Daily Integral

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1 Easy

Problem 1. *Evaluate the following*

$$\int_0^1 x^2\sqrt{1-x^2}\mathrm{d}x$$

Consider the u sub:

$$\begin{aligned} x &= \sin u \\ \mathrm{d}x &= \cos u \mathrm{d}u \end{aligned}$$

When $x = 0, u = 0$ and $x = 1, u = \dfrac{\pi}{2}$

$$\begin{aligned} \int_0^1 x^2\sqrt{1-x^2}\mathrm{d}x &= \int_0^{\pi/2} \sin^2 u \cos^2 u \mathrm{d}u \\ &= \int_0^{\pi/2} \frac{1}{4}(1-\cos 2u)(1+\cos 2u)\mathrm{d}u \\ &= \frac{1}{4}\int_0^{\pi/2} 1-\cos^2 2u \mathrm{d}u \\ &= \frac{1}{4}\int_0^{\pi/2} 1-\frac{1}{2}(1+\cos 4u)\mathrm{d}u \\ &= \frac{1}{8}\int_0^{\pi/2} 1-\cos 4u \mathrm{d}u \\ &= \frac{1}{8}\Big[u-\frac{\sin 4u}{4}\Big]_0^{\pi/2} \\ &= \frac{\pi}{16} \end{aligned}$$

2 Medium

Problem 2. *Evaluate the following*

$$\int_{\int_{-1/2}^{1/2} |x| \mathrm{d}x}^{\int_{-1}^{0} \frac{1}{x^2+2x+2} \mathrm{d}x} (5 + 6^x + 7^2 x) \mathrm{d}x$$

Let us evaluate the top bound first:

$$\int_{-1}^{0} \frac{1}{x^2+2x+2} \mathrm{d}x = \int_{-1}^{0} \frac{1}{(x+1)^2+1} \mathrm{d}x$$

Use the following trigonometric substitution

$$\begin{aligned} x + 1 &= \tan u \\ \mathrm{d}x &= \sec^2 u \mathrm{d}u \end{aligned}$$

When $x = 0, u = \dfrac{\pi}{4}$ and $x = -1, u = 0$

$$\begin{aligned} \int_{-1}^{0} \frac{1}{(x+1)^2+1} \mathrm{d}x &= \int_{0}^{\pi/4} \frac{\sec^2 u}{\tan^2 u + 1} \mathrm{d}u \\ &= \int_{0}^{\pi/4} \frac{\sec^2 u}{\sec^2 u} \mathrm{d}u \\ &= \int_{0}^{\pi/4} \mathrm{d}u \\ &= [u]_0^{\pi/4} \\ &= \frac{\pi}{4} \end{aligned}$$

Now let us evaluate the lower bound: