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In[3]:= Clear["Global`*"]
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In[4]:= gramschmidt[w_, var_, interval_, n_] :=
  (* Density function, variable, interval and number of terms *)
  Module[{a, gram, ψ, ϕ},
    a[i_, j_] :=
      Integrate[w * var^(i + j), Join[{var}, interval], GenerateConditions → False];
    gram[m_] := Det[Table[a[i, j], {i, 0, m}, {j, 0, m}]]; gram[-1] = 1;
    ψ[m_] :=
      Det[Append[Table[a[i, j], {i, 0, m - 1}, {j, 0, m}], var^Range[0, m]] // Simplify];
    ϕ[m_] := ψ[m]/Sqrt[gram[m - 1] * gram[m]]; Table[{k, ϕ[k]}, {k, 0, n}]
```

```
In[5]:= gramschmidt[Exp[-x^2], x, {-∞, ∞}, 6] // TableForm
```

Out[5]//TableForm=

$$\begin{array}{l}
 0 \quad \frac{1}{\pi^{1/4}} \\
 1 \quad \frac{\sqrt{2} x}{\pi^{1/4}} \\
 2 \quad \frac{-1+2 x^2}{\sqrt{2} \pi^{1/4}} \\
 3 \quad \frac{x(-3+2 x^2)}{\sqrt{3} \pi^{1/4}} \\
 4 \quad \frac{3-12 x^2+4 x^4}{2 \sqrt{6} \pi^{1/4}} \\
 5 \quad \frac{x(15-20 x^2+4 x^4)}{2 \sqrt{15} \pi^{1/4}} \\
 6 \quad \frac{-15+90 x^2-60 x^4+8 x^6}{12 \sqrt{5} \pi^{1/4}}
 \end{array}$$

```
In[6]:= gramschmidt[1, x, {-1, 1}, 3] (* Legendre *) // TableForm
```

Out[6]//TableForm=

$$\begin{array}{l}
 0 \quad \frac{1}{\sqrt{2}} \\
 1 \quad \sqrt{\frac{3}{2}} x \\
 2 \quad \frac{1}{2} \sqrt{\frac{5}{2}} (-1+3 x^2) \\
 3 \quad \frac{1}{2} \sqrt{\frac{7}{2}} x (-3+5 x^2)
 \end{array}$$

```
In[7]:= gramschmidt[1/Sqrt[1-x^2], x, {-1, 1}, 3] (* Chebyshev I *) // TableForm
```

Out[7]//TableForm=

$$\begin{array}{l}
 0 \quad \frac{1}{\sqrt{\pi}} \\
 1 \quad \sqrt{\frac{2}{\pi}} x \\
 2 \quad \sqrt{\frac{2}{\pi}} (-1+2 x^2) \\
 3 \quad \sqrt{\frac{2}{\pi}} x (-3+4 x^2)
 \end{array}$$

```
In[8]:= gramschmidt[Sqrt[1 - x^2], x, {-1, 1}, 3] (* Chebyshev II *) // TableForm
```

```
Out[8]//TableForm=
```

$$\begin{array}{l} 0 \quad \sqrt{\frac{2}{\pi}} \\ 1 \quad 2 \sqrt{\frac{2}{\pi}} x \\ 2 \quad \sqrt{\frac{2}{\pi}} (-1 + 4 x^2) \\ 3 \quad 4 \sqrt{\frac{2}{\pi}} x (-1 + 2 x^2) \end{array}$$

```
In[9]:= gramschmidt[Exp[-x], x, {0, ∞}, 3] (* Laguerre *) // TableForm
```

```
Out[9]//TableForm=
```

$$\begin{array}{l} 0 \quad 1 \\ 1 \quad -1 + x \\ 2 \quad \frac{1}{2} \times (2 - 4 x + x^2) \\ 3 \quad \frac{1}{6} \times (-6 + 18 x - 9 x^2 + x^3) \end{array}$$

```
In[12]:= gramschmidt[x Exp[-x], x, {0, ∞}, 3] (* Arfken 10.3.4 - 6th E. *) // TableForm
```

```
Out[12]//TableForm=
```

$$\begin{array}{l} 0 \quad 1 \\ 1 \quad \frac{-2+x}{\sqrt{2}} \\ 2 \quad \frac{6-6 x+x^2}{2 \sqrt{3}} \\ 3 \quad \frac{1}{12} \times (-24 + 36 x - 12 x^2 + x^3) \end{array}$$

```
In[11]:=
```