LISTA TEÓRICA II

THIAGO MALTA COUTINHO - 2014123335

1) $x \in \mathbb{R}^{2\times1}$, $\boxed{z^{[1]} \in \mathbb{R}^{4\times1}}$; $z^{[1]} = w^{[1]}x \Rightarrow \boxed{w^{[1]} \in \mathbb{R}^{4\times2}}$

$\Downarrow$

$\boxed{b^{[1]} \in \mathbb{R}^{4\times1}}$

$f(z^{[1]}) = A^{[1]} \Rightarrow \boxed{A^{[1]} \in \mathbb{R}^{4\times1}}$

$\underset{1\times1}{z^{[2]}} = w^{[2]} \underset{4\times1}{A^{[1]}}$

$\Downarrow$ $\boxed{w^{[2]} \in \mathbb{R}^{1\times4}}$

$\boxed{b^{[2]} \in \mathbb{R}}$

---

2) a) $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$; $f'(x) = \left[\underbrace{(e^x+e^{-x})(e^x+e^{-x})}_{(e^x+e^{-x})^2} - \underbrace{(e^x-e^{-x})(e^x-e^{-x})}_{(e^x-e^{-x})^2}\right] \Big/ (e^x+e^{-x})^2$

$= 1 - \dfrac{(e^x-e^{-x})^2}{(e^x+e^{-x})^2} = \boxed{1 - \tanh^2(x)}$; $\boxed{\arg\max_x f'(x) = 0}$

b) $f(x) = \max(0,x)$; p/ $x<0$, $f(x) = 0 \Rightarrow f'(x) = 0$

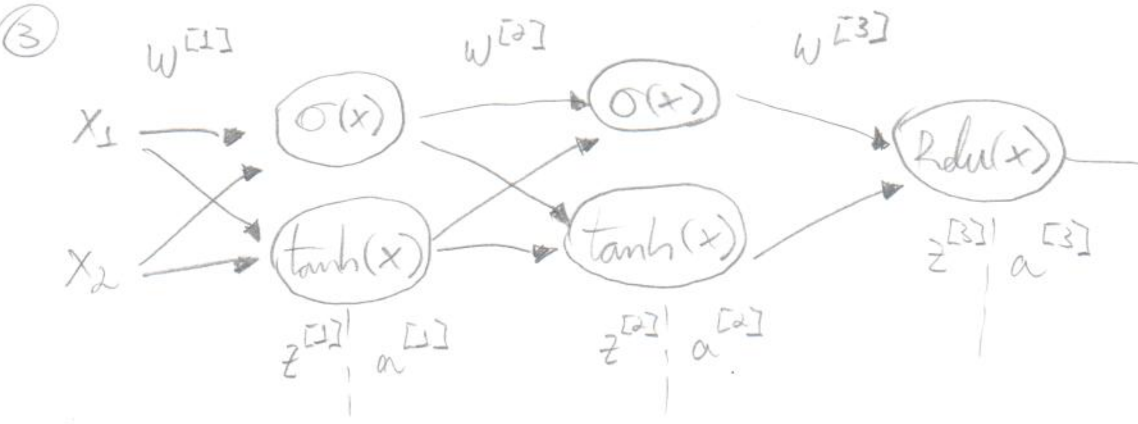
p/ $x>0$, $f(x) = x \Rightarrow f'(x) = 1$

c) $f(x) = a\tanh(bx) = a \cdot \dfrac{e^{bx} - e^{-bx}}{e^{bx} + e^{-bx}}$

Utilizando a resposta da letra a) ~

$f'(x) = a\left(1 - \tanh^2(bx)\right) = \boxed{a - a\tanh^2(bx)}$

(3)



$$Y = a^{[3]} = Relu(z^{[3]}) = Relu(W^{[3]} \underbrace{a^{[2]}})$$

$$a^{[2]} = \begin{bmatrix} \sigma(z_1^{[2]}) \\ \tanh(z_2^{[2]}) \end{bmatrix}$$

$\sigma(z_1^{[2]}) \to \sigma(W_1^{[2]} a^{[1]})$

$\tanh(z_2^{[2]}) \to \tanh(W_2^{[2]} a^{[1]})$

$$a^{[1]} = \begin{bmatrix} \sigma(z_1^{[1]}) \\ \tanh(z_2^{[1]}) \end{bmatrix}$$

