Search-Then-Commit: Multitask Belief Space Planning

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***Abstract*—Robotic manipulation in real-world environments is fundamentally constrained by perceptual uncertainty. Whether in autonomous driving or cluttered bin picking, sensors are imperfect, and state estimation is rarely exact. Traditional deterministic planners, such as RRT*, often fail when the target object's location is imperfectly known, leading to missed grasps or unintended collisions. In this project, we address the problem of picking a target object under joint discrete and continuous uncertainty within the Drake simulation framework.**

We implement a hierarchical planning framework leveraging the Rapidly-Exploring Random Belief Tree (RRBT) structure to explicitly model both the discrete belief of the target bin and the continuous uncertainty of the object's position. Our planner reasons between informative ("Light") and uninformative ("Dark") sensing regions. The algorithm prioritizes two distinct search phases: first, navigating to resolve bin identity using a discrete Bayes filter, and second, refining the grasp pose using a Kalman filter. We benchmark this strategy against a deterministic RRT baseline. Our results demonstrate that coupling active information gathering with motion planning enables the robot to autonomously manage uncertainty, resulting in higher pick-and-place success rates and improved safety margins compared to uncertainty-agnostic methods. This paves the way for designing more robust service robots where external sensors could be highly unreliable. The code is available in this repository https://github.com/thiago-jvds/mit-6.4212-belief-space-rrt. Our video presentation is available on YouTube https://youtu.be/j6uy53jciCc.

***Index Terms*—TAMP, uncertainty, RRBT**

I. INTRODUCTION

Advances in robotic manipulation and planning have made complex autonomous vehicle tasks increasingly attainable due to high levels of reliability. However, real-world settings remain highly unstructured, posing a challenge for robot deployment. Among these issues, perceptual uncertainty is a primary bottleneck. Traditional sensors are inherently noisy and have been shown to be susceptible to noise and domain shifts. LiDAR detection performance is particularly sensitive to adverse weather environments [10, 13]. Camera-based systems also suffer from similar uncertainty problems, such as those related to occlusion and object re-identification [5, 17].

Traditional sampling-based motion planners, such as Rapidly-exploring Random Trees (RRT) [12], operate in the configuration space assuming the robot's knowledge of the world is exact. When applied to scenarios with significant pose uncertainty, these deterministic planners often generate trajectories that are theoretically valid but practically unsafe. For example, a robot might plan a path that grazes an obstacle or attempt a grasp based on a noisy estimate, leading to collisions or failed manipulation attempts. To ensure robustness, a planner must not only avoid obstacles but also account for the belief state. Recent work has focused mostly on uncertainty in proprioception, that is, the robot's position, speeds, and forces. Rapidly Exploring Random Belief Trees [4] lay a framework to explore this problem under uncertainty in position.

In this work, we focus on a robotic manipulation task using a Kuka iiwa arm, where the system must identify and grasp a target object. We assume the robot's proprioception (joint encoders) is precise, but its perception of the external world is subject to significant hybrid uncertainty. Specifically, the robot faces discrete uncertainty regarding which bin contains the object and continuous uncertainty regarding the object's precise pose within that bin. To model real-world perceptual limitations (e.g., occlusion or lighting conditions), we employ a spatially dependent sensor model: the workspace is divided into "Light" regions, where visual sensors provide high-fidelity observations, and "Dark" regions, where measurements are noisy or unavailable.

We propose a belief-space planning framework based on RRBT that explicitly addresses this hybrid belief state—comprising a discrete probability distribution for bin identification and a Gaussian covariance matrix for position refinement. Unlike standard planners that optimize solely for path length, our approach generates trajectories that actively navigate to these informative "Light" regions to gather necessary data before attempting a grasp. We implement a sequential multi-stage planning strategy:

1) **Discrete Search Phase:** The robot plans a trajectory to resolve the discrete bin belief (Bayes Filter) by navigating to informative regions to reduce misclassification risk.
2) **Refinement Phase:** After minimizing discrete uncertainty and initializing a pose estimate via ICP, the robot plans a second trajectory to actively reduce an introduced high continuous position variance (Kalman Filter) below a safety threshold.
3) **Commit Phase:** Once confidence in both beliefs is sufficiently high, the robot computes and executes a trajectory to pick-and-place the target object.

This project utilizes the Drake [16] framework to simulate the Kuka Iiwa and validate the benefits of our approach. To address the limitations of traditional methods, our system designs three goals: **1) Maximizing Task Completion Rate:** Achieve high success rate compared to deterministic planners by reducing uncertainty; **2) Path Feasibility:** Maintain a collision-free trajectory in highly uncertain states; **3) High Modularity:** Formulate the framework to be easily extendable to complementary and/or sequential tasks. Our specific contributions are:

- A framework implementation of an RRBT planner that updates environmental belief states based on the robot's spatial proximity to informative sensing regions.
- A sequential multi-stage algorithm that autonomously balances path length with information gain cost functions (misclassification risk and covariance trace) for both discrete and continuous uncertainty.
- A quantitative evaluation demonstrating that our belief-aware planner achieves higher success rates and safety margins compared to a deterministic RRT-Connect baseline in uncertain environments.

II. Related Work

Planning under uncertainty in robotic manipulation intersects several major research areas, including sampling-based motion planning, belief-space planning, active perception, and pose estimation. This section situates our work within these threads, highlights limitations of existing approaches, and motivates our design decisions.

A. Deterministic Motion Planning Under Uncertainty

Sampling-based planners such as RRT, RRT-Connect, and RRT* have demonstrated strong performance in high-dimensional configuration spaces, particularly for navigation and manipulation tasks [7, 9, 12, 14]. However, these methods fundamentally assume perfect knowledge of the robot's state and environment. In cluttered manipulation scenarios, where object poses are uncertain and sensor observations are noisy, deterministic planners often produce nominally valid but practically unsafe trajectories. Small errors in pose estimation can lead to collisions, mis-grasps, or failed task completion. Extensions that enlarge obstacles or add safety margins introduce conservatism but still lack the ability to reason about how uncertainty evolves as the robot moves. These limitations motivate the need for planners that operate directly in belief space rather than configuration space.

