FIAP

MBA

Computação Quântica

Medidas e Bases

Lecture Overview

Today's Topics

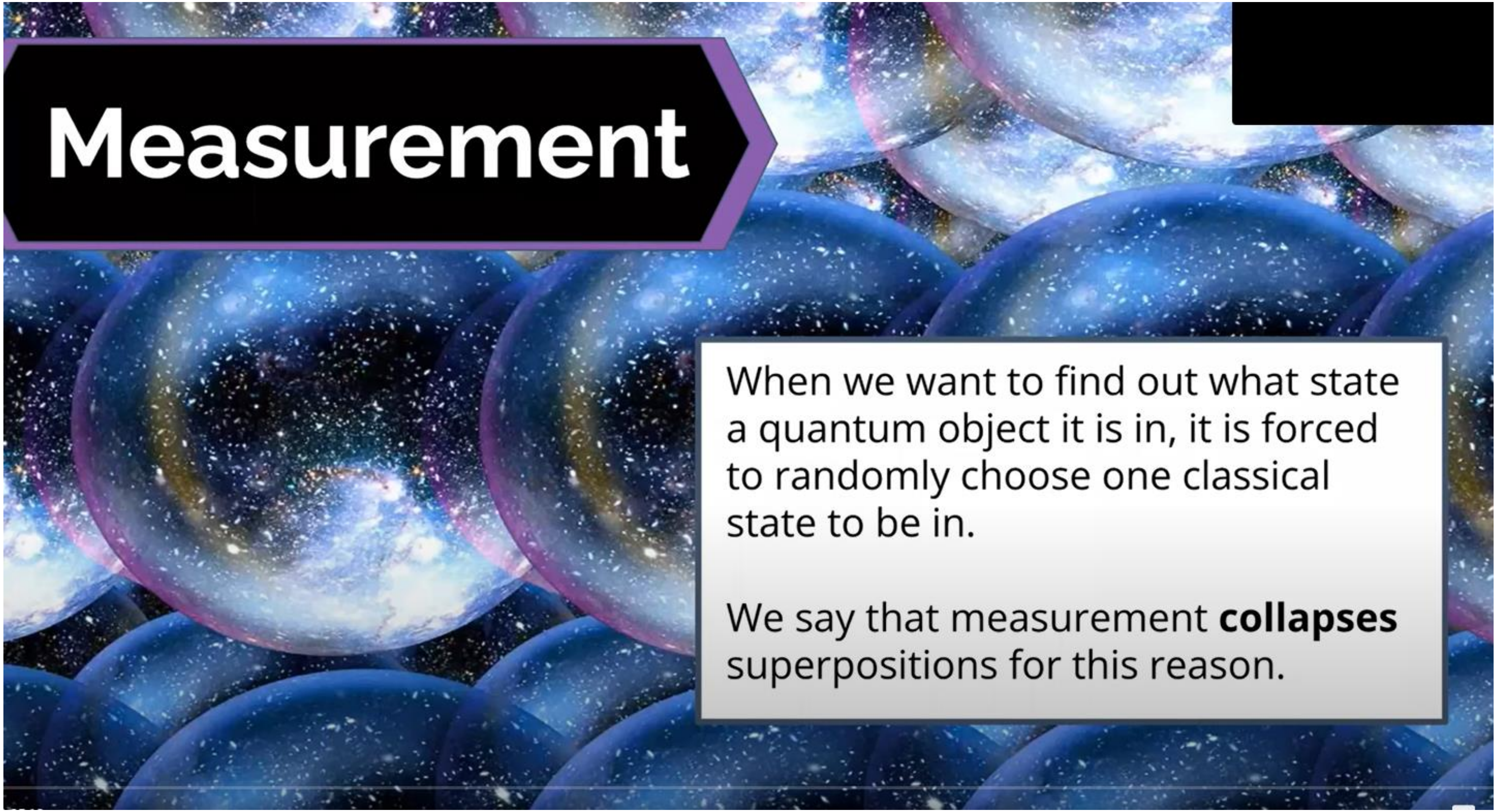
- **Measurement**
  - What is it?
  - How does it affect quantum states?
- **Bases**
  - What are they?
  - How does this relate some different states?

