TOV

$$\begin{aligned}\frac{\mathrm{d}P}{\mathrm{d}r} &= -\frac{G\,[P+\mathcal{E}(r)]\,[M(r)+4\pi r^3 P/c^2]}{c^2 r^2[1-2GM(r)/(c^2 r)]} \\ \frac{\mathrm{d}M}{\mathrm{d}r} &= 4\pi r^2 \frac{\mathcal{E}(r)}{c^2}\end{aligned} \tag{1}$$

An interpretation of these equations can be more readily seen by multiplying the first equation by $4\pi r^2 \mathcal{E} \mathrm{d}r/c^2 = \mathrm{d}M$ and cancelling $\mathcal{E}$ on both sides:

$$\begin{aligned}4\pi r^2 \mathrm{d}P = -\frac{GM\mathrm{d}M}{r^2}\left(1+\frac{P}{\mathcal{E}(r)}\right)\left(1+\frac{4\pi r^3 P}{Mc^2}\right) \\ \times\left(1-\frac{2GM}{c^2 r^2}\right)^{-1}\end{aligned} \tag{2}$$

The term on the left hand side is the force exerted on a infinitesimal shell at radius r. The first factor on the right hand side is the newtonian gravitational force from the interior acting on this shell.

Numerical set-up

Making the substitutions $r = R_0 x$, $P = P_0 p$, $\mathcal{E} = P_0 \varepsilon$ and $M = M_0 m$, we can write

$$\begin{aligned}&\left(\frac{P_0}{R_0}\right)\frac{\mathrm{d}p}{\mathrm{d}x} = \\ &-\left(\frac{GP_0 M_0}{c^2 R_0^2}\right)\frac{[p+(\mathcal{E}_0/P_0)\varepsilon]\,[m+4\pi R_0^3 x^3 P_0 p/c^2]}{x^2\,[1-2GM_0 m/(c^2 R_0 x)]}\end{aligned} \tag{3}$$

and thus

$$\frac{\mathrm{d}p}{\mathrm{d}x} = -\left(\frac{G_0 M_0}{c^2 R_0}\right)\frac{[p+(\mathcal{E}_0/P_0)\varepsilon]\,[m+4\pi R_0^3 x^3 P_0 p/c^2]}{x^2\,[1-2GM_0 m/(c^2 R_0 x)]} \tag{4}$$

Now, we set

$$1 = \frac{\mathcal{E}_0}{P_0} = \frac{GM_0}{c^2 R_0} = \frac{4\pi R_0^3 P_0}{c^2} \tag{5}$$

and find

$$\begin{aligned}\frac{\mathrm{d}p}{\mathrm{d}x} &= -\frac{(\varepsilon+p)(m+x^3 p)}{x(x-2m)} \\ \frac{\mathrm{d}m}{\mathrm{d}x} &= x^2\varepsilon\end{aligned} \tag{6}$$

The initial value for the first equation can either be $p(0) = p_0$ or $p(x_0) = 0$ where p_0 is the pressure at the center and x_0 is the radius of the star. The initial value for the second equation is $m(0) = 0$. We wish to solve this equation with a given equation of state $\epsilon(p)$.

Notes

- Compare with Newton
- Also do white dwarfs