B. Belief-Space Planning and RRBT

Full belief-space planning, often formulated as a POMDP, provides a theoretically optimal framework for decision-making under uncertainty. However, POMDP solvers scale poorly with dimensionality, making them computationally prohibitive for 6–7 DOF manipulators operating in continuous domains. Approximate methods such as online POMDP solvers alleviate some burden but remain too slow for real-time manipulation in cluttered environments.

Rapidly-Exploring Random Belief Trees (RRBT) [4] offer a sampling-based alternative by extending the structure of RRT into belief space. RRBT propagates uncertainty along sampled trajectories, maintaining beliefs over the robot's state while preserving many of the computational advantages of RRT. Prior applications of RRBT have focused primarily on localization problems, where uncertainty arises from proprioception or robot pose drift. In contrast, our work adapts RRBT to manipulation tasks that involve hybrid belief states—a discrete probability distribution over which bin contains the target object and a continuous Gaussian distribution over the object's pose. This extension allows the planner to reason about and actively reduce both forms of uncertainty during execution.

C. Active Perception and Information-Gathering Motions

Traditional robotic manipulation pipelines typically separate perception and planning: the robot takes an initial observation, estimates object poses once, and plans accordingly. However, this "sense-then-plan" paradigm breaks down in environments where occlusions or poor lighting degrade measurement quality. Active perception methods address this by jointly optimizing motion and sensing, driving the robot to selected viewpoints to maximize information gain, reduce entropy, or disambiguate hypotheses about the environment. Such strategies have proven particularly effective in object search, viewpoint selection, and grasp disambiguation tasks [1, 2, 8, 11].

Our work follows this general principle by incorporating expected information gain into the planning loop. For each candidate action, the planner simulates (mock) observations using a spatially dependent sensor model and updates the belief state to evaluate the resulting uncertainty. In our setup, "Light" regions correspond to configurations where observations are more informative, so actions that move the end-effector through these areas tend to yield greater expected reduction in both bin-classification uncertainty and continuous pose variance. By embedding these expected belief updates into an RRBT framework, the planner can generate trajectories that purposefully gather information before attempting the pick-and-place, improving robustness in settings where passive perception alone would be unreliable.

D. Pose Estimation and Manipulation Under Continuous Uncertainty

Point cloud registration methods such as Iterative Closest Point (ICP) [3] are widely used in robotic manipulation to estimate object poses from depth observations. While these approaches can provide accurate alignments under favorable conditions, their performance degrades in the presence of partial views, clutter, or sensor noise. In many manipulation systems, the estimated pose is treated as a fixed quantity and passed directly to a downstream planner, which can lead to brittle behavior when the estimate carries significant residual error. Recent work in uncertainty-aware manipulation emphasizes the importance of modeling and reducing pose uncertainty rather than relying solely on a single best estimate [6].

Our system adopts this perspective by representing the object pose as a Gaussian belief and updating it throughout the task. After the initial ICP registration, the planner leverages a Kalman filter to refine this estimate as new observations become available. Incorporating this continuous uncertainty into the planning process helps ensure that the robot does not attempt a grasp until the position estimate is sufficiently reliable, reducing the likelihood of misalignment, unintended contact, and missed grasps.

E. Hybrid Planning Architectures

Fully integrated belief-space planners, while expressive, often exhibit overly conservative behavior because they attempt to satisfy strict safety guarantees under all modeled uncertainties. In practice, such planners may halt or refuse to commit to actions when no trajectory satisfies a stringent global risk bound. Conversely, purely deterministic planners do not incorporate any mechanism for reducing uncertainty during execution, and therefore may proceed even when the belief state is too ambiguous to ensure success.

Hybrid planning architectures offer a middle ground by applying uncertainty-aware reasoning only during phases where information gathering is essential, and switching to deterministic planning once confidence is sufficiently high [15]. Following this philosophy, our project uses RRBT during the bin-identification and pose-refinement stages, when both discrete and continuous uncertainties dominate decision-making. Once these uncertainties have been reduced to acceptable levels, a standard RRT planner is employed to execute the final pick-and-place. This division of responsibility provides a practical balance: information is collected when needed to ensure safety and feasibility, while final execution remains efficient and computationally lightweight.

III. Problem Formulation

We define the robot's mission as a strictly ordered sequence of N decoupled tasks $\mathcal{T} = (\tau_1, \tau_2, \ldots, \tau_N)$. The system executes these tasks sequentially, where each task τ_k is active only during a specific time interval $[T_k^{\text{start}}, T_k^{\text{end}}]$. The start time of a task coincides with the completion of the previous one, such that $T_k^{\text{start}} = T_{k-1}^{\text{end}}$. The system operates in the configuration space $\mathcal{C}$.

A. Task State Space

While the robot's fully observable configuration $q_t \in \mathcal{C}$ is continuous across the entire mission, the hidden state variables are unique to the active task. For the current task τ_k, the system state x_t^k is defined as:

$$x_t^k = \begin{bmatrix} q_t \\ \theta_k \end{bmatrix}, \quad \text{for } t \in [T_k^{\text{start}}, T_k^{\text{end}}] \tag{1}$$

where $\theta_k \in \Theta_k$ is the task-specific hidden variable.

B. Observation

Crucially, observations are task-specific and are only collected and processed while the task is active. The robot receives observations z_t^k according to:

$$z_t^k = \begin{cases} h_k(q_t, \theta_k) + \nu_k(q_t) & \text{if } t \geq T_k^{\text{start}} \\ \emptyset & \text{otherwise} \end{cases} \tag{2}$$

This implies that any sensor data collected prior to T_k^{start} is irrelevant to the estimation of θ_k and is discarded.

