Assignment 1: Alpha Decay

deadline 10.15 Tuesday March 29 2022.

- **Background** Many heavier nuclei decay through α - particle emission; $A(Z,N) \Rightarrow A(Z-2,N-2) + {}^4_2He$. The stability against α - decay varies tremendously from nucleus to nucleus. Some have very long lifetimes, e.g. ^{238}U has a half life of $4 \cdot 10^9$ years, while other nuclei have an extremely short lifetime, e.g. ^{212}Po has a half life of 0.3μs. It was early noticed that the kinetic energy of the α - particle was lower for the nuclei with long lifetimes. However, while the lifetime range was over many orders of magnitude - the kinetic energy range was very modest. This puzzle was solved 1928 when Gamov showed that quantum mechanical tunneling could explain the difference in half life.
Alpha decay is energetically possible if a nucleus $A(Z,N)$ has a higher mass than the α decay products, i.e. if $M(Z,A) > M(Z-2,A-4)+M(\alpha)$. The α - particle still does not leave the nucleus very easily because it is lying in a well (provided by the strong force) and has to overcome a *Coulomb* barrier (provided by the electromagnetic force). The only way to get free is indeed to tunnel through the barrier. *Advice: read through the chapter on tunneling in your quantum mechanics text book.*
- **Your task** Use a simple one dimensional model (see the extra page for details) of the potential for the α - particle. In your model you can assume that the α - particle has zero angular momentum with respect to the daughter nucleus. Many nuclei, but not all, decay in this way. The kinetic energy available in the decay can be calculated from the masses involved. Think through how it is divided between the α - particle and the daughter nucleus and the implications of this. Divide the barrier into segments. In each segment you can assume that the potential is constant. This is an approximation, but it gets better and better the more narrow the segments are. Since you now know the solution to the Schrödinger equation in each segment you can set up a system of equations by requiring that the wave function and its derivative are continues at every segment border. This system of equations can be solved for example with Gauss elimination. Another possibility is to use an iterative scheme. Remember that your program should verify that you really have found a solution.
- **Test your model** on a few different nuclei. Some suggestions are found below. You should test the numerical stability of your calculation, but you should also remember to investigate the reliability of your model. How sensitive is the result to input data for example? The physical model itself is approximate in many ways. Try to list which simplifications you have done and how important they might be. You have to assume something about the extension, R, of the well in your model. Pick a reasonable value (see the additional paper) and check the life time or vary R to find a the particular R that gives the experimental lifetime. How sensitive are the qualitative/quantitative results for the choice of potential well depth (V_0)? If you have time left you can investigate how you could use a more realistic nuclear potential (see information on the home page).
- **Nuclear data** You are welcome to look for nuclear data yourself. A site with a lot of information is: http://www.nndc.bnl.gov/chart/. Note, however, that some decays are not to the ground state of the daughter nuclei and that some nuclei decay through several paths. Also, if the decay is not from one zero angular momenta state to another there will be an extra angular momentum barrier which your model does not account for. A straight forward case to try is ^{212}Po which only decays to the ground state of ^{208}Pb. The ^{238}U nuclei on the other hand decays in 79% of the cases to the ground state of ^{234}Th, keep this in mind when you analyze the result. The listed mass excess, ΔM(Z,N), is

ΔMc^2	^{238}U	+ 47.3089	MeV
	^{234}Th	+ 40.6140	MeV
	^{212}Po	-10.3694	MeV
	^{208}Pb	-21.7480	MeV
	^{4}He	+ 2.4249	MeV

defined as: ΔM(Z,N)= Mass(Z,N) [amu] - A $\times$ (1 amu). ^{222}Rn is also interesting to check.
- **Visualize** your result in some appropriate way (one possibility is to plot the logarithm of the probability density (square modulus of the wave function) as a function of distance from the nuclear center).