulsar traps and accelerates charged particles. As these particles move they are confined to move along the strong magnetic field lines, and this causes their trajectories to curve. Any charged particle that accelerates, as the particle must to move in a curved trajectory, emits radiation, and this produces a beam of radiation from the North and South poles of the star. The emitted radiation turns out to be in radio waves.

[Slide 2 – pulsar schematic]

If the pulsar rotates and the magnetic and rotation axes are not perfectly aligned, this beam will sweep through space, and, if it happens to pass over the Earth, we will see a pulse of radio waves once per rotation period. These objects are therefore called pulsars.

The first pulsar was discovered by Jocelyn Bell as a graduate student at Cambridge in 1967, quite by accident. She was building a radio telescope as part of her thesis, and discovered an extremely regular signal coming from a spot on the sky. Due to its regularity, she at first thought it might be a beacon from an alien civilization, and she actually labelled the signal on the paper record "LGM", with the LGM standing for Little Green Men.

[Slide 3 – pulsar discovery record]

In 1974, the Nobel Prize for physics was awarded for the discovery of pulsars, but it was *not* given to Jocelyn Bell. Instead, it went to her (male) PhD advisor Anthony Hewish. She went on to become a very successful astronomer and a university president. She is now the president of the Institute of Physics in the UK.

Pulsars are extremely regular because their "clock" is the rotation of the neutron star, which has a huge amount of inertia to keep it spinning steadily. However, pulsars do slow down. A rotating magnetic dipole such as a pulsar emits radiation (which is not the same as the radio beam we see), and this radiation reduces the kinetic energy of the pulsar, causing it to slow. The slowdown is very slow: a typical pulsar requires of order ten million years to slow down significantly.

The magnetic dipole radiation is deposited in the material around the pulsar, leading to formation of a structure called a pulsar wind nebula, and these are often observed at the center of supernova remnants. A famous example is the Crab nebular supernova remnant, which hosts a pulsar, and which the x-ray telescope Chandra showed to host a pulsar wind nebula as well. In other cases we also see supernova remnants with pulsar wind nebulae at their centers.

[Slides 4, 5 – crab and G292.0+1.8 pulsar wind nebulae]

II. Black holes

A. The Schwarzschild Radius

We have seen that there is a maximum possible mass for neutron stars. Usually the stellar core of a star that explodes as a supernova is smaller than this limit, and the result is a neutron star. However, it is possible for the core to be pushed above the maximum neutron star mass in rare cases. One way this may happen is if not all of the stellar envelope is ejected, and some of it falls back onto the proto-neutron star. In this case the neutron star may accrete the material and exceed its maximum mass. Another possibility is that a very massive star may encounter the pair instability region in the $(\log \rho, \log T)$ plane before it gets to the iron photodisintegration instability region. In this case the collapsing core may be more massive than $2-3\ M_\odot$, and the result will again be a core that exceeds the maximum possible neutron star mass.

If such a core is created, there is, as far as we know, nothing that can stop it from collapsing indefinitely. A full description of what happens in such a collapsing star requires general relativity, which we will not cover in this class. However, we can make some rough estimates of what must happen using general arguments.

As the star collapses, the escape velocity from its surface rises:

$$v_{\rm esc} = \sqrt{\frac{2GM}{R}}.$$

Once the radius is small enough, this velocity exceeds the speed of light. The critical velocity at which this happens is called the Schwarzchild radius:

$$R_{\rm Sch} = \frac{2GM}{c^2} \approx 3 \frac{M}{M_\odot}\ \mathrm{km}.$$

Thus a neutron star is roughly $2-3$ Schwarzschild radii in size, and it doesn't take much additional compression to push it over the edge.

