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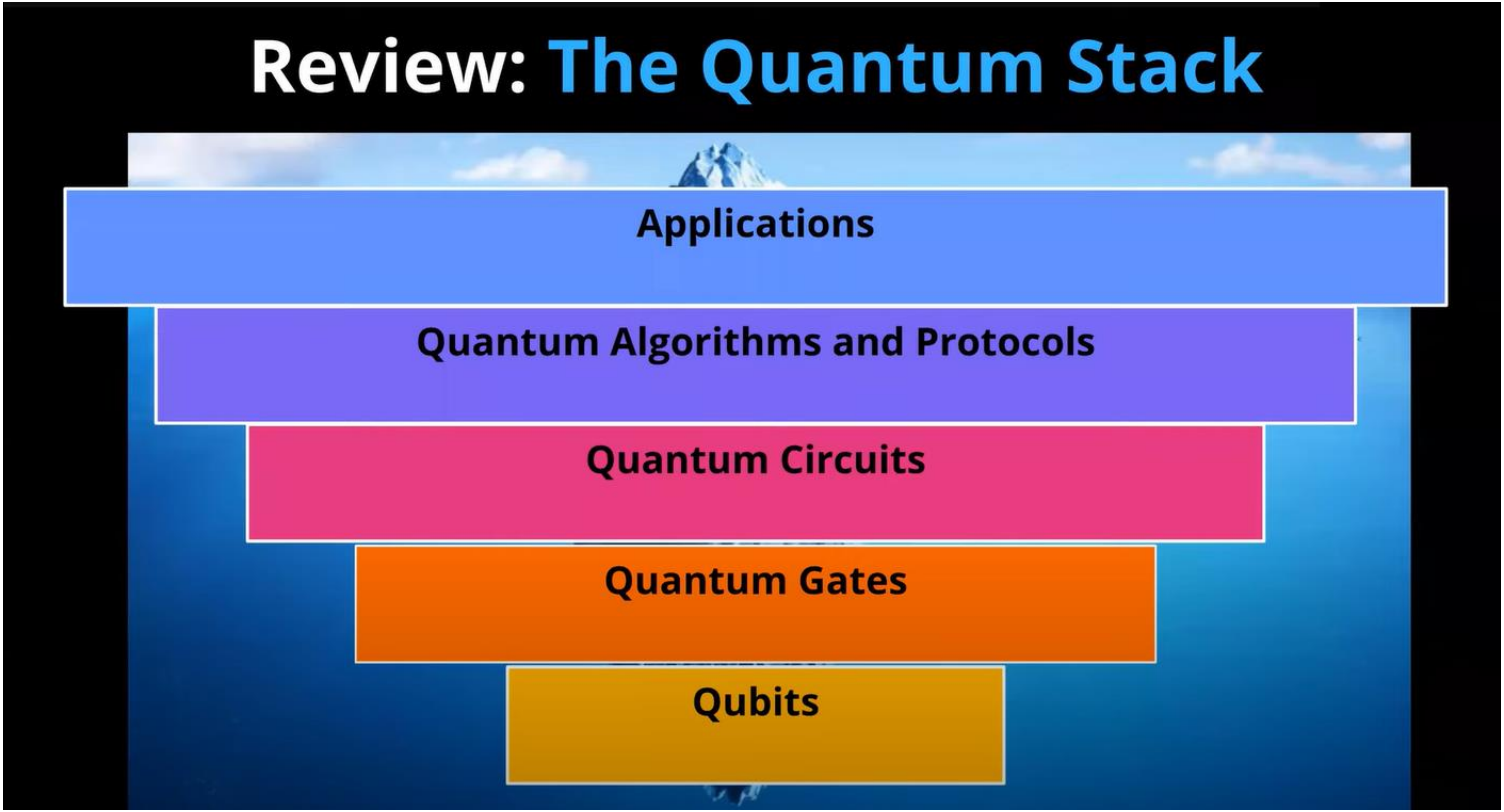
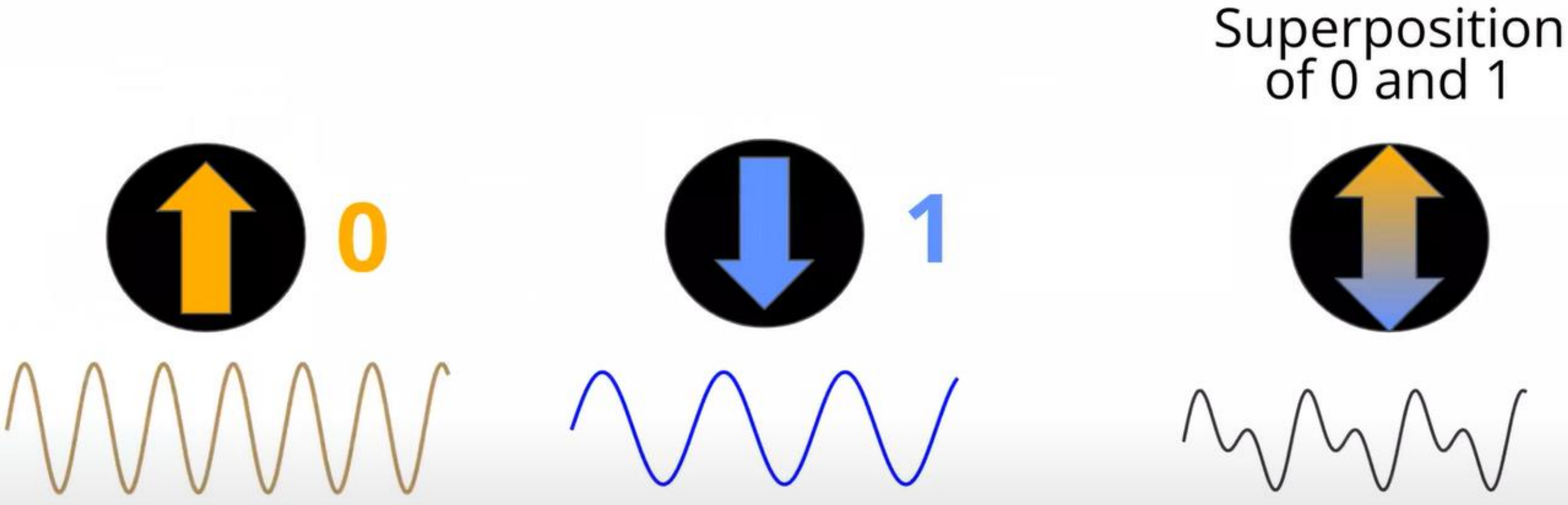
Computação Quântica

Qubits, portas e circuitos


Review: The Quantum Stack
Applications
Quantum Algorithms and Protocols
Quantum Circuits
Quantum Gates
Qubits

Review: Superposition of a qubit



A qubit has both wave and particle qualities. A superposition is the combination of the waves that represent the 0 and 1 states.

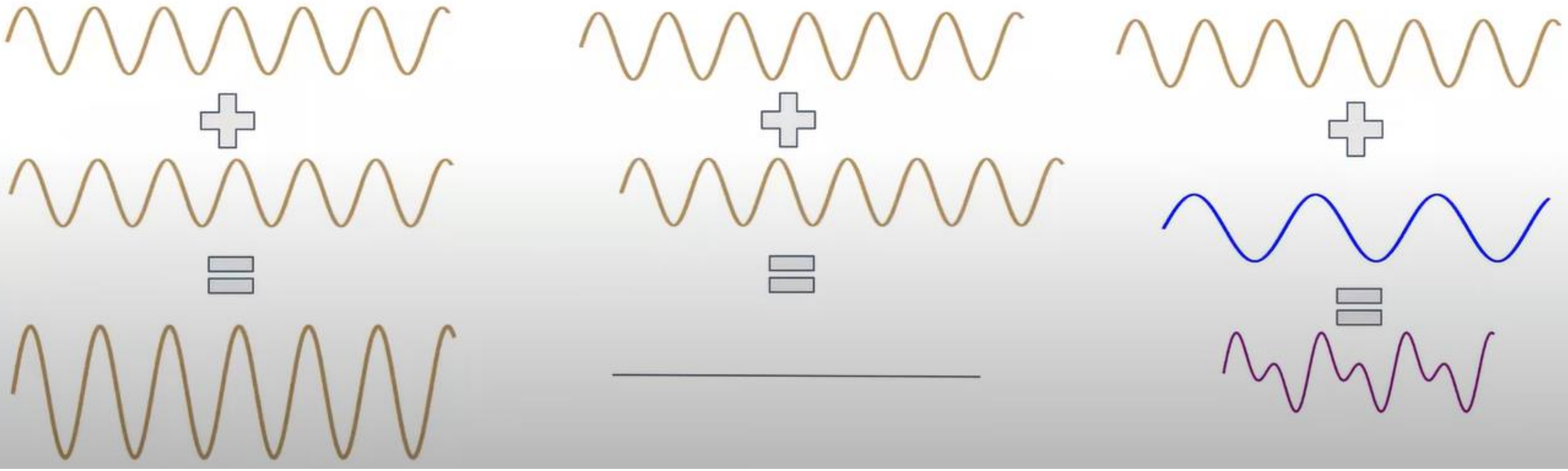
Review: Interference of a qubit

Different states of the qubit can interfere with each other

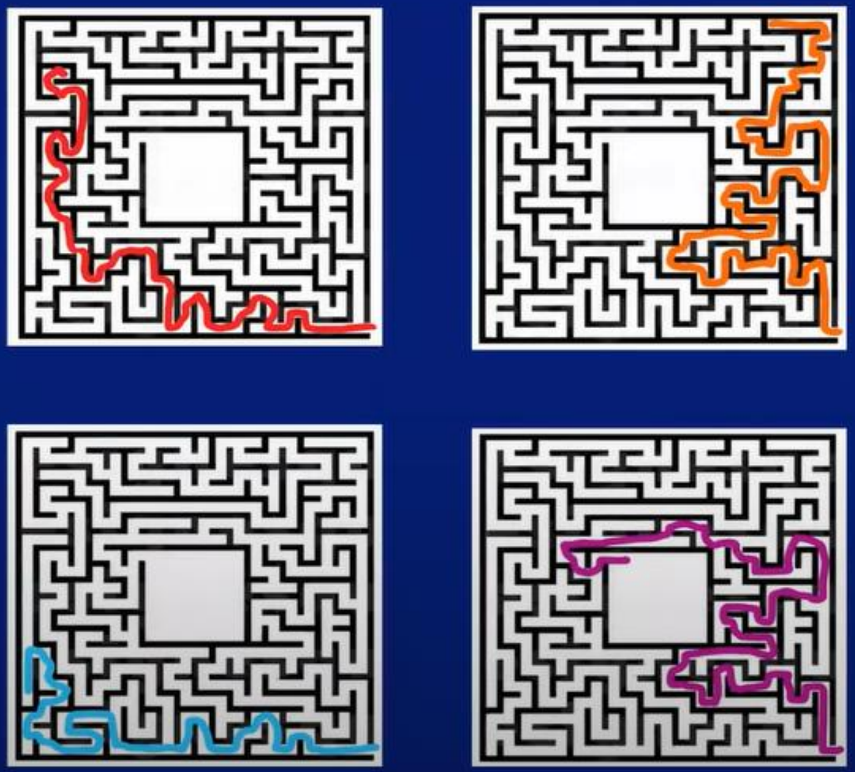
0 state constructively interfering with 0 state

0 state destructively interfering with 0 state

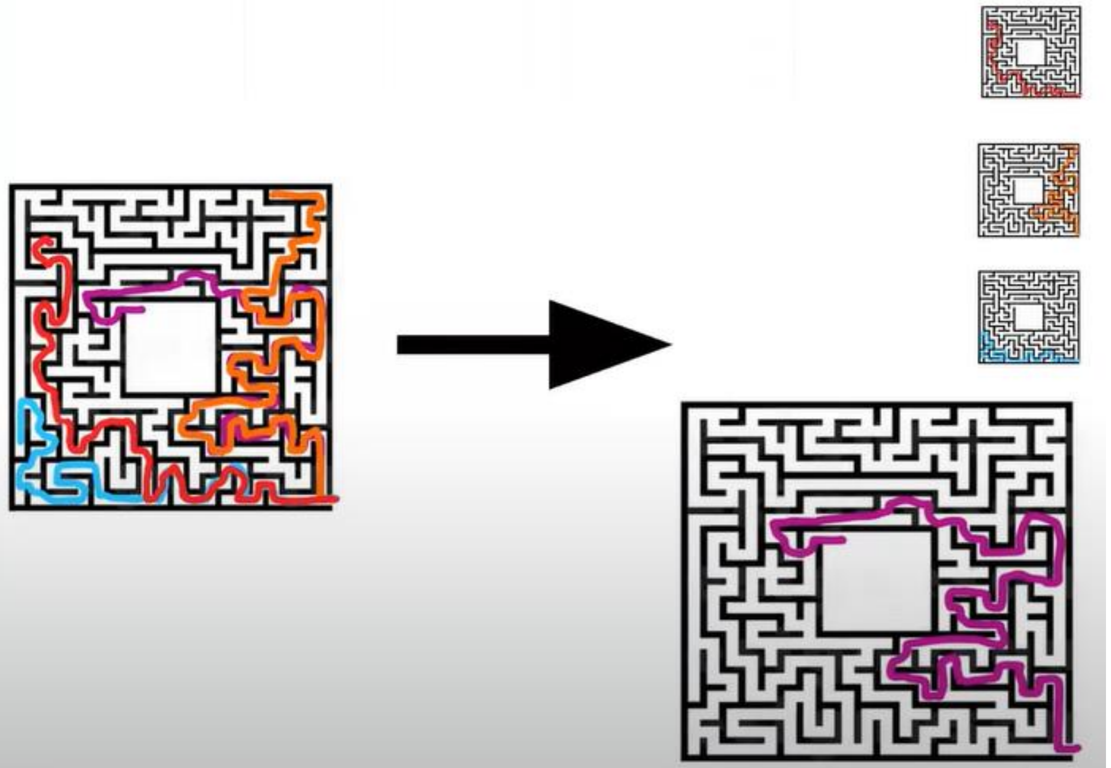
0 state interfering with 1 state



To solve a maze classically, we have to try every option one at a time



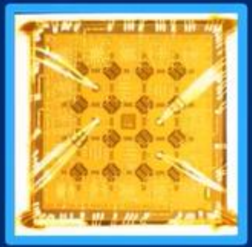
To solve a maze quantum mec
we can **create a superpositi**
options and then **use interference to rule out incorrect paths**



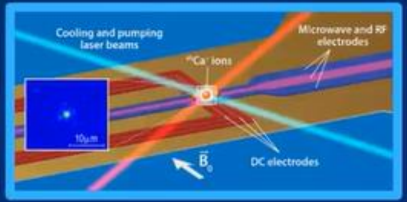
- Review
- **Representing Qubits**
- Quantum Gates
- Quantum Circuits

Representing Qubits

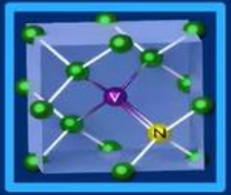
Different types of qubits



Superconducting

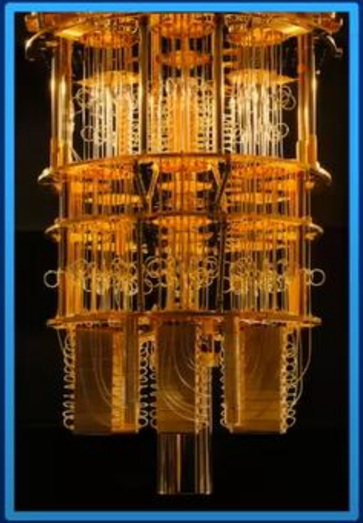


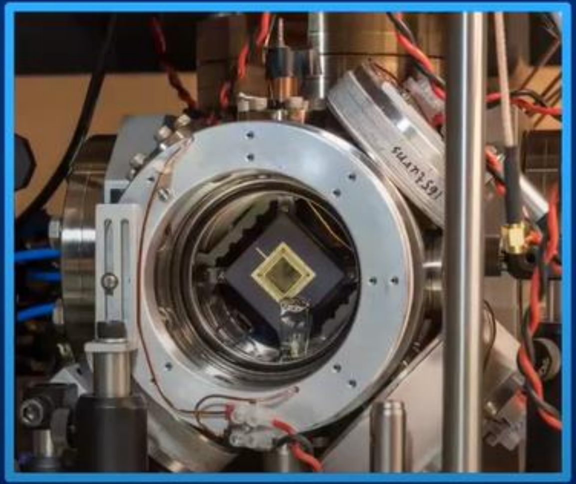
Trapped Ions



Diamond NV Centers

Photonics





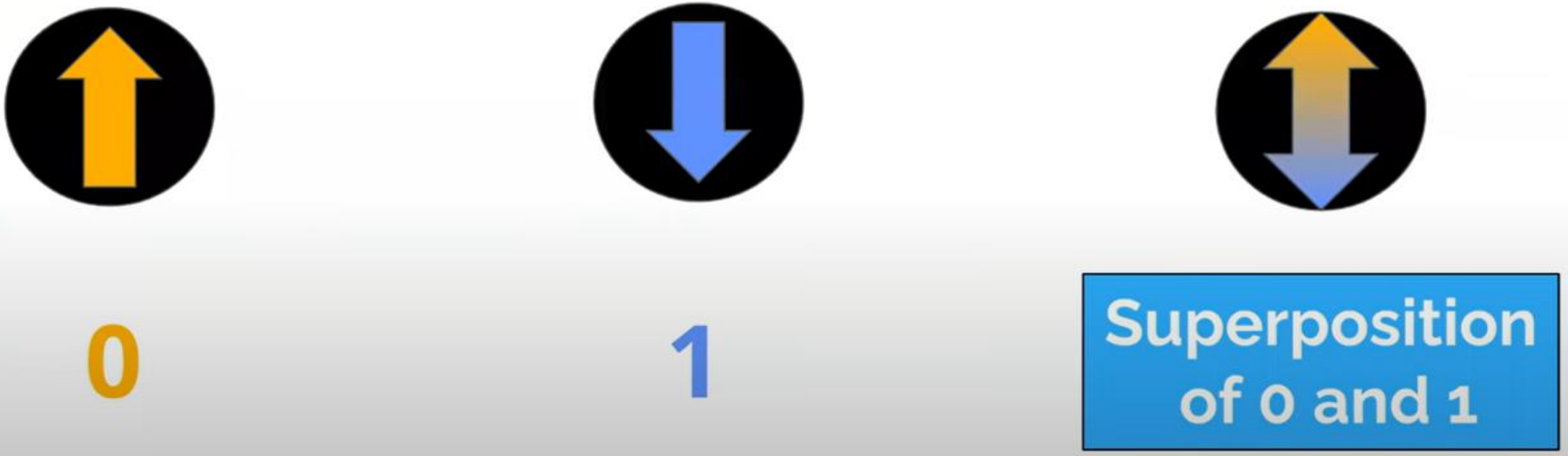
There are many different ways to build quantum computers using different types of qubits.

