

Daily Integral

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1 Easy

Problem 1. Evaluate the following

$$\begin{aligned}\int_0^{\pi/2} \frac{\cos x}{2 - \cos^2 x} dx \\ \int_0^{\pi/2} \frac{\cos x}{2 - \cos^2 x} dx &= \int_0^{\pi/2} \frac{\cos x}{2 - (1 - \sin^2)x} dx \\ &= \int_0^{\pi/2} \frac{\cos x}{1 + \sin^2 x} dx\end{aligned}$$

Now we consider the u sub

$$\begin{aligned}u &= \sin x \\ du &= \cos x \, dx\end{aligned}$$

when $x = 0, u = 0$ and $x = \frac{\pi}{2}, u = 1$

$$\begin{aligned}\int_0^{\pi/2} \frac{\cos x}{1 + \sin^2 x} dx &= \int_0^1 \frac{\cos x}{1 + \sin^2 x} dx \\ &= \int_0^1 \frac{1}{1 + u^2} du\end{aligned}$$

Now consider the trig sub

$$\begin{aligned}u &= \tan \theta \\ du &= \sec^2 \theta \, d\theta\end{aligned}$$

when $u = 0, \theta = 0$, and $u = 1, \theta = \frac{\pi}{4}$, thus our final integral is transformed

$$\begin{aligned}\int_0^1 \frac{1}{1 + u^2} du &= \int_0^{\pi/4} \frac{\sec^2 \theta}{1 + \tan^2 \theta} d\theta \\ &= \int_0^{\pi/4} \frac{\sec^2 \theta}{\sec^2 \theta} d\theta \\ &= [\theta]_0^{\pi/4} \\ &= \frac{\pi}{4}\end{aligned}$$

2 Medium

Problem 2. Evaluate the following

$$\int_0^{\pi/4} \frac{(x+1)\tan x}{(1+\tan^2 x)} dx$$

$$\begin{aligned} \int_0^{\pi/4} \frac{(x+1)\tan x}{(1+\tan^2 x)} dx &= \int_0^{\pi/4} \frac{(x+1)\tan x}{1+\tan^2 x} dx \\ &= \int_0^{\pi/4} \frac{(x+1)\tan x}{\sec^2 x} dx \\ &= \int_0^{\pi/4} (x+1) \sin x \cos x dx \\ &= \frac{1}{2} \int_0^{\pi/4} x \sin 2x + \sin 2x dx \\ &= \frac{1}{2} \int_0^{\pi/4} \sin 2x dx + \frac{1}{2} \int_0^{\pi/4} x \sin 2x dx \\ &= -\frac{1}{4} [\cos 2x]_0^{\pi/4} + \frac{1}{2} \int_0^{\pi/4} x \sin 2x dx \\ &= 0 - -\frac{1}{4} + \frac{1}{2} \int_0^{\pi/4} x \sin 2x dx \\ &= \frac{1}{4} + \frac{1}{2} \int_0^{\pi/4} x \sin 2x dx \end{aligned}$$

Now we use Integrations by Parts:

$$\int_a^b uv' dx = [uv]_a^b - \int_a^b u'v dx \quad (1)$$

Now we select $u = x$ and $v = -\frac{1}{2} \cos 2x$ such that $v' = \sin 2x$

$$\begin{aligned} \int_0^{\pi/4} x \sin 2x dx &= \left[(x) \left(-\frac{1}{2} \cos 2x \right) \right]_0^{\pi/4} + \int_0^{\pi/4} \frac{1}{2} \cos 2x dx \\ &= \frac{\pi}{4} \cdot 0 - 0 + \left[\frac{\sin 2x}{4} \right]_0^{\pi/4} \\ &= \frac{1}{4} \end{aligned}$$

Thus we have the final result of:

$$\begin{aligned} \int_0^{\pi/4} \frac{(x+1)\tan x}{(1+\tan^2 x)} dx &= \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4} \\ &= \frac{3}{8} \end{aligned}$$

3 Hard

Problem 3. Evaluate the following

$$\int_0^1 \frac{\pi e^{i\pi x/2}}{e^{i\pi x} + 2e^{i\pi x/2} + 1} dx$$

$$\int_0^1 \frac{\pi e^{i\pi x/2}}{e^{i\pi x} + 2e^{i\pi x/2} + 1} dx = \int_0^1 \frac{\pi e^{i\pi x/2}}{(1 + e^{i\pi x/2})^2} dx$$

Simplify the expression

$$\begin{aligned} u &= 1 + e^{i\pi x/2} \\ du &= \frac{i\pi}{2} e^{i\pi x/2} dx \\ dx &= \frac{2}{(i\pi) e^{i\pi x/2}} du \end{aligned}$$

when $x = 0, u = 1 + i$ and $x = 1, u = 2$

$$\begin{aligned} \int_0^1 \frac{\pi e^{i\pi x/2}}{(1 + e^{i\pi x/2})^2} dx &= \int_{1+i}^2 \frac{\pi e^{i\pi x/2}}{(u)^2} \frac{2}{(i\pi) e^{i\pi x/2}} du \\ &= \frac{2}{i} \int_{1+i}^2 \frac{1}{u^2} du \\ &= -\frac{2}{i} \left[\frac{1}{u} \right]_{1+i}^2 \end{aligned}$$