Assignment 7: The Earth Temperature

deadline Thursday May 12 2022

In this exercise we will try to estimate the average temperature of the earth, and if you have time you can also try to use your model on Mars.

The prediction of the climate, on both short and long time scales, is of course a very hard and involved problem. This should be clear from the shortcomings of the weather forecasts as well as from the debate on the effects of the increased amount of CO_2 in the atmosphere. The goal of the present exercise is thus only to get a rough estimate of the average temperature and to be able to see how it would vary with certain parameters. We will thus make a number of approximations when we build our model. First, we will only be concerned with the average temperature. The input energy here comes of course from the sun, and what we will do is a model of the *radiation flux*.

We will use the beginning of chapter 19 from *Introduction to geophysical fluid dynamics : physical and numerical aspects*, by Benoit Cushman-Roisin and Jean-Marie Beckers, as an introduction to the subject. The book is available as an e-book from SUB :
`http://www.sciencedirect.com.ezp.sub.su.se/science/bookseries/00746142/101`, or to chapter 19 directly:
`http://www.sciencedirect.com.ezp.sub.su.se/science/article/pii/B9780120887590000195`

The average incoming flux from the sun is 344 W/m^2, and (following Cushman-Roisin & Beckers) a simple calculation shows that without an atmosphere this should only give us an average temperature of $\sim -20°$C. Without the presence of the atmosphere and the resulting *greenhouse effect* the earth should thus be very much colder than it is. Chapter 19 from *Introduction to geophysical fluid dynamics* then presents a very simplified "calculation" of the greenhouse effect. The purpose here is to make a more detailed model. You will look at an atmospheric "pillar" from ground level to the outer atmosphere and calculate how radiation flux is transmitted in this pillar. Dividing it up in segments you can follow the absorption, transmission and reflection in each segment. This will still be a very simplified model since we will only consider the radiation balance, but you can view it as a first step towards a realistic model which can be refined step-by-step. As is briefly discussed in Cushman-Roisin-Beckers the hydrological cycle should be the most important aspect that we are neglecting

The radiation that enters the atmosphere can be absorbed, transmitted and reflected. First, some of the 344 W/m^2 flux that hits the earth will never penetrate down in the lower atmosphere, but will be reflected at high altitudes, e.g. by clouds. This reflectivity of the earth is called the *albedo* and is usually assumed to be 30% (that is the reflected amount). In your simulation you can start with just subtracting this part. Second, the absorption depends strongly on the wavelength of the radiation, see Fig. 1. While the atmosphere is rather transparent for visible light it absorbs most of the ultraviolet (UV)- and infrared (IR) -radiation. The amount of absorption can be quantified through the formula for the intensity that penetrates to x:

$$I(x) = I_0 e^{-\sigma_a \rho x}, \tag{1}$$

where $I(x)$ is the intensity reaching x if I_0 is the incoming intensity at $x = 0$, i.e. the light intensity decreases exponentially with the distance travelled in the medium (here the air). The rate of the decrease is governed by σ_a, the absorption cross section (which has the dimension of area) and ρ the density of absorbing particles. Alternatively σ_a can be given in area/mass, and then ρ should be in mass/volume. We expect σ_a to depend on the absorbing molecule as well as on the radiation wavelength. To simply our treatment we will here work with an average absorption cross section for visible light(and one for IR radiation, see below). Often people give instead of the cross section the so-called attenuation coefficient, α, which is an averaged cross section times the density, i.e.