$\sigma(z_1^{[1]}) \to \sigma(W_1^{[1]} x)$

$\tanh(z_2^{[1]}) \to \tanh(W_2^{[1]} x)$

→ EXPANDINDO $a^{[2]}$:

$$a^{[2]} = \begin{bmatrix} \sigma\left(W_1^{[2]} \begin{bmatrix} \sigma(W_1^{[1]} x) \\ \tanh(W_2^{[1]} x) \end{bmatrix}\right), & \tanh\left(W_2^{[2]} \begin{bmatrix} \sigma(W_1^{[1]} x) \\ \tanh(W_2^{[1]} x) \end{bmatrix}\right) \end{bmatrix}$$

$$a^{[2]} = \Big\{ \sigma\left[W_{11}^{[2]} \sigma(W_{11}^{[1]} x_1 + W_{12}^{[1]} x_2) + W_{12}^{[2]} \tanh(W_{21}^{[1]} x_1 + W_{22}^{[1]} x_2)\right],$$
$$\tanh\left[W_{21}^{[2]} \sigma(W_{11}^{[1]} x_1 + W_{12}^{[1]} x_2) + W_{22}^{[2]} \tanh(W_{21}^{[1]} x_1 + W_{22}^{[1]} x_2)\right] \Big\}$$

$$Y = Relu\Big(W_{11}^{[3]} \Big\{ \sigma\left[W_{11}^{[2]} \sigma(W_{11}^{[1]} x_1 + W_{12}^{[1]} x_2) + W_{12}^{[2]} \tanh(W_{21}^{[1]} x_1 + W_{22}^{[1]} x_2)\right] \Big\} +$$
$$W_{12}^{[3]} \Big\{ \tanh\left[W_{21}^{[2]} \sigma(W_{11}^{[1]} + W_{12}^{[1]} x_2) + W_{22}^{[2]} \tanh(W_{21}^{[1]} + W_{22}^{[1]} x_2)\right] \Big\} \Big)$$

(4) XOR:

x_1	x_2	S
0	0	0
1	0	1
0	1	1
1	1	0

p/ h_1:

$$\left.\begin{aligned} 1-0&=1\\ 0-0&=0\\ 0-1&=0\\ 1-1&=0 \end{aligned}\right\} \rightarrow x_1 - x_2 > 0 \rightarrow \vec{S_1}$$

p/ h_2:

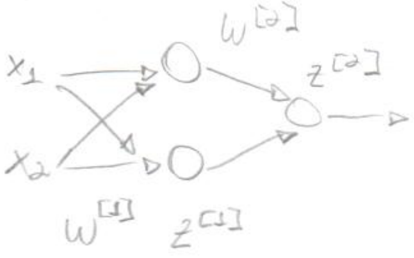
$$\left.\begin{aligned} 0-1&=0\\ 0-0&=0\\ 1-0&=1\\ 1-1&=0 \end{aligned}\right\} \rightarrow x_2 - x_1 = 0 \rightarrow \vec{S_2}$$

$S = \vec{S_1} + \vec{S_2} \Rightarrow$

x_1	x_2	S
1	0	1
0	0	0
1	0	1
1	1	0

$$\therefore \boxed{\begin{aligned} h_1 &= [0 \;\; 1 \; -1] \\ h_2 &= [0 \; -1 \;\; 1] \\ OUT &= [0 \;\; 1 \;\; 1] \end{aligned}}$$

(5)



$$Y = z^{[2]} = z_1^{[1]} W_{11}^{[2]} + z_2^{[2]} W_{12}^{[2]}$$

$$z_1^{[1]} = W_{11}^{[1]} x_1 + W_{12}^{[1]} x_2$$

$$z_2^{[1]} = W_{21}^{[1]} x_1 + W_{21}^{[1]} x_2$$

$$Y = W_{11}^{[2]} W_{11}^{[1]} x_1 + W_{11}^{[2]} W_{12}^{[1]} x_2 + W_{12}^{[2]} W_{21}^{[1]} x_1 + W_{12}^{[2]} W_{22}^{[1]} x_2$$

$$Y = \underbrace{\left(W_{11}^{[2]} W_{11}^{[1]} + W_{12}^{[2]} W_{21}^{[1]}\right)}_{k_1} x_1 + \underbrace{\left(W_{11}^{[2]} W_{12}^{[1]} + W_{12}^{[2]} W_{22}^{[1]}\right)}_{k_2} x_2$$

$Y = k_1 x_1 + k_2 x_2 \rightarrow$ Combinação linear (C.L.). C.L. de C.L. será uma C.L.