Objectives

After today, you will:

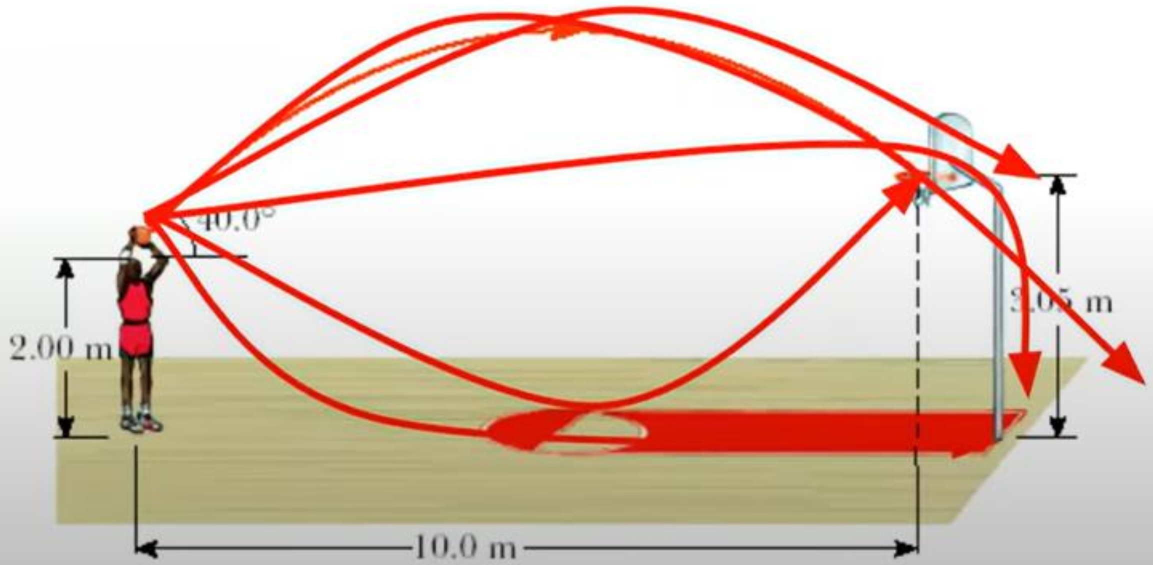
- Understand measurement as a means of extracting information
- Understand the idea of bases as different coordinate systems
- See how we relate the states $|+\rangle$ and $|-\rangle$ to $|0\rangle$ and $|1\rangle$


Review
Measurement
Bases
Review


Measurement
When we want to find out what state a quantum object it is in, it is forced to randomly choose one classical state to be in.
We say that measurement **collapses** superpositions for this reason.

Quantum Measurements

As we've learned, quantum objects can be in a superposition of states. So what happens when we ask the quantum basketball what path it's taking?



Quantum Measurements

Quantum measurement is unique in two key ways:

1. **The outcome of a quantum measurement is often random.**

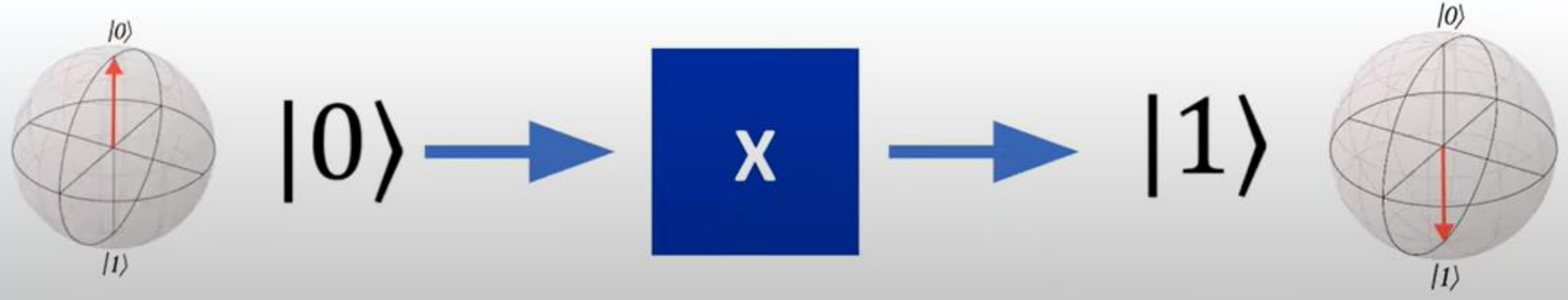
2. **By observing (measuring) a quantum state, we can change it.**


Review
Measurement
Bases
Measurement

How do we know the state?