C. Belief Dynamics

Upon the initialization of task τ_k at time $t = T_k^{\text{start}}$, the belief system is reset. The robot maintains a belief b_t^k conditioned strictly on the history of observations collected *since the task began*:

$$b_t^k = p(\theta_k \mid z^k_{T_k^{\text{start}}:t}, q_{T_k^{\text{start}}:t}) \tag{3}$$

The belief evolves via a task-specific filter Ψ_k:

$$b_{t+1}^k = \Psi_k(b_t^k, z_{t+1}^k, u_t) \tag{4}$$

D. Termination Condition

Task τ_k concludes at time T_k^{end} when the belief b_t^k satisfies a specific confidence criterion S_k, triggering the immediate transition to task τ_{k+1}:

$$T_k^{\text{end}} = \min\{t > T_k^{\text{start}} \mid S_k(b_t^k) = 1\} \tag{5}$$

E. Control Objective

The goal is to compute a control policy π that generates a valid trajectory $\zeta = \{q_0, q_1, \ldots, q_T\}$ such that the termination conditions for all sequential tasks are satisfied. Formally, we seek to find a finite sequence of controls $u_{0:T}$ that ensures:

$$\forall k \in \{1, \ldots, N\}, \quad \exists t \text{ such that } S_k(b_t^k) = 1 \tag{6}$$

This implies the successful resolution of uncertainty for every task in the sequence $\mathcal{T}$.

F. Search Tasks

We classify a task τ_k as a *Search Task* when the initial uncertainty of the hidden variable θ_k is high. The primary objective of these tasks is belief concentration rather than physical actuation.

In this regime, the planner must synthesize trajectories that maximize information gain. The task concludes when the belief b_t^k converges to a confident estimate $\hat{\theta}_k$.

G. Commit Tasks

We classify a task τ_k as a *Commit Task* when the relevant hidden variables are assumed known (either a priori or as the output of a preceding Search Task).

IV. Approach & Methods

We implement our belief-space planning framework within Drake, using the `LeafSystem` abstraction to create a modular, event-driven architecture. The system orchestrates a sequential pipeline of three distinct phases: (1) discrete bin localization via RRBT with a Bayes filter, (2) continuous position refinement via RRBT with a Kalman filter, and (3) deterministic grasp execution. Each phase is designed as a separate "task" that terminates when a confidence threshold is satisfied, at which point the system transitions to the next phase.

A. System Architecture

Our architecture consists of interconnected Drake `LeafSystems` organized into perception, estimation, planning, and visualization subsystems. The central component is a `PlannerSystem` that implements a finite state machine with nine states:

1) **IDLE**: Awaiting configuration with ground truth.
2) **RRBT_PLANNING** (Bin): Computing belief-space trajectory for bin localization.
3) **RRBT_EXECUTING** (Bin): Executing trajectory while belief updates in real-time.
4) **POSE_ESTIMATION**: Running ICP on camera point clouds.
5) **RRBT2_PLANNING** (Position): Computing trajectory for position uncertainty reduction.
6) **RRBT2_EXECUTING** (Position): Executing trajectory while covariance shrinks.
7) **GRASP_PLANNING**: Sampling from covariance, antipodal grasp selection, IK validation.
8) **GRASP_EXECUTING**: Trajectory through pregrasp, grasp, lift, transfer, and release.
9) **COMPLETE**: Task finished, holding final configuration.

This state machine ensures sequential execution of search phases before any commit action is attempted.

B. Light and Dark Sensing Regions

We model spatially-dependent observation quality through designated "Light" and "Dark" regions in the workspace. When the robot's end-effector is within a Light region, sensors provide informative observations; in Dark regions, measurements are uninformative (equivalent to random noise).

For the bin belief task, we use a True Positive Rate / False Positive Rate (TPR/FPR) sensor model:

$$\text{Sensor Model} = \begin{cases} (\text{TPR} = 0.80, \text{FPR} = 0.15) & \text{if } q \in \mathcal{L}_{\text{bin}} \\ (\text{TPR} = 0.50, \text{FPR} = 0.50) & \text{otherwise} \end{cases} \tag{7}$$

where $\mathcal{L}_{\text{bin}}$ denotes the bin-sensing Light region. In the Dark region, TPR = FPR = 0.5 corresponds to a coin flip, yielding zero information gain.

For the position belief task, we use a variance-based model:

$$R(q) = \begin{cases} \sigma^2_{\text{light}} \cdot I_2 & \text{if } q \in \mathcal{L}_{\text{pos}} \\ \sigma^2_{\text{dark}} \cdot I_2 & \text{otherwise} \end{cases} \tag{8}$$

where $\sigma_{\text{light}} \ll \sigma_{\text{dark}}$ (in our implementation, $\sigma_{\text{light}} = 0.01$m and $\sigma_{\text{dark}} = 10^9$m, effectively infinite).

C. Phase 1: Discrete Bin Localization (RRBT with Bayes Filter)

The first search task resolves the discrete uncertainty over which bin ($\{0, 1\}$) contains the target object. We initialize with a uniform prior $b_0 = [0.5, 0.5]$ and use a discrete Bayes filter to update the belief based on binary detections.

1) Belief Update: When the robot's end-effector enters the Light region, we simulate observations for each bin hypothesis and apply Bayes' rule:

$$b_{t+1}(i) \propto P(z_t|\theta = i) \cdot b_t(i) \tag{9}$$

where the likelihood $P(z_t|\theta = i)$ depends on whether bin i is being measured and whether a detection occurred:

$$P(z = \text{detect}|\theta = i) = \begin{cases} \text{TPR} & \text{if measuring bin } i \text{ (object present)} \\ \text{FPR} & \text{otherwise (object absent)} \end{cases} \tag{10}$$

2) RRBT Cost Function: We define the cost of a belief node n as:

$$C_{\text{bin}}(n) = \ell(n) + \lambda_{\text{bin}} \cdot (1 - \max_i b_n(i)) \tag{11}$$

where $\ell(n)$ is the path length from the start configuration, b_n is the belief at node n, and $(1 - \max_i b_n(i))$ is the misclassification risk. This cost trades off path efficiency against uncertainty reduction.

3) Termination Condition: The task terminates when misclassification risk drops below a threshold:

$$S_{\text{bin}}(b_t) = \mathbf{1}\left[1 - \max_i b_t(i) \leq \epsilon_{\text{bin}}\right] \tag{12}$$

with $\epsilon_{\text{bin}} = 0.01$ in our implementation (99% confidence).