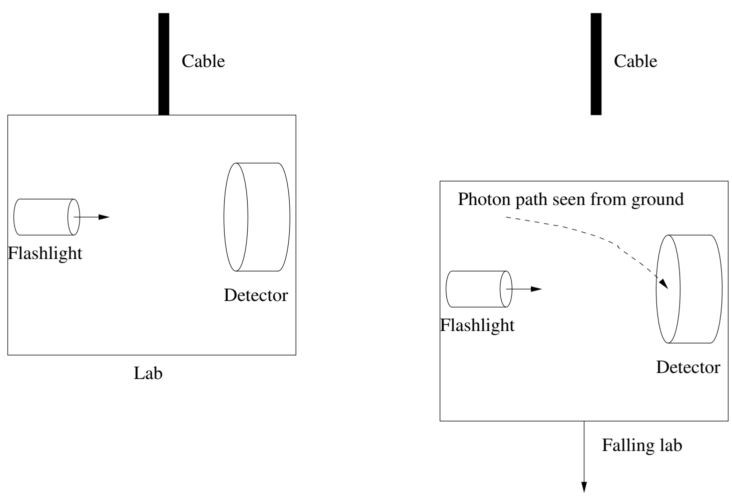
The Schwarzschild radius is the effective size of the black hole. Nothing that approaches within that distance of the mass can escape, since nothing can move faster than light. Because nothing that happens inside the Schwarzschild radius can ever influence events outside it, the Schwarzschild radius is called an event horizon.

B. Light deflection, gravitational redshift, and time dilation

A full description of what happens near the event horizon of a black hole is a subject for a GR class, but we can sketch some basic phenomenology here. Our main tool to do so will be Einstein's equivalent principle, which states that any physical experiment must give identical results in all local, freely-falling, non-rotating laboratories. In other words, if I take a laboratory and put it in deep space anywhere in the universe, or allow it to freely orbit or fall in a gravitational field, I have to get the same results. This seems like a simple statement, but it has profound implications for the effects of gravity.

Our basic tool to understand this will be a simple thought experiment: consider a laboratory inside a sealed elevator, which is suspended from a cable in a gravitational field. The laboratory contains flashlights capable of emitting one photon, and detectors capable of detecting them.

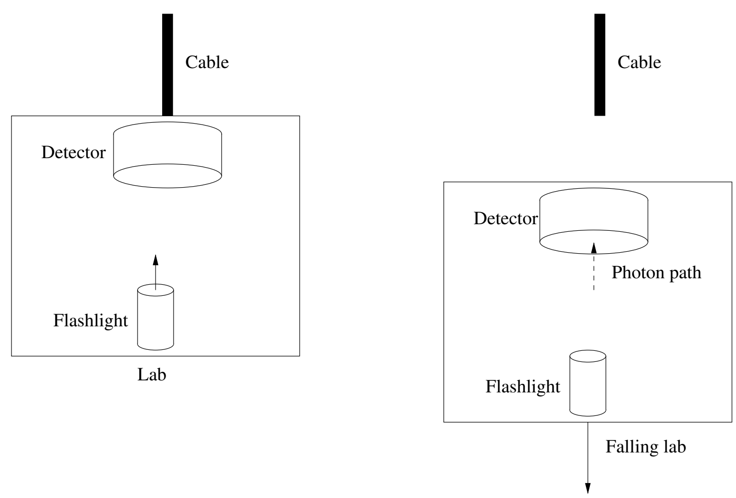
In our first experiment, the flashlight is attached to one of the vertical walls of the lab, and the detector is attached to the opposite horizontal wall. The lab is rigged so that, at the moment the flashlight emits its photon, the cable detaches and the lab is allowed to fall freely.



Since the lab is in free-fall, the physicist in the lab must get the same result he would get if the lab were in free-fall in deep space: the photon travels in a straight horizontal line across the lab to the detector. However, consider what an observer on the ground would see. The light still has to hit the detector – when they compare the reading on the detector after the experiment is over, the physicist on the ground and the one in the lab must read off the same result for whether a photon was detected or not. Since the lab is falling, however, this requires that the photon follow a curved rather than straight path. Otherwise the photon would miss the detector. From the perspective of the physicist on the ground, the gravitational field that accelerated the lab must have also caused the photon's trajectory to bend. The conclusion: gravity bends light.

