Comparing Theoretical Models (by Thiago Amin)

Attempting to create a theoretical model that predicts the expected value of participation in climate activism, Lubell developed a collective interest model. His multivariable model contains the following parameters: two probability variables which represent the group's probability of success (p_g) and an agent's marginal contribution to this probability of success (p_i); selective costs (C) and benefits (B) to the individual; and the value of the collective public good (V). The relationship between these variables is described in the following equation:

$$\mathrm{M}_0 = \mathrm{EV(Global\ Warming\ Activism)} = \left[\left(p_g \times p_i\right)V\right] - C + B$$

However, we propose the following alternative model, where the V variable was substituted by two other parameters (B_d and C_d), and the term p_g was substituted for $1 - p_g$:

$$\mathrm{M}_1 = \mathrm{EV(Global\ Warming\ Activism)} = \left[p_i(1 - p_g)(B_d - C_d)\right] - C_i + B_i$$

The key difference between the two models can be established graphically. To do that, the additive parameters of the models ($[-C + B]$ for M_0 and $[-C_i + B_i]$ for M_1) were set to zero. Then, V and $(B_d - C_d)$ were both set to 1 in M_0 and M_1 respectively. Therefore, the following graphs exemplify the relationship between the probability parameters (p_g and p_i) and the expected value of participation (EV).

Figure 1: Relationship of Probability Variables and EV (1)

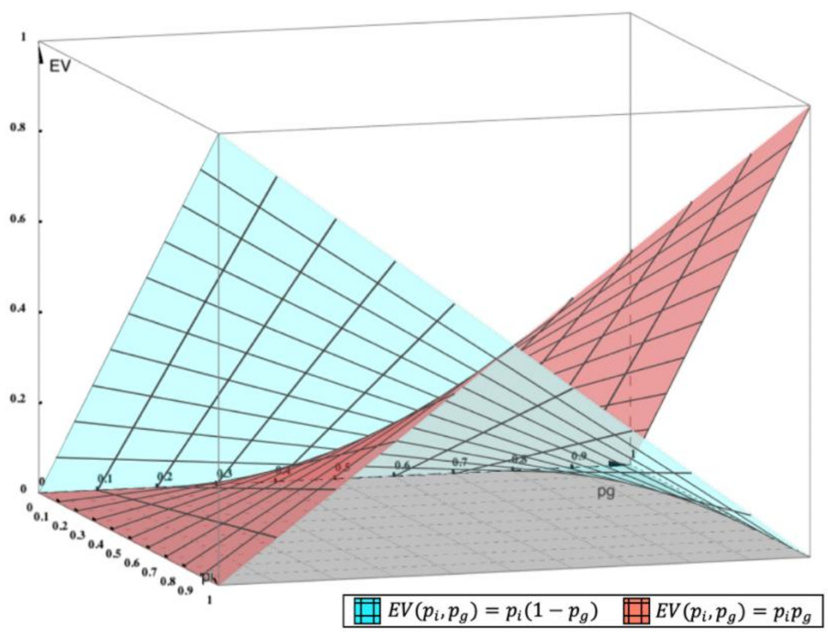
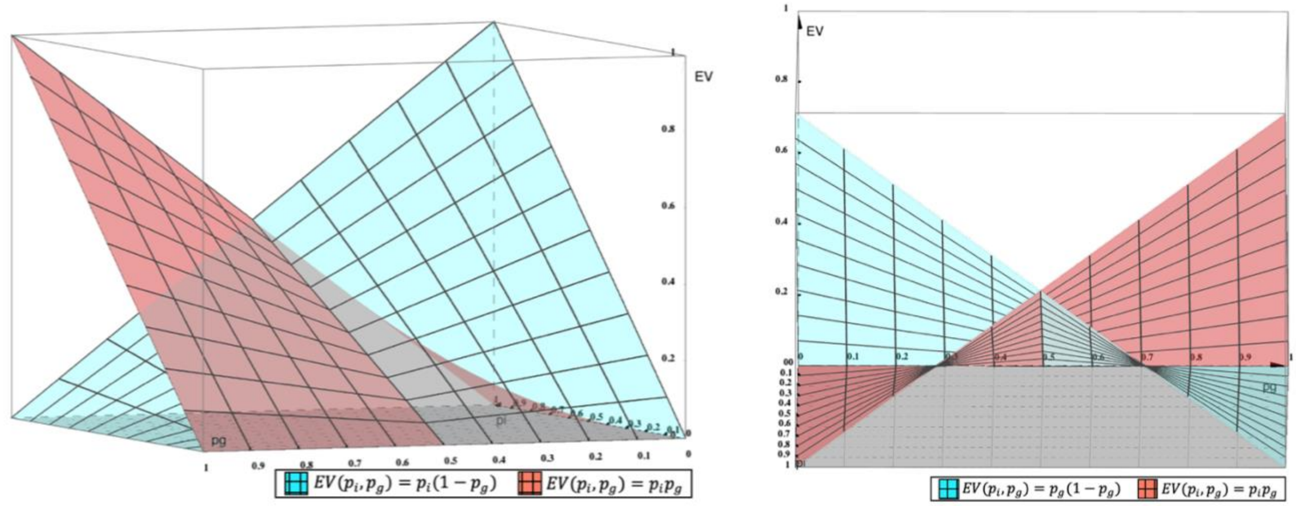


Figure 2&3: Probability Variables and EV (2) (3)



There are a few mathematical implications that the graphs above highlight. Firstly, M_0 and M_1 are equal along the line $p_g = 0.5$. Furthermore, the minimum and maximums of the graphs occur in opposite locations. While in M_0 the EV is at a minimum when p_g or p_i equal to zero, in M_1 the EV is at a minimum when $p_i = 0$ or $p_g = 1$. In terms of maximum values, M_0 reaches its global maximum at point (1, 1), while M_1 reaches its global maximum at (1, 0).