4) RRBT Algorithm: Our RRBT implementation uses an anytime RRT*-style approach. For each iteration:

1) Sample configuration q_{rand} with biases toward (a) the goal region and (b) the Light region
2) Find nearest node and extend tree along collision-free path
3) Propagate belief using expected posterior (planning uses expected observations, not stochastic)
4) If node satisfies S_{bin} and has lower cost than current best, update best solution

The algorithm runs for all $N_{\max}$ iterations (anytime behavior) and returns the best path found.

D. Pose Estimation via ICP

Upon completing the bin localization task, the system triggers pose estimation. Based on the Maximum A Posteriori (MAP) bin estimate, we select cameras facing that bin (cameras 0–2 for bin 0, cameras 3–5 for bin 1), fuse their point clouds, segment yellow-colored points corresponding to the mustard bottle, and run Iterative Closest Point (ICP) registration against a known object model.

The ICP output provides an initial pose estimate $\hat{X}_{WM}$ that serves as the mean for the subsequent position belief. This estimate is treated as fixed during Phase 2; only the covariance is refined.

E. Phase 2: Continuous Position Refinement (RRBT with Kalman Filter)

After ICP, we introduce an artificial initial covariance $\Sigma_0 = \sigma_0^2 I_2$ over the object's X-Y position to model residual uncertainty from a noisy ICP estimate. The Z coordinate is fixed (object rests on the bin floor). The robot then plans a second RRBT trajectory to reduce this 2D covariance by navigating to the position-sensing Light region.

1) Kalman Filter Update: The position mean μ is held fixed at the ICP estimate. When the robot enters the position Light region, we update the covariance via the standard Kalman filter equations:

$$S = C\Sigma C^\top + R(q) \tag{13}$$

$$K = \Sigma C^\top S^{-1} \tag{14}$$

$$\Sigma_{t+1} = (I - KC)\Sigma_t \tag{15}$$

where $C = I_2$ (direct observation of X-Y) and $R(q)$ is the position-dependent measurement noise.

2) RRBT Cost Function: For this phase, the cost function uses the trace of the covariance:

$$C_{\text{pos}}(n) = \ell(n) + \lambda_{\text{pos}} \cdot \text{tr}(\Sigma_n) \tag{16}$$

3) Termination Condition:

$$S_{\text{pos}}(\Sigma_t) = \mathbf{1}\left[\text{tr}(\Sigma_t) \leq \epsilon_{\text{pos}}\right] \tag{17}$$

with $\epsilon_{\text{pos}} = 0.001$ in our implementation.

F. Phase 3: Grasp Planning and Execution

Once both search tasks are complete, the system transitions to the commit phase. Grasp planning proceeds as follows:

1) Position Sampling: We sample a grasp target position from the refined 2D Gaussian belief:

$$p_{\text{sample}} \sim \mathcal{N}(\mu_{XY}, \Sigma) \tag{18}$$

truncated at $k\sigma$ (we use $k = 2$) to ensure the sample remains within a high-probability region. The Z coordinate is fixed at the ICP estimate.

2) Antipodal Grasp Selection: Using the sampled position and ICP-estimated orientation, we transform a known object model into the world frame and generate antipodal grasp candidates. Each candidate is evaluated for:

- Collision with the environment and point cloud
- Gripper orientation (preferring vertical approach)
- Normal alignment (antipodal criterion)

The top candidates are validated with inverse kinematics to ensure reachability.

3) Trajectory Execution: The grasp trajectory consists of six phases: (1) pregrasp (30cm above grasp), (2) descend to grasp, (3) close gripper and hold, (4) lift, (5) transfer to drop bin, (6) open gripper and release. We introduced these intermediate waypoints to ensure smooth, collision-free motion during lift and transfer.

G. Baseline Comparison

To demonstrate the necessity of belief-space planning, we implement a baseline that makes immediate decisions without information gathering:

- **Bin Prediction**: Random selection from uniform prior (coin flip, 50% accuracy)
- **Position Uncertainty**: Uses initial large covariance without reduction
- **No RRBT Trajectories**: Skips both search phases entirely

This baseline is expected to fail approximately 50–75% of the time due to incorrect bin predictions and large position sampling errors, providing a control comparison for our RRBT approach.

V. Evaluation & Discussion

Figure 1 illustrates our experimental scenario. A mustard bottle is randomly placed in one of two candidate bins (bin0 on the left or bin1 on the right), and the robot's objective is to locate the bottle, grasp it, and place it in a goal bin at the center of the workspace. To introduce variability across trials, the bottle's initial pose is randomized: the X-Y position is sampled uniformly within the bin bounds (±2cm in X, ±15cm in Y relative to bin center), and the yaw orientation is sampled uniformly from $[0, 2\pi)$. The setup includes two spatially distinct Light sensing regions: a green box where the robot obtains informative bin measurements (for the discrete Bayes filter), and an orange box where the robot obtains informative position measurements (for the Kalman filter).

We evaluate our RRBT-based belief-space planner against a deterministic baseline to assess performance relative to our three design goals: (1) maximizing task completion rate, (2) maintaining path feasibility under uncertainty, and (3) demonstrating high modularity through sequential task execution.

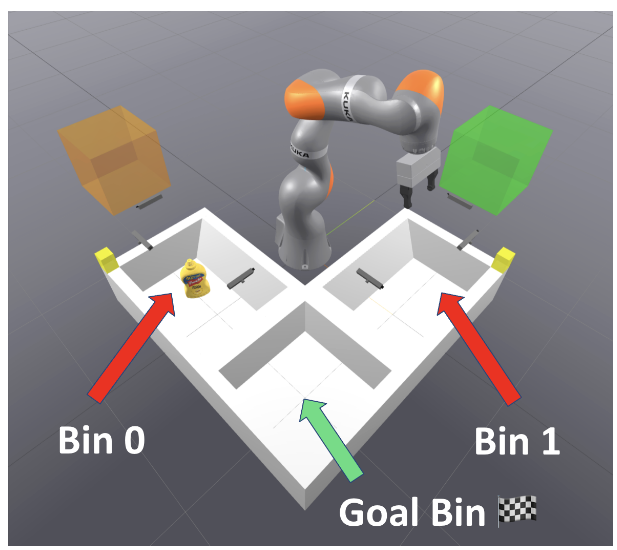


Fig. 1. Experiment setup: Kuka iiwa arm with WSG gripper, two candidate bins, six depth cameras, and Light sensing regions (green and orange boxes).

