Wavelet Analysis on Images

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Wavelet Analysis on Images

- Forms
  - Continuous WT
  - Discrete WT
  - Wavelet Packet Transform

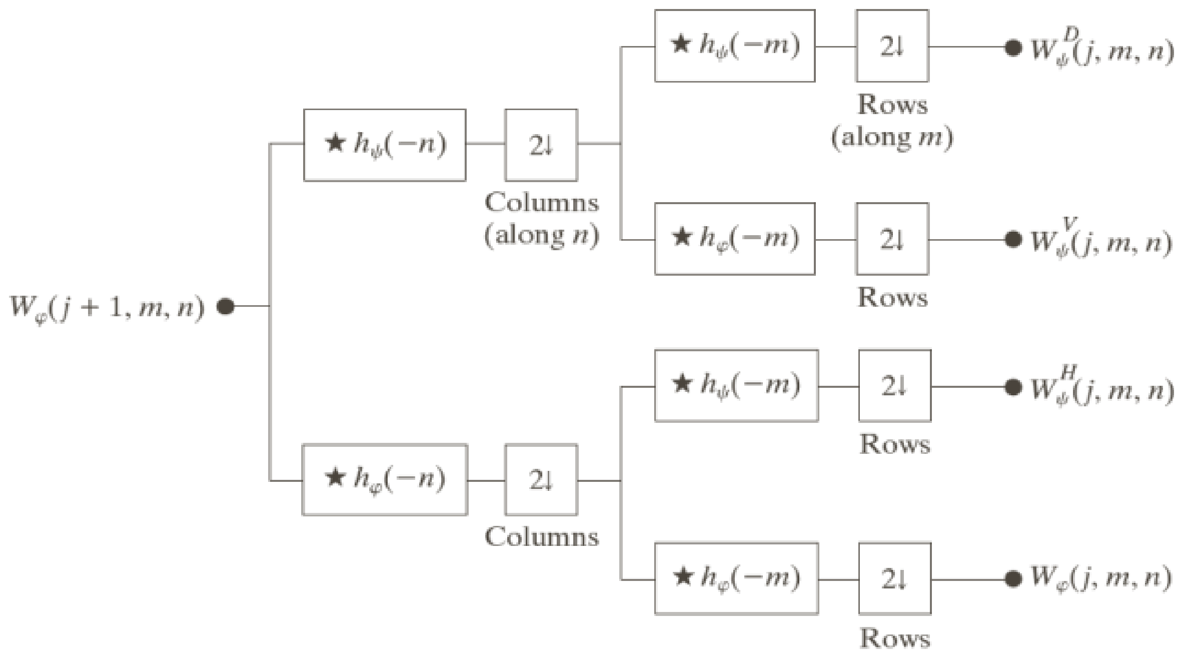
Continuous Wavelet Transform

- Pick a wavelet and correlate it in 2D with the image

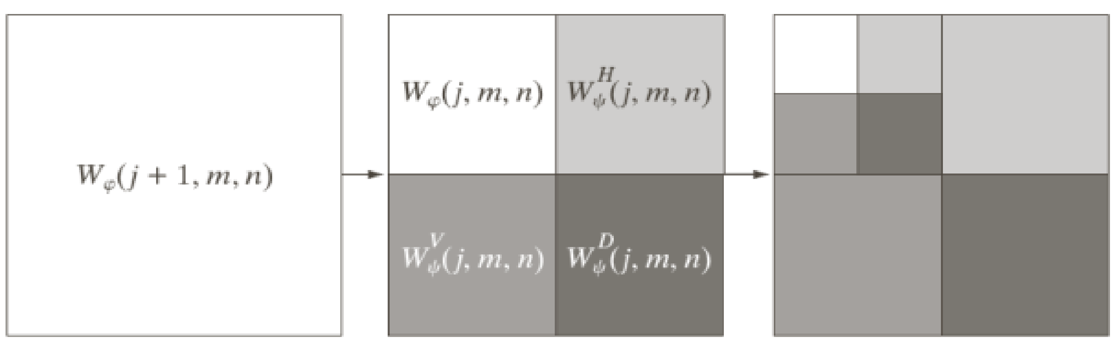
$$cwt(s,a,b) = \frac{1}{\sqrt{s}} \int \int f(x,y)\psi\left(\frac{x-a}{s}, \frac{y-b}{s}\right) dxdy$$

$$CWT(s,\omega_1,\omega_2) = \sqrt{s}F(\omega_1,\omega_2)\Psi(s\omega_1,s\omega_2)$$

DWT



$\star h_\psi(-m)$
$2\downarrow$
$W_\psi^D(j, m, n)$
Rows (along m)
$\star h_\psi(-n)$
$2\downarrow$
Columns (along n)
$\star h_\varphi(-m)$
$2\downarrow$
$W_\psi^V(j, m, n)$
Rows
$W_\varphi(j + 1, m, n)$
$\star h_\psi(-m)$
$2\downarrow$
$W_\psi^H(j, m, n)$
Rows
$\star h_\varphi(-n)$
$2\downarrow$
Columns
$\star h_\varphi(-m)$
$2\downarrow$
$W_\varphi(j, m, n)$
Rows



$W_\varphi(j + 1, m, n)$
$W_\varphi(j, m, n)$
$W_\psi^H(j, m, n)$
$W_\psi^V(j, m, n)$
$W_\psi^D(j, m, n)$

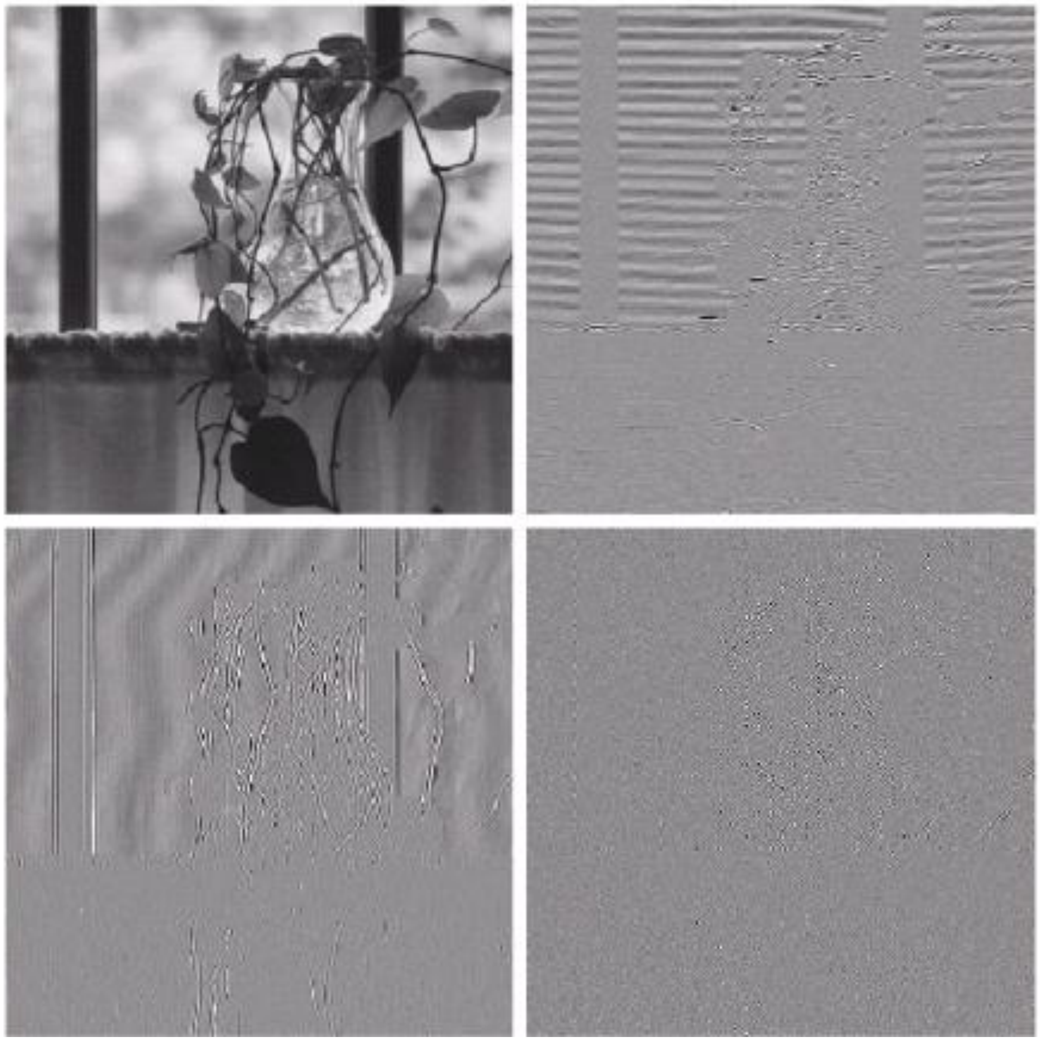


FIGURE 7.7 A four-band split of the vase in Fig. 7.1 using the subband coding system of Fig. 7.5.