Different approaches can involve *very different physics*.

So, we need a *common* way to talk about all of these!

Representing Qubits

So far, we've represented qubits like this:

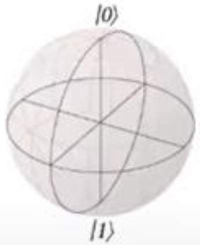


Representing Qubits

Today, we're going to learn more sophisticated ways of representing qubits through two approaches:

$$|0\rangle \quad |1\rangle$$

ket notation



Bloch sphere

ket notation

These symbols make it hard to tell when we're talking about classical versus quantum bits, even though we know they behave very differently.

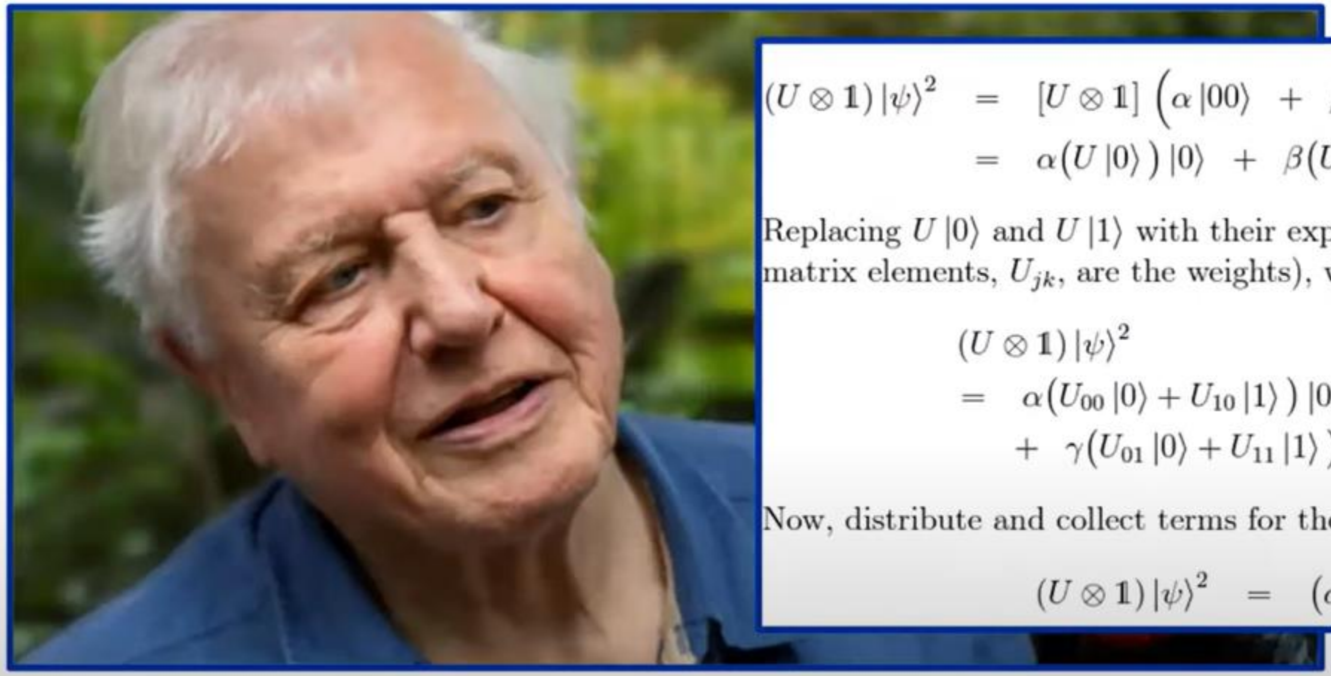
ket notation is how physicists indicate a state is quantum:

$|0\rangle$ $|1\rangle$

Superposition of:

$|0\rangle$ $|1\rangle$

Kets in their natural habitat



$$(U \otimes \mathbb{1})\,|\psi\rangle^2 \;=\; [U \otimes \mathbb{1}]\Big(\alpha\,|00\rangle \;+\; \beta\,|01\rangle \;+\; \gamma\,|10\rangle \;+\; \delta\,|11\rangle\Big)$$
$$\;=\; \alpha\big(U\,|0\rangle\,\big)\,|0\rangle \;+\; \beta\big(U\,|0\rangle\,\big)\,|1\rangle \;+\; \gamma\big(U\,|1\rangle\,\big)\,|0\rangle \;+\; \delta\big(U\,|1\rangle\,\big)\,|1\rangle\;.$$

Replacing $U\,|0\rangle$ and $U\,|1\rangle$ with their expansions along the CBS (and noting that the matrix elements, U_{jk}, are the weights), we get

$$(U \otimes \mathbb{1})\,|\psi\rangle^2$$
$$=\; \alpha\big(U_{00}\,|0\rangle + U_{10}\,|1\rangle\,\big)\,|0\rangle \;+\; \beta\big(U_{00}\,|0\rangle + U_{10}\,|1\rangle\,\big)\,|1\rangle$$
$$+\; \gamma\big(U_{01}\,|0\rangle + U_{11}\,|1\rangle\,\big)\,|0\rangle \;+\; \delta\big(U_{01}\,|0\rangle + U_{11}\,|1\rangle\,\big)\,|1\rangle\;.$$

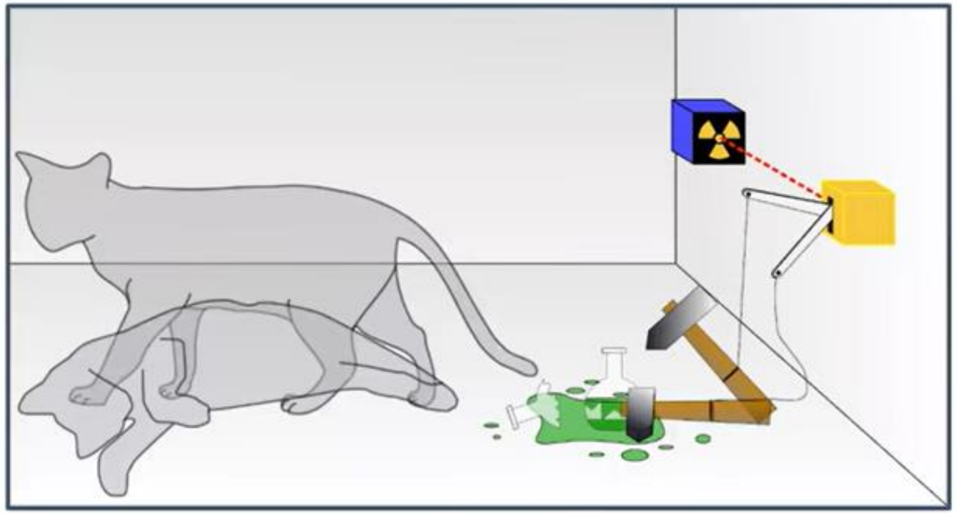
Now, distribute and collect terms for the four CBS tensor basis, to see that

$$(U \otimes \mathbb{1})\,|\psi\rangle^2 \;=\; \big(\alpha U_{00} + \gamma U_{01}\big)\,|00\rangle \;+\; \cdots$$

This notation is also known as the bra-ket notation or Dirac notation

Schrodinger's "Ket"

How would you write the state of Schrodinger's cat as a ket?



Visualizing superpositions

Another issue with our arrow notation is that it makes it very **hard to distinguish superpositions** or **visualize** anything the qubits are doing.

Ket notation can help with superpositions, but require some math first...

Superposition of:

50% $|1\rangle$ **50%** $|0\rangle$

versus

Superposition of:

10% $|1\rangle$ **90%** $|0\rangle$

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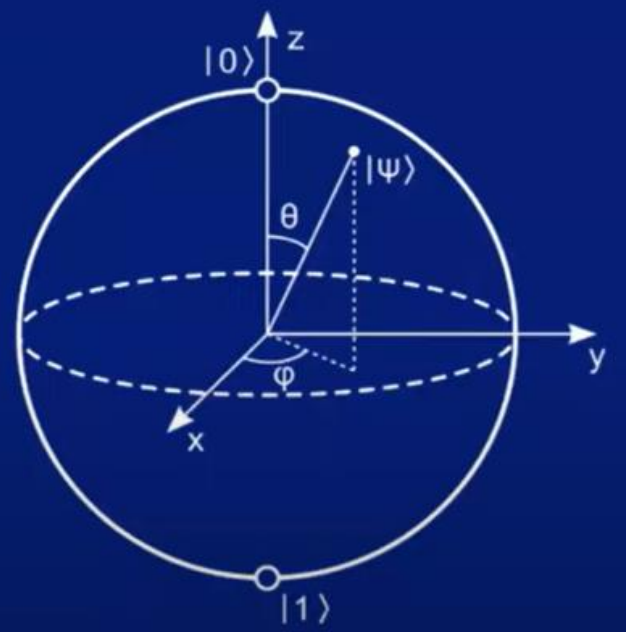
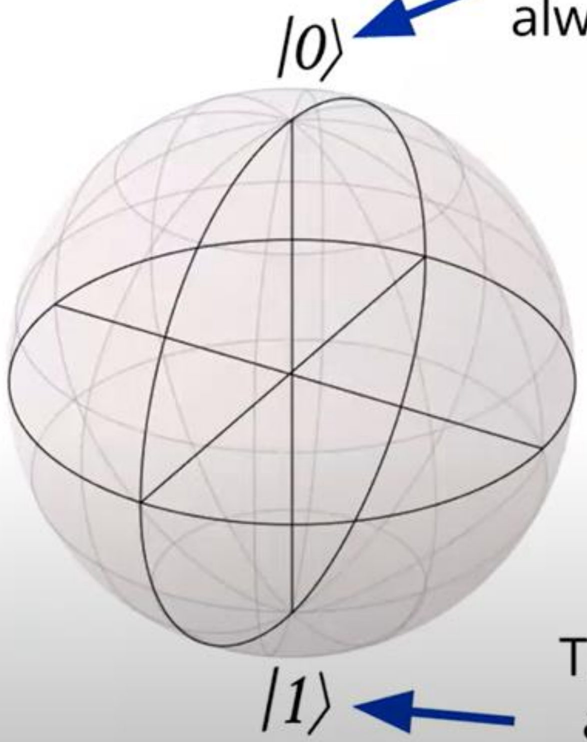
The Bloch sphere lets us represent all possible qubit states.



Kind of like how a globe lets us represent all possible places on Earth!

Landmarks on the Bloch sphere

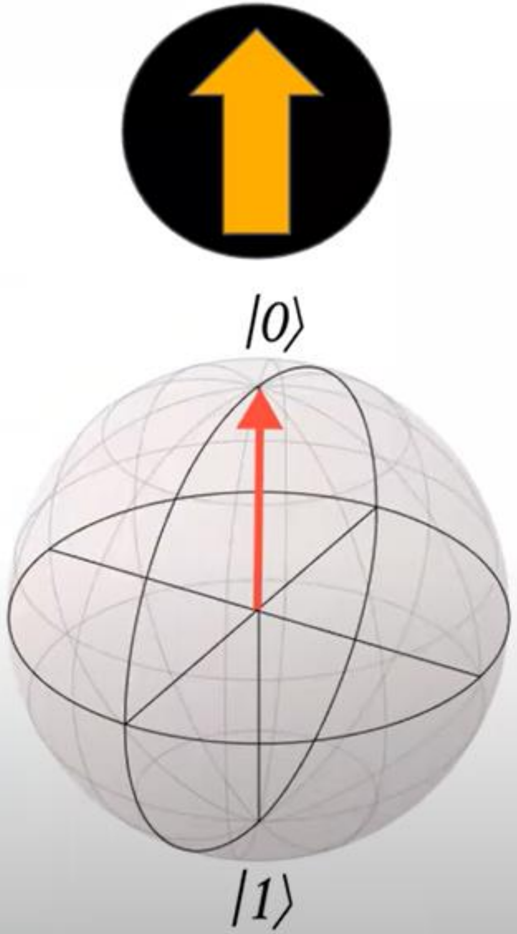
Just like we use the North and South pole of the Earth to orient ourselves, we can use the |0> and |1> states to orient a qubit on the Bloch Sphere.



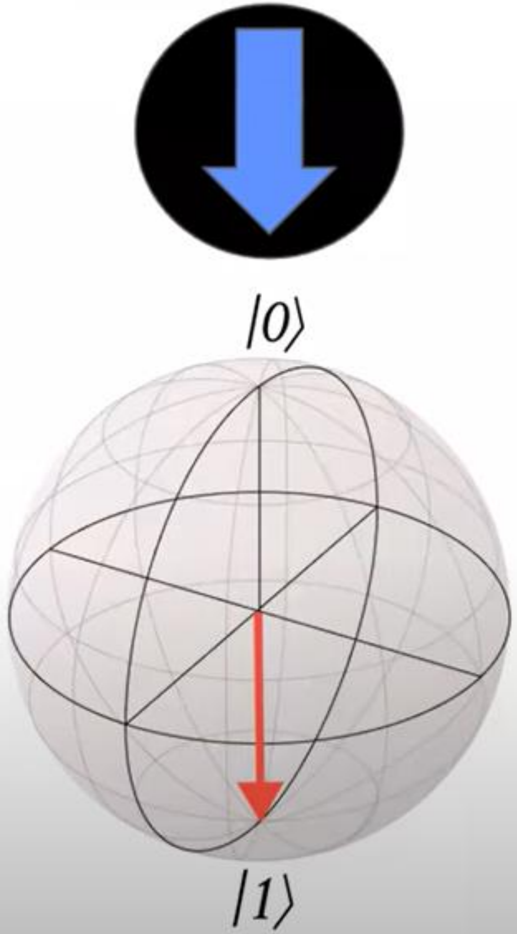
The |0> state is always on the top

The |1> state is always on the bottom

Locating a qubit on the Bloch sphere

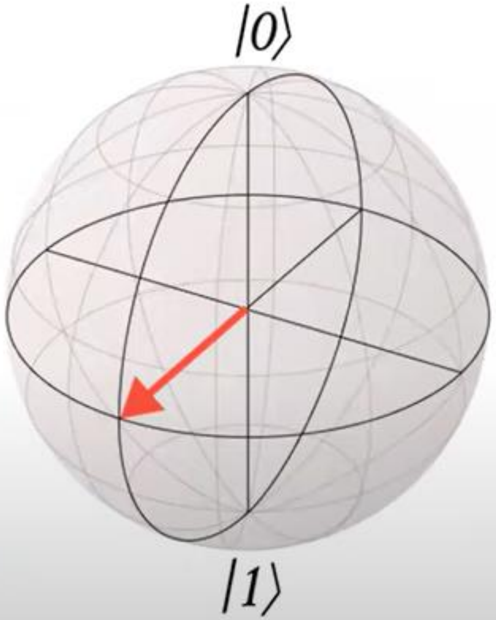


We draw an arrow from the center to the state the qubit is in.

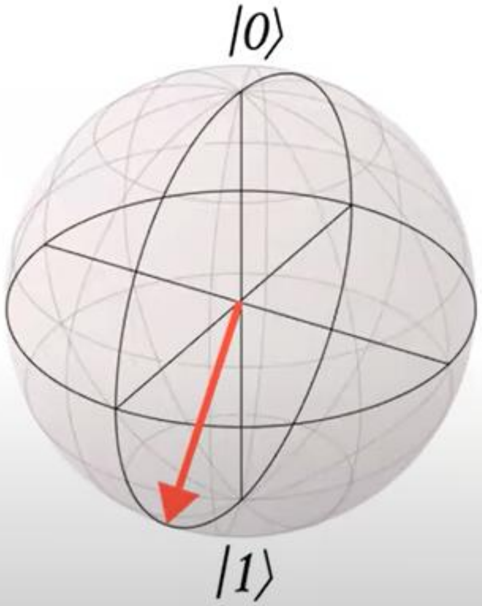


Locating a qubit on the Bloch sphere

Then what are these states?