A. Evaluation Protocol

To assess the benefits of belief-space planning, we conduct a controlled comparison between our RRBT-based system and a baseline planner that makes immediate decisions without information gathering. For each method, we execute 10 independent trials with distinct random seeds controlling:

- The true bin placement (randomly selected as bin 0 or bin 1)
- The mustard bottle's X-Y position within the selected bin
- The bottle's yaw orientation
- RRBT sampling and grasp candidate generation

This controlled randomization ensures that performance differences arise from the planning strategy rather than favorable initial conditions. We record each trial's outcome as one of three categories:

1) **Success**: The robot correctly identifies the bin, estimates the pose, grasps the object, and places it in the goal bin.
2) **Grasp-Related Failure**: The robot correctly identifies the bin but fails during grasping (e.g., misses the object, knocks it over, or drops it during transfer).
3) **Incorrect Bin Prediction**: The robot predicts the wrong bin, causing ICP to fail due to the absence of yellow points in the selected cameras' field of view.

B. Results: Task Completion Rate

Figure 2 presents the overall task completion rates for both methods. Our RRBT-based planner achieves a **90% success rate** (9 out of 10 trials), while the baseline achieves only **10% success rate** (1 out of 10 trials). This nine-fold improvement directly validates our first design goal of maximizing task completion through uncertainty-aware planning.

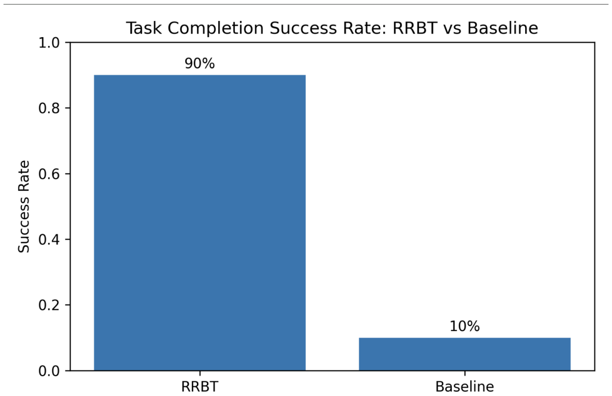


Fig. 2. Task completion rate comparison. The RRBT-based planner completes 90% of pick-and-place tasks, compared to only 10% for the uncertainty-agnostic baseline.

C. Qualitative Results: RRBT Execution Progression

Figure 3 illustrates the sequential phases of a successful RRBT execution. The progression demonstrates how the system methodically reduces uncertainty before committing to manipulation:

1) **Start Position** (Fig. 3a): The robot begins at its home configuration with uniform belief over the two bins ($b = [0.5, 0.5]$).
2) **Bin Light Region** (Fig. 3b): The RRBT-planned trajectory guides the end-effector into the green Light region. Here, the Bayes filter receives informative observations (TPR=0.80, FPR=0.15), rapidly concentrating belief on the correct bin.
3) **Position Light Region** (Fig. 3c): After ICP provides an initial pose estimate, the second RRBT trajectory moves the robot into the orange Light region. The Kalman filter shrinks the position covariance as low-noise measurements are incorporated.
4) **Grasp Execution** (Fig. 3d): With both discrete and continuous uncertainties below their thresholds, the system commits to grasping. The gripper approaches from above and securely grasps the mustard bottle.
5) **Goal Placement** (Fig. 3e): The robot transfers the bottle to the goal bin at the center of the workspace and releases it, completing the pick-and-place task.

D. Belief State Visualization

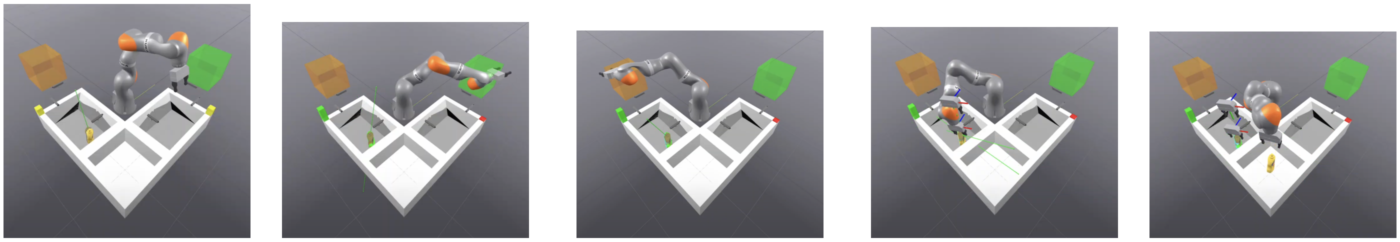
Figure 4 illustrates the discrete bin belief before and after the robot enters the green Light region. At the start (Fig. 4a), the belief is uniform ($b = [0.5, 0.5]$), reflecting complete uncertainty about which bin contains the object. After executing the RRBT-planned trajectory through the bin Light region (Fig. 4b), the Bayes filter has concentrated belief on the correct bin, satisfying the 99% confidence threshold required to proceed to pose estimation.

Figure 5 shows the initial position uncertainty after ICP registration, before the robot enters the orange position Light region. This visualization applies to both RRBT (at the start of Phase 2) and the baseline (throughout execution, since it skips uncertainty reduction). The green point cloud represents the true segmented bottle location, the blue point cloud shows the model placed at the ICP-estimated position, and the red ellipse indicates the 2D covariance in the X-Y plane. For RRBT, this covariance shrinks as the robot gathers position measurements; for the baseline, it remains at this magnitude during grasp planning, leading to the failures analyzed below.

E. Results: Failure Mode Analysis

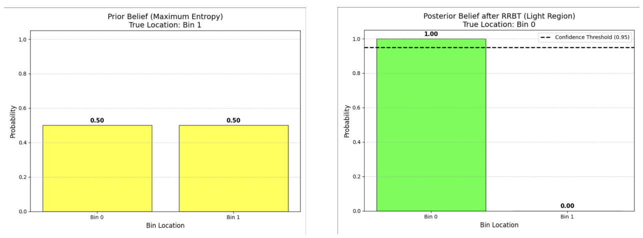
Figure 6 provides a detailed breakdown of outcomes for both methods. The failure modes reveal fundamental differences between the two approaches.