a	d^V
d^H	d^D

$a(m,n)$: approximation

$d^V(m,n)$: detail in vertical

$d^H(m,n)$: detail in horizontal

$d^D(m,n)$: detail in diagonal

Wavelets Applications

- Compression
- Fusion
- Feature Extraction

Compression

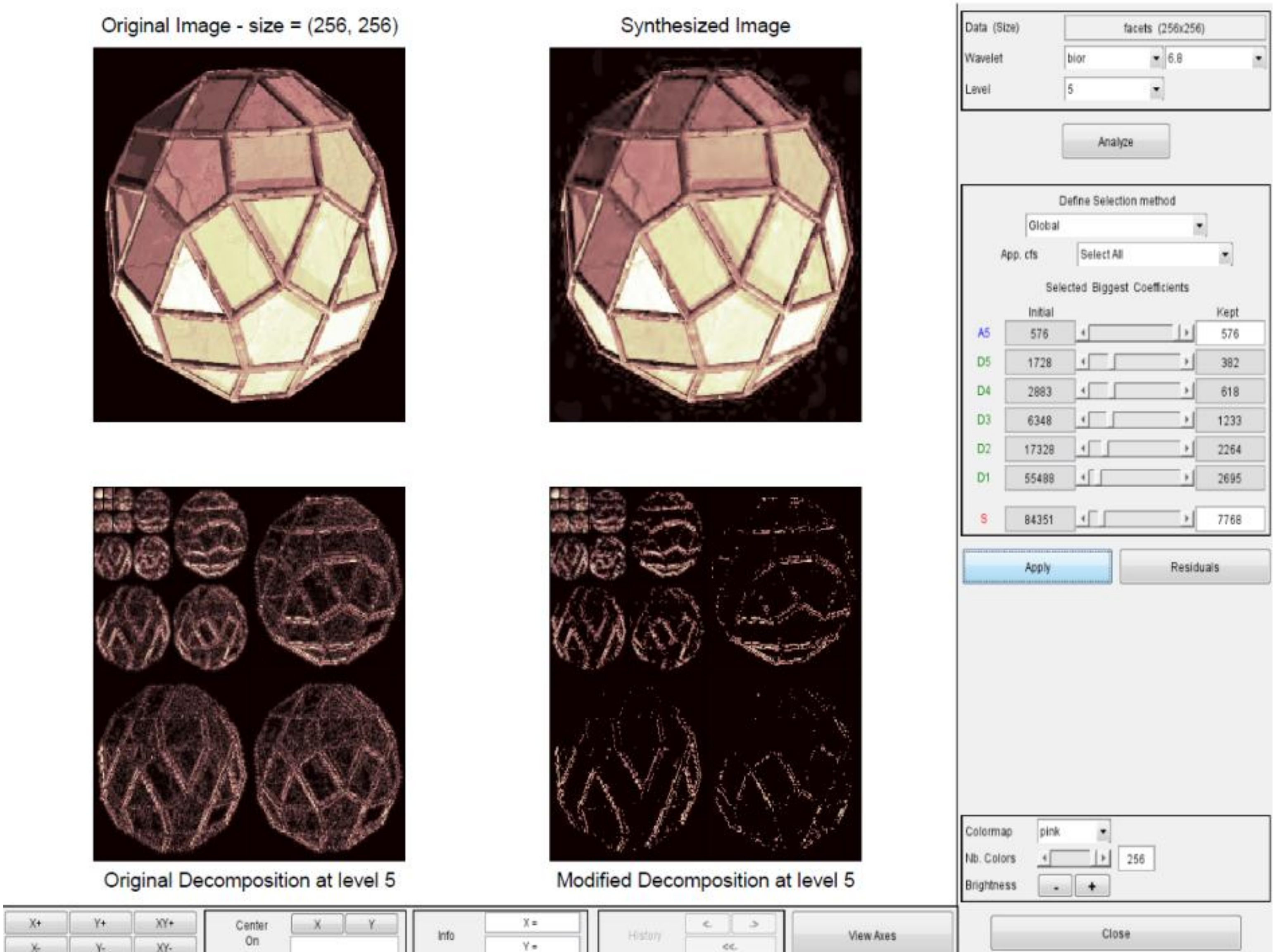
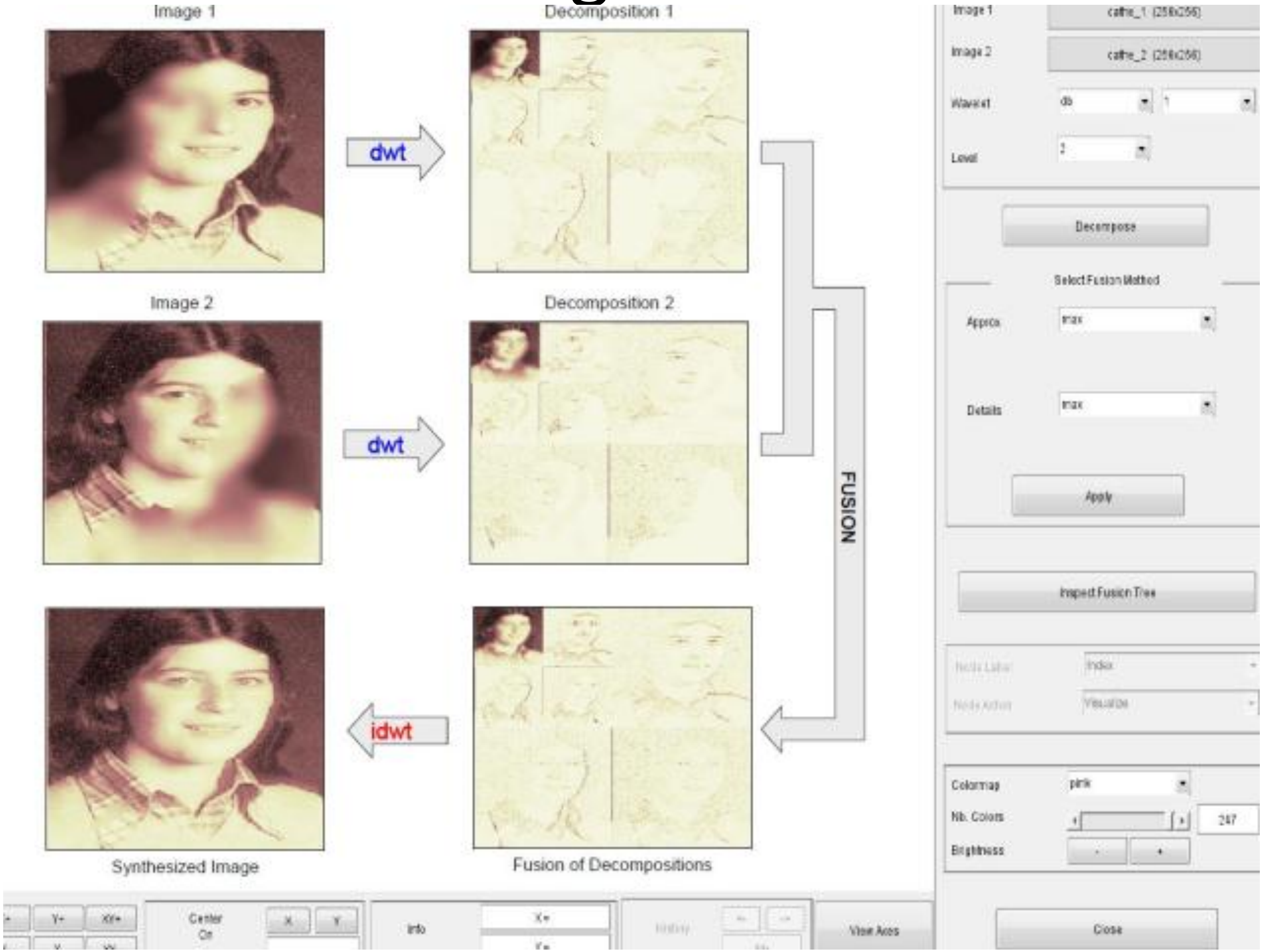


Image Fusion



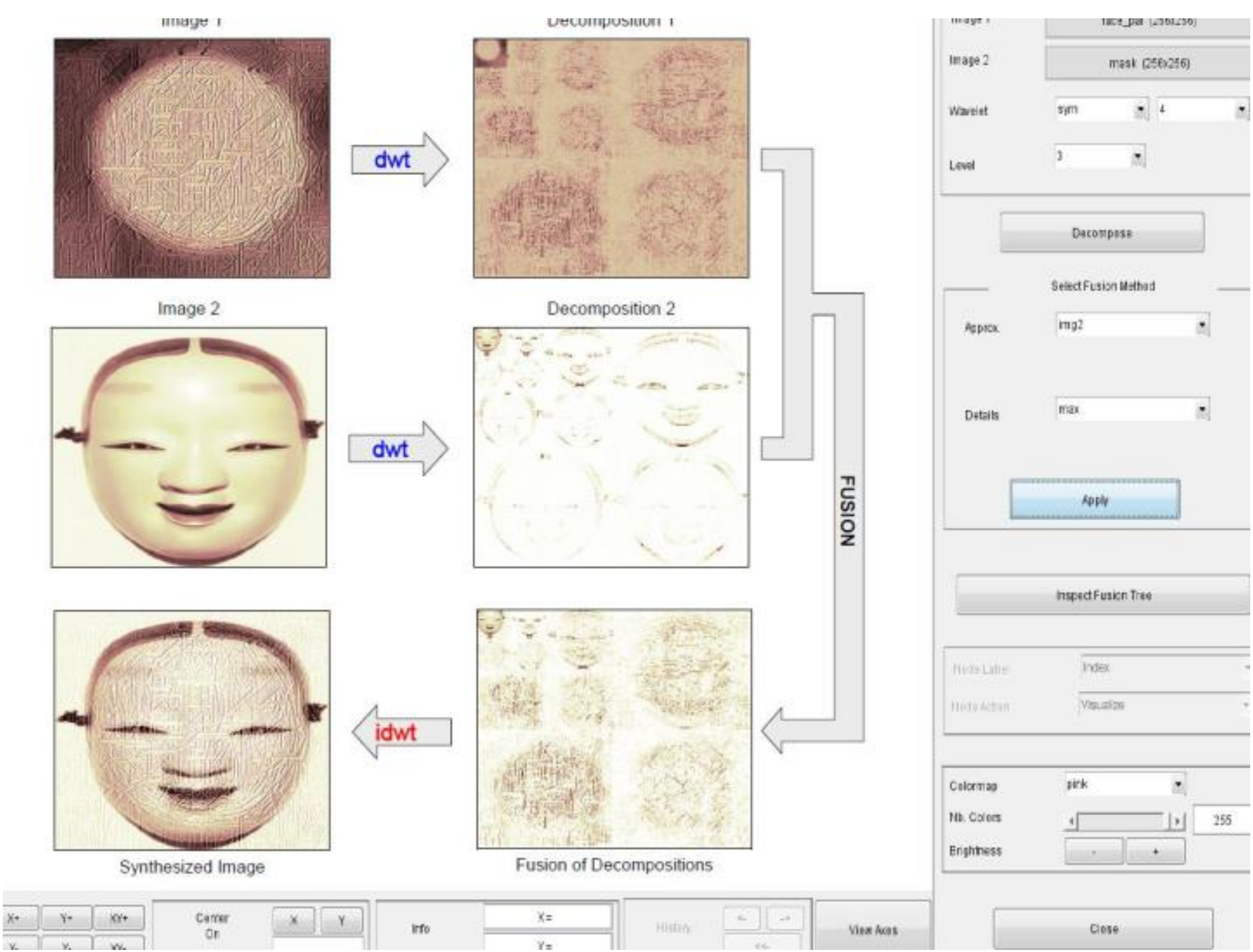
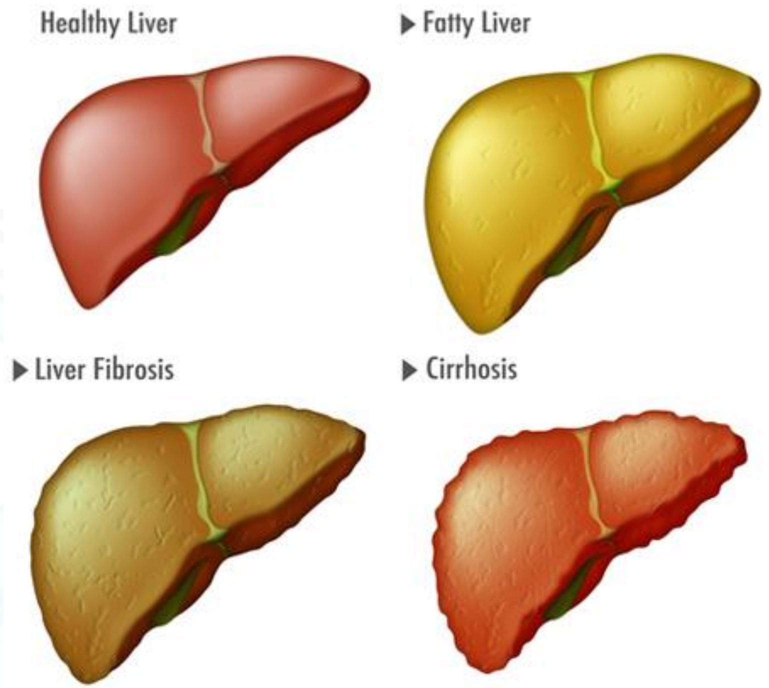
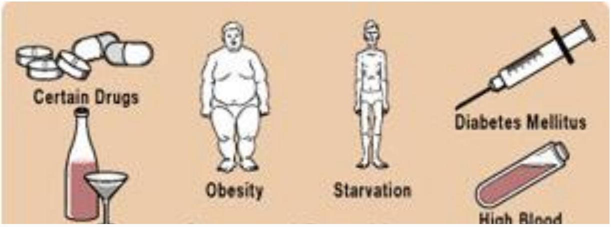


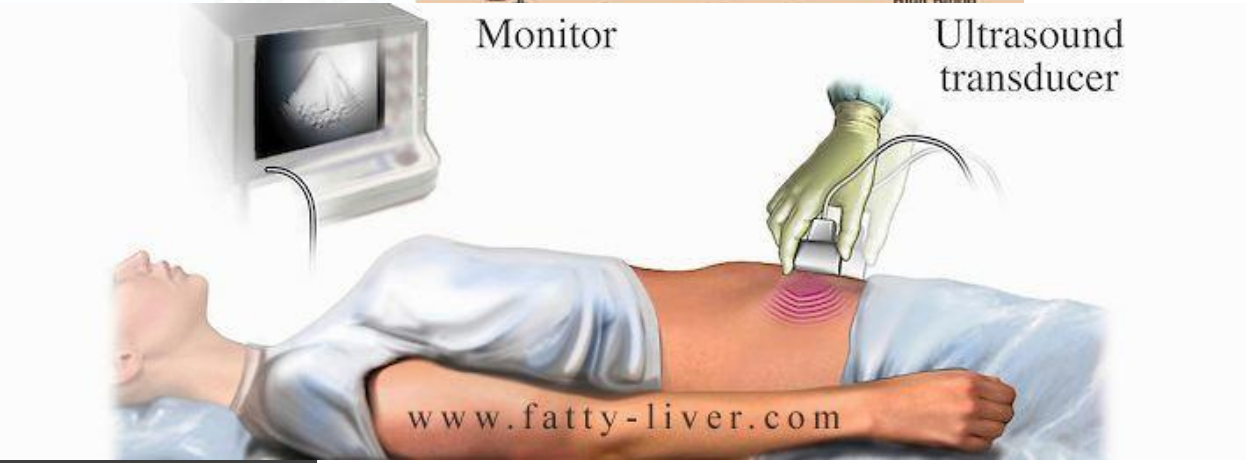
Image 1
Decomposition 1
dwt
Image 2
Decomposition 2
dwt
FUSION
idwt
Synthesized Image
Fusion of Decompositions
Image 2
mask (256x256)
Wavelet
sym
4
Level
3
Decompose
Select Fusion Method
Approx.
img2
Details
max
Apply
Inspect Fusion Tree
Index
Visualize
Colormap
pink
Nb. Colors
255
Brightness
Center On
X
Y
Info
X=
Y=
View Axes
Close

A Case Study

- Predicting Liver Texture Type
- Given
  - Ultrasound Images
- To do
  - Identify what type
    - Normal
    - Fatty
    - Heterogenous








