Daily Integral

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1 Easy

Problem 1. *Evaluate the following*

$$\int_0^{\pi/2} \frac{\cos x}{2-\cos^2 x}\mathrm{d}x$$

$$\begin{aligned}\int_0^{\pi/2} \frac{\cos x}{2-\cos^2 x}\mathrm{d}x &= \int_0^{\pi/2} \frac{\cos x}{2-(1-\sin^2)x}\mathrm{d}x \\ &= \int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\mathrm{d}x\end{aligned}$$

Now we consider the u sub

$$\begin{aligned}u &= \sin x \\ \mathrm{d}u &= \cos x \ \mathrm{d}x\end{aligned}$$

when $x = 0, u = 0$ and $x = \dfrac{\pi}{2}, u = 1$

$$\begin{aligned}\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\mathrm{d}x &= \int_0^{1} \frac{\cos x}{1+\sin^2 x}\mathrm{d}x \\ &= \int_0^{1} \frac{1}{1+u^2}\mathrm{d}u\end{aligned}$$

Now consider the trig sub

$$\begin{aligned}u &= \tan\theta \\ \mathrm{d}u &= \sec^2\theta \ \mathrm{d}\theta\end{aligned}$$

when $u = 0, \theta = 0$, and $u = 1, \theta = \dfrac{\pi}{4}$, thus our final integral is transformed

$$\begin{aligned}\int_0^{1} \frac{1}{1+u^2}\mathrm{d}u &= \int_0^{\pi/4} \frac{\sec^2\theta}{1+\tan^2\theta}\mathrm{d}\theta \\ &= \int_0^{\pi/4} \frac{\sec^2\theta}{\sec^2\theta}\mathrm{d}\theta \\ &= \Big[\theta\Big]_0^{\pi/4} \\ &= \frac{\pi}{4}\end{aligned}$$

2 Medium

Problem 2. *Evaluate the following*

$$\int_0^{\pi/4} \frac{(x+1)\tan x}{(1+\tan^2 x)} \mathrm{d}x$$

$$\begin{aligned}
\int_0^{\pi/4} \frac{(x+1)\tan x}{(1+\tan^2 x)} \mathrm{d}x &= \int_0^{\pi/4} \frac{(x+1)\tan x}{1+\tan^2 x} \mathrm{d}x \\
&= \int_0^{\pi/4} \frac{(x+1)\tan x}{\sec^2 x} \mathrm{d}x \\
&= \int_0^{\pi/4} (x+1)\sin x \cos x \mathrm{d}x \\
&= \frac{1}{2}\int_0^{\pi/4} x\sin 2x + \sin 2x \mathrm{d}x \\
&= \frac{1}{2}\int_0^{\pi/4} \sin 2x \mathrm{d}x + \frac{1}{2}\int_0^{\pi/4} x \sin 2x \mathrm{d}x \\
&= -\frac{1}{4}\Big[\cos 2x\Big]_0^{\pi/4} + \frac{1}{2}\int_0^{\pi/4} x\sin 2x \mathrm{d}x \\
&= 0 - -\frac{1}{4} + \frac{1}{2}\int_0^{\pi/4} x\sin 2x \mathrm{d}x \\
&= \frac{1}{4} + \frac{1}{2}\int_0^{\pi/4} x\sin 2x \mathrm{d}x
\end{aligned}$$

Now we use Integrations by Parts:

$$\int_a^b uv' \mathrm{d}x = \Big[uv\Big]_a^b - \int_a^b u'v \mathrm{d}x \tag{1}$$

Now we select $u = x$ and $v = -\frac{1}{2}\cos 2x$ such that $v' = \sin 2x$

$$\begin{aligned}
\int_0^{\pi/4} x\sin 2x \mathrm{d}x &= \Big[(x)(-\frac{1}{2}\cos 2x)\Big]_0^{\pi/4} + \int_0^{\pi/4} \frac{1}{2}\cos 2x \mathrm{d}x \\
&= \frac{\pi}{4}\cdot 0 - 0 + \left[\frac{\sin 2x}{4}\right]_0^{\pi/4} \\
&= \frac{1}{4}
\end{aligned}$$

Thus we have the final result of:

$$\begin{aligned}
\int_0^{\pi/4} \frac{(x+1)\tan x}{(1+\tan^2 x)} \mathrm{d}x &= \frac{1}{4} + \frac{1}{2}\frac{1}{4} \\
&= \frac{3}{8}
\end{aligned}$$

3 Hard

Problem 3. *Evaluate the following*

$$\int_0^1 \frac{\pi e^{i\pi x/2}}{e^{i\pi x} + 2e^{i\pi x/2} + 1} \mathrm{d}x$$

$$\int_0^1 \frac{\pi e^{i\pi x/2}}{e^{i\pi x} + 2e^{i\pi x/2} + 1} \mathrm{d}x = \int_0^1 \frac{\pi e^{i\pi x/2}}{(1 + e^{i\pi x/2})^2} \mathrm{d}x$$

Simplify the expresion

$$\begin{aligned} u &= 1 + e^{i\pi x/2} \\ \mathrm{d}u &= \frac{i\pi}{2} e^{i\pi x/2} \mathrm{d}x \\ \mathrm{d}x &= \frac{2}{(i\pi)e^{i\pi x/2}} \mathrm{d}u \end{aligned}$$

when $x = 0, u = 1 + i$ and $x = 0, u = 2$

$$\begin{aligned} \int_0^1 \frac{\pi e^{i\pi x/2}}{(1 + e^{i\pi x/2})^2} \mathrm{d}x &= \int_2^{1+i} \frac{\pi e^{i\pi x/2}}{(u)^2} \frac{2}{(i\pi)e^{i\pi x/2}} \mathrm{d}u \\ &= \frac{2}{i} \int_2^{1+i} \frac{1}{u^2} \mathrm{d}u \\ &= -\frac{2}{i} \left[\frac{1}{u} \right]_2^{2+i} \end{aligned}$$