Q05

April 26, 2025

[1]:
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import numpy as np
import matplotlib.pyplot as plt
import smplotlib
from scipy.constants import h, k, c
```

[2]:
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T_dust = 20
M_dust = 1e-2 * 1.989e30
d = 150
d_m = d * 3.086e16
theta_arcmin = 5
theta_rad = np.radians(theta_arcmin / 60)

kappa_0 = 0.1  # cm^2/g at 1000 GHz
nu_0 = 1e12
beta = 1.7

wavelengths = np.logspace(-5, -2, 100)  # meters (from 10 μm to 3 mm)
frequencies = c / wavelengths

kappa_nu = kappa_0 * (frequencies / nu_0) ** beta  # Dust absorption␣
↪coefficient (cm^2/g
kappa_nu_m2kg = kappa_nu * 0.1

A = np.pi * (theta_rad * d_m) ** 2  # Approximate projected area in m^2
Sigma_dust = M_dust / A

tau_nu = kappa_nu_m2kg * Sigma_dust

B_nu = (2 * h * frequencies**3 / c**2) / (np.exp(h * frequencies / (k *␣
↪T_dust)) - 1)

S_nu = B_nu * (1 - np.exp(-tau_nu))

Omega = (theta_rad / 2) ** 2 * np.pi  # Approximate solid angle

F_nu = S_nu * Omega
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B_nu_blackbody = B_nu * Omega
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We model the dust emission assuming a fixed dust mass and effective temperature. First, we take some values for the dust temperature and its mass, along with the source's distance and angular size on the sky in arcminutes.

We then convert this angular size into radius to properly estimate the projected physical area A of the source as $A = \pi(\theta_{\text{rad}} \times d)^2$.

After that, we define the dust opacity as κ_ν that scales with frequency as $\kappa_\nu = \kappa_0(\nu/\nu_0)^\beta$, where β is the dust emissivity index and κ_0 is a reference opacity at ν_0. We then convert κ_ν from cgs units (cm2/g) to SI units (m2/kg). Using the dust mass and the physical area, we estimate the dust surface density $\Sigma_{\text{dust}} = M_{\text{dust}}/A$. The optical depth at each frequency is then given by $\tau_\nu = \kappa_\nu \Sigma_{\text{dust}}$.

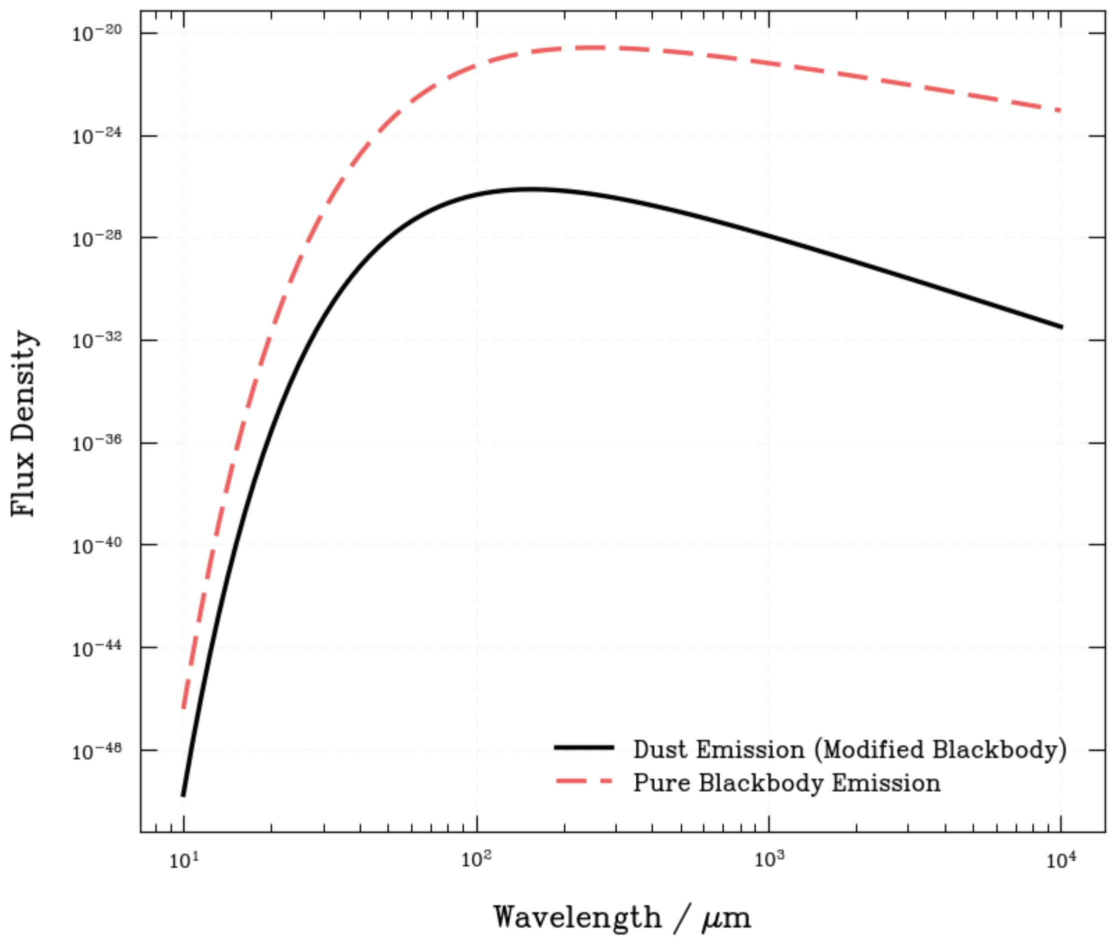
Next, we compute the blackbody emission $B_\nu(T_{\text{dust}})$ at each frequency using the Planck function. However, since the emission is absorbed and re-emitted by dust, the actual specific intensity from the dust is reduced by a factor $(1 - e^{-\tau_\nu})$, accounting for the finite optical depth. Therefore, the source function is $S_\nu = B_\nu(1 - e^{-\tau_\nu})$.

At the end, we calculate the solid angle Ω subtended by the source, assuming it's approximately circular. The observed flux density F_ν is then the product of the modified intensity S_ν and the solid angle $\Omega : F_\nu = S_\nu \Omega$. We also estimate $B_\nu \times \Omega$, which would represent the flux if the source behaved as a perfect blackbody without any dust opacity effects.

The results are plotted below.

[3]:
```
fig, ax = plt.subplots(figsize = (7, 6))
ax.grid(alpha = 0.05)
ax.tick_params(axis = 'both', which = 'major', labelsize = 10)
ax.set_xlabel('Wavelength / $\mu$m', fontsize = 15, labelpad = 12)
ax.set_ylabel('Flux Density', fontsize = 15, labelpad = 12)
ax.loglog(wavelengths * 1e6, F_nu, label = 'Dust Emission (Modified␣
 ↪Blackbody)', lw = 2)
plt.loglog(wavelengths * 1e6, B_nu_blackbody, '--', label = 'Pure Blackbody␣
 ↪Emission', lw = 2, alpha = 0.7)
plt.legend(fontsize = 12, loc = 'lower right')

plt.tight_layout()
plt.savefig('Q05_SED.pdf', dpi = 500)
```



10^{-20}
10^{-24}
10^{-28}
10^{-32}
10^{-36}
10^{-40}
10^{-44}
10^{-48}
Flux Density
10^{1}
10^{2}
10^{3}
10^{4}
Wavelength / μm
Dust Emission (Modified Blackbody)
Pure Blackbody Emission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