70 FATTY
52 FATTY
FATTY
35 YR NORMAL
65 YR NORMAL
35 YR NORMAL

Strategy

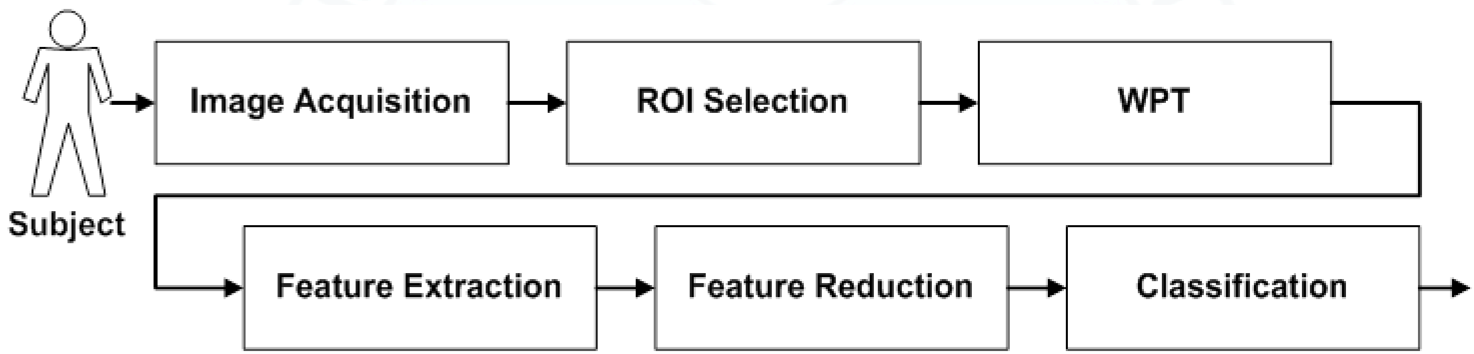
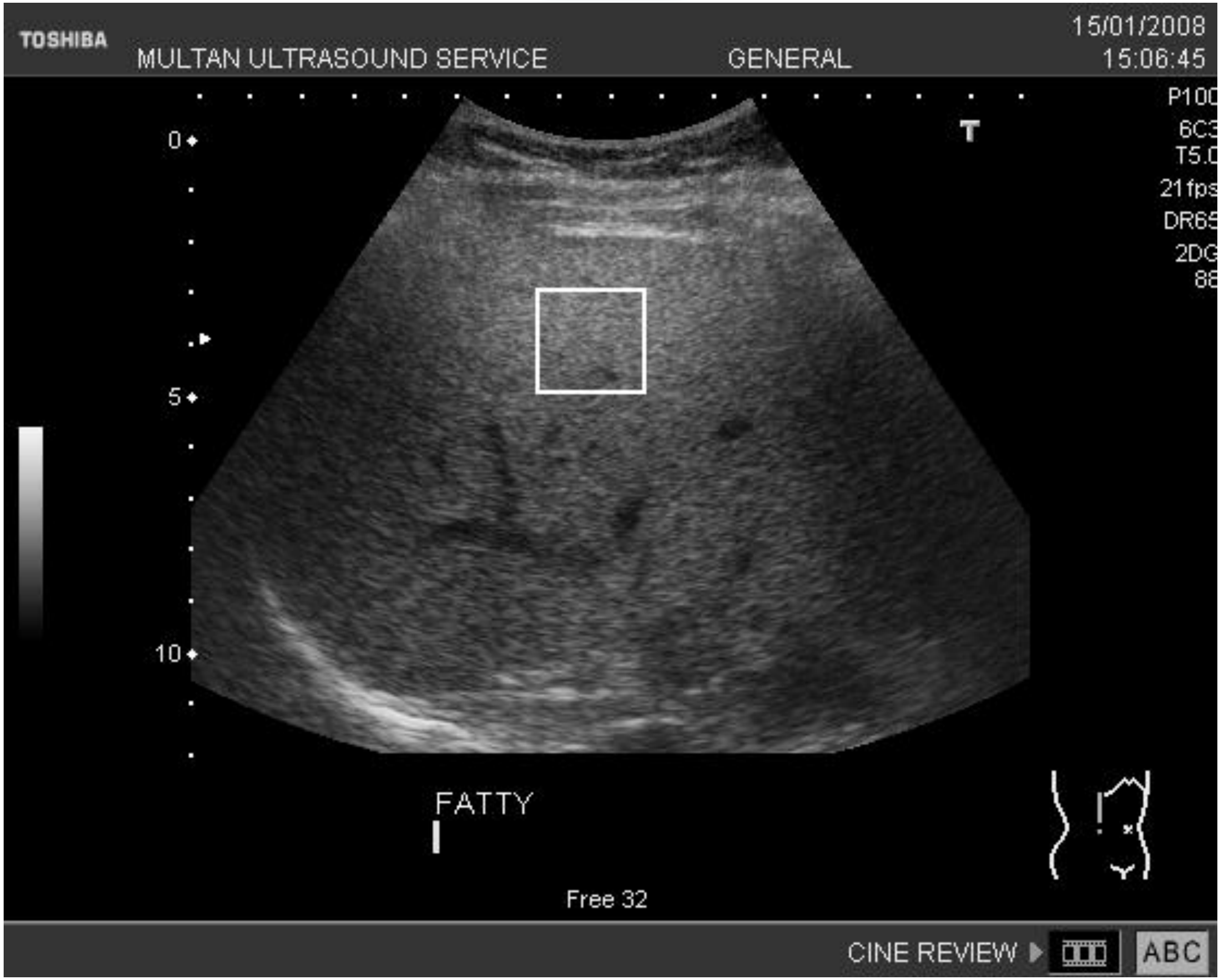


Image Acquisition
ROI Selection
WPT
Subject
Feature Extraction
Feature Reduction
Classification

Acqusition

- acquired at Multan Institute of Nuclear Medicine and Radiotherapy (MINAR), Multan, Pakistan by trained professional medical experts.
- acquired on a Toshiba SSA 550 digital ultrasound machine with a convex probe;
  - 5MHz Tissue Harmonic Imaging frequency.
  - An intercostal section showing segment 7 was used for analysis, care being taken to exclude any vessels, bile ducts or areas of echo inhomogeneity. The images are of size 560x450 and are saved as BMP files.
  - 30 with FLD,
  - 39 normal
  - 19 heterogeneous
- A 64x64 ROI was marked by radiologists


TOSHIBA
MULTAN ULTRASOUND SERVICE
GENERAL
15/01/2008
15:06:45
FATTY
Free 32
CINE REVIEW
ABC

Automatic Segmentation

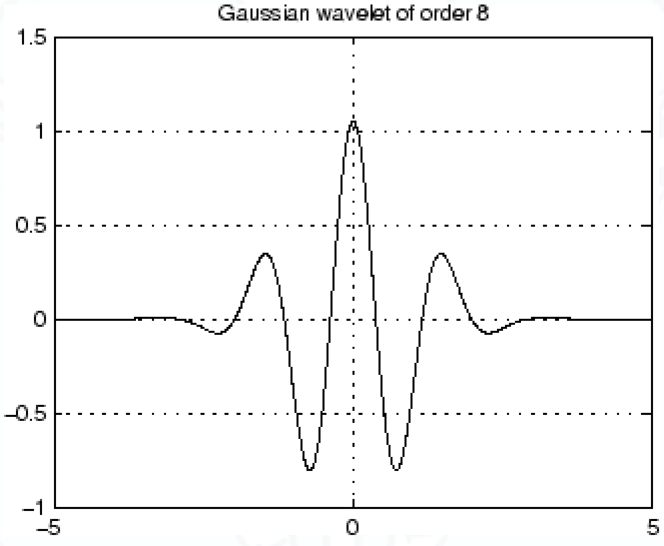
- Extract an ROI automatically
- Using Continuous Wavelet Transform

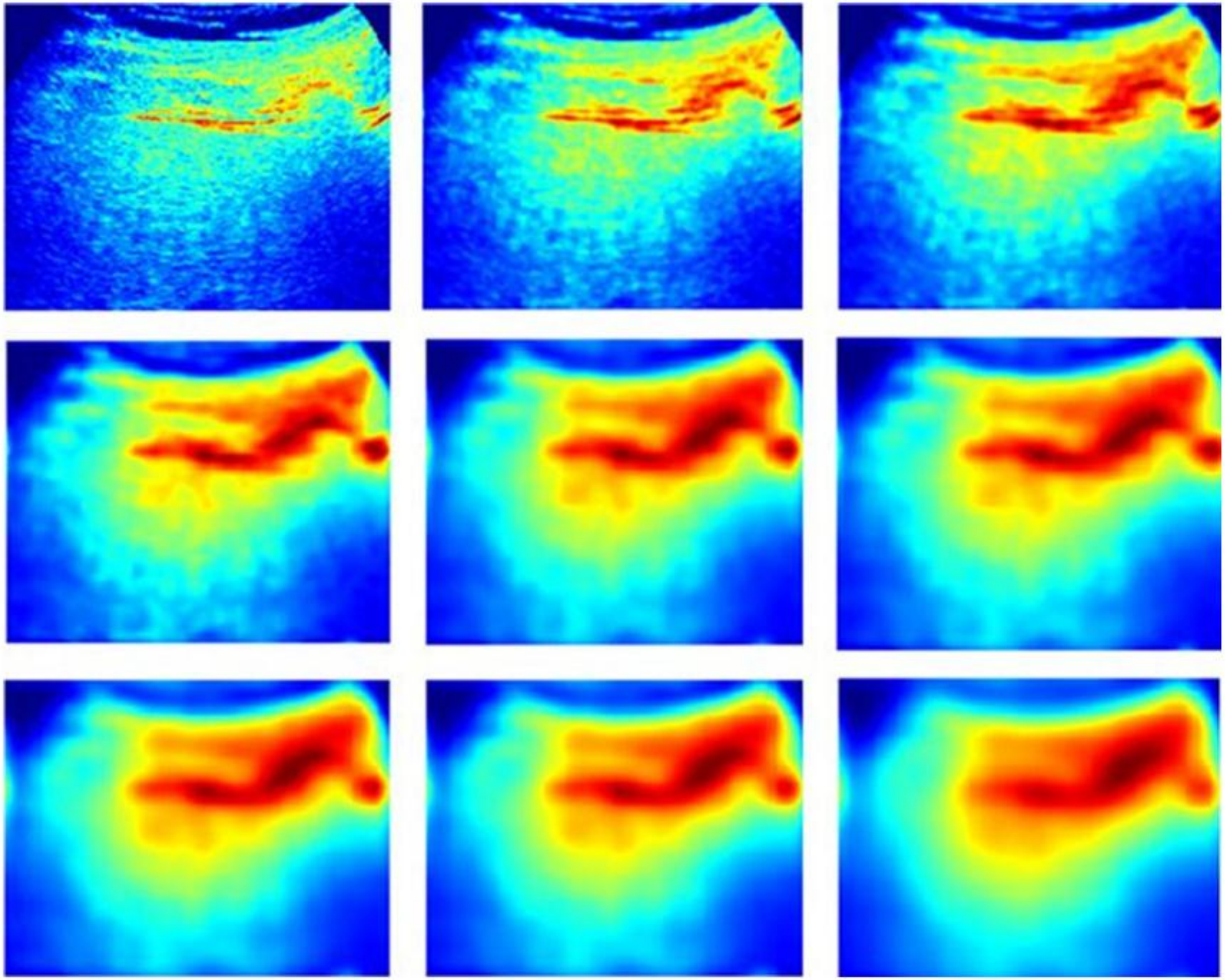
$$cwt(s,a,b) = \frac{1}{\sqrt{s}} \int\int f(x,y)\psi\left(\frac{x-a}{s}, \frac{y-b}{s}\right) dxdy$$

$$CWT(s,\omega_1,\omega_2) = \sqrt{s}F(\omega_1,\omega_2)\Psi(s\omega_1, s\omega_2)$$

- Gaussian wavelet with eight different scale parameter values (1.0, 1.6, 2.6, 3.9, 4.0, 5.0, 5.4 and 7.0)