Of course the amount by which the light bends depends on how strong the gravitational field is. Near a black hole, at 1.5 Schwarzschild radii the gravity is strong enough that photons can go in circular orbits. Light can still escape from this radius if travels radially outward from the black hole, but if it is emitted tangentially, it will instead orbit forever.

Now consider another experiment. This time the flashlight is affixed to the bottom of the experimental chamber, and the detector is affixed to the top. As before, the chamber is rigged so that, at the same moment the flashlight emits its photon, the cable is released and the apparatus begins to fall freely.



Again, we invoke the principle that the physicist in the falling laboratory must obtain the same result as if the lab were in deep space. Thus, the frequency of the light detected by the detector must match the frequency of light emitted by the flashlight, since that is what would happen in deep space.

For the physicist on the ground, this creates a problem. The detector is falling toward the photon, so there should be a Doppler shift that makes the photon appear bluer. However, when that physicist examines the reading on the detector later, it will not show a shift in frequency. The conclusion is that the photon, in climbing out of the gravity well, must have undergone a redshift that counters the blue Doppler shift. Thus, gravity not only bends light, it shifts the frequency of light. Photons that climb out of gravity wells become redder. Reversing the thought experiment so that the flashlight is mounted on the ceiling and the detector is on the floor shows that the converse is also true: photons falling into gravity wells become bluer.

The general relativistic result is that the frequency of a photon of frequency ν_0 emitted at a radius r_0 around a black hole and received at infinity will have frequency

$$\nu_\infty = \nu_0 \left(1 - \frac{2GM}{r_0 c^2}\right)^{1/2} = \nu_0 \left(1 - \frac{R_{\rm Sch}}{r_0}\right)^{1/2}.$$

Note that this formula has the property that, as $r_0 \to R_{\rm Sch}$, the observed frequency $\nu_\infty \to 0$. Thus, light emitted near the event horizon becomes more and more redshifted, until finally at the event horizon it becomes infinitely redshifted and can no longer be observed by the outside world.

An important corollary of this is gravitational time dilation. Consider constructing a clock based around a monochromatic light source. For every crest of a light wave that passes, the clock records one tick. Now consider constructing two such clocks and lowering one near the surface of a black hole. The light coming out of the clock near the black hole will be redshifted, so its frequency will diminish as seen from an observer at infinity. This means that fewer wave crests pass the detector on that clock than pass the detector at infinity in the same amount of time. When the clock near the black hole is pulled back up, it will have recorded fewer ticks than the clock at infinity. The conclusion is that time must slow down near the black hole.

Time dilation follows the same formula as frequency shifting, just in reverse. If a clock at infinity records the passive of a time Δt_∞, then one near the black hole will record a time

$$\Delta t_0 = \Delta t_\infty \left(1 - \frac{R_{\rm Sch}}{r_0}\right)^{1/2}.$$

Thus objects near $R_{\rm Sch}$ appear to outside observers to slow down, until at $R_{\rm Sch}$ they become entirely frozen in time.

III. Accretion power

A. Luminosity

How do we observe neutron stars and black holes? The answer is that, when they're all alone, for the most part we don't. A bare black hole is, by definition, completely free of any kind of emission. A bare neutron star does radiate, but only very weakly. It luminosity is

$$L = 4\pi R^2 \sigma T^4.$$

Neutron stars are born very hot, $T > 10^{10}$ K, but after ~ 1 Myr the star cools and the temperature drops to $\sim 10^6$ K. Plugging in $R = 10$ km with that temperature gives $L = 0.2\ L_\odot$. This is dim enough to make it quite hard to detect any but the nearest neutron stars by thermal emission, particularly since, at this temperature, the emission peaks in the x-ray, and must therefore be studied from space. We have indeed identified some of the nearest and youngest neutron stars, such as the Crab pulsar, by their thermal x-ray emission. However, this is not an option for most neutron stars.