1) RRBT Performance: The RRBT-based system succeeded in 9 out of 10 trials, with only 1 grasp-related failure. Notably, there were zero incorrect bin predictions. This confirms that the discrete Bayes filter, combined with active information-gathering trajectories into the Light region, reliably resolves bin uncertainty before committing to perception. The single grasp failure occurred due to an edge case in antipodal grasp selection where the sampled grasp candidate, while valid for the model point cloud, resulted in an unstable grasp on the physical object during lift.



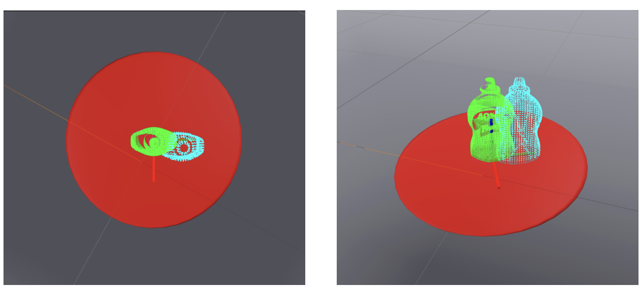
(a) Start (b) Bin Light Region (c) Position Light Region (d) Grasp (e) Goal Placement

Fig. 3. Progression of a successful RRBT execution. (a) Robot at home position with uniform bin belief. (b) End-effector enters the green Light region to gather bin information via Bayes filter updates. (c) After ICP, the robot enters the orange Light region to reduce position covariance via Kalman filter. (d) With uncertainty thresholds satisfied, the robot executes the grasp. (e) The bottle is transferred and released into the goal bin.



(a) Prior (uniform) (b) Posterior (confident)

Fig. 4. Discrete bin belief evolution. (a) Initial uniform prior before information gathering. (b) Concentrated posterior after the robot traverses the green Light region, corresponding to Figs. 3a and 3b respectively.



(a) Top-down view (b) Side view

Fig. 5. Initial position uncertainty after ICP. Green: true bottle location (segmented). Blue: model at estimated position. Red ellipse: X-Y covariance. RRBT reduces this uncertainty before grasping; the baseline uses it directly, causing frequent grasp failures.

2) Baseline Performance: The baseline exhibited close to the expected failure distribution:

- **5 incorrect bin predictions (50%)**: Consistent with the theoretical expectation of random guessing from a uniform prior. When the baseline predicts the wrong bin, the ICP algorithm receives point clouds from cameras that cannot observe the mustard bottle, causing segmentation to return zero yellow points and the pipeline to halt.
- **4 grasp-related failures (40%)**: Even when the baseline correctly guessed the bin, the large unreduced position covariance ($\sigma_0^2 = 0.005$) led to grasp positions sampled far from the true object location. These failures manifested as the gripper closing on empty space, knocking the bottle over during approach, or grasping at an unstable angle.

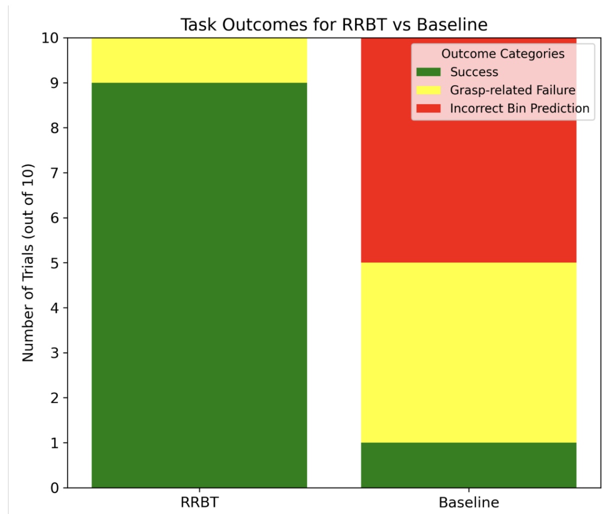


Fig. 6. Task outcome breakdown showing the distribution of successes and failure modes for each method.

- **1 success (10%)**: This single success required the baseline to (a) correctly guess the bin (50% chance) and (b) sample a grasp position sufficiently close to the true location despite the large covariance. This outcome represents a "lucky" trial and is not reproducible under typical conditions.

F. Grasp Failure Visualization

Figure 7 illustrates the primary failure mode for the baseline when position uncertainty is high. The visualization shows:

- **Green point cloud**: The segmented yellow points representing the true mustard bottle location
- **Blue point cloud**: The model point cloud placed at the sampled position (offset from truth due to large covariance)
- **Red ellipse**: The 2D covariance ellipse in the X-Y plane, showing the magnitude of position uncertainty
- **Gripper**: Positioned to grasp the blue (sampled) model, missing the green (true) bottle entirely

This visualization demonstrates why uncertainty reduction is essential: without shrinking the covariance before grasp planning, the believed position may lie well outside the object's true location, leading to inevitable grasp failure.

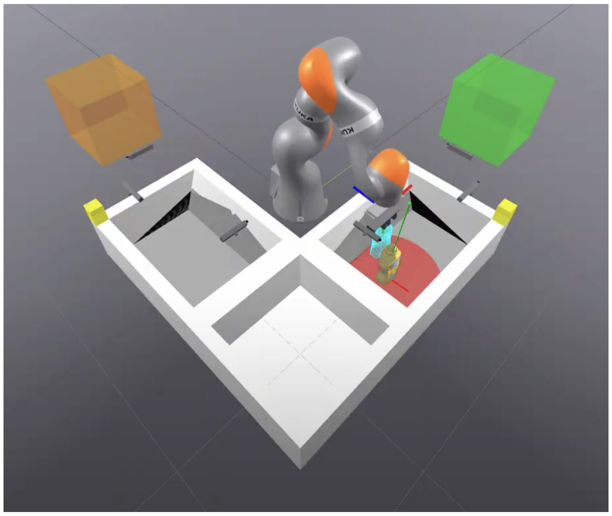


Fig. 7. Visualization of a grasp failure caused by high position uncertainty. The green points show the true bottle location (segmented from cameras), the blue points show the model placed at the sampled position, and the red ellipse indicates the X-Y covariance. The gripper plans a grasp for the blue model, missing the true bottle.