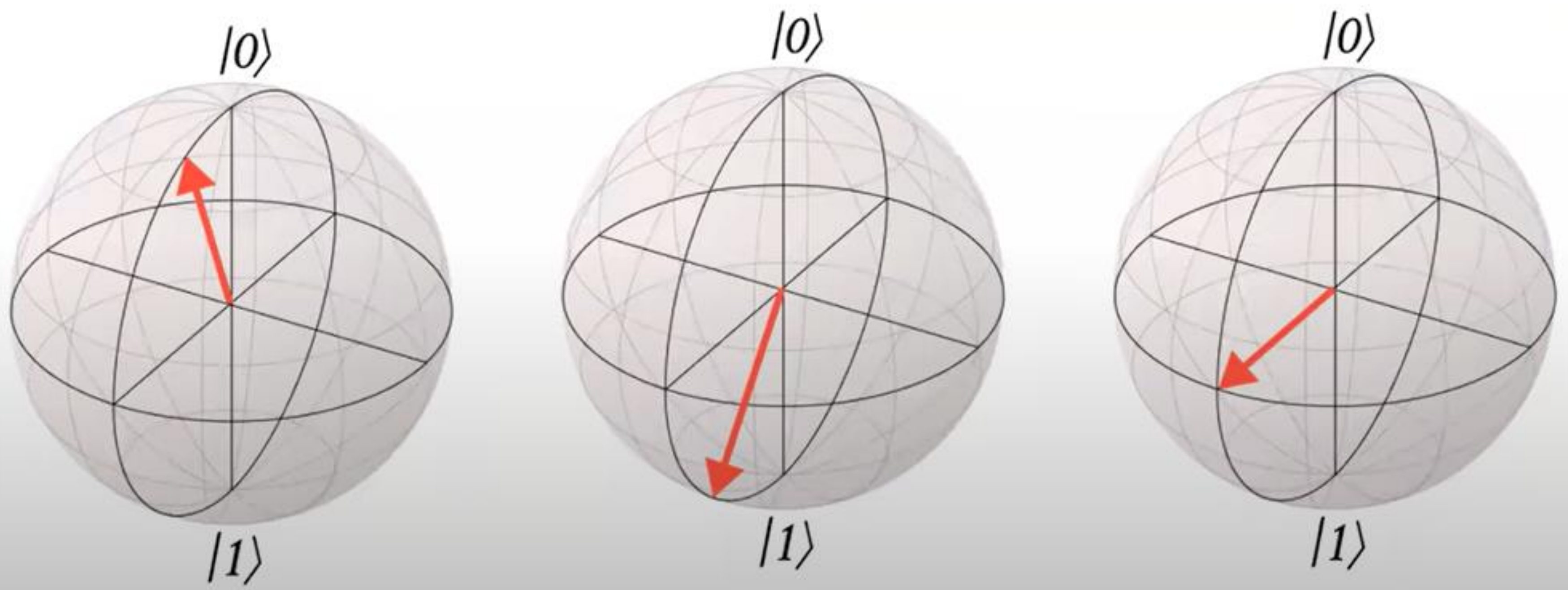


Anything that is not |0> or |1> is a **superposition**.



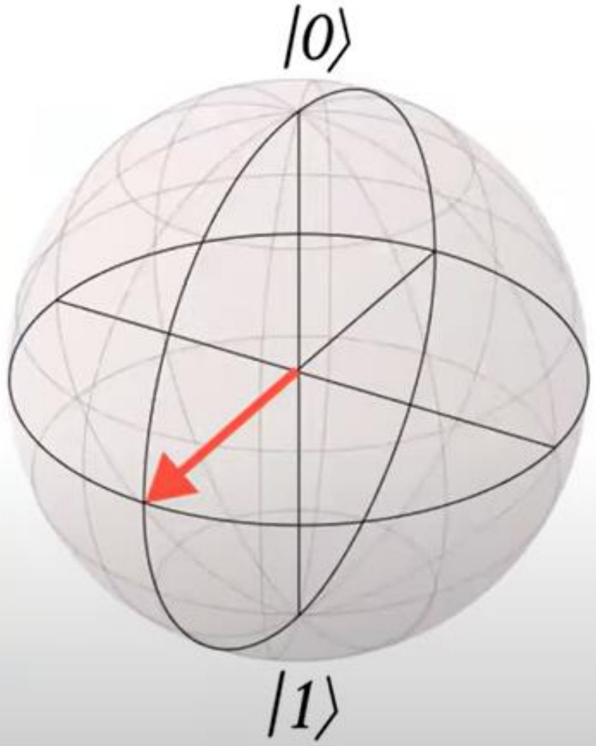
Possible superpositions

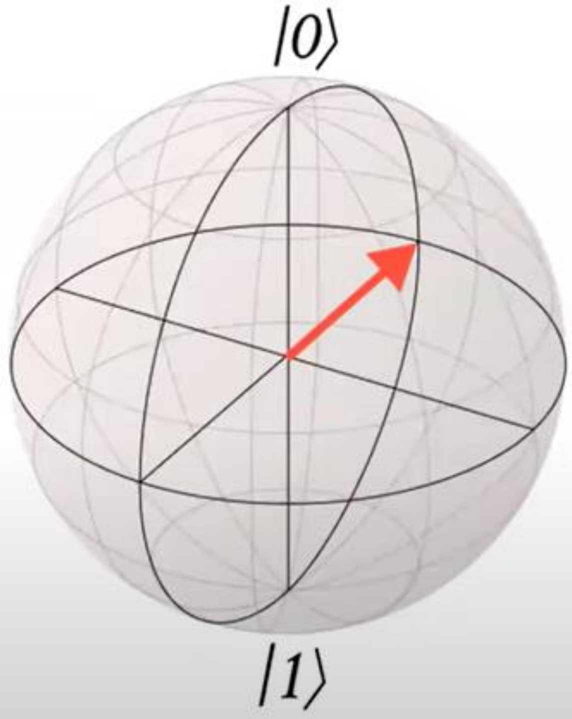
There are technically an infinite number of possible superpositions (points on the Bloch sphere).



Equal superposition

Equal superposition: 50% |0> and 50% |1>.

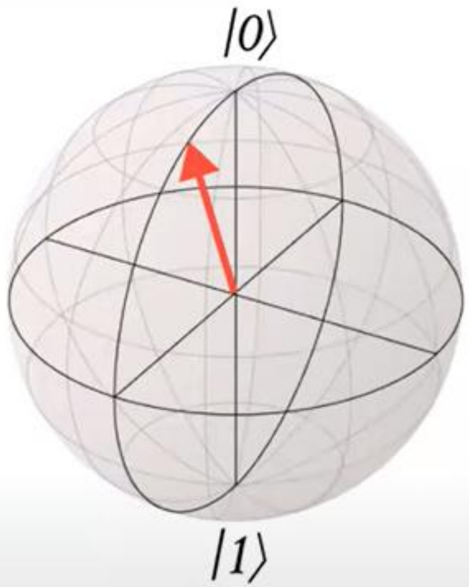




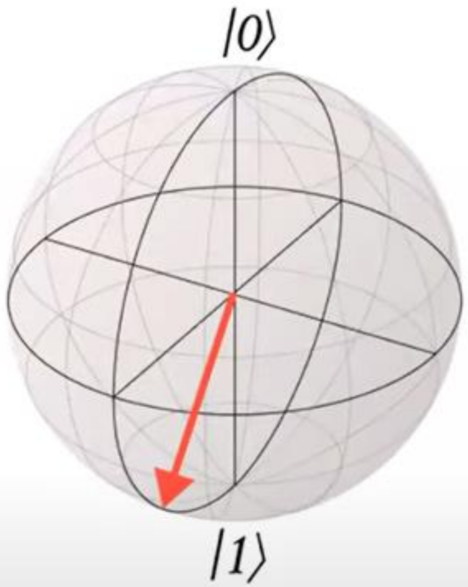
Plus state: $|+\rangle$

Minus state: $|-\rangle$

Unequal superposition examples



This state has a bigger contribution of |0> than |1>



This state has a bigger contribution of |1> than |0>

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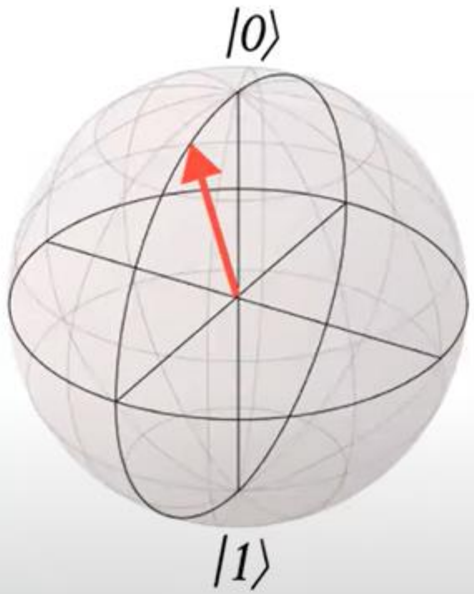
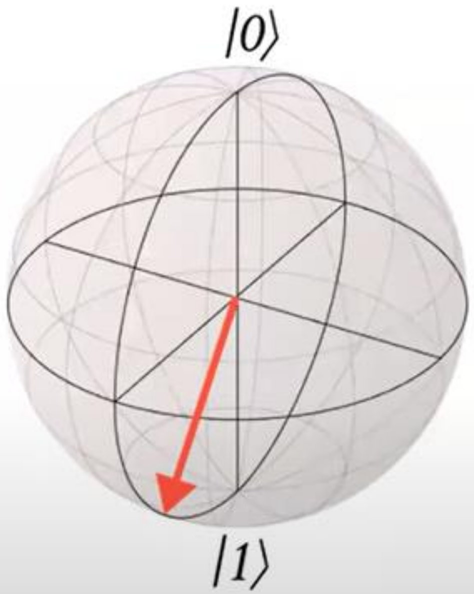
Unequal superposition examples

Unequal superpositions are like an unfair coin.





They're more likely to give us a certain value at measurement.

Practice

Which Bloch sphere shows a state that has a 20% contribution of |0> and a 80% contribution of |1> ?

a b c d

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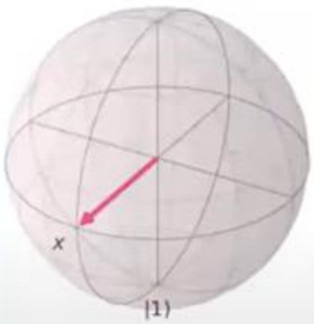
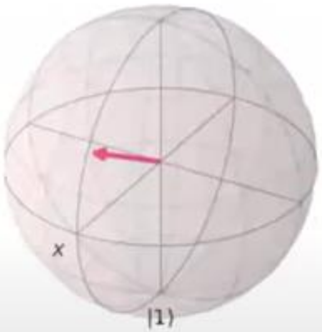
Practice

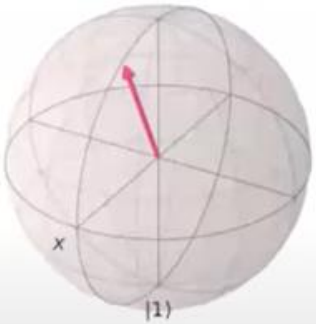
Which Bloch sphere shows a state that has a 70% contribution of |0> and a 30% contribution of |1> ?



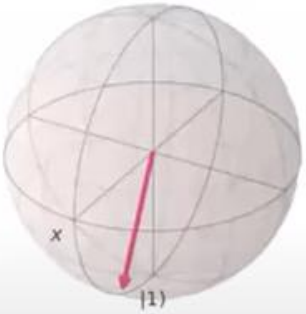
a



b



c



d

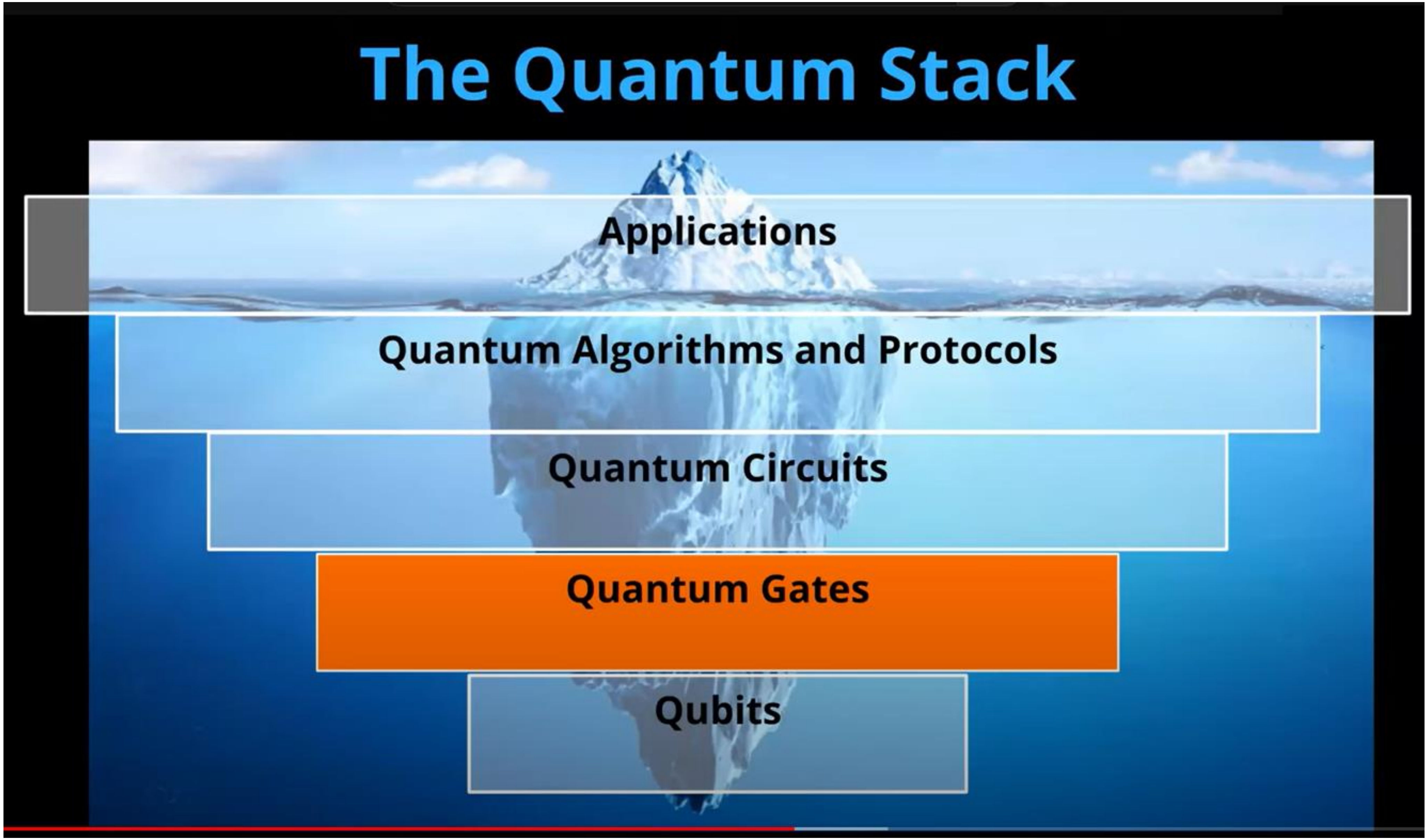
Review

Representing Qubits

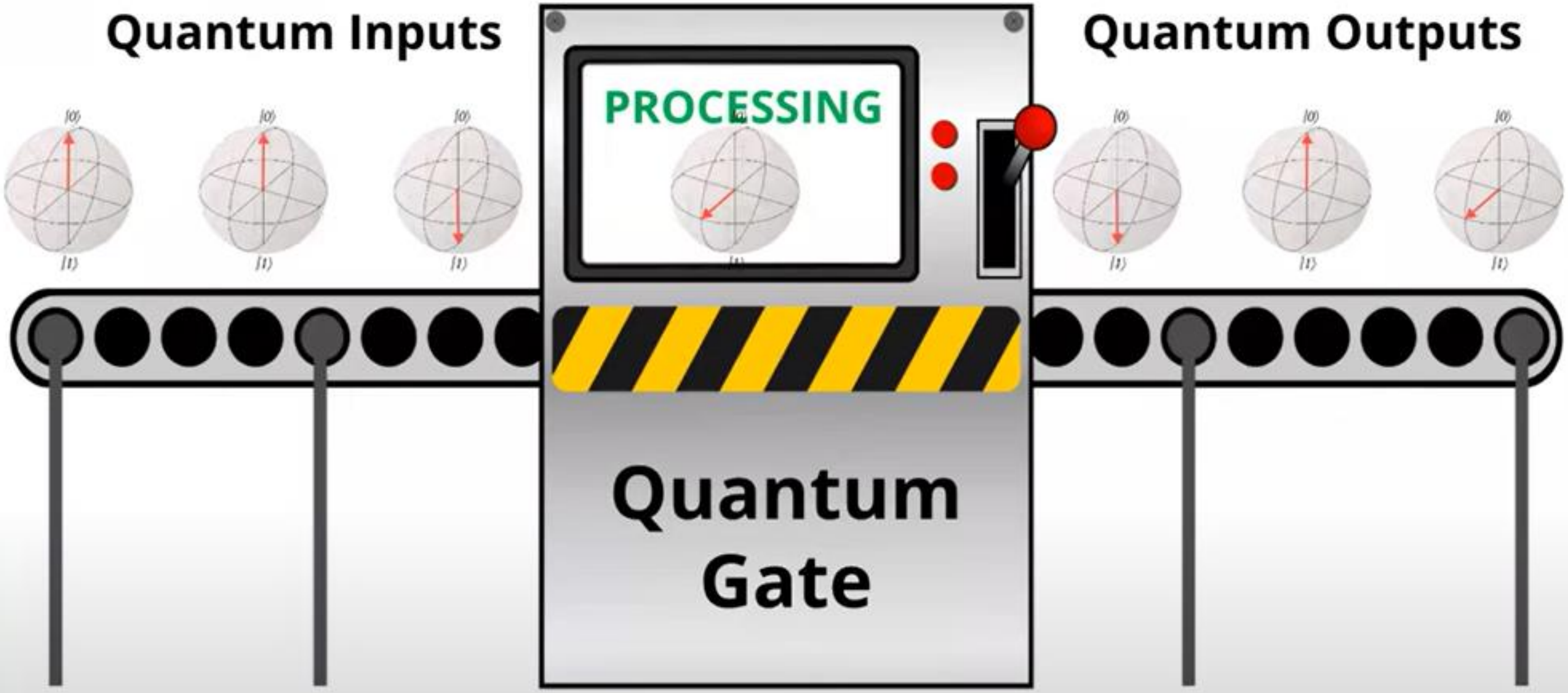
Quantum Gates

Quantum Circuits

Quantum Gates


The Quantum Stack
Applications
Quantum Algorithms and Protocols
Quantum Circuits
Quantum Gates
Qubits