$$I(x) = I_0 e^{-\alpha x}. \tag{2}$$

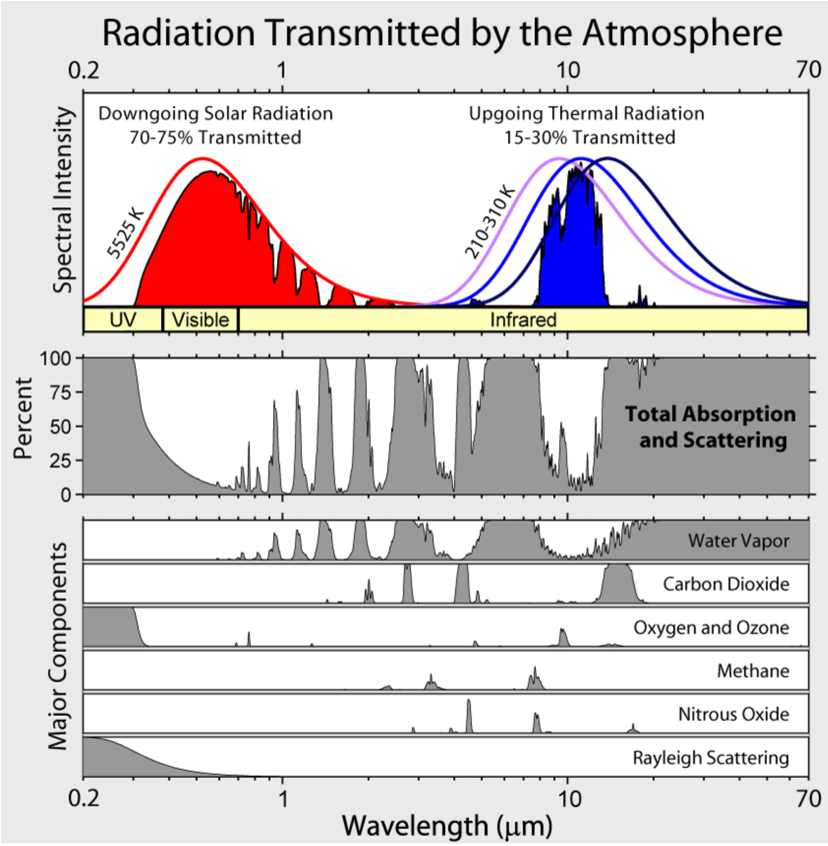

Figure 1: The absorption bands in the Earth's atmosphere (middle panel) and the effect that this has on both solar radiation and upgoing thermal radiation (top panel). Individual absorption spectrum for major greenhouse gases plus Rayleigh scattering are shown in the lower panel, from Wikimedia Commons.

($\alpha = \sigma_a^{average}\rho$) For visible light $\alpha = (10^{-5} - 10^{-4})\mathrm{m}^{-1}$ in air at the earth's surface (can be measured). Although only $\sim 25\%$ of the radiation from the sun is in the visible range it, nearly half of the radiation that reaches the earth is.

We note also that the absorption depends on the density, cf. Eq.1. This means that we need to know the density at different heights above the earth. Start with calculating this density. Since we know the air pressure at sea-level, we know the weight of the air-pillar with a cross section of for example 1 m^2. We also know the density of air at sea-level and the variation of the gravitational force with height. If we divide the air-pillar in segments of height h we can estimate the density in each such segment with the help of the ideal gas law $pV = nRT$ if we (rather bravely) assume that the temperature is constant. Indeed the temperature variations with height is much weaker than that of the density, see Fig. 2 below. Some of you will undoubtedly stumble on the *barometric formula* when you google around, but it is more fun to realize that you is able to calculate the pressure and density at high heights by yourselves! The barometric formula is very simple and gives no better result than what you can calculate.

At this point you should be able to estimate the flux in the form of visible light that hits the earth surface. Compare with the values given in Cushman-Roisin- Beckers.

The flux that reaches the earth's surface will mostly be absorbed (there is a small amount of reflection, see Cushman-Roisin- Beckers Fig.19.2, and this is a parameter you can play with in your model). Now since the average temperature is constant the earth must emit the same energy per time and area (flux) as it absorbs. Assuming that the earth is a *black-body* radiator, we know that the emitted flux (F) depends, according to the Stefan-Boltzmann law, on the temperature (T) as

$$F = \sigma T^4, \text{ where } \sigma = 5.67 \times 10^{-8} \ \mathrm{Wm}^{-2}\mathrm{K}^{-4} \tag{3}$$

In this way a certain absorbed flux will lead to a specific temperature on the surface. At this point you will probably find a much too low temperature.

The next, and very important, step is to consider what happens with the radiation that earth emits upwards. The temperature of the earth is such that it will emit in the IR-spectrum, cf. Fig.1. This radiation will to a much larger extent than the visible light be absorbed in the atmosphere. We expect thus a significant amount to be absorbed in each layer of air, and again since the temperature is constant, the same radiation will be re-emitted. On average half of the radiation is emitted upwards and half downwards, see Fig. 19.2 in Cushman-Roisin- Beckers. Thus the amount of radiation that reaches the earth's surface becomes much larger than what would have been the case without an atmosphere. Similarly, every air segment in the model will absorb energy not only from the sun directly (visible light), but also IR -radiation from the air segments below and above, and it emits radiation both upwards and downwards. This leads to a set of coupled equations. The simplest way to solve this system of equations is through iterations.

