$$\left(-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{\hbar^2\ell(\ell+1)}{2\mu r^2}-\frac{Ze^2}{4\pi\epsilon_0 r}\right)P_{n\ell}(r)=EP_{n\ell}(r). \tag{1}$$

Set

$$\begin{aligned} E' &= E/\left(Z^2\mu e^4/(32\pi^2\epsilon_0^2\hbar^2)\right) \\ &= E/\left(Z^2\mu\hbar^2/(2m_e^2a_0^2)\right) \end{aligned} \tag{2}$$

with this defintion, E' should take the values $-1,-4,-9,\ldots$. Then we get

$$\hat{H}u=-\beta^2\left(\frac{\mathrm{d}^2}{\mathrm{d}\xi^2}-\frac{\ell(\ell+1)}{\xi^2}+\frac{2}{\beta\xi}\right)u=E'u \tag{3}$$

where $\beta=m_e/(Z\mu\alpha)$. We write the numerical solution to (3) as a linear combination of B-splines of degree k:

$$\hat{u}(\xi)=\sum_{j=0}^{n-1}c_jB_{j,k}(\xi). \tag{4}$$