Gaussian Wavelet

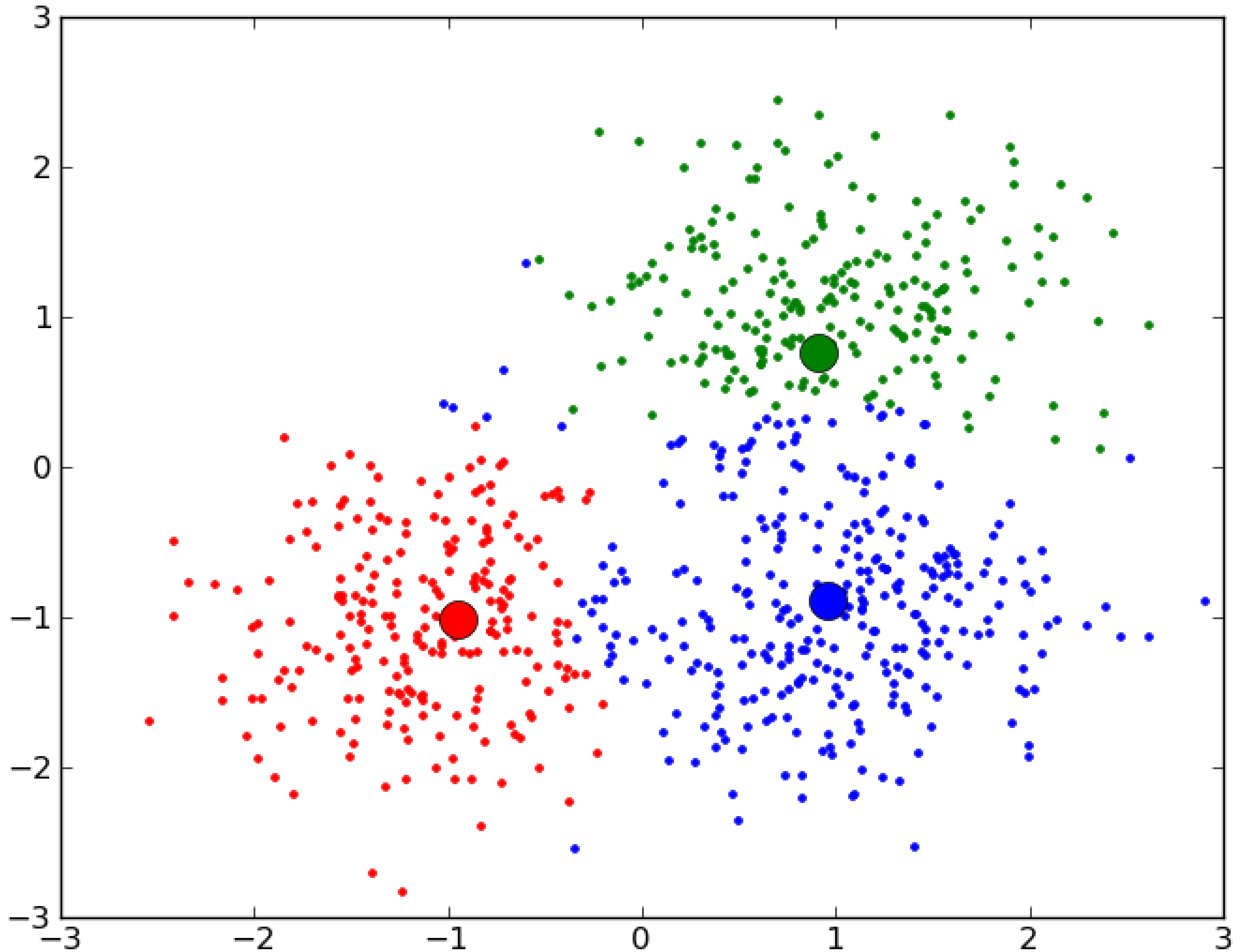




Supervised Segmentation

- Each given 64x64 ROI has 4096 pixels
- Each pixel is represented by a 9 dimensional vector
- Total 88 images
- Thus, total number of examples is 88x4096
- We used Clustering to reduce the number of data ponts to 4096 training examples

Clustering



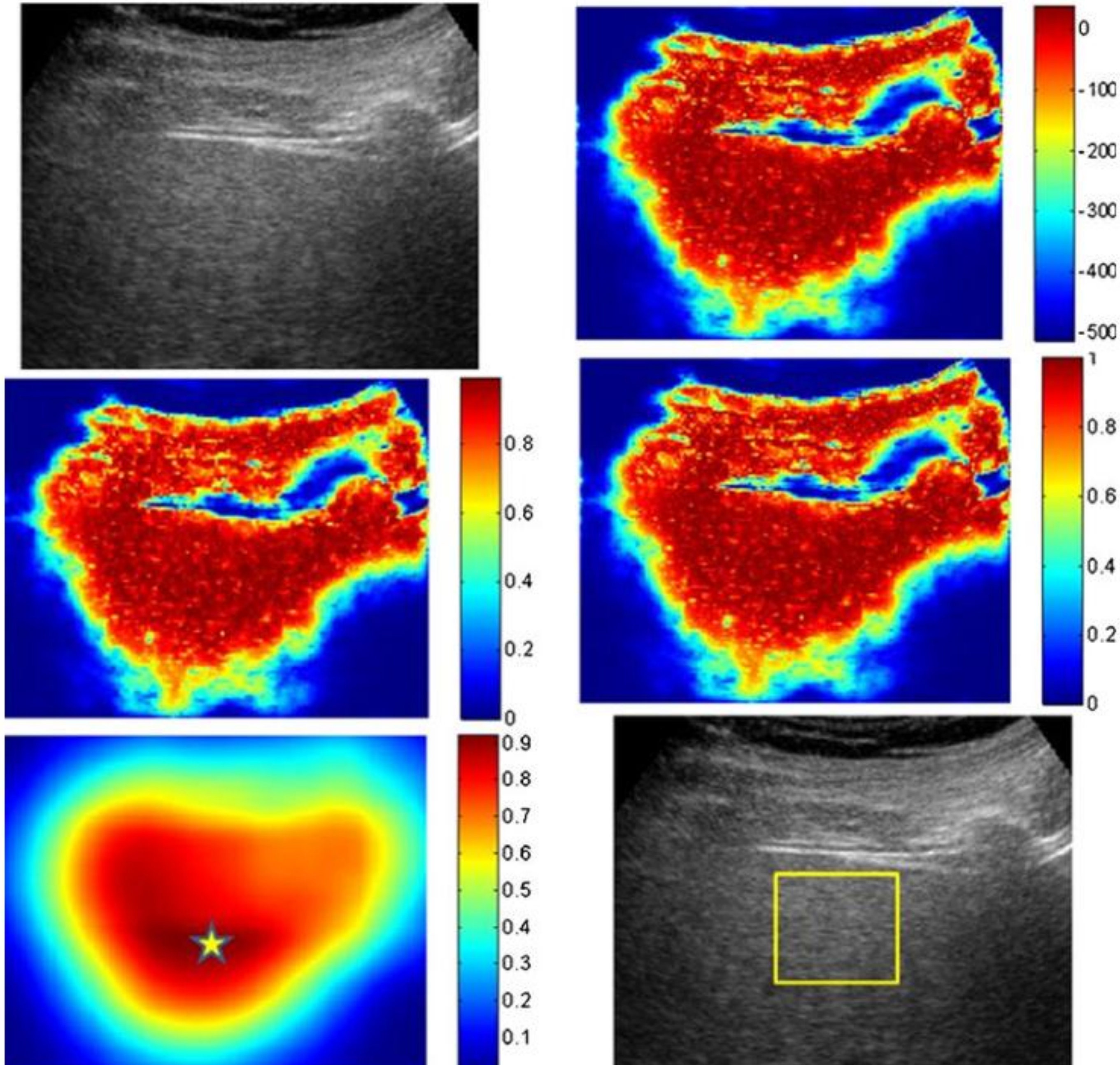
Supervised Segmentation

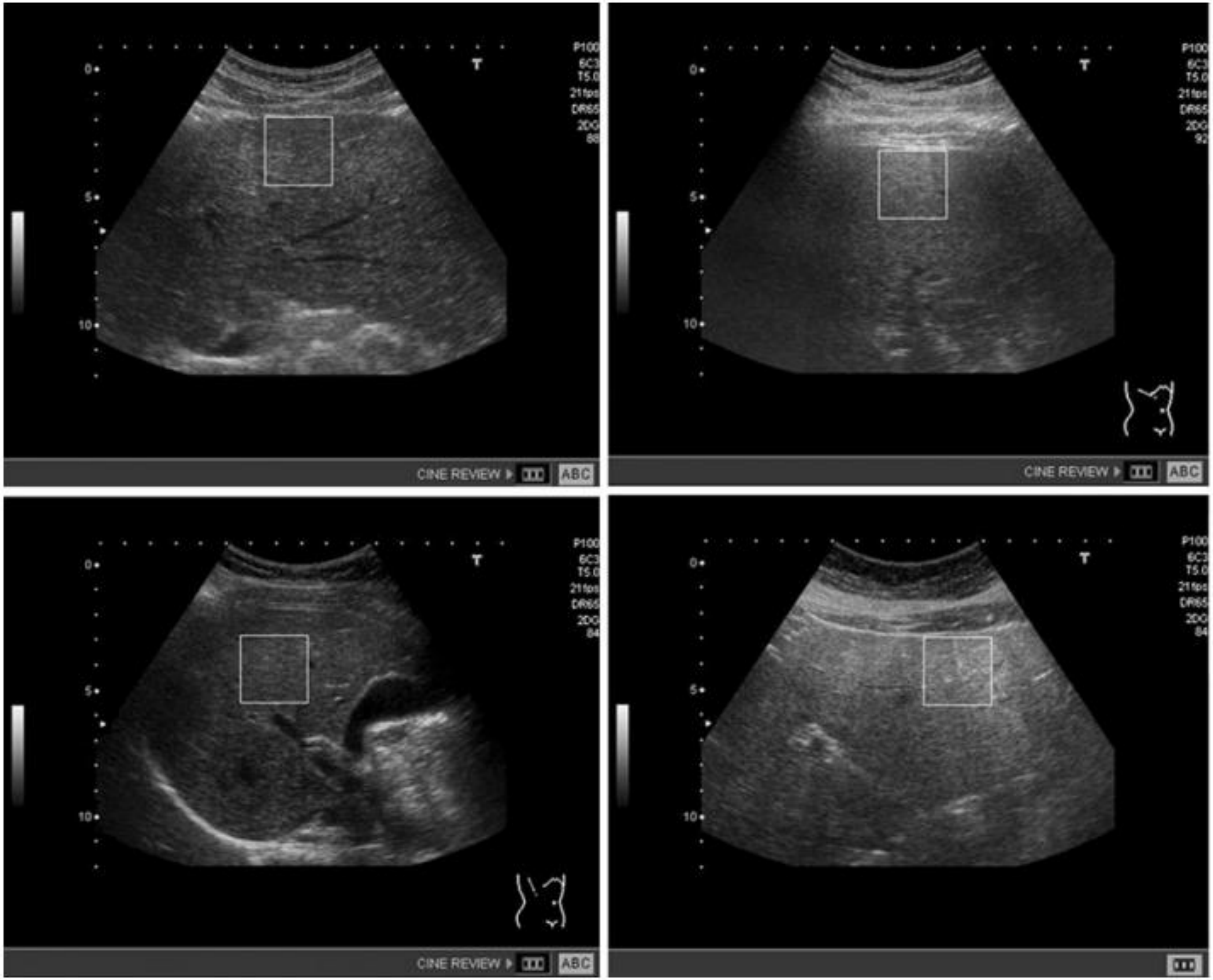
- Now we have 4096 training examples with 9 features each
- However, we only had labels for the positive examples (ROIs)
  - No information about negative examples
- So, we used a One-Class Classifier

One-Class vs. Binary Classification

- Binary classification
  - Apple or Orange given training data for both
- One-Class Classification
  - Apple or not given just training data for apples
  - Uses density estimation or likelihood estimation for the target class
- One-Class SVM
  - A large-margin One-Class Classifier
- ROI Extraction
  - Finding the maximum value after testing on a given image

Different steps in ROI detection (segmentation). Row-wise from the *top-left*: original image, the corresponding Z-image, normalized Z-image (note the change in the range on the color bar), after morphological opening, after averaging filter and finding the point with the maximum value (indicated by the *star*) and the resulting ROI *square*





P100
6C3
T5.0
21fps
DR65
2DG
0
5
10
CINE REVIEW
ABC

Texture Classification using WPT

- Wavelet Packet Transform
  - ows the WPT of an ultrasound image ROI at level 2 using the Daubechies 3 wavelet.