Exactly how much that is absorbed in a certain segment of your atmospheric pillar, and how much that is just transmitted to the next layer, will depend on the concentration of the absorbing gases, especially water and CO_2. It will also depend on the specific IR wave length. This is well illustrated in the figures in the Physics Today article by Raymond T. Pierrehumbert(Physics Today, January 2011, page 33), a direct link through which you might not get access (but it works if you are recognized as belonging to SU):
`http://physicstoday.scitation.org/doi/pdf/10.1063/1.3541943` alternatively go to `http://su.se/biblioteket/` choose E-tidskrifter, search for Physics Today and go on from there. One approach for your model is to use the IR absorption as an input parameter which you can vary within reasonable values. To estimate what is reasonable, note that the concentration of CO_2 is 0.039% while the amount of water vapor is strongly varying; 0.25% of the mass of the atmosphere, but locally varying between 0.001% and 5%.

For testing your numerical implementation of the model, note that the flux leaving the earth should be the same as the flux reaching the top of the atmosphere. Your model should also not depend strongly on the height of the segments you use.

A last comment on the approximations in the proposed model: In principle we could correct our original assumption that the temperature is not strongly varying with hight when we know how the energy flux varies with height. However we must then remember that there is also convection - warm air that has been in contact with the surface is transported upwards. You will probably find that your model indicates that the air above the earth is much colder than you would expect; the neglect of convection is one reason for this.

Background information:

• Introduction to geophysical fluid dynamics : physical and numerical aspects, by Benoit Cushman-Roisin, Jean-Marie Beckers, available as an e-book from SUB.
• Infrared radiation and planetary temperature, Raymond T. Pierrehumbert, Physics Today, January 2011, page 33 `http://physicstoday.scitation.org/doi/pdf/10.1063/1.3541943`, the link will work from Stockholm University. Here there is also some discussion on the situation for other planets which you should read if you want to try the model on Mars.

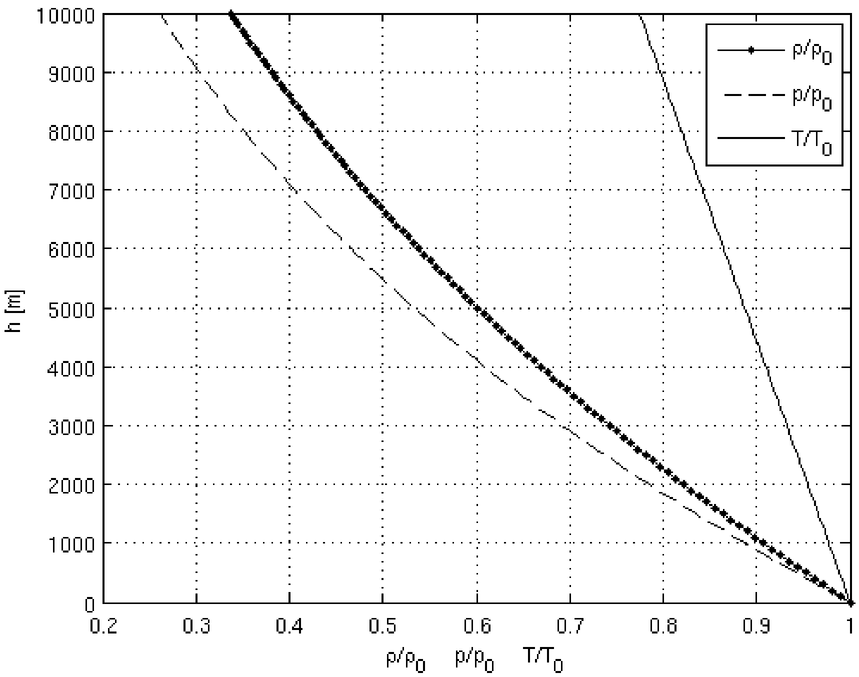

Figure 2: The variation of density, pressure and temperature as a function of height, in units of the values at the earth's surface.