## Some title

The time independent Schrödinger equation for the harmonic oscillator is

$$-\frac{\hbar}{2m}\frac{\mathrm{d}^2u}{\mathrm{d}x^2}+\frac{m\omega^2}{2}x^2=Eu \tag{1}$$

By changing the variables with $\xi = x/\sqrt{\hbar/m\omega}$ and $E' = \hbar\omega$, it can be written as

$$-\frac{\mathrm{d}^2u}{\mathrm{d}\xi^2}+\xi^2u=2E'u \tag{2}$$

### Numerical solution to eigenvalue problem

We wish to solve (2) numerically by discretizing the real line and the second derivative of $u$. In order to do so, we introduce the gridpoints $\xi_j$ and the approximation of the function $u$ at these gridpoints, $u_j \approx u(\xi_j)$. Furthermore, we assume that $\xi_j \in [-L, L]$, where $L$ is chosen such that $u(\xi) \approx 0$ for $|\xi| > L$. Since the eigenfunctions of an even potential can be taken to be even or odd, we can focus on the interval $[0, L]$ and apply the boundary conditions

$$\begin{aligned} u'(0)=0 \quad &\text{and} \quad u(L)=0 \quad \text{for even } u,\\ u(0)=0 \quad &\text{and} \quad u(L)=0 \quad \text{for odd } u. \end{aligned} \tag{3}$$

The second derivative is approximated up to fourth order:

$$\begin{aligned} u''(x_j) \approx \frac{1}{12h^2}\big(&-u_{j-2}+16u_{j-1}-30u_j\\ &+16u_{j+1}-u_{j+2}\big) \end{aligned} \tag{4}$$

Special care had to be taken at the boundaries. For the even case, two ghost points, $\xi_{-1}$ and $\xi_{-2}$, were introduced to the left of zero and two fourth order approximations of the first derivative used to (... words ...) :

$$\begin{aligned} u'(0) &\approx \frac{1}{12h}\left(u_{-2}-8u_{-1}+8u_1-u_2\right)\\ u'(0) &\approx \frac{1}{12h}\left(-3u_{-1}-10u_0+18u_1-6u_2+u_3\right) \end{aligned} \tag{5}$$