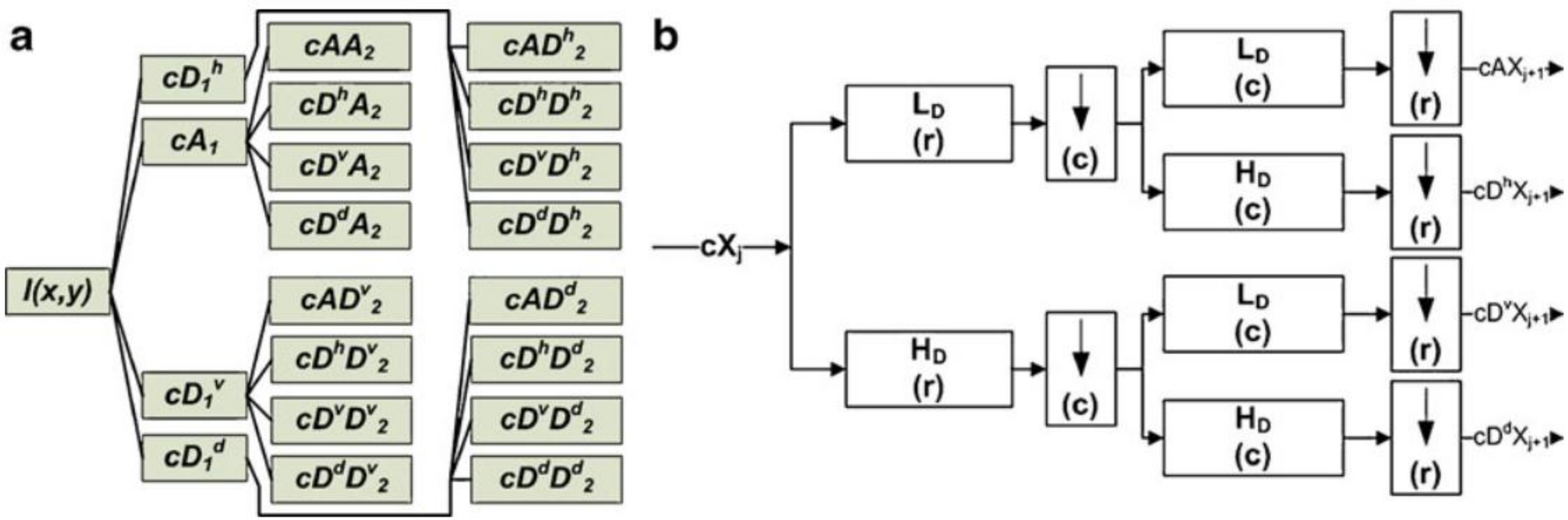
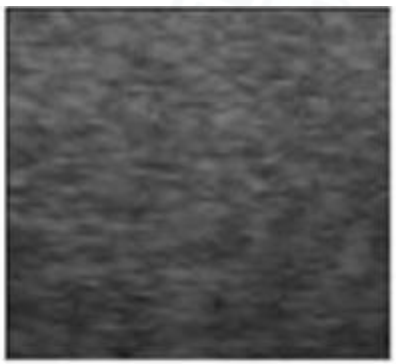
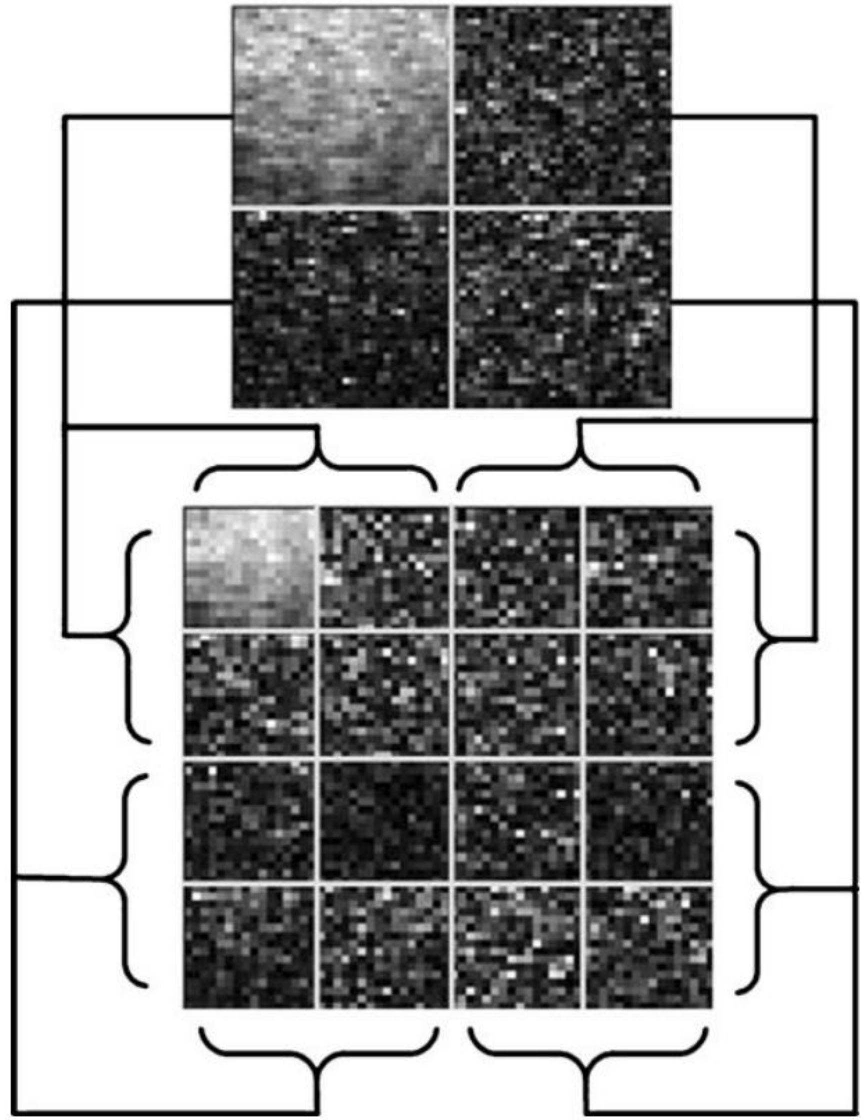


Fig. 7 **a** The 2 level wavelet packet decomposition of image I(x,y) into approximation (cA_1) and details coefficients in horizontal, vertical and diagonal directions (cD_1^h, cD_1^v, cD_1^d). Each of these coefficients is then decomposed further to level 2 yielding a total of 16 sets of coefficients at level 2. **b** The coefficients of the j^{th} level are decomposed further to produce the corresponding approximations and details. The symbols (r) and (c) indicate whether the down-sampling (by 2, indicated by the *downward arrow*) and the filtering operation in the indicated box is performed on the rows (r) or columns (c) of the input





Feature Extraction

- The following statistical features are extracted from each of the WPT sub images:
  a. Median (m)
  b. Standard deviation (s)
  c. Inter-quartile range (q)
- Total: 61 Features

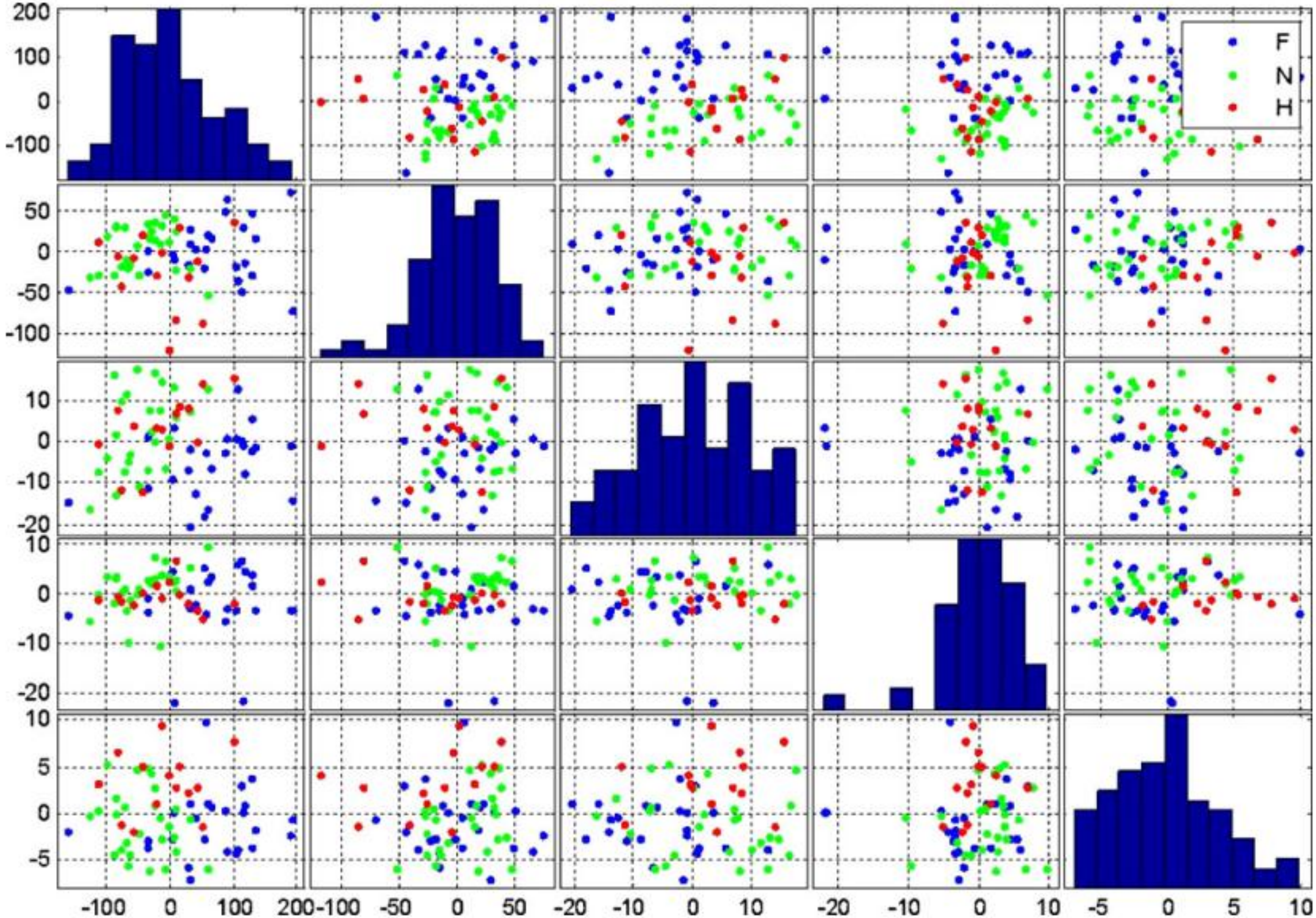


Fig. 9 The scatter-matrix plot of the first five principal components (obtained only for visualization) of the 61 features extracted from each sub-image

Classification

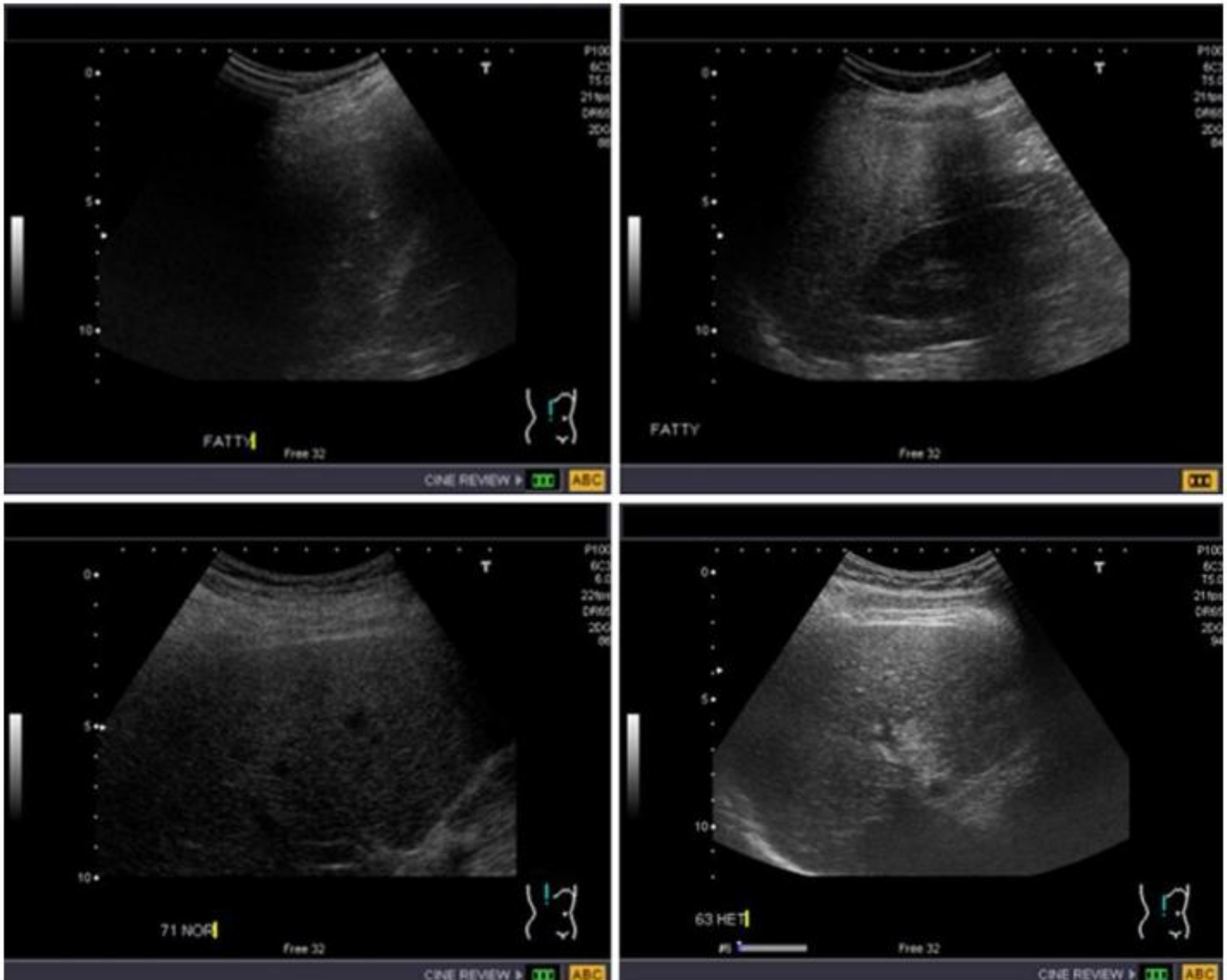
- Using a Binary SVM
  - Se = TP / (TP+FN) aka Recall
  - PPV = TP / (TP+FP) aka Precision

		Classifier labels			Sensitivity
		Fatty	Normal	Heterogeneous	
True labels	Fatty	28	2	0	93.3%
	Normal	1	38	0	97.4%
	Heterogeneous	1	0	18	94.7%
Positive predictive value		93.3%	95.0%	100.0%	
Accuracy		95.4%			