O problema XOR não é linearmente separável, portanto esse tipo de rede não consegue classificar corretamente todos os inputs.

⑥ a) $x \in \mathbb{R}^{d \times 1}$ 1ª camada: d neurônios $\rightarrow d^2 + d = d(d+1)$

2ª camada: m_H neurônios $\rightarrow d \cdot mH + mH = mH(d+1)$

3ª camada: c neurônios $\rightarrow c \cdot mH + c = c(mH+1)$

TOTAL: $d(d+1) + mH(d+1) + c(mH+1) = (d+mH)(d+1) + c(mH+1)$

b) $\vec{z} = f(W^{[2]}y)$; $f(x) = f(-x)$

$\vec{y} = f(W^{[1]}x)$

$\rightarrow$ TROCANDO O SINAL DOS PESOS:

$f(-W^{[1]}x) = f(W^{[1]}x) = y$

$f(-W^{[2]}f(-W^{[1]}x)) = f(-W^{[2]}y) = f(W^{[2]}y) = z$

---

⑦ a) $f(x,y,z) = (x+y) \cdot z = zx + zy$; $q(x,y) = (x+y)$

$\frac{\partial f}{\partial x} = z$; $\frac{\partial f}{\partial y} = z$; $\frac{\partial f}{\partial z} = x+y$; $\frac{\partial q}{\partial x} = 1$; $\frac{\partial q}{\partial y} = 1$

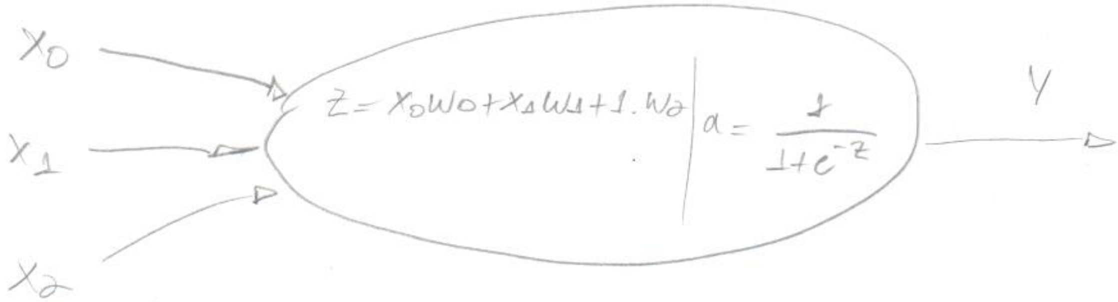
b) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x} = z \cdot 1$

$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial y} = z \cdot 1$

$\frac{\partial f}{\partial z} = x + y$

$$\boxed{\begin{aligned} \nabla f &= \{z, z, x+y\} \\ &= \{-2, 5, -4\} \end{aligned}}$$

(8) $f(w, x) = \dfrac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$



$$\frac{\partial y}{\partial w_0} = \frac{\partial a}{\partial z} \cdot \frac{\partial z}{\partial w_0}$$

$$\frac{\partial y}{\partial w_1} = \frac{\partial a}{\partial z} \cdot \frac{\partial z}{\partial w_2}$$

$$\frac{\partial y}{\partial w_2} = \frac{\partial a}{\partial z} \cdot \frac{\partial z}{\partial w_2}$$

$$\frac{\partial a}{\partial z} = \sigma(z) \cdot (1 - \sigma(z))$$

$$\frac{\partial z}{\partial w_0} = x_0 \; ; \; \frac{\partial z}{\partial w_1} = x_1 \; ; \; \frac{\partial z}{\partial w_2} = 1$$

$$\nabla f = \sigma(z)(1 - \sigma(z)) \left\langle x_0, x_1, 1 \right\rangle$$

$$z = -2 + 6 - 3 = 1$$

$$\nabla f = \frac{e^{-1}}{(1 + e^{-1})^2} \left\langle -1, -2, 1 \right\rangle$$