Figure 8 shows this failure mode during actual execution: the gripper closes on empty space because the planned grasp target (based on the offset model in Figure 7) does not coincide with the true bottle position.

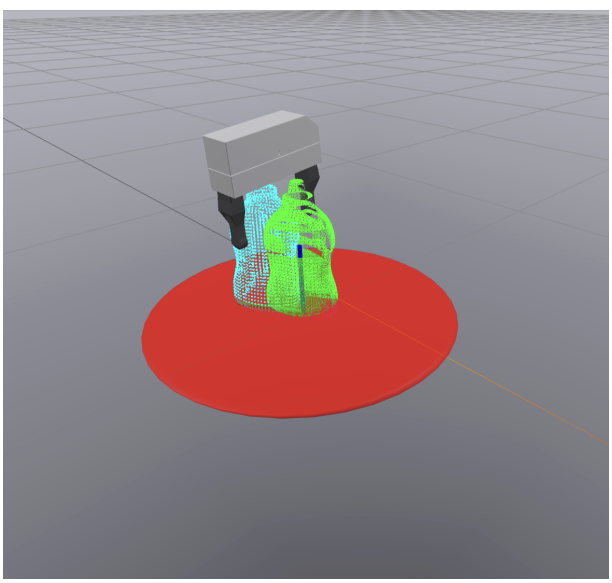


Fig. 8. Baseline execution failure: the gripper misses the mustard bottle entirely due to the position error illustrated in Figure 7. The large unreduced covariance causes the sampled grasp position to be offset from the true object location.

G. Interpretation and Relation to Prior Work

Our experimental results illustrate the practical benefits of the concepts discussed in our Related Work section. The baseline's behavior, random bin guessing followed by grasping under high uncertainty, exemplifies the limitations of deterministic planners described in Section II-A: without reasoning about belief, the system proceeds even when state knowledge is insufficient for reliable execution.

By contrast, our RRBT-based approach follows the active perception paradigm outlined in Section II-C. Rather than taking a single observation and committing immediately, the planner generates trajectories that move through informative Light regions to reduce uncertainty before acting. This information-gathering behavior, which emerges naturally from the RRBT cost function balancing path length against belief uncertainty, is what enables the system to achieve reliable bin identification.

The grasp failure analysis reinforces the motivation from Section II-D on pose estimation under uncertainty. As Chen et al. note [6], treating pose estimates as fixed values can lead to brittle manipulation. Our results show this concretely: even when the baseline correctly identified the bin, the large position covariance caused grasp targets to be sampled far from the true object location. The RRBT-based system's use of a Kalman filter to shrink this covariance before grasp planning, while simpler than full probabilistic grasp reasoning, provides a practical way to delay commitment until confidence is sufficient.

Finally, our sequential task structure reflects the hybrid planning philosophy from Section II-E [15]: we apply belief-space reasoning during the search phases when uncertainty dominates, then switch to deterministic execution once confidence thresholds are met. This separation kept our implementation tractable while still capturing the core benefits of uncertainty-aware planning.

H. Anomalies and Limitations

1) Single RRBT Grasp Failure: The one failure in the RRBT trials occurred during the grasp execution phase, not during belief estimation. Post-hoc analysis revealed that the antipodal grasp selector chose a candidate that, while geometrically valid on the model point cloud, approached the bottle at an angle susceptible to slippage. This suggests that our grasp cost function, which prioritizes vertical gripper orientation and antipodal alignment, may benefit from additional terms penalizing unstable approach directions.

2) Single Baseline Success: The baseline's single success is a statistical artifact of the small sample size (10 trials). With a 50% bin prediction accuracy and approximately 20–30% chance of a valid grasp given correct bin identification and large covariance, we would expect roughly 10–15% success rate. The observed 10% is consistent with this estimate. Importantly, this success is not indicative of reliable behavior—the baseline cannot be trusted for deployment.

3) Limitations: Several limitations should be noted:

- **Synthetic sensor model**: Our Light/Dark regions provide idealized observations. Real sensors exhibit spatially varying noise, occlusions, and calibration errors that may degrade both ICP and Bayes filter performance.

- **Known object model**: We assume access to a precise CAD model for ICP registration and grasp planning. In practice, model uncertainty would compound position uncertainty.
- **Static environment**: The mustard bottle does not move during execution. Dynamic objects would require re-planning or continuous belief updates during the commit phase.
- **Limited sample size**: Ten trials per method provide statistical significance for large effect sizes (90% vs. 10%) but may not capture rare failure modes. Larger-scale studies would strengthen confidence in the results.

I. Significance Relative to Design Goals

We assess significance against each design goal:

1) Goal 1: Maximizing Task Completion Rate: The RRBT-based planner achieves $9\times$ higher success rate than the baseline (90% vs. 10%). This dramatic improvement validates the core hypothesis that explicit uncertainty reasoning enables robust manipulation in perceptually challenging environments. The elimination of incorrect bin predictions (0 vs. 5) demonstrates that information-gathering trajectories are essential when the prior is uninformative.

2) Goal 2: Path Feasibility: All planned trajectories remained collision-free across both methods. The RRBT planner's additional trajectories (information gathering for bin belief, uncertainty reduction for position belief) did not introduce collision risks, confirming that belief-space planning can be safely integrated with existing motion planning infrastructure.

3) Goal 3: High Modularity: The sequential task formulation proved effective in practice. Each phase (bin localization, pose estimation, position refinement, grasp execution) operated independently with well-defined inputs and outputs. The state machine cleanly transitioned between phases upon meeting confidence thresholds, demonstrating that the architecture can be extended to additional tasks (e.g., multi-object manipulation, re-grasping) without modifying existing components.

VI. Conclusion

A. Summary of Findings

This project implemented and evaluated a belief-space planning framework for robotic manipulation under hybrid uncertainty. Our key findings are:

1) **Information gathering is essential**: The RRBT-based planner achieved a 90% task completion rate compared to only 10% for the uncertainty-agnostic baseline. The baseline's 50% bin prediction failure rate demonstrates that deterministic planners cannot reliably operate when prior knowledge is uninformative.
2) **Sequential uncertainty reduction works**: By decomposing the problem into discrete bin localization (Bayes filter) followed by continuous position refinement (Kalman filter), the system reliably resolved both forms of uncertainty before committing to manipulation. The RRBT cost functions, balancing path length against misclassification risk and covariance trace respectively, successfully guided the robot toward informative Light regions.
3) **Modular architecture enables clean implementation**: The nine-state finite state machine, built using Drake's `LeafSystem` abstraction, cleanly separated planning, estimation, and execution phases. Each component operated independently with well-defined interfaces, making the system straightforward to debug and extend.

B. Significance

Our results demonstrate that the principles of belief-space planning, originally developed for robot localization, transfer effectively to manipulation tasks involving external object uncertainty. The dramatic performance gap between RRBT and baseline ($9\times$ improvement) validates the core hypothesis that coupling active information gathering with motion planning yields substantially more robust behavior in perceptually uncertain environments.

For the scope of this class project, we showed that relatively simple estimators (discrete Bayes filter, linear Kalman filter) combined with the RRBT framework can achieve reliable manipulation in the Light-Dark domain. This suggests that belief-space planning is a practical approach for service robots operating in environments with spatially varying observation quality (that they can be designed to reason about), such as cluttered shelves, occluded workspaces, or areas with inconsistent lighting.

C. Limitations

Several limitations constrain the generality of our conclusions:

- **Idealized sensor model**: The Light/Dark regions provide binary observation quality (informative vs. uninformative), whereas real sensors exhibit continuous, spatially varying noise characteristics.
- **Known object model**: ICP registration and grasp planning assume access to a precise object model. Unknown or deformable objects would require shape estimation as an additional uncertainty source.
- **Static scene assumption**: The mustard bottle remains stationary throughout execution. Dynamic environments would necessitate replanning or continuous belief updates during the commit phase.
- **Limited experimental scale**: Ten trials per method suffice for detecting large effect sizes but may miss rare failure modes. The single RRBT grasp failure hints at edge cases in antipodal grasp selection that warrant further investigation.

D. Future Directions

Several extensions could address these limitations and broaden the framework's applicability:

- **Non-parametric belief representations**: Replacing the Gaussian position belief with a particle filter would better capture multi-modal uncertainty from ambiguous ICP solutions or partial occlusions.

- **Improved sensor models**: As a preliminary exploration, we developed a voxel-based occupancy observation model (Figure 9) that infers occupancy from depth images using distance-dependent true positive and true negative rates. A wrist-mounted camera observes the workspace, and voxels are updated via Bayesian log-odds: uncertain regions appear yellow, confidently occupied voxels appear red, and confidently free voxels become transparent green. Integrating such observation models into the RRBT planner could replace the idealized Light/Dark regions with more realistic, spatially varying uncertainty estimates derived directly from sensor data.

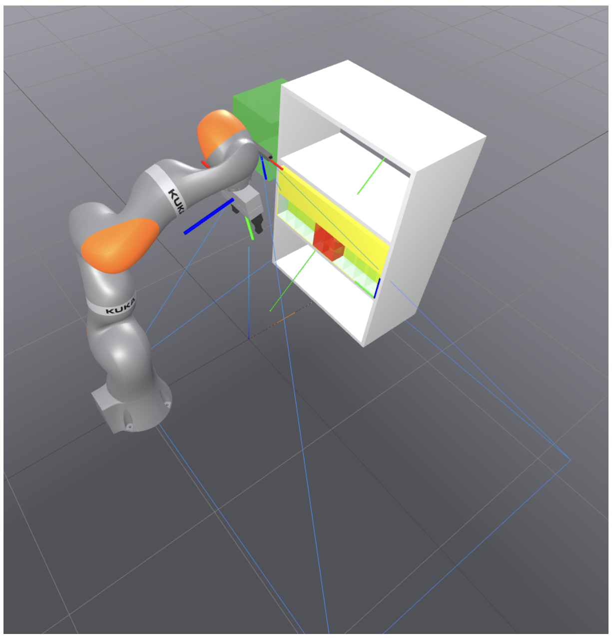


Fig. 9. Voxel-based occupancy observation model. A wrist-mounted camera observes the workspace, updating voxel occupancy probabilities via Bayesian log-odds. Yellow voxels indicate uncertain occupancy, red indicates confident occupancy, and transparent green indicates confident free space.

- **Sim-to-real transfer**: Validating the framework on physical hardware with real depth cameras would test robustness to calibration errors, sensor noise, and unmodeled dynamics.
- **Extended cost functions**: Incorporating safety margins, grasp stability metrics, or energy consumption into the RRBT cost function could improve performance on tasks where path length and uncertainty alone are insufficient objectives.

In summary, this project demonstrates that belief-space planning via RRBT provides a principled and effective approach to manipulation under uncertainty. By treating information gathering as an explicit planning objective, the robot autonomously navigates to reduce uncertainty before acting, a capability essential for deploying manipulation systems in uncertain, real-world environments.

VII. Contributions

- **TV**: Set up project infrastructure, defined problem with TAs, iterated over problem definitions, created first RRBT implementations, set scenario, and standard RRT implementations, designed slide deck.
- **BE**: Adapted RRBT for 2D position covariance (Kalman filter) and discrete bin belief (Bayes filter). Designed and implemented the sequential planning pipeline: 9-state FSM, Drake `LeafSystem` architecture, and transitions. Created custom scenario (square goal bin, 6-camera placement). Implemented Light/Dark sensing regions (TPR/FPR for bin, variance for position), ICP integration with MAP-based camera selection, antipodal grasp planning with covariance sampling, and baseline RRT system. Ran experiments, generated figures and quantitative results, created demo videos. Developed voxel-based observation model prototype. Contributed to the slide deck. Edited and posted our presentation video. Authored report.

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