Analysis of Results

Performance metric (%)		K=5 Fold CV	K=10 Fold CV
Positive predictive value	Fatty	93.8 (4.5)	92.4 (3.1)
	Normal	87.2 (3.8)	92.3 (2.8)
	Heterogeneous	92.9 (7.3)	100 (0.0)
Sensitivity	Fatty	87.3 (4.1)	92.4 (2.5)
	Normal	94.9 (2.7)	96.3 (2.0)
	Heterogeneous	85.3 (7.8)	91.0 (5.0)
Accuracy		90.2 (2.6)	93.7 (2.0)

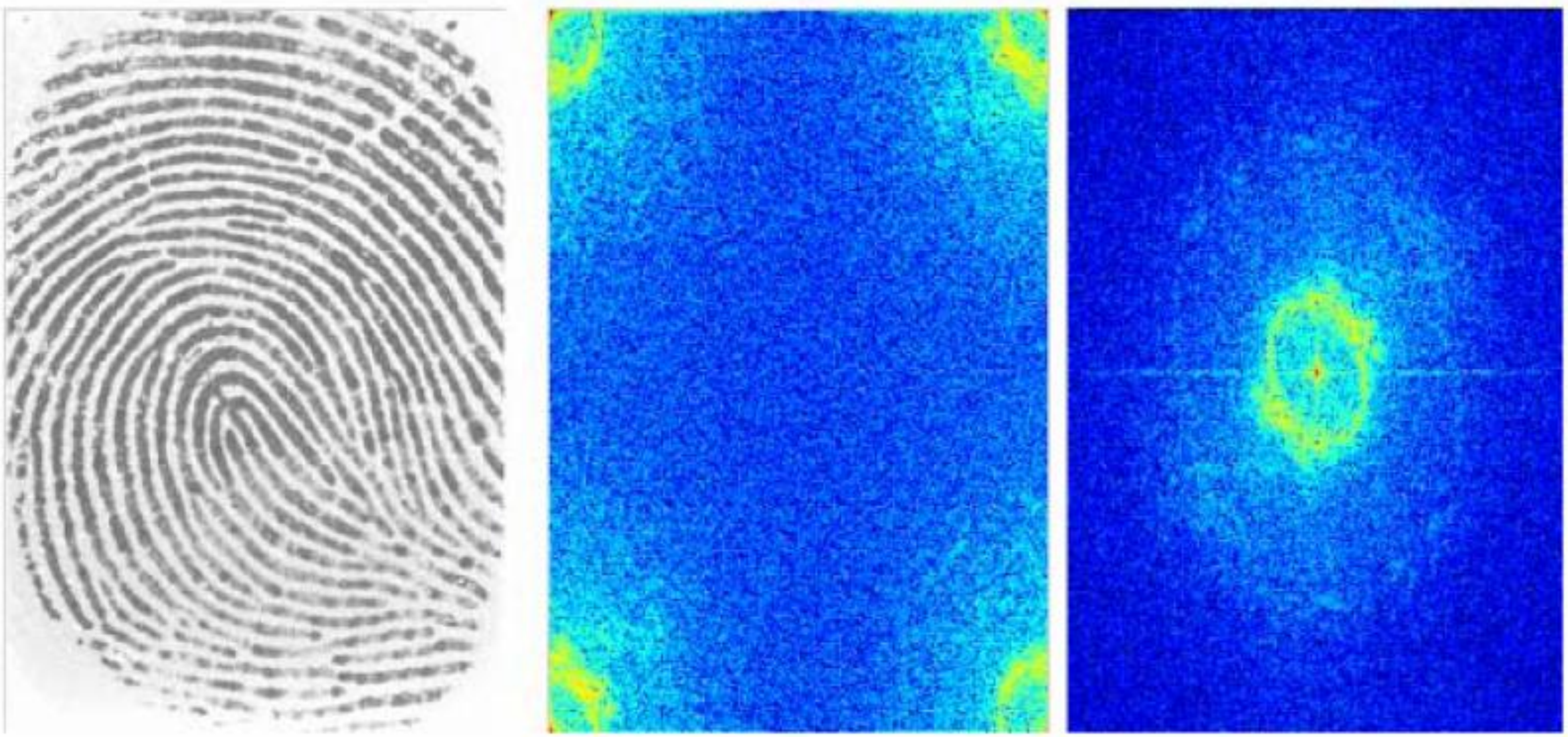
Misclassified Cases



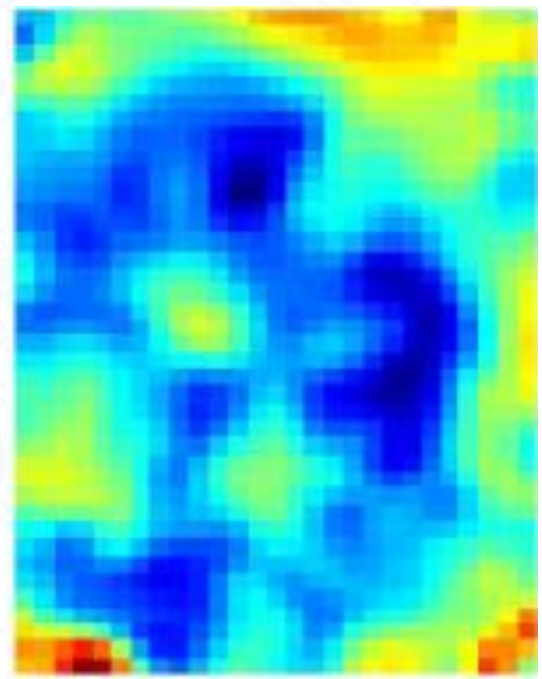
Analysis

- Why did it work?
  - Texture granularity changes in FLD
  - Which is essentially a change in the local frequency characteristics, esp. for Heterogenous liver
  - CWT and WPT allow us to capture this information
  - The rest is machine learning

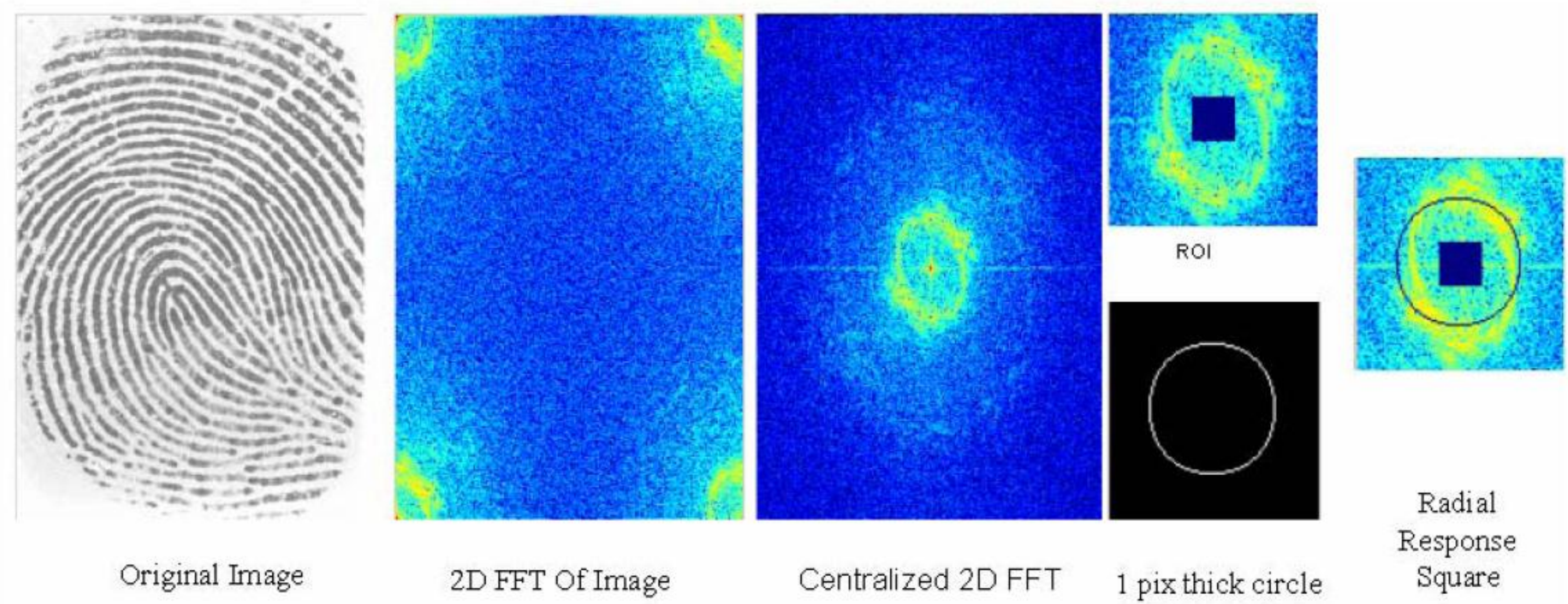
More Fourier Analysis



Results of Ridge Frequency Estimation



Global Frequency



- Hough Transform to find the circle corresponding to the global frequency
- Or use the average of the block-wise frequency estimates

Enhancement using Gabor Filters

- Model an image using the same mechanism used in the human visual system

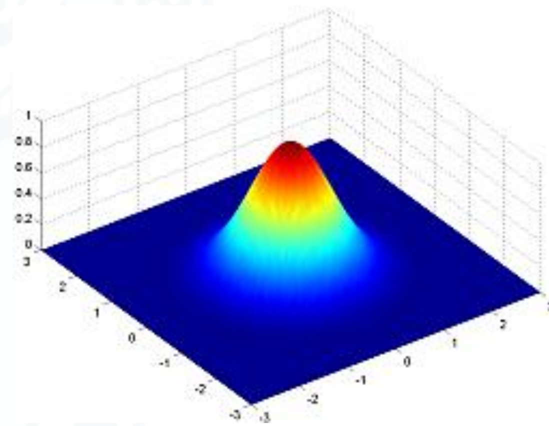
$$g(x,y:\theta,f)=exp\left\{-\frac{1}{2}\left[\frac{{x_\theta}^2}{{\sigma_x}^2}+\frac{{y_\theta}^2}{{\sigma_y}^2}\right]\right\}\cdot cos(2\pi f\cdot x_\theta),$$

$$\begin{bmatrix} x_\theta \\ y_\theta \end{bmatrix}=\begin{bmatrix} cos(90^\circ-\theta) & sin(90^\circ-\theta) \\ -sin(90^\circ-\theta) & cos(90^\circ-\theta) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} sin\,\theta & cos\,\theta \\ -cos\,\theta & sin\,\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

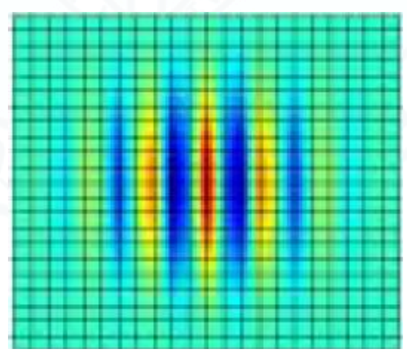
Analysis of the Gabor Filter

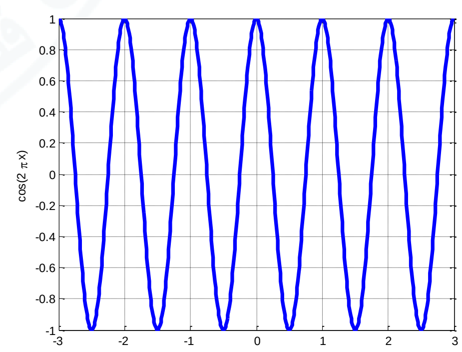
- Assume $\boldsymbol{\theta = 90, \sigma_x = \sigma_y = \frac{1}{\sqrt{2}}, f = 1}$

$$g(x, y, 90, 1) = exp\left(-(x^2 + y^2)\right)cos(2\pi x)$$



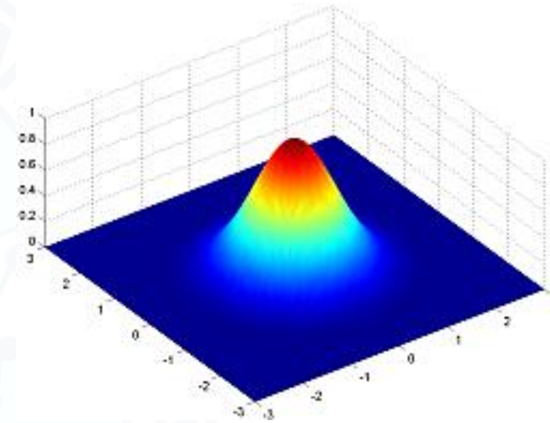
- It has two components
  - $exp\left(-(x^2 + y^2)\right)$
  - $cos(2\pi x)$