Instead, we tend to find neutron stars and black holes only when they emit non-thermally (e.g. pulsars) or if they are powered by accretion of material from another body. The energetics of this work exactly as they do for protostars, as you worked out on your last problem set. Consider a neutron star of radius R that accretes an amount of mass dM in a time dt. The material falls from rest at infinity, so it has zero energy initially. Just before it arrives at the surface, its potential and kinetic energies must add up to zero, so

$$\frac{1}{2}v^2\,dM - \frac{GM\,dM}{R} = 0 \qquad \Longrightarrow \qquad \frac{1}{2}v^2 = \frac{GM}{R}.$$

When the material hits the surface and stops, its kinetic energy is converted into heat, and then it is radiated away. In steady state all the extra energy must be radiated, so the amount of energy released is

$$dE = \frac{1}{2}v^2\,dM = \frac{GM\,dM}{R}.$$

The resulting luminosity is just the energy per unit time emitted via this process:

$$L_{\rm acc} = \frac{dE}{dt} = \frac{GM}{R}\dot{M}.$$

Note that this is slightly different than the protostar case in that not all of the thermal energy of the material that falls onto the surface of a protostar has to be radiated – about half of it is retained and is used to heat up the star instead. Here, if the temperature of the star is fixed, it will all be radiated. That explains the factor of 2 difference from the protostar case.

Accretion luminosity increases as the radius of the star decreases, which means that it can be a much more potent energy source for compact things like neutron stars than it is for protostars. For example, suppose a star accretes at a rate of $10^{-10}\ M_\odot\ {\rm yr}^{-1}$, so that it gains roughly 1 $M_\odot$ of mass over the age of the universe. This is $4-5$ orders of magnitude slower accretion than in the protostar case. For the Sun, $L_{\rm acc} \approx 10^{-3}\ L_\odot$, unnoticeably small. For a white dwarf, $R = 0.01\ R_\odot$, it would be $L \approx 0.1\ L_\odot$, high enough to be brighter than just an isolated white dwarf normally is. For a neutron star, $R = 10$ km, it would be 100 $L_\odot$, and for a black hole, $R \approx 3$ km, it approaches 1000 $L_\odot$! (Black holes don't have surfaces for matter to crash into, but we'll see that they still emit nearly as much as if they did.)

Of course this process cannot produce arbitrarily high luminosities, for the same reason that stars cannot have arbitrarily high luminosities: the Eddington limit. If the luminosity is too high, radiation forces are stronger than gravity, so material will be pushed away from the accreting object rather than attracted to it. The Eddington limit is

$$L_{\rm Edd} = \frac{4\pi c G M}{\kappa},$$

and if we require that $L_{\rm acc} < L_{\rm Edd}$, then we have

$$\frac{GM}{R}\dot{M} < \frac{4\pi cGM}{\kappa} \qquad \Longrightarrow \qquad \dot{M} < \frac{4\pi cR}{\kappa}.$$

Thus there is a maximum accretion rate onto compact objects. The value of κ that is relevant is usually $\kappa_{\rm es}$, since usually the accreting material is hot and fairly low density.

To figure out the wavelength of the emission, we need to estimate the surface temperature of the accreting object. This is given by the normal result

$$L = L_{\rm acc} = 4\pi R^2 \sigma T^4 \qquad \Longrightarrow \qquad T = \left(\frac{GM\dot{M}}{4\pi R^3 \sigma}\right)^{1/4}.$$

If we plug in the maximum possible accretion rate, we get the maximum temperature, which will apply to the brightest objects:

$$T = \left(\frac{GMc}{R^2 \sigma \kappa}\right)^{1/4}$$

Thus smaller radii also lead to higher temperatures. Plugging in $R = 0.01 R_\odot$ for a white dwarf, $R = 10$ km for a neutron star, or $R = 3$ km for a black hole gives about 10^6 K for a white dwarf, 2×10^7 K for a neutron star, and 4×10^7 K for a black hole. Thus compact objects accreting near the maximum rate should emit primarily in the ultraviolet or x-ray.

