Bayesian A/B Testing: Methodology Notes

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Overview

These notes summarize the methodology for Bayesian A/B testing on both proportions and means. We present a step-by-step description of prior definitions, posterior inference via conjugacy and MCMC, and highlight normalization applied for mean-based MCMC.

1 Proportion Metric

1.1 Step 1: Prior Definition

Define a Beta prior for the success probability:

$$p \sim \mathrm{Beta}(\alpha_0, \beta_0)$$

where α_0, β_0 encode prior knowledge about the expected proportion.

1.2 Step 2: Posterior Inference

1.2.1 Method 1: Conjugacy

Given k successes in n trials:

$$p \mid k, n \sim \mathrm{Beta}(\alpha_0 + k, \beta_0 + n - k)$$

This uses the Beta-Binomial conjugacy for closed-form posterior sampling.

1.2.2 Method 2: MCMC

For MCMC, define the likelihood using individual Bernoulli observations:

$$x_i \sim \mathrm{Bernoulli}(p), \quad i = 1, \dots, n$$

$$\log \mathcal{L}(p \mid x_1, \dots, x_n) = \sum_{i=1}^{n} \big(x_i \log p + (1 - x_i) \log(1 - p) \big)$$

Use Metropolis-Hastings sampling to obtain posterior draws of p.

2 Mean Metric

2.1 Step 1: Prior Definition

Define a Normal prior for the group mean:

$$\theta \sim \mathcal{N}(\mu_0, \sigma_0^2)$$

2.2 Step 2: Posterior Inference

2.2.1 Method 1: Conjugacy (Normal-Normal)

Given sample mean $\bar{x}$ and sample variance s^2:

$$\theta \mid \bar{x}, n \sim \mathcal{N}\left(\frac{\mu_0/\sigma_0^2 + n\bar{x}/s^2}{1/\sigma_0^2 + n/s^2}, \frac{1}{1/\sigma_0^2 + n/s^2}\right)$$

2.2.2 Method 2: MCMC with Normalization

Likelihood is defined as:

$$x_i \sim \mathcal{N}(\theta, \sigma^2), \quad \log \mathcal{L}(\theta \mid x) = \sum_{i=1}^{n} \log \mathrm{NormalPDF}(x_i; \theta, \hat{\sigma})$$

where

$$\hat{\sigma} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Normalizing the likelihood using the sample standard deviation $\hat{\sigma}$ stabilizes MCMC sampling and prevents numerical issues.

3 Posterior Summary and Lift

For each group:

- Posterior mean: $\mathbb{E}[\theta]$
- Posterior standard deviation: $\mathrm{SD}[\theta]$
- Credible interval (e.g., 95%)

Note: Although MCMC for the mean uses normalized data for numerical stability, all posterior summaries are converted back to the original data scale when reporting results, ensuring direct interpretability.

Lift and probability of treatment superiority:

$$\mathrm{Lift} = \frac{\theta_{\text{treatment}}}{\theta_{\text{control}}} - 1$$

$$\Pr(\theta_{\text{treatment}} > \theta_{\text{control}}) = \text{fraction of posterior samples where } \theta_{\text{treatment}} > \theta_{\text{control}}$$

References

[1] Gelman, A., Carlin, J. B., Stern, H. S., & Rubin, D. B. (2013). *Bayesian Data Analysis (3rd ed.)*. CRC Press. `https://www.stat.columbia.edu/~gelman/book/`

[2] PyMC Documentation (2024). *Bayesian A/B Testing Example.* Open-source resource demonstrating Bayesian inference workflows. `https://www.pymc.io/projects/docs/en/stable/`

[3] Stan Development Team (2024). *Stan User's Guide.* Documentation on Bayesian modeling, conjugate priors, and MCMC implementation. `https://mc-stan.org/users/documentation/`

Contact and Repository

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