Frequency Response of the Gabor Filter

- Fourier transform of a Gaussian

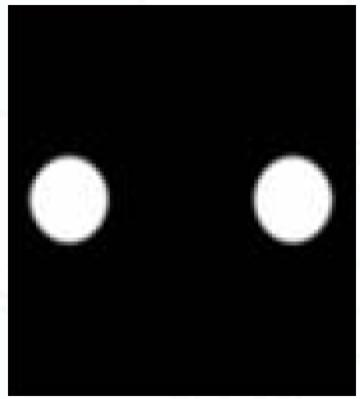


- Fourier transform of a cosine
- Multiplication in time domain becomes
-

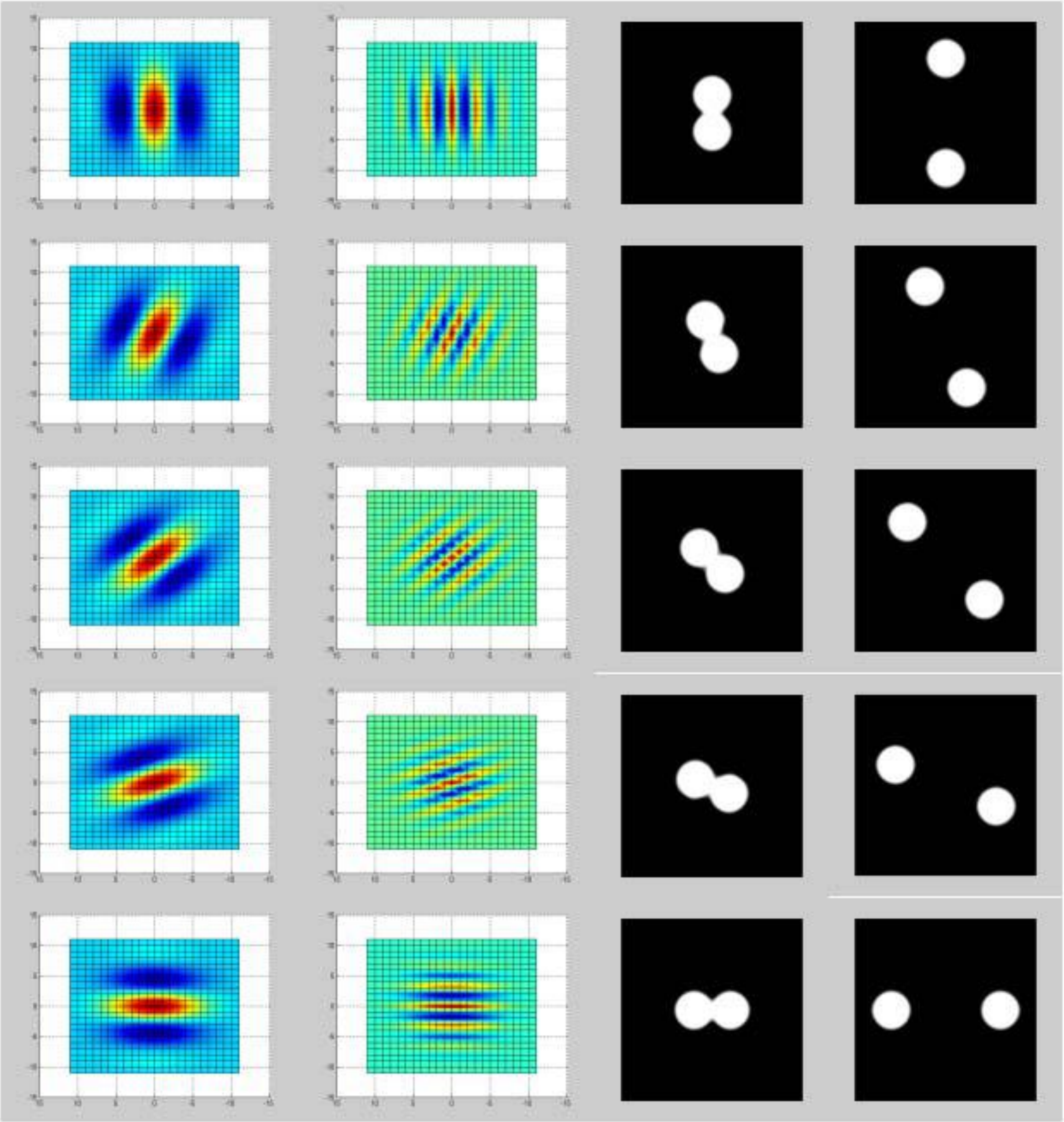
Frequency Response of the Gabor Filter

- Convolution of a Gaussian with an impulse at 0 will be
- Convolution of a Gaussian with an impulse at f will be

Frequency Response of the Gabor Filter



- The spread of the “eyes” is determined by
- The distance of the eyes from the origin is determined by
- The angle the line of the “eyes” is determined by



Gabor filters for 0, 22.5, 45, 67.5 & 90 degrees at f=0.1, dx=dy=4 in the spatial domain

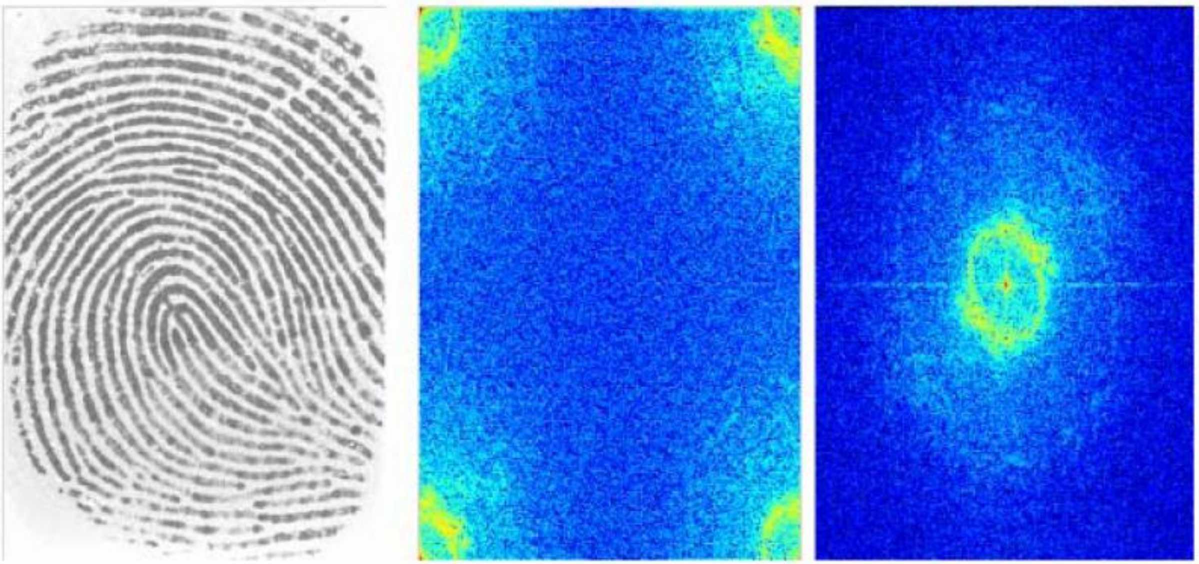
Gabor filters for 0, 22.5, 45, 67.5 & 90 degrees at f=0.3, dx=dy=4 in the spatial domain

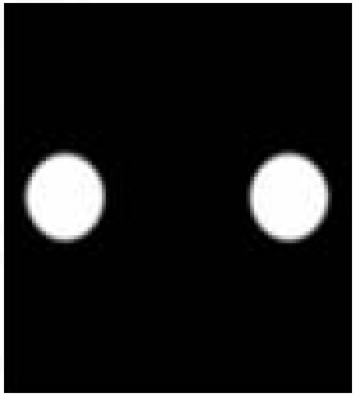
Gabor filters for 0, 22.5, 45, 67.5 & 90 degrees at f=0.1, dx=dy=4 in frequency domain

Gabor filters for 0, 22.5, 45, 67.5 & 90 degrees at f=0.3, dx=dy=4 in frequency domain

Properties of a Gabor Filter

- If we perform Gabor filtering of an image

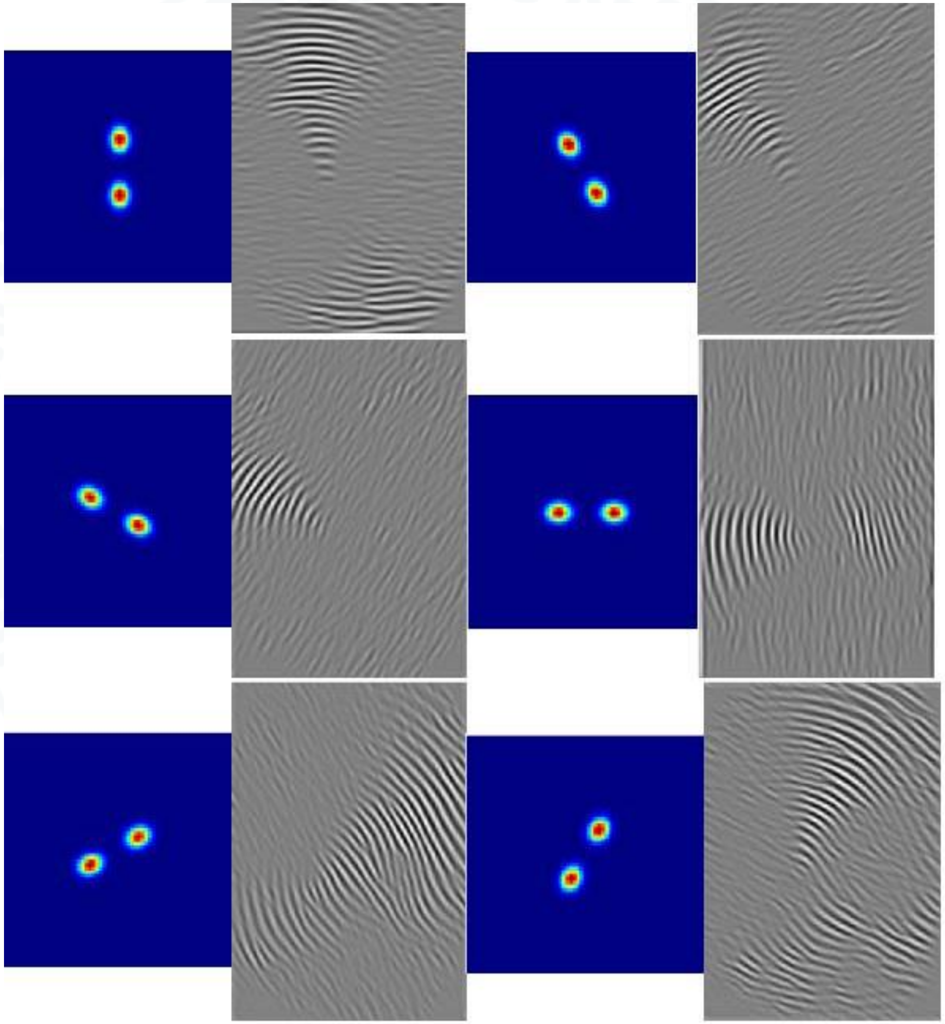




Properties of the Gabor Filter

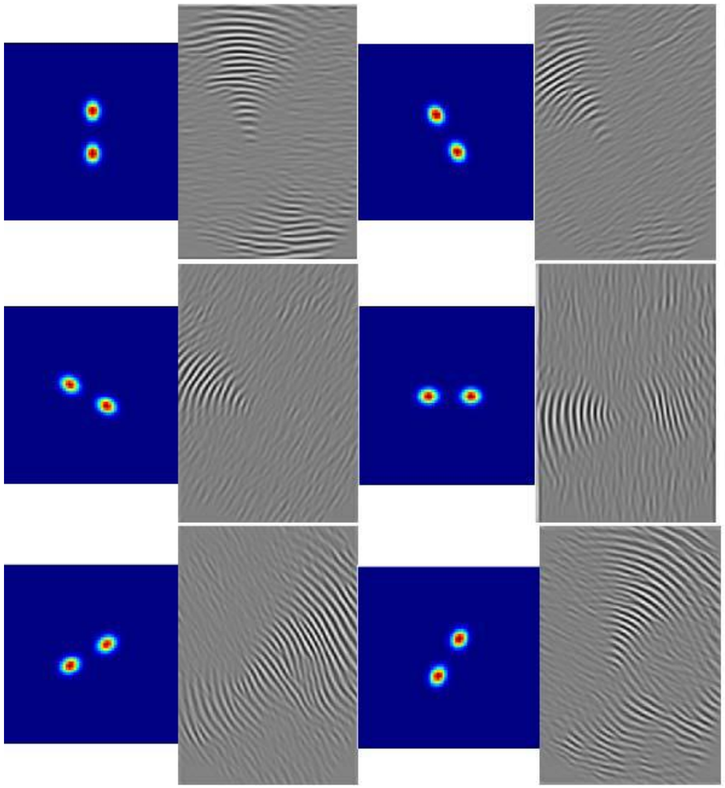
- Thus a Gabor Filter acts as a:
  - Frequency specific orientation selector

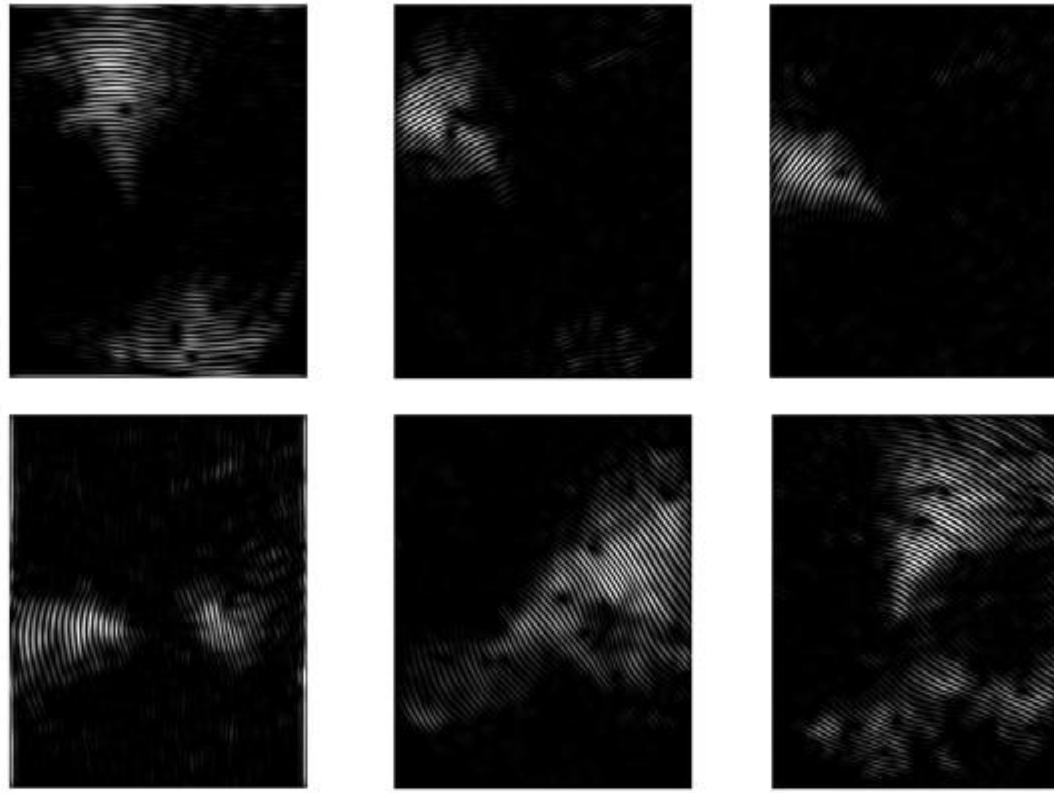
Response of Gabor Filters of a single frequency



How do we use this for enhancement?

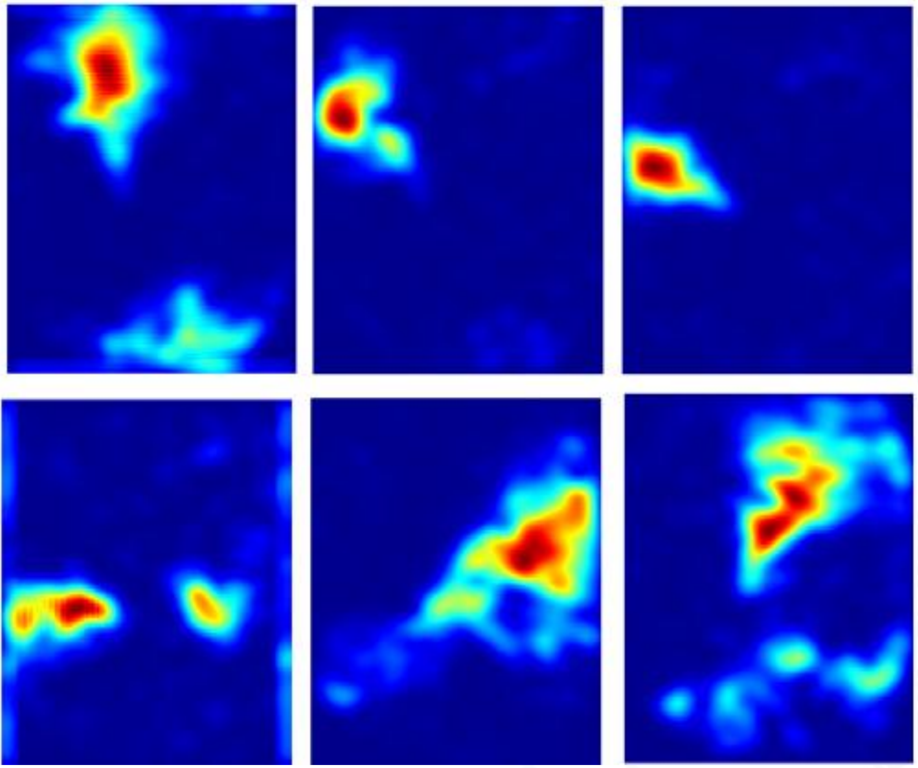
- Obtain Gabor Filters for different orientations tuned at the global ridge frequency of the fingerprint
- Convolve those Gabor filters with the fingerprint to obtain a filter response
- Obtain energy of the filter response
- Smooth the energy response using a Gaussian filter
- Multiply the smoothed energy response with the original response
- Add all weighted responses

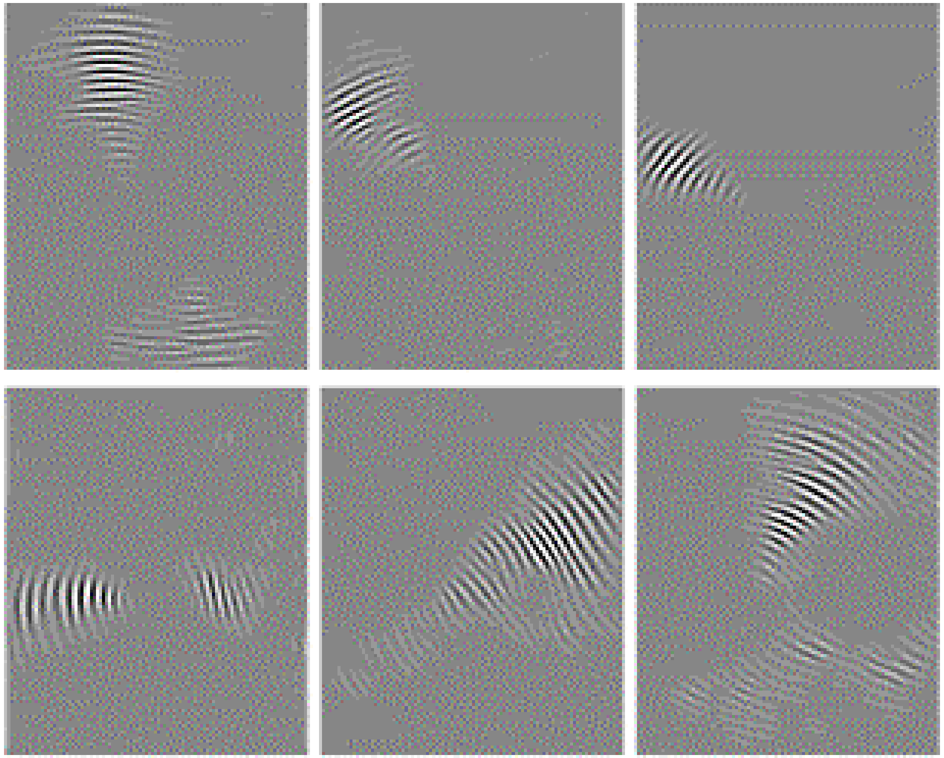




$f_i = I \otimes g_i \qquad i = 1, 2, \ldots, K$

$e_i = f_i^2 \qquad i = 1, 2, \ldots, K$



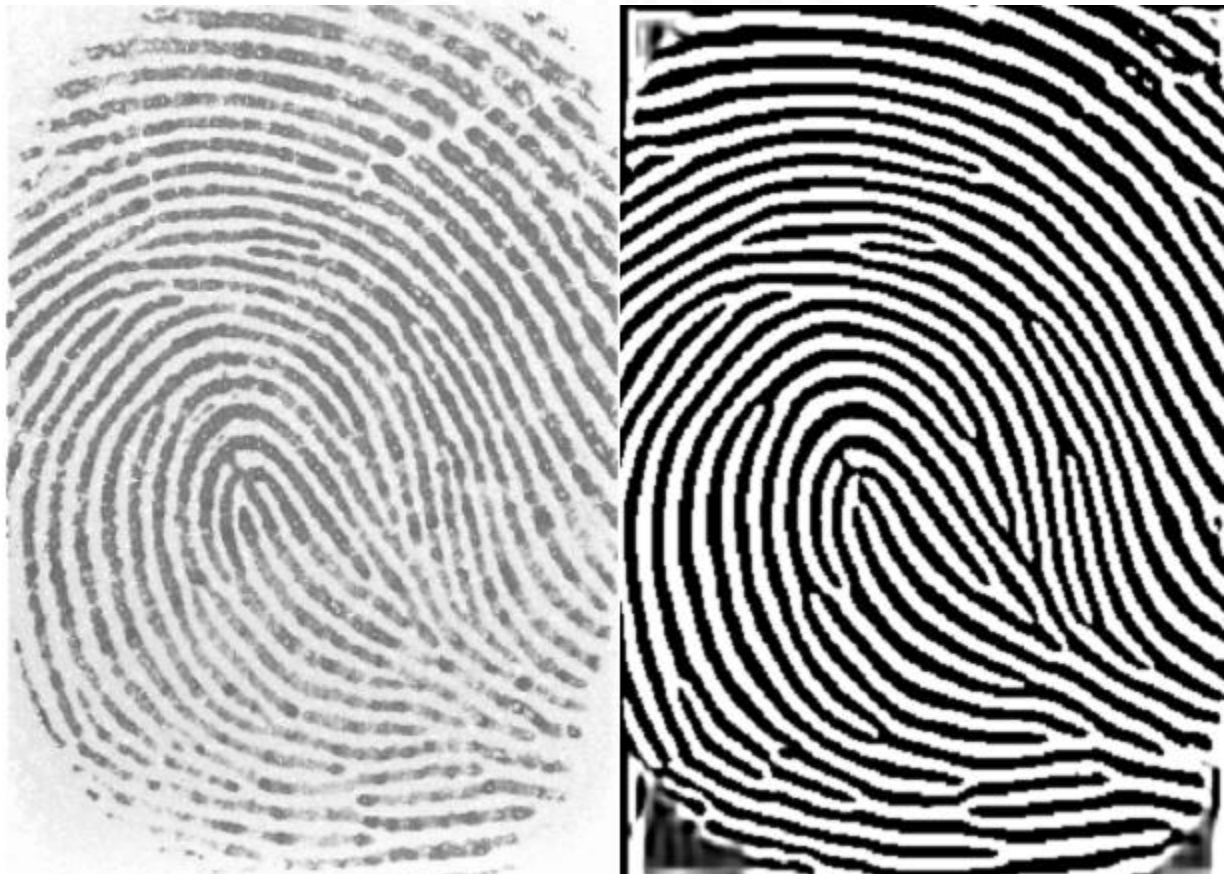


$$e_i' = e_i \otimes \Gamma \qquad i = 1, \ldots, K$$

$$\Gamma = \frac{1}{2\pi\sigma} \exp\left[-\frac{(x^2 + y^2)}{2\sigma^2}\right]$$

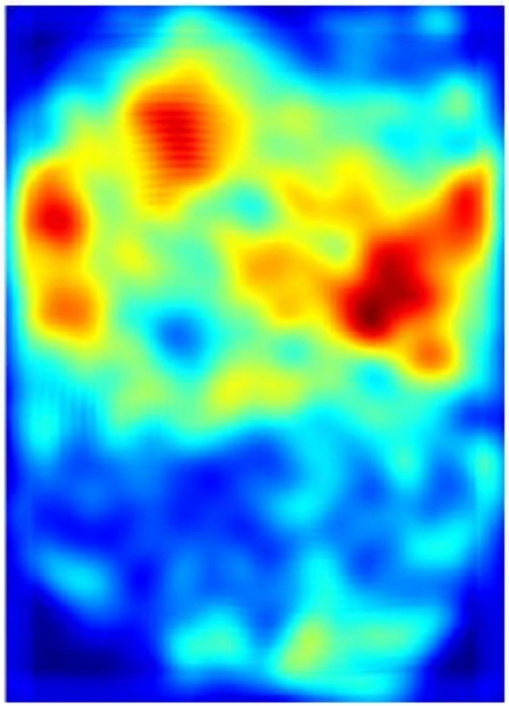
$$f_i'(x, y) = f_i'(x, y) \times e_i'(x, y)$$

Output



$$I_e(x,y) = \sum_{i=1}^{K} f_i^{'}(x,y)$$

Segmentation Mask



$$I_s = \sum_{i=1}^{K} e'_i$$

Binarized

End of Presentation

Like as the waves make towards the pebbled shore,
So do our minutes hasten to their end.

William Shakespeare (1564–1616), English poet and playwright.
Sonnet 60 (1609).