B. Binaries, Roche Lobe Overflow, and Disks

We've worked out the energetics, but how does material actually get onto a compact object like a neutron star or black hole? The answer is generally that it must be donated by a companion. Fortunately for us, most stars massive enough to produce neutron stars or black holes are born as members of binary systems. In such a system, the more massive member will evolve off the main sequence first, while the other star is still on the main sequence.

Many binaries are disrupted by the supernova that creates a neutron star or white dwarf, but some remain bound – it's a matter of how much mass is ejected and how asymmetric the explosion is. If the binary remains bound, the result is a main sequence star with a neutron star or black hole companion.

Some time later the companion will begin to evolve off the main sequence. When it does, it will swell into a giant star. However, this may bring the outer parts of its envelope very close to its companion – close enough that they can be gravitationally captured by the companion and accrete onto it. The region around each star where mass is safely bound to that star is known as the star's Roche lobe. (Lobe because it has a teardrop-like shape.) As a star swells into a giant, its outer layers may overflow its Roche lobe.

[Slides 6, 7 – Roche lobe overflow diagram and animation]

The overflowing material falls onto the compact companion. However, it cannot fall directly onto the star because it has too much angular momentum. Instead, it goes into a rotating disk around the compact object, where it is in Keplerian rotation – just like a planet going around a star. The resulting object is called an accretion disk.

If the material in the disk were free of viscosity, it would simply orbit happily forever, just like planets around stars. However, the gas in a disk has some viscosity, due to mechanisms that we won't discuss in this class. The viscosity acts like a frictional drag: blobs of gas somewhat closer to the compact object rub against those somewhat further out, and this friction slows down the inner blobs so that they lose angular momentum and spiral ever close to the central object.

As material in the disk rubs against other material and moves inward, it must heat up to conserve energy: gravitational potential energy is being lost, so it must be converted to heat. In turn, this heating causes the material to radiate. As a result, half the gravitational potential energy of infalling material is radiated away in the accretion disk even before the gas gets to the surface of the compact object. This is why we can see accreting black holes even though accreting material that actually gets to the event horizon simply plunges on through without radiating further – half the energy has already come out in the accretion disk.

Energy release in an accretion disk around a black hole is probably the most efficient form of energy release in the universe. Let's put this in perspective by comparing it to nuclear burning. Consider a single proton. If we burn it to helium in a star, the energy release is $\epsilon = 0.007$ of its total rest energy, where

$$\Delta E = \epsilon m_p c^2$$

is the total energy released, and $m_p c^2$ is the proton's total energy content. If we burn it all the way to iron, the efficiency increases to $\epsilon = 0.009$.

In contrast, consider the same proton being accreted onto a black hole. The energy released is

$$\Delta E = \frac{GMm_p}{2R_{\mathrm{Sch}}},$$

where the factor of 2 assumes that we only get the half of the energy that comes out in the accretion disk, while the rest is swallowed by the black hole. Plugging in $R_{\mathrm{Sch}} = 2GM/c^2$, this is

$$\Delta E = \frac{m_p c^2}{4} \qquad \Longrightarrow \qquad \epsilon = \frac{\Delta E}{m_p c^2} = \frac{1}{4}.$$

This calculation is quite approximate, since we have neglected a number of important general relativistic effects that occur near the Schwarzschild radius. A more sophisticated treatment gives $\epsilon \approx 0.1$.

Nonetheless, this means that accreting a proton onto a black hole releases roughly 10 times as much energy as burning the same proton to iron. It releases 10%

as much energy as the maximum possible amount, which would be released by annihilating the proton with an anti-proton. For this reason, accreting black holes are some of the brightest objects in the universe – indeed, early in the history of the universe, they dominated the total light output of the cosmos. These objects are called quasars. They are powered by black holes much larger than that created by any star, with masses up to $10^9\ M_\odot$.

[Slide 8 – radio / optical image of NGC 4261]

The image shows jets of material being ejected from the galaxy by the quasar. Notice the scale, and how the jet compares in size to the galaxy: each jet is about 100,000 light years (30 kpc) long, larger than the entire galaxy. That's what accretion power can do.