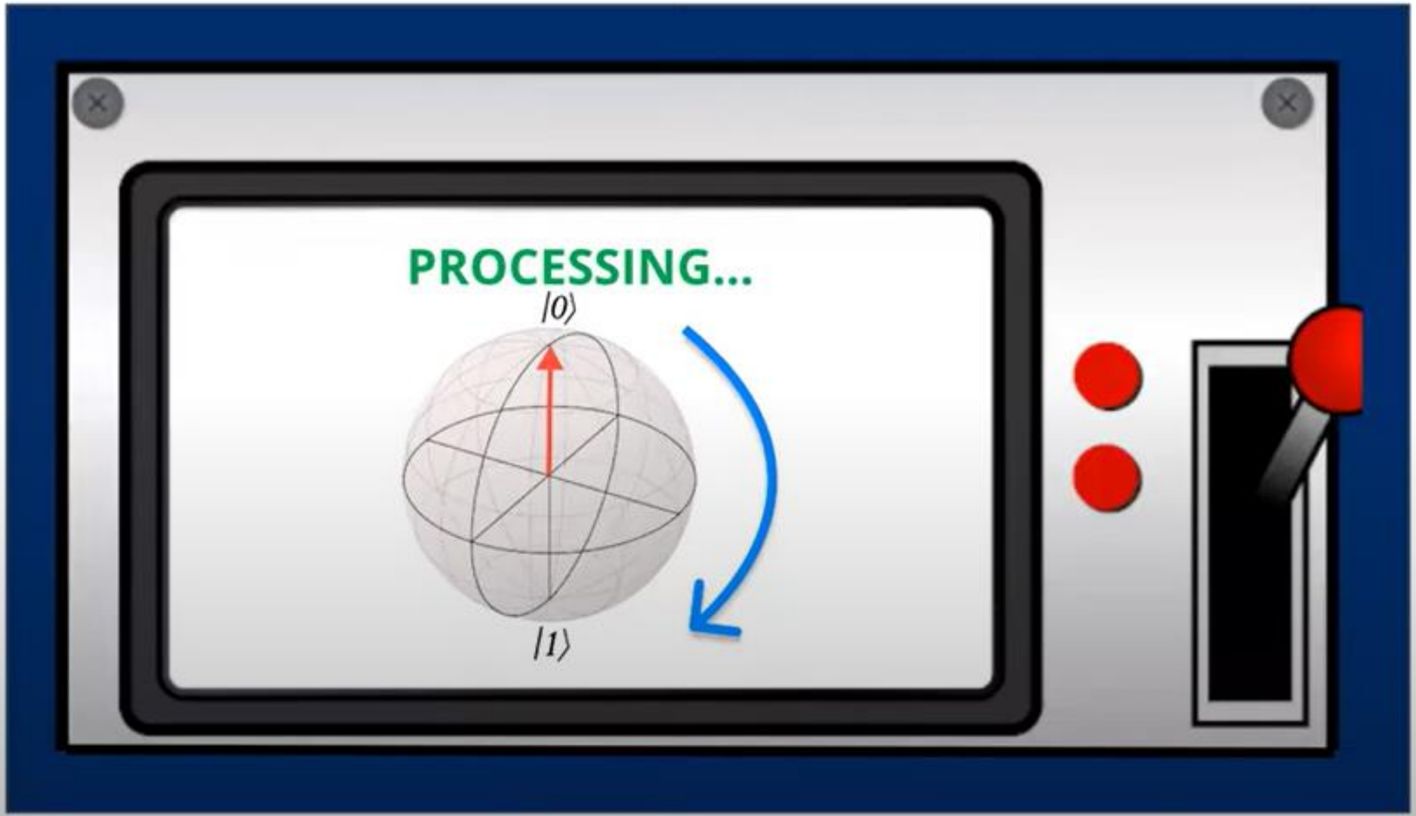
Quantum Gates



Much like how classical gates manipulate classical bits, **quantum gates** manipulate **qubits**.

Quantum Gates = Rotations

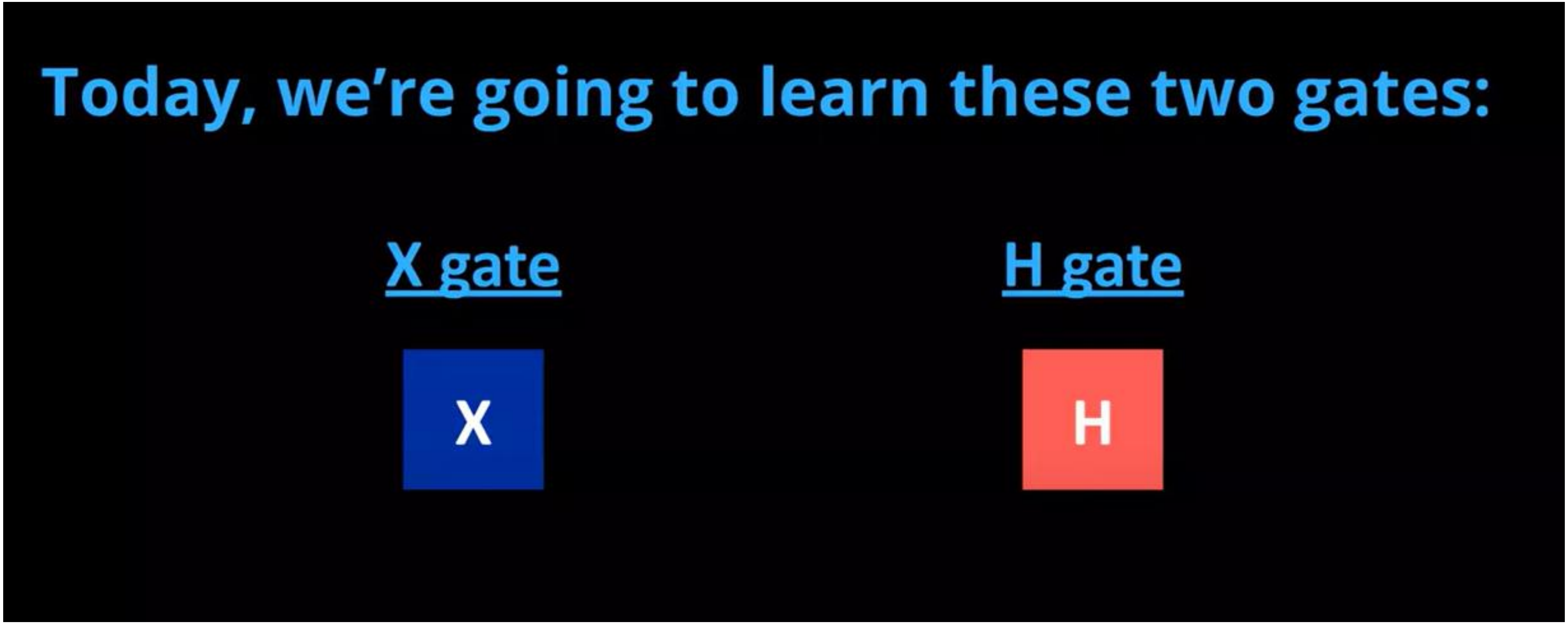
Quantum gates rotate on the Bloch sphere.



Quantum Gates = Rotations

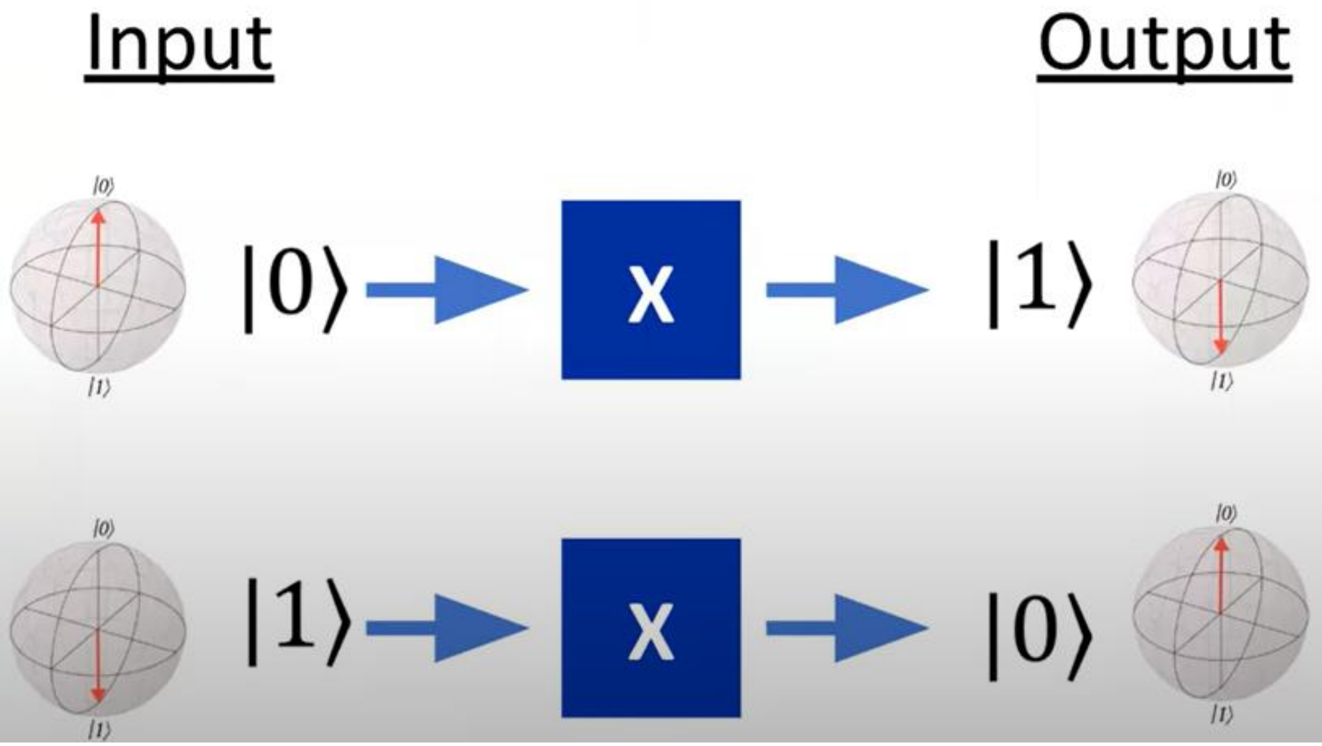
There are many quantum gates. Every one changes qubit states differently.

Every gate performs a different kind of rotation on the Bloch sphere.


Today, we're going to learn these two gates:
X gate
X
H gate
H

X Gate

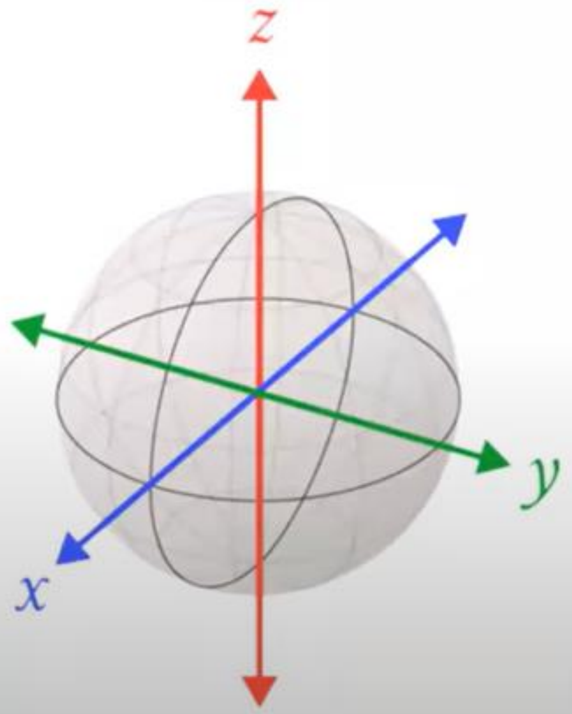
Rule: the X gate gives the opposite of the input

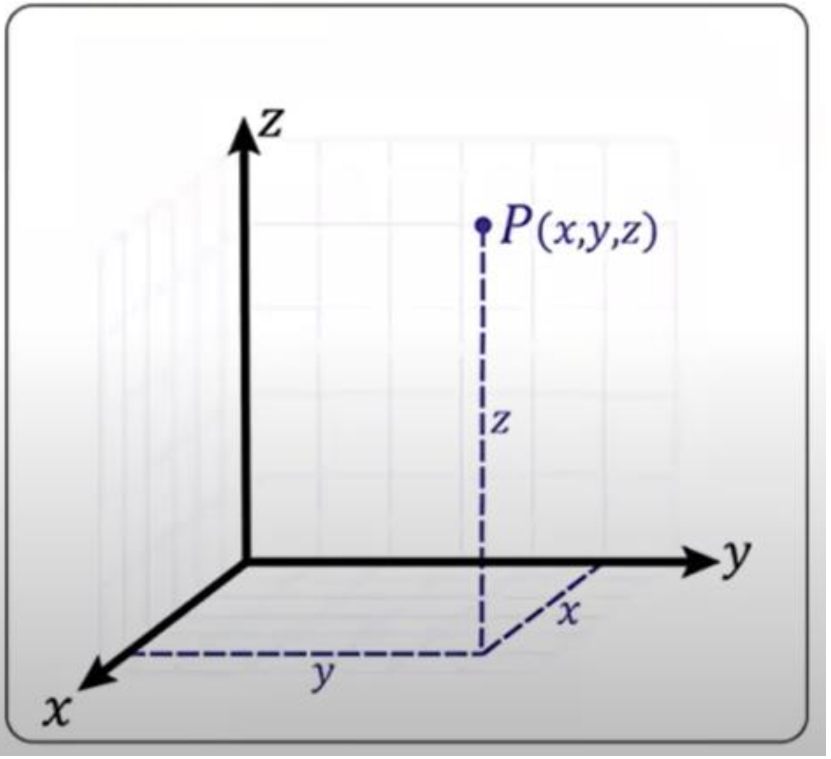


Bloch sphere coordinates: X, Y, Z

We can place the Bloch sphere on a coordinate system: X, Y, Z.
Any point can be described by 3 numbers: (x, y, z)

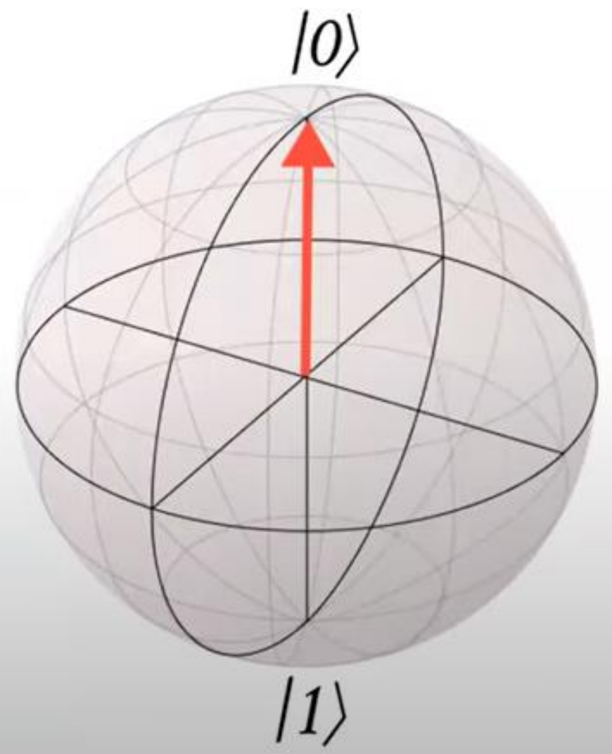
The z-axis runs vertically.
The x- and y-axes are coming in and out of the page.

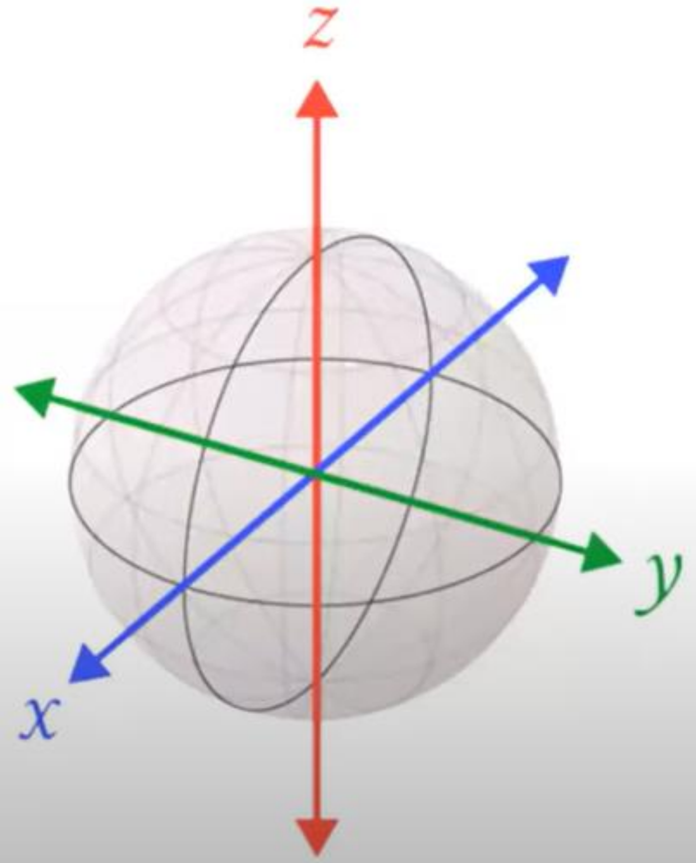




Bloch sphere coordinates: X, Y, Z

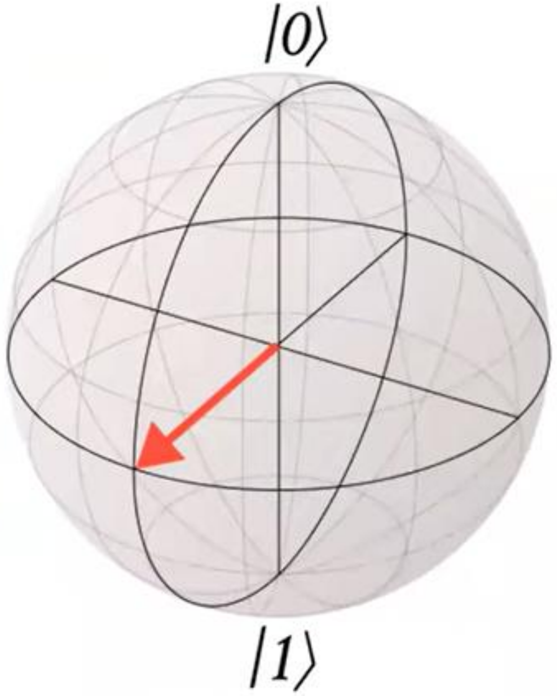
What axis does the $|0\rangle$ state lie along?

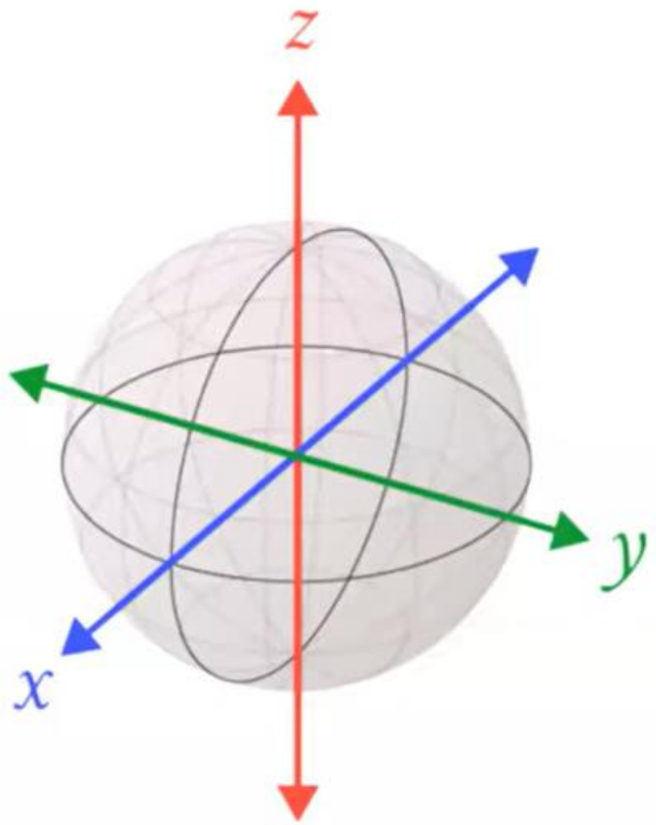




Bloch sphere coordinates: X, Y, Z

What about this state?





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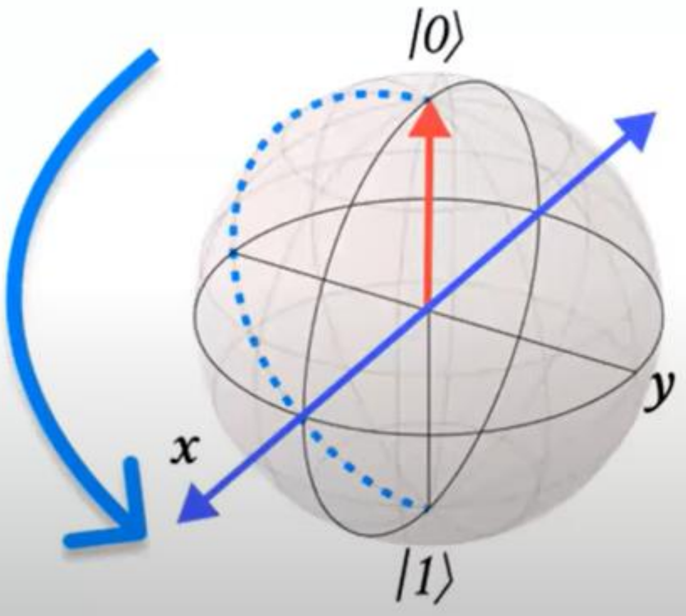
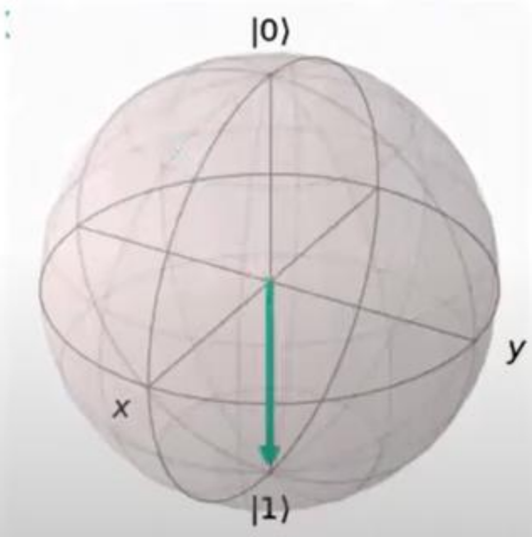
Why is it called an X Gate?

The X gate does a **180° rotation around the X axis**.

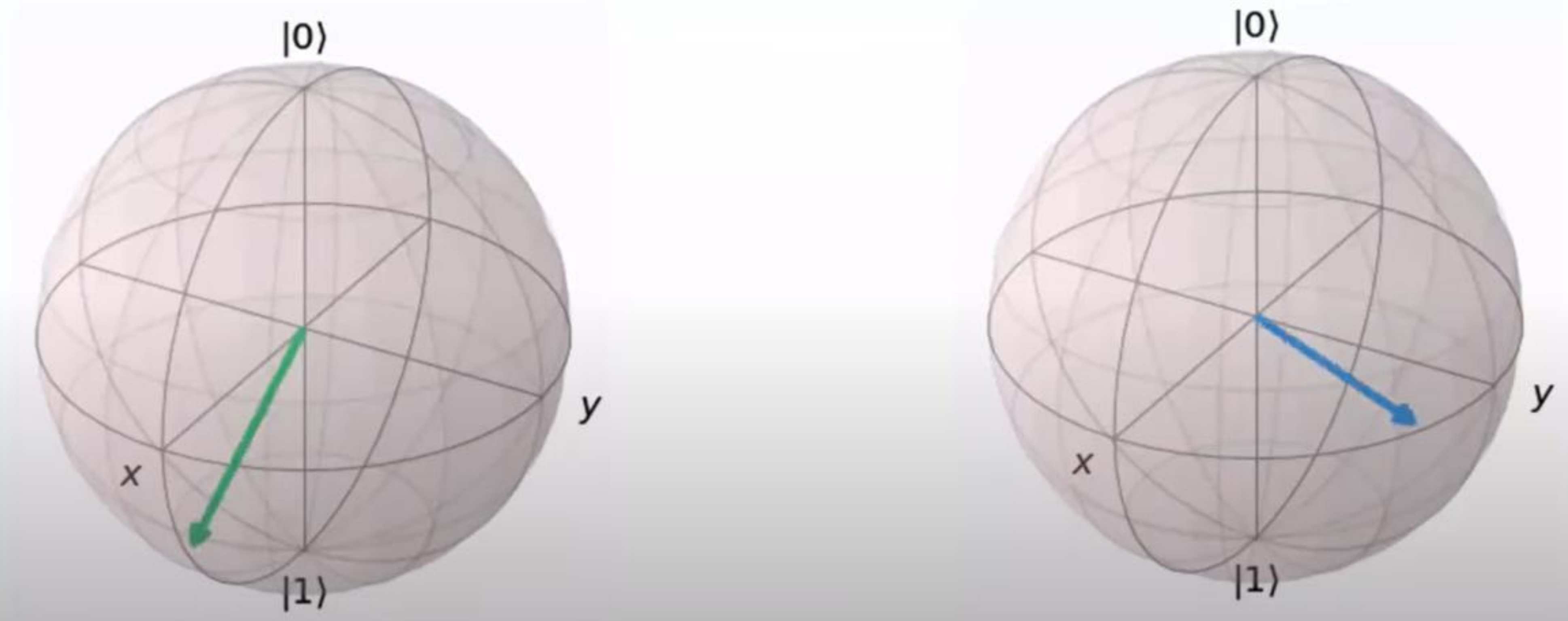


In order for $|0\rangle$ to turn to $|1\rangle$ and vice versa, the state would have to rotate around the x axis.



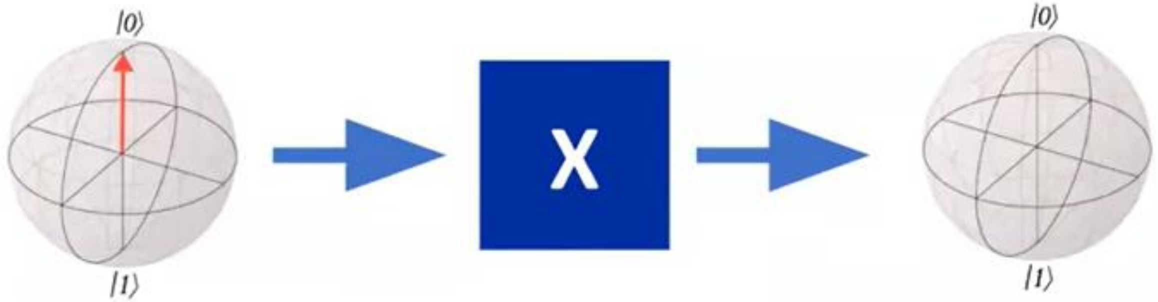
Y and Z Gates

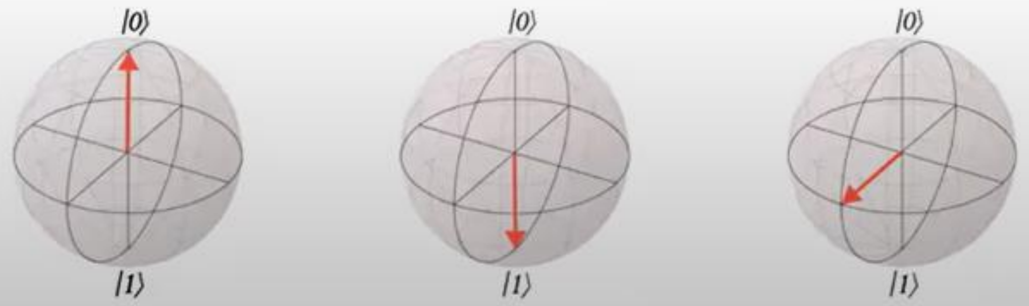
We won't look at these more now, but this means we also have Y and Z Gates.



Practice with the X Gate

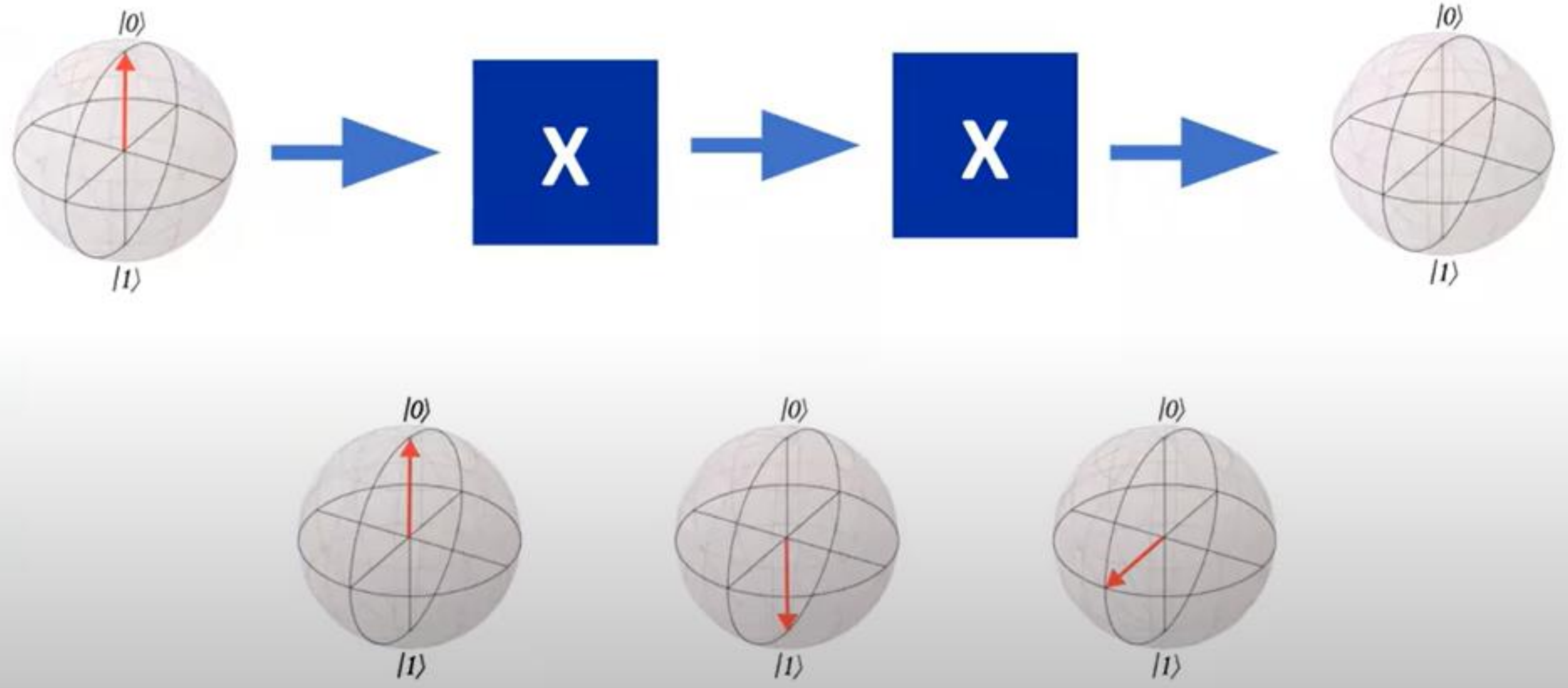
What do we get when we use the following gate?





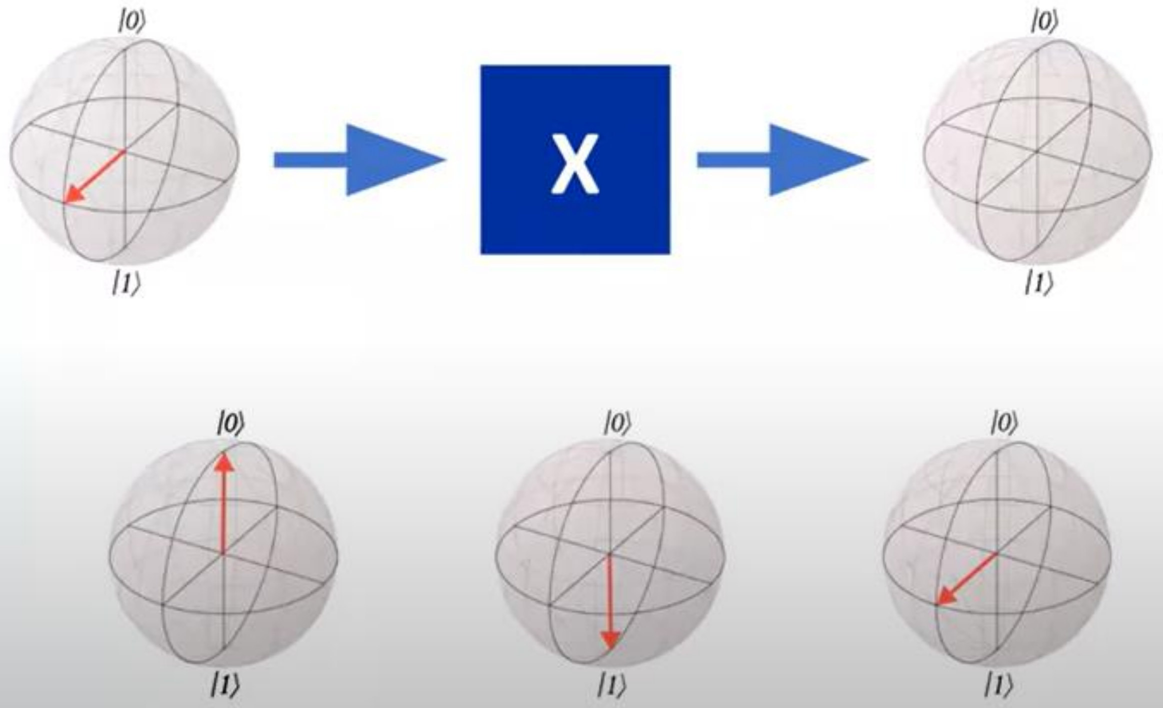
Practice with the X Gate

What do we get when we use the following gates?



Practice with the X Gate

What do we get when we use the following gates?




Today, we're going to learn these two gates:
X gate
X
H gate
H

H Gate: Hadamard Gate

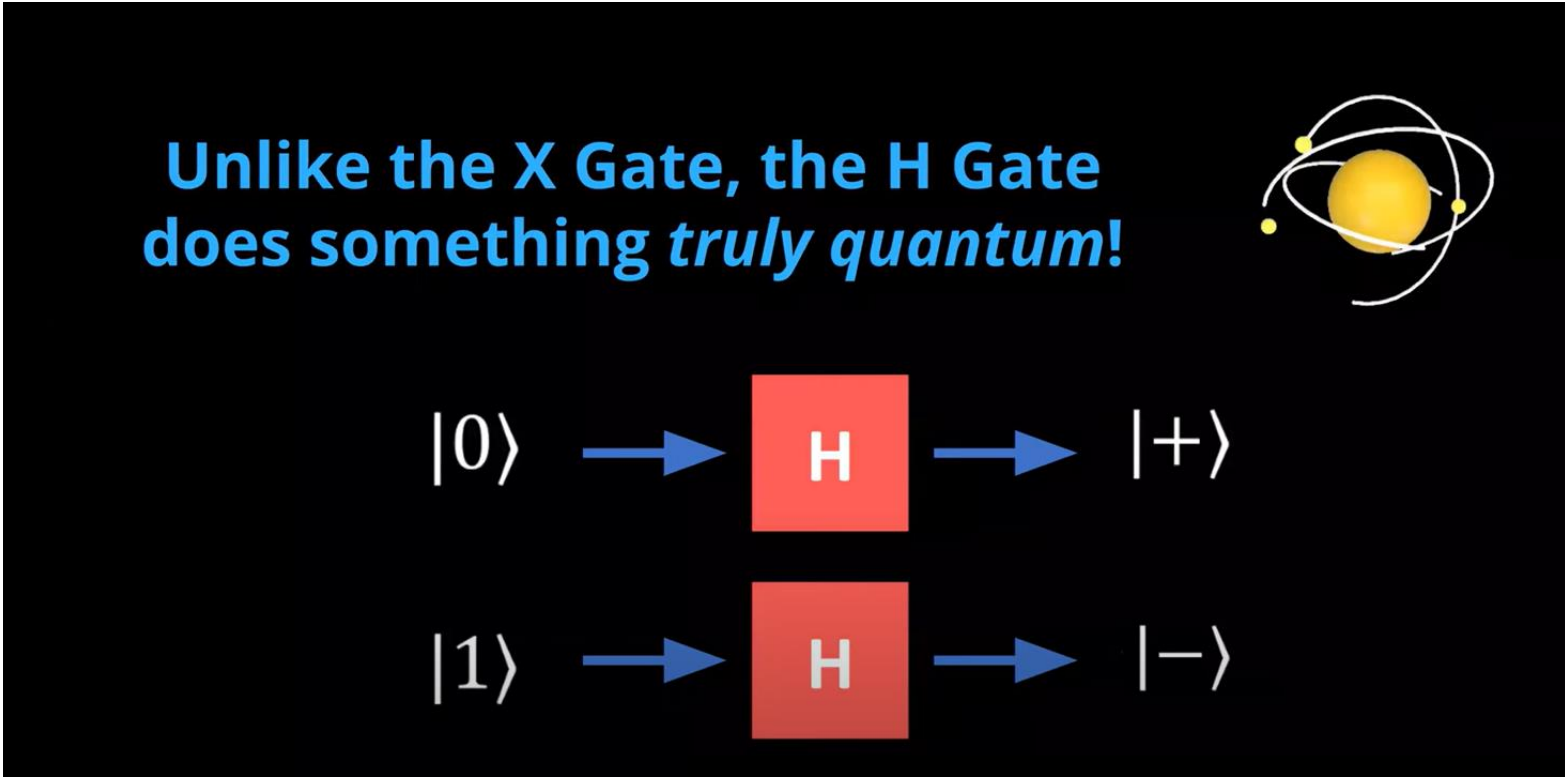
Rule: the H gate creates superposition

$|0\rangle \rightarrow \boxed{H} \rightarrow |+\rangle$

Applying the H gate to the $|0\rangle$ state creates the $|+\rangle$ state.

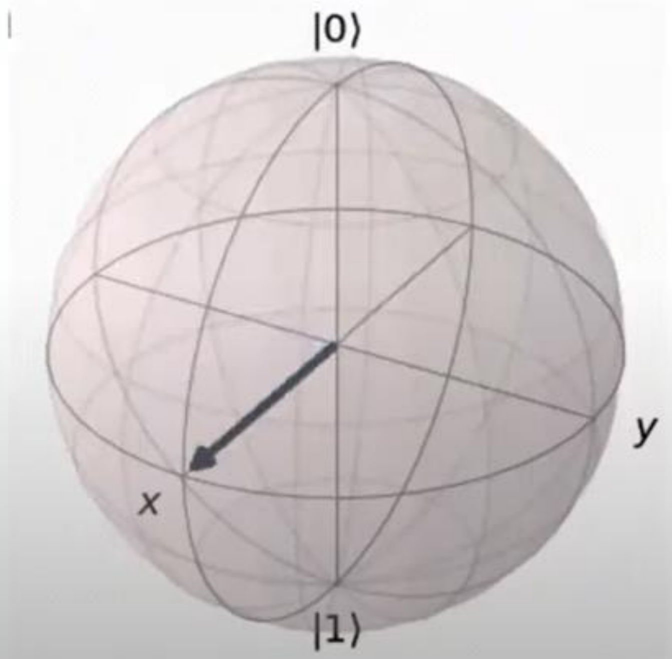
$|1\rangle \rightarrow \boxed{H} \rightarrow |-\rangle$

Applying the H gate to the $|1\rangle$ state creates the $|-\rangle$ state.


Unlike the X Gate, the H Gate
does something *truly quantum*!
$|0\rangle$ → H → $|+\rangle$
$|1\rangle$ → H → $|-\rangle$

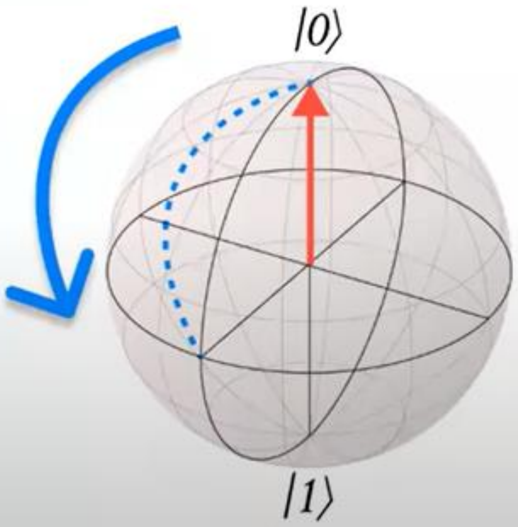
H Gate Rotation

Since this is a gate, it has its own unique rotation.
However, it's not just around the X, Y, or Z axis:



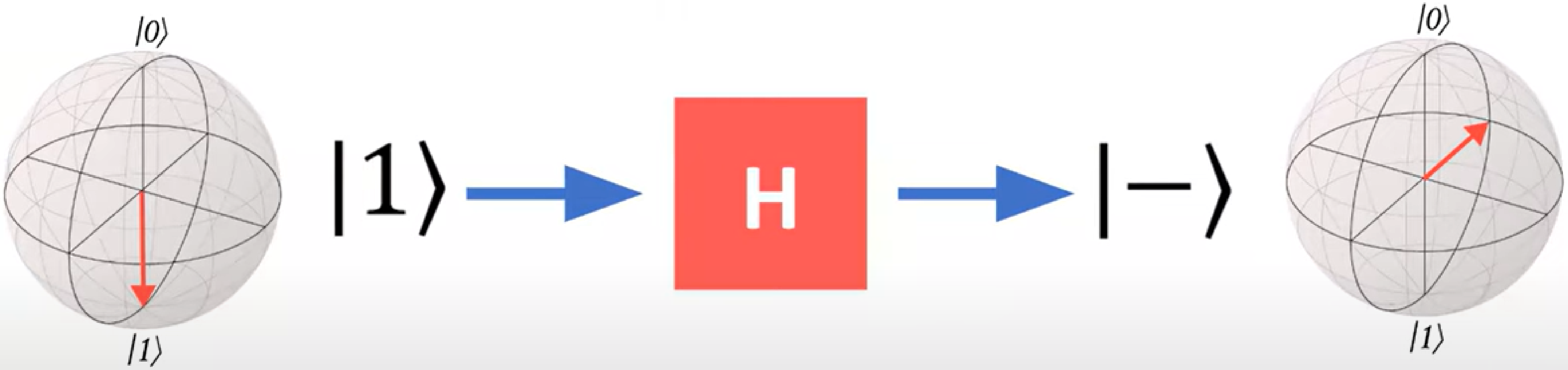
H Gate Rotation: starting from 0 state

The H gate rotates the $|0\rangle$ state to create an equal superposition of $|0\rangle$ and $|1\rangle$.



H Gate Rotation: starting from 1 state

When we start from $|1\rangle$, we get a different kind of superposition!

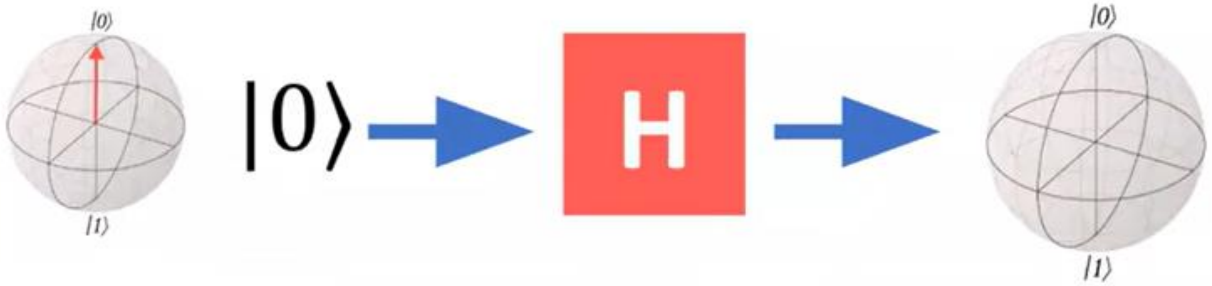


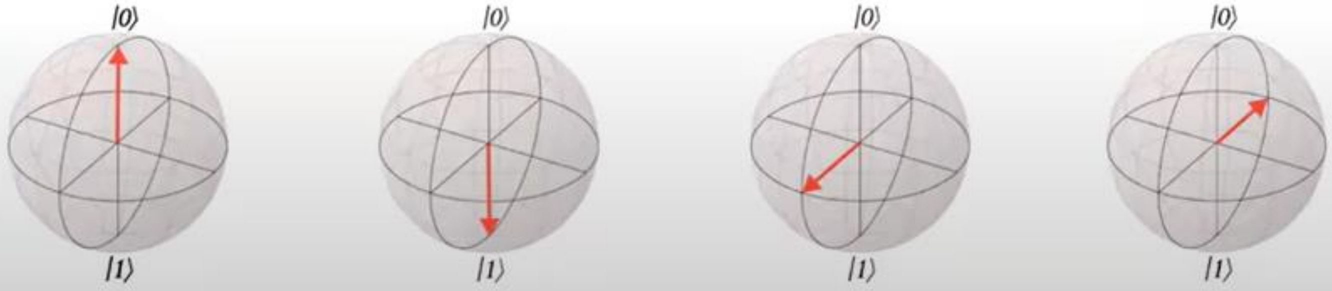
We use the notation $|-\rangle$ to represent this superposition state.


Let's Practice!

Practice with the H Gate

What do we get when we use the following gate?

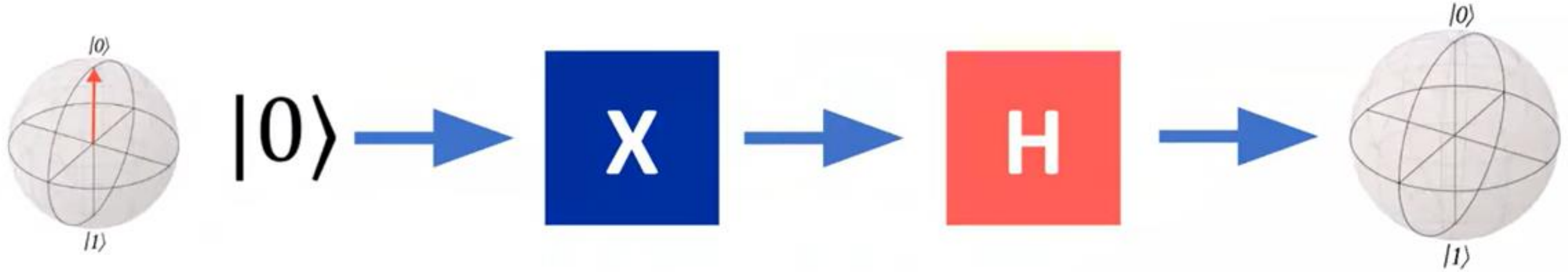


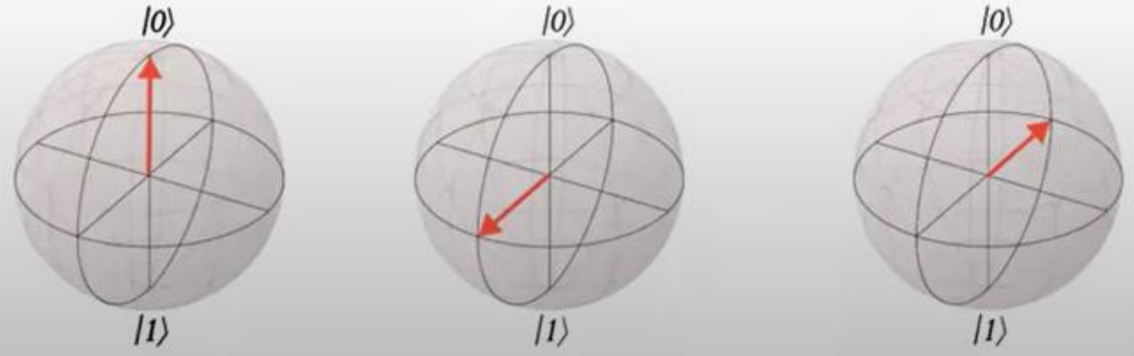


a b c d

Practice with the H Gate

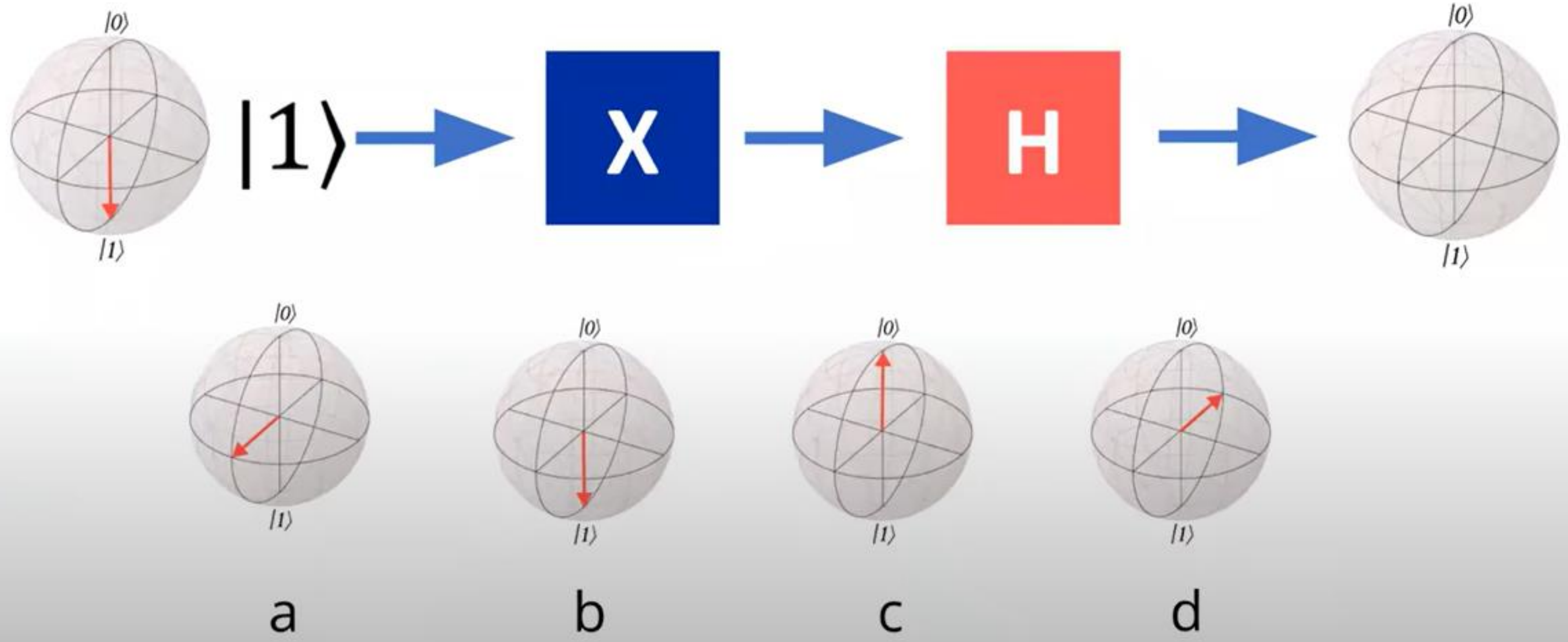
What do we get when we use the following gates?





Practice with the H Gate

What do we get when we use the following gates?



Some questions to ponder...

- What would be the result of applying the X gate to a qubit in the |+> state?
- How can we relate the |0> and |1> states with the |+> and |-> states?
- Does the order in which gates are applied matter? Is the final state the same if we first apply an X gate and then an H gate, and if we reverse the order of the gates?

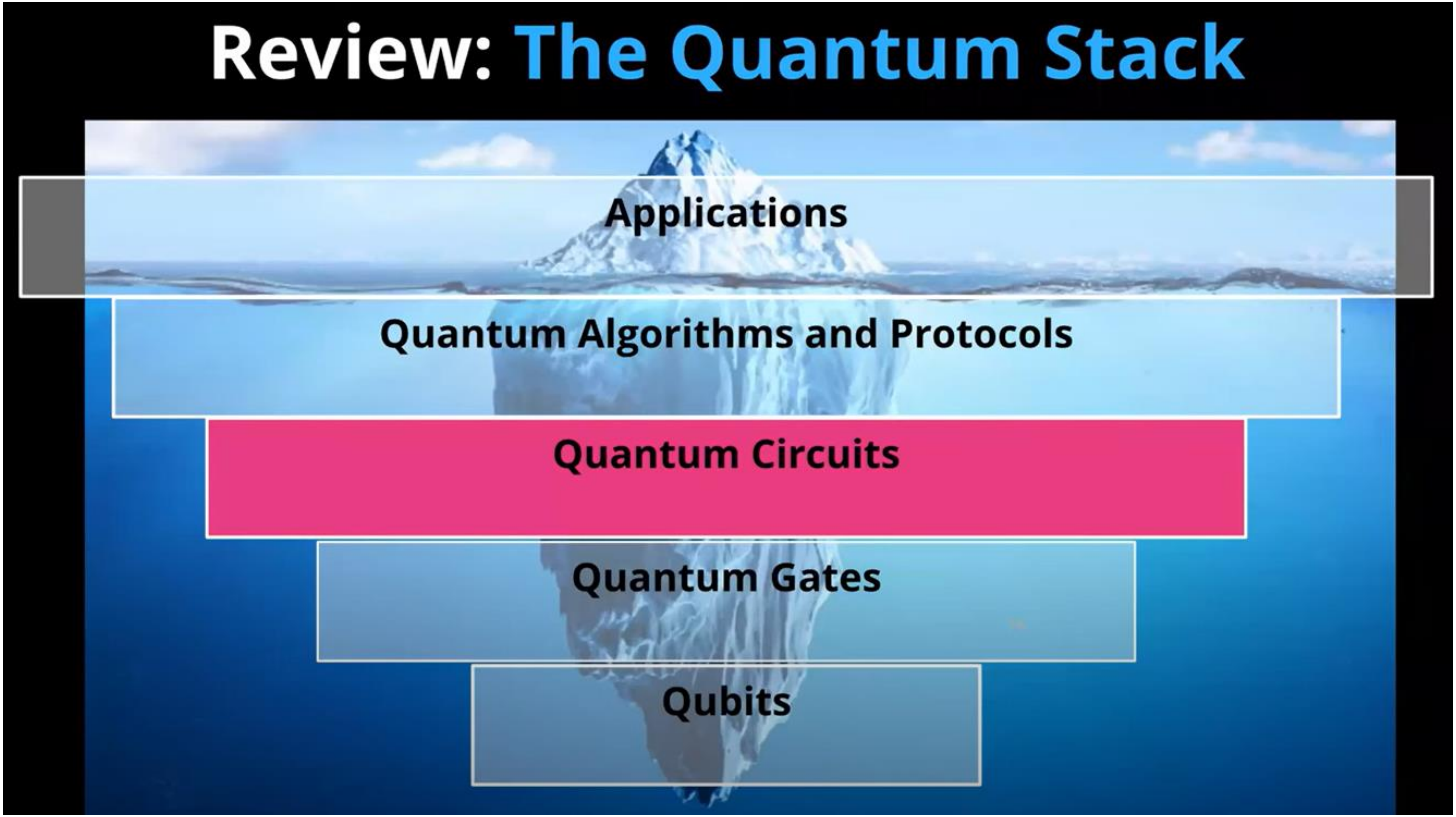
Review

Representing Qubits

Quantum Gates

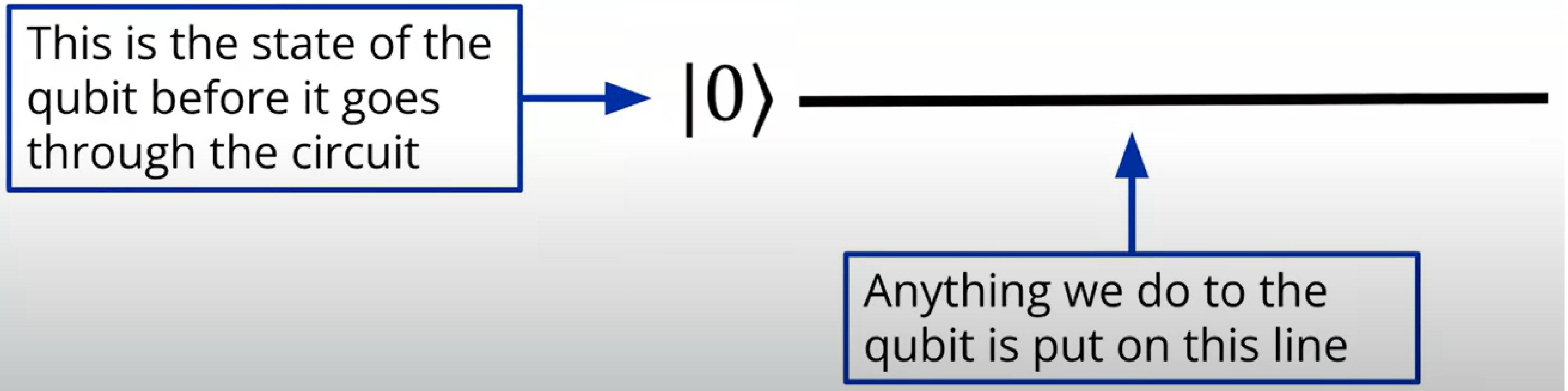
Quantum Circuits

Quantum Circuits


Review: The Quantum Stack
Applications
Quantum Algorithms and Protocols
Quantum Circuits
Quantum Gates
Qubits

Quantum Circuits

Here is how we draw a **quantum circuit**:



Quantum Circuits

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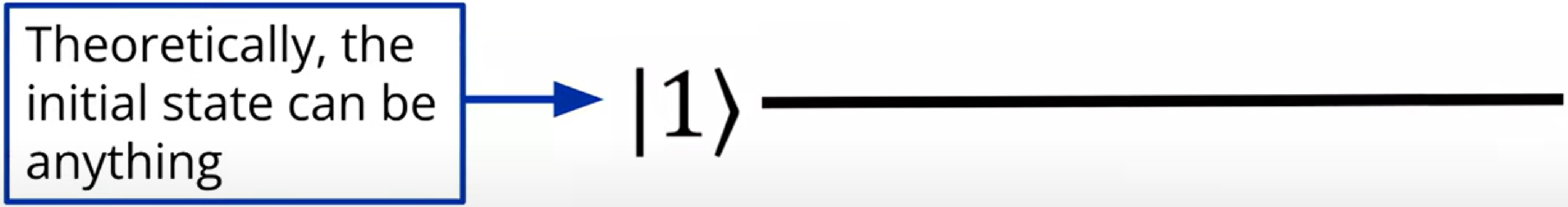

Quantum Circuits

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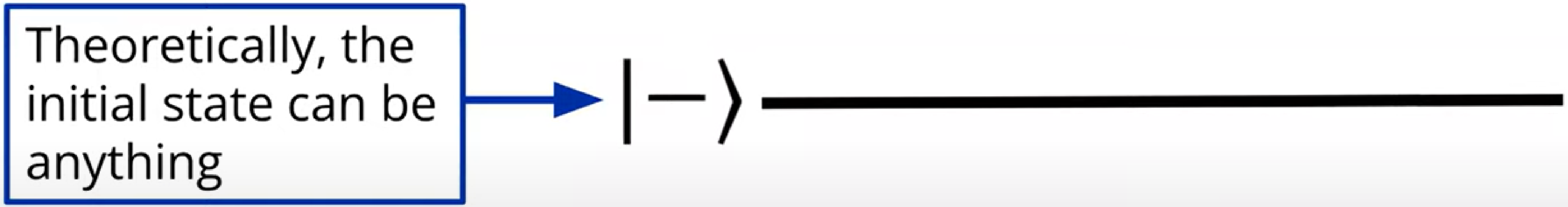

Quantum Circuits

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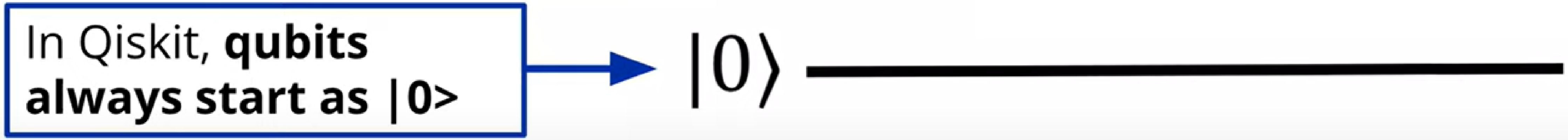


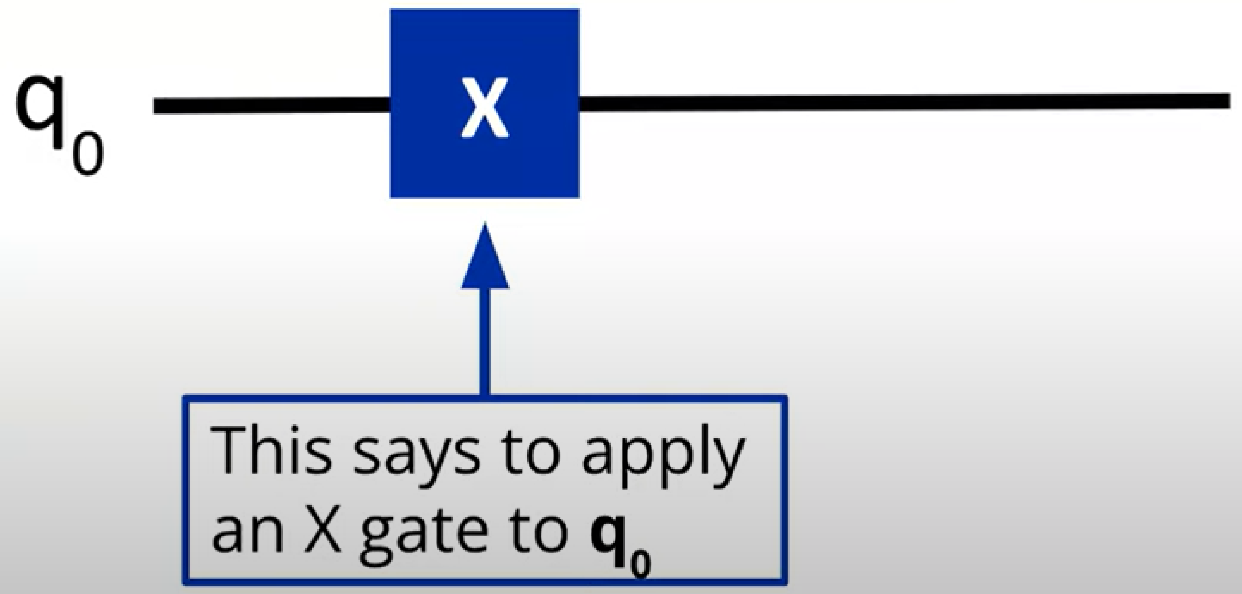
Quantum Circuits

Each qubit is given a label (or index) starting at 0. So the first qubit is $\mathbf{q_0}$, the second is $\mathbf{q_1}$, then there's $\mathbf{q_3}$, $\mathbf{q_4}$,...

NOTE: In Qiskit, qubits always start in the $|0\rangle$ state.

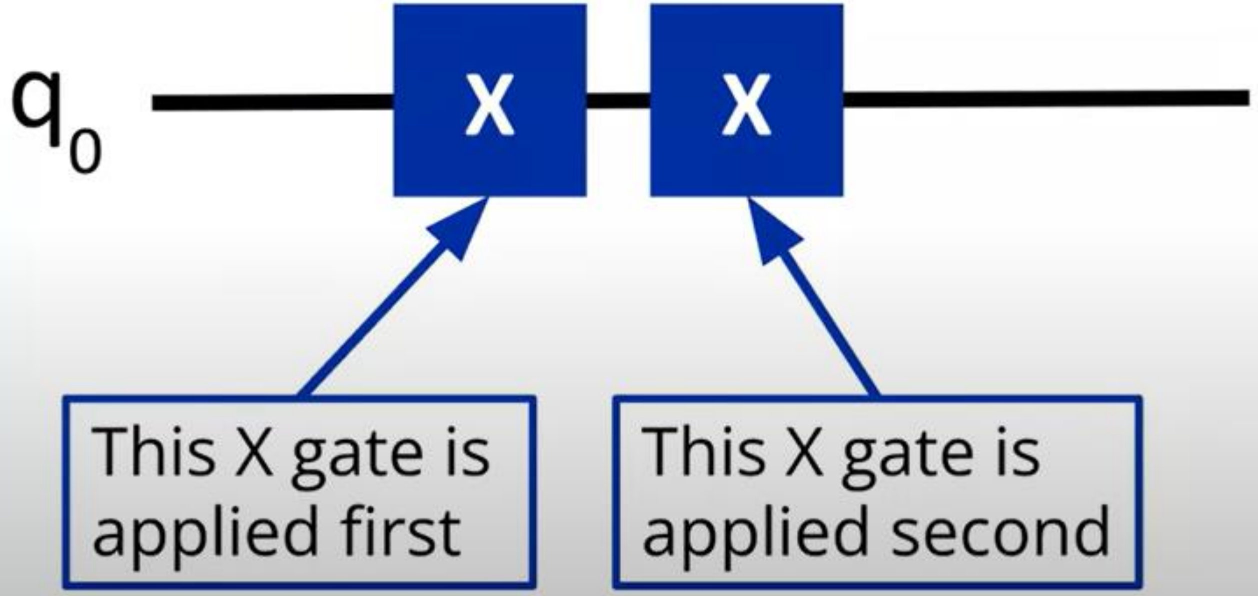
Quantum Circuits

Here is a one gate circuit acting on one qubit:



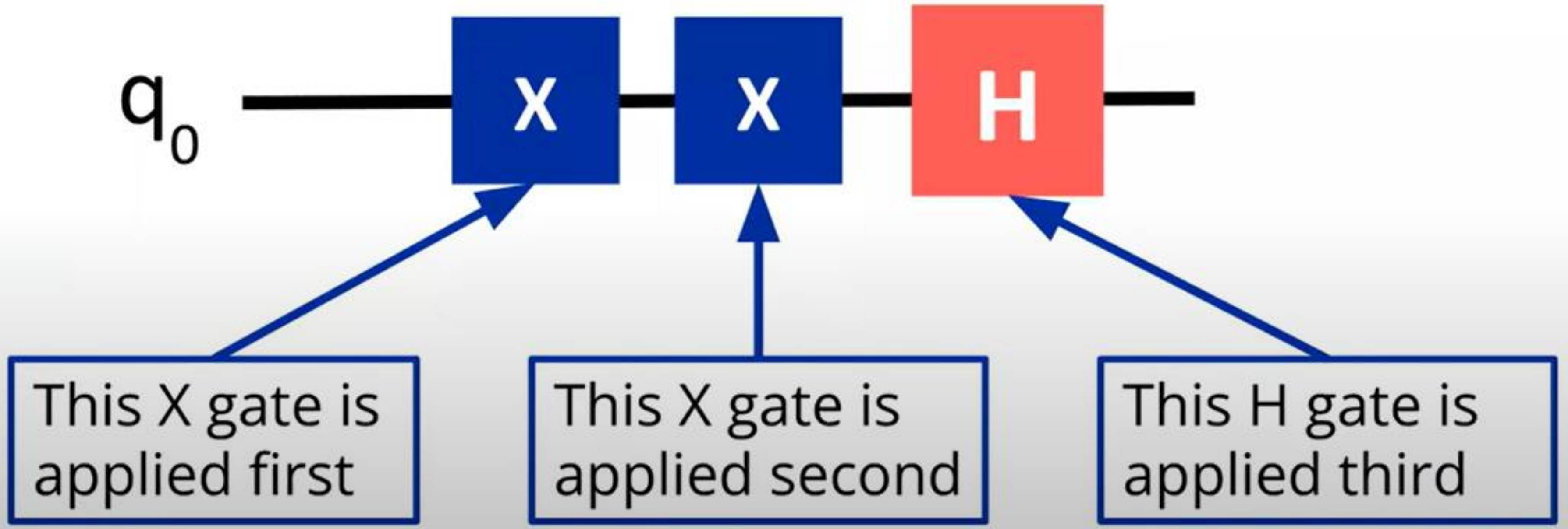
Quantum Circuits

We can have as **many gates** as we want. We apply them one at a time from left to right:



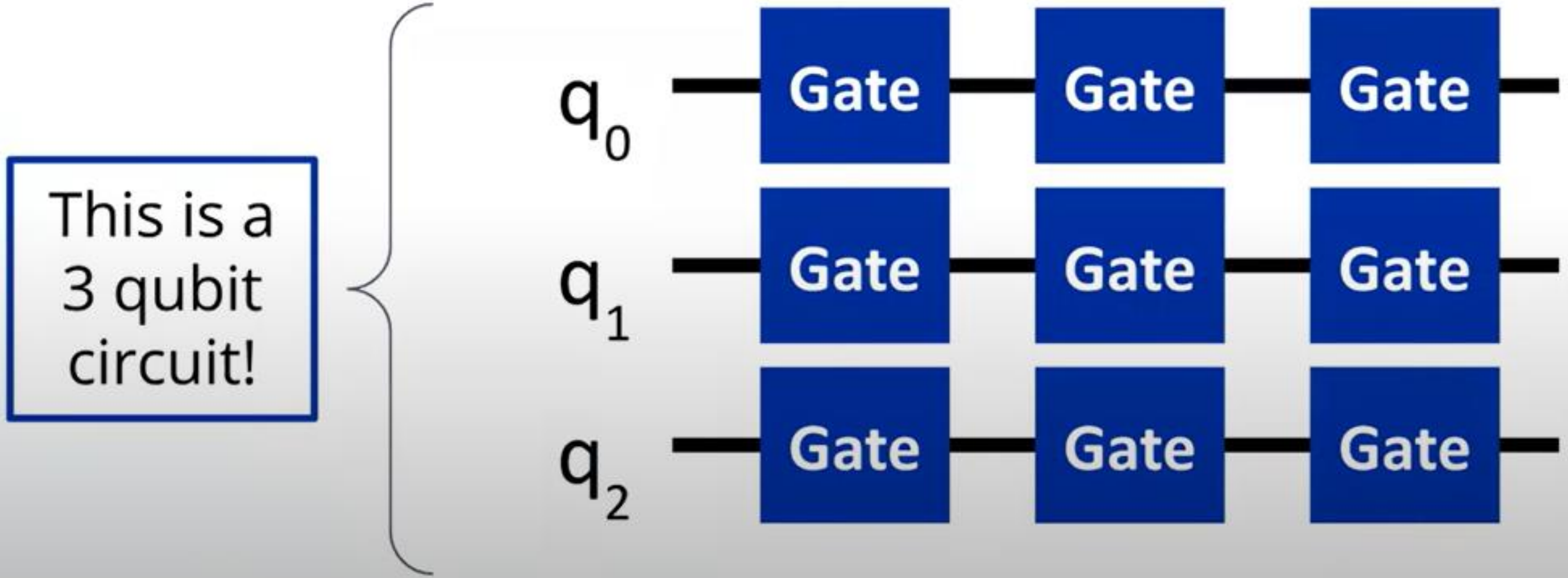
Quantum Circuits

We can have as **many gates** as we want. We apply them one at a time from left to right:



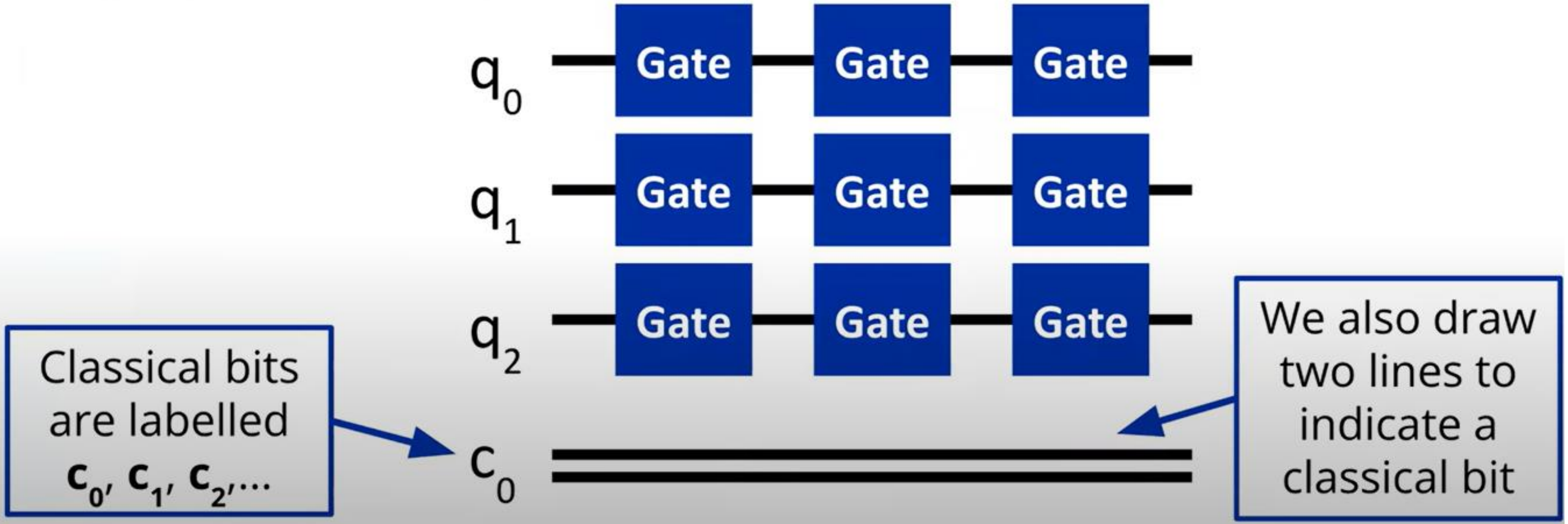
Quantum Circuits

We can also have as **many qubits** as we want, by adding more qubits with their own lines below:



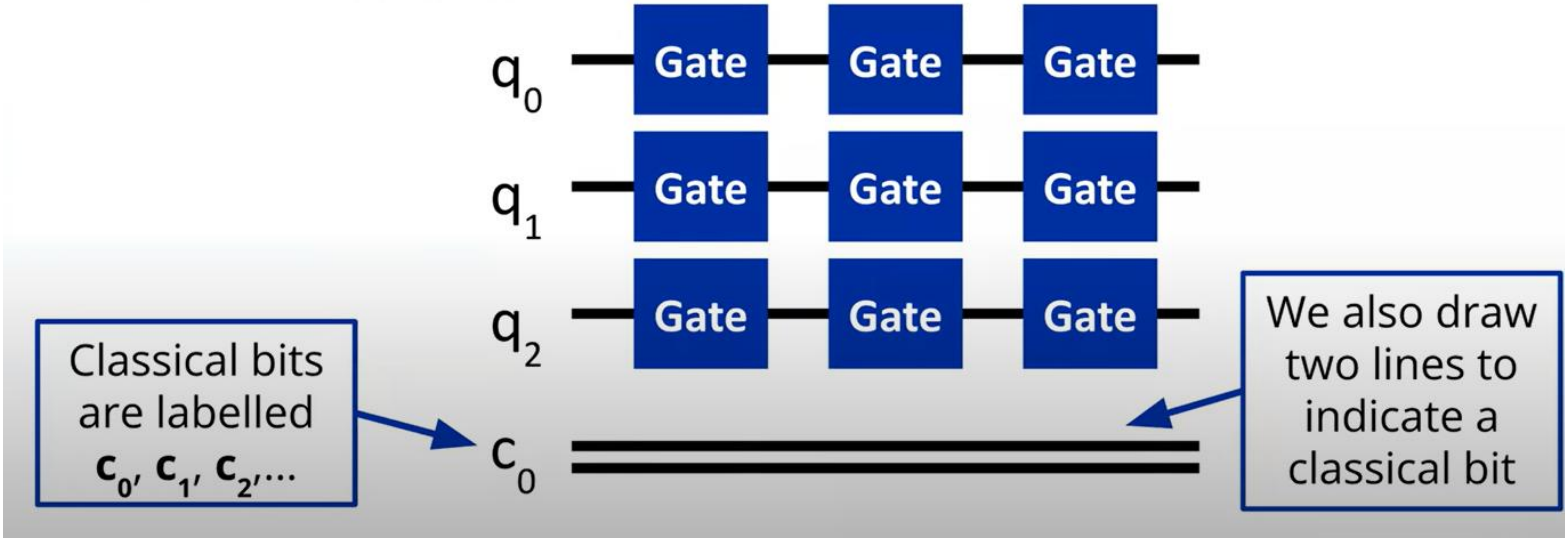
Quantum Circuits

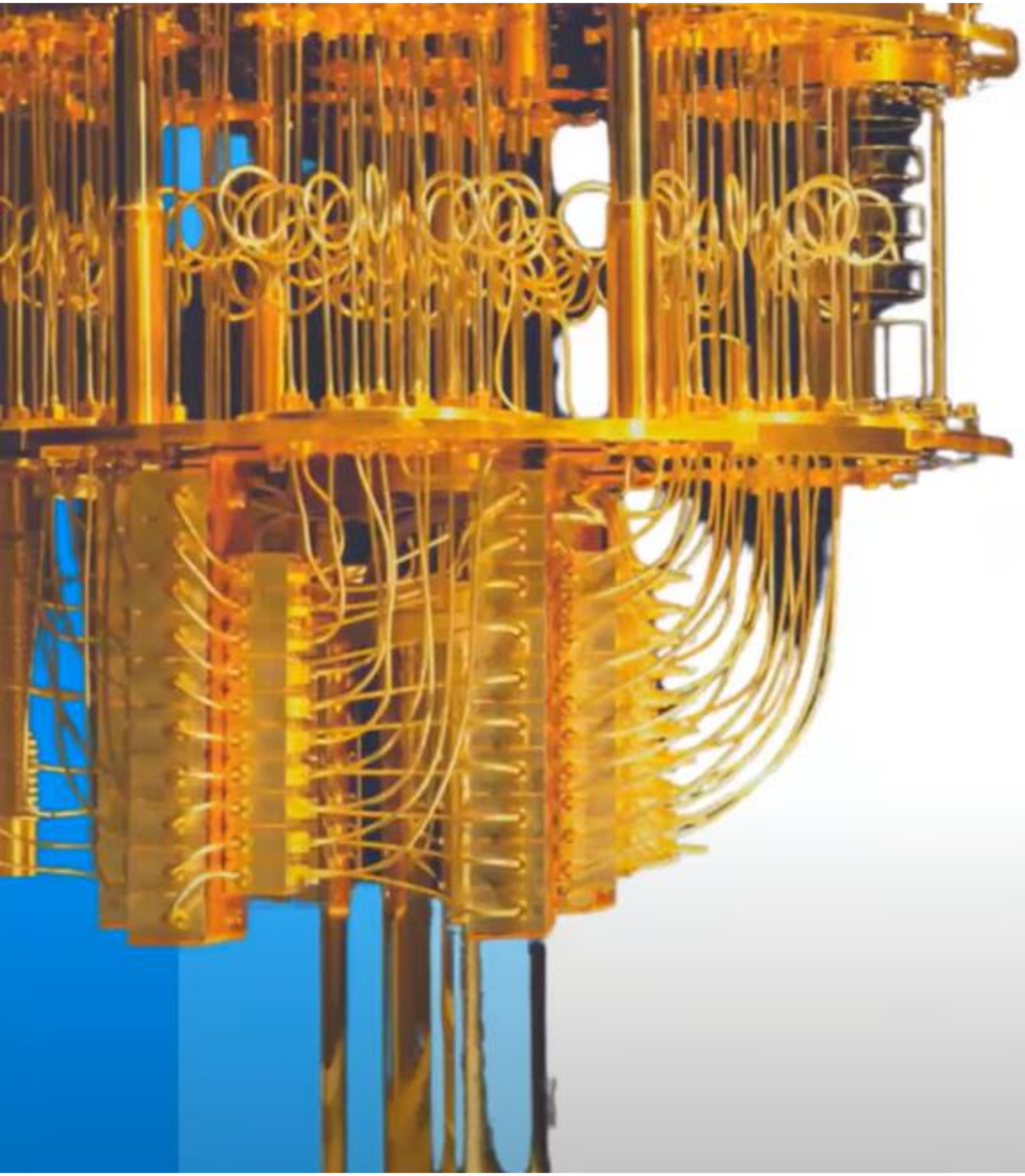
We can even have **classical bits**:



Quantum Circuits

We can even have **classical bits**:



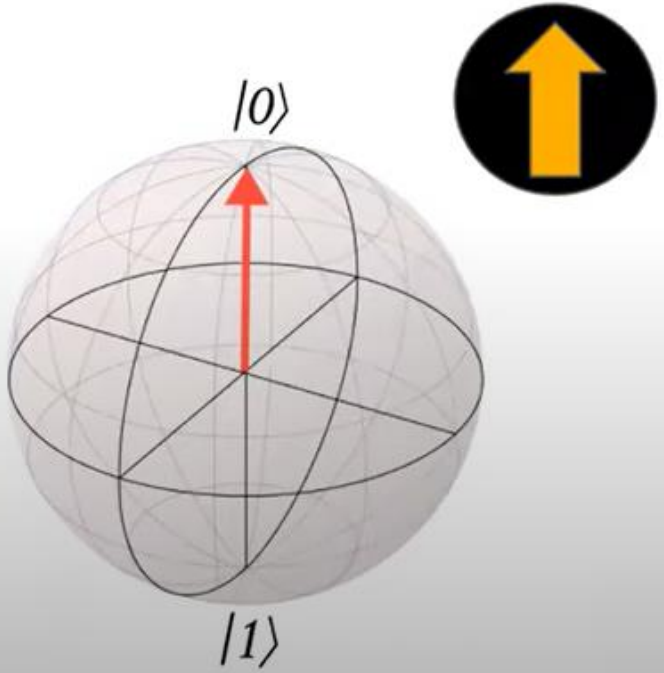


Lesson Recap

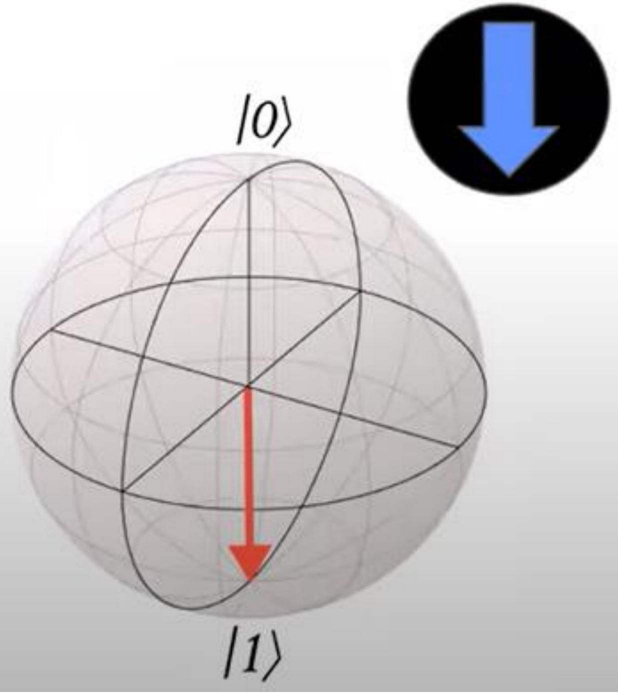
Key Lecture Takeaways

Bloch sphere and ket representations:

We define this to be the $|0\rangle$ state.

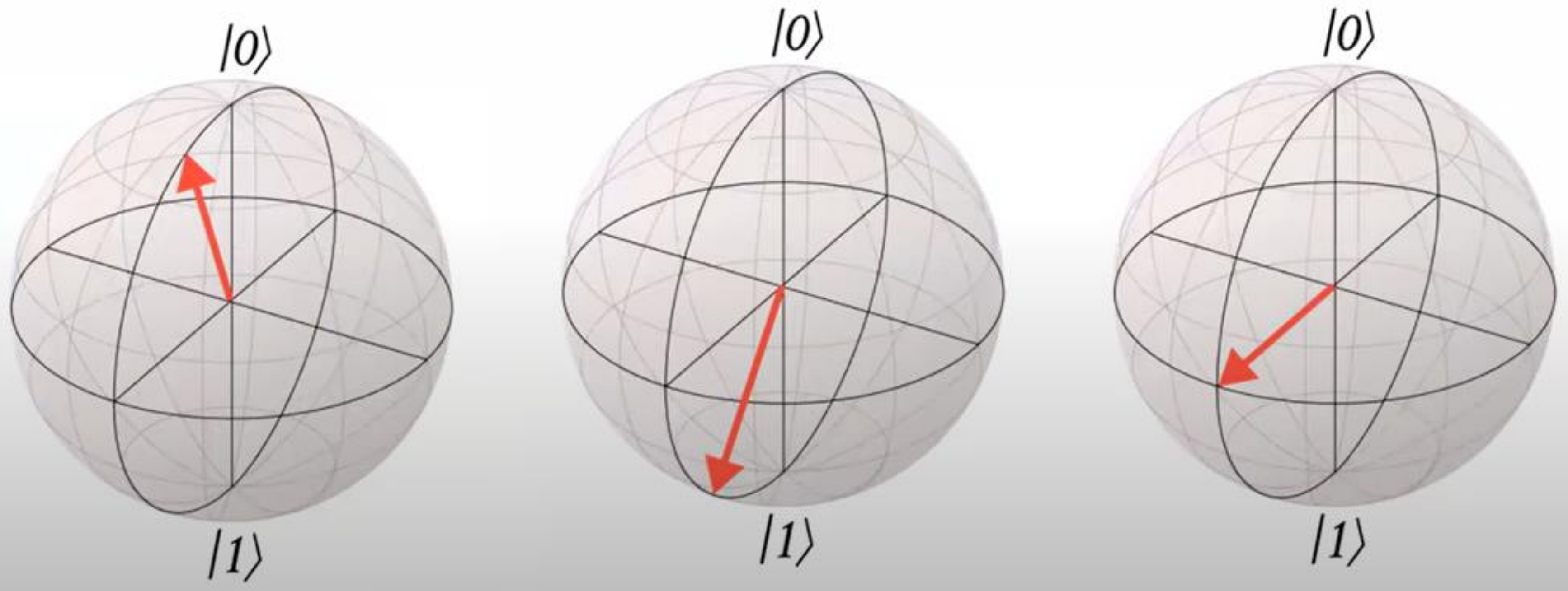


And this to be the $|1\rangle$ state.



Key Lecture Takeaways

All other points on the Bloch sphere are superpositions:



Key Lecture Takeaways

Gates change the states of qubits:

X gate

X

180° rotation around X axis

Hadamard gate

H

Creates superposition

Lab Preview

In lab, we'll use what we learned today to implement quantum circuits using a Python library by IBM called Qiskit.

Qiskit