Other important mathematical differences are highlighted by the value given by the partial derivative values obtained throughout its domain. We find that the partial derivatives of M_0 is given by:

$$EV_{p_i} = \frac{\partial EV}{\partial\, p_i} = p_g, \text{ and } EV_{p_g} = \frac{\partial EV}{\partial\, p_g} = p_i$$

Meanwhile, the partial derivatives found for M_1 are given by:

$$EV_{p_i} = \frac{\partial EV}{\partial\, p_i} = 1 - p_g, \text{ and } EV_{p_g} = \frac{\partial EV}{\partial\, p_g} = -p_i$$

The partial derivatives of M_0 tells us that the rate of change of EV with respect to either probability variable will be the other probability variable (and will always be positive). Meanwhile, for M_1, the rate of change of EV with respect to p_i is decreasing as p_g increases (but like the partial derivative for M_0, always remains positive). And the rate of change of EV with respect to p_g is always negative (and equal to $-p_i$).

The mathematical characteristics lead to important differences between the models. Firstly, Lubell's model predicts that as the value of p_g increases, the rate of change of EV will be positive. In other words, as the group's probability of success (without considering the individual's contribution) increases, the individual's expected value of participation would also increase. This would be counterintuitive since it seems to not account for the free-rider issue, which would predict that participation would be less likely as p_g increases. This is because the agent will still yield the benefits of the group's success even without participating in the movement and experiencing its costs).

Another implausible feature of Lubell's model is that it predicts the maximum EV of participation occurs when both p_g and p_i are equal to one. This raises the question: if the movement is almost certain to succeed ($p_g = 1$), how could an agent believe that his contribution will have a significant impact in the group's probability of success ($p_i = 1$)? Therefore, the coordinate (1, 1), where the maximum occurs for M_0, would not occur for rational agents. This leads to the conclusion that $p_g + p_g \leq 1$.

One might say that if we optimize (find the maximum) Lubell's function under the constraint $p_g + p_g \leq 1$, then we will obtain the value that Lubell intended to be its model's maximum. However, when doing that, we find that the maximum occurs at (0.5, 0.5).

$$\max\{EV(p_i, p_g) \big| (p_i \geq 0) \wedge (p_g \geq 0) \wedge (p_i + p_g \leq 1)\} = 0.25\ at\ (0.5, 0.5)$$

In other words, according to Lubell, an agent is the most likely to join a movement when its marginal contribution to the group's success is 50% and the group's base probability of success is 50%. But why would that be the case? I would argue that a rational agent is more likely to join a movement where his p_i is 80% and p_g is 20%, since his personal contribution would be higher and the group's total probability of success (consider his contribution) would be equal to the aforementioned scenario of (0.5, 0.5).

The problems presented highlight the need for the development of a model that correct these counterintuitive predictions. The model developed in this paper (M_1) addresses these issues and develops a new model that accounts for them. Firstly, our model calculates that as the value of p_g increases, $\frac{\partial EV}{\partial\, p_g} \leq 0$. In other words, as the group's probability of success (without considering the individual's contribution) increases, the individual's expected value of participation would decrease. This would account for the free-rider issue. Furthermore, in this new developed model, the maximum value of EV occurs at (1, 0):

$$\max\{EV(p_i, p_g) | (p_i \geq 0) \wedge (p_g \geq 0) \wedge (p_i + p_g \leq 1)\} = 1\ at\ (1,0)$$

The maximum value obtained would seem to be rational. Let's say for example that a group has no chance of succeeding without a given agent ($p_g = 0$). However, let us assume that if this certain agent joins the movement, then it has a guaranteed chance of success ($p_i = 100\%$). It would only be rational for this to represent the maximum value of EV, since free riding would not occur, but the total chances of success (considering agent's participation) are 100% (success and yielding of benefits is guaranteed).

Another interesting feature of our new model is that it states that $\frac{\partial EV}{\partial\, p_i} = 1 - p_g$, while Lubell model argues that $\frac{\partial EV}{\partial\, p_i} = p_g$. To put it another way, our model states that the rate of change of EV with respect to p_i decreases as the base group probability of success (p_g) increases. The explanation behind this prediction again lies within the free rider problem. As the probability of group success (p_g) increases, the free-rider issue starts affecting the model. This means that an increase in p_i when p_g is high (free-riding) will lead to a smaller increase in EV than it would have otherwise led to if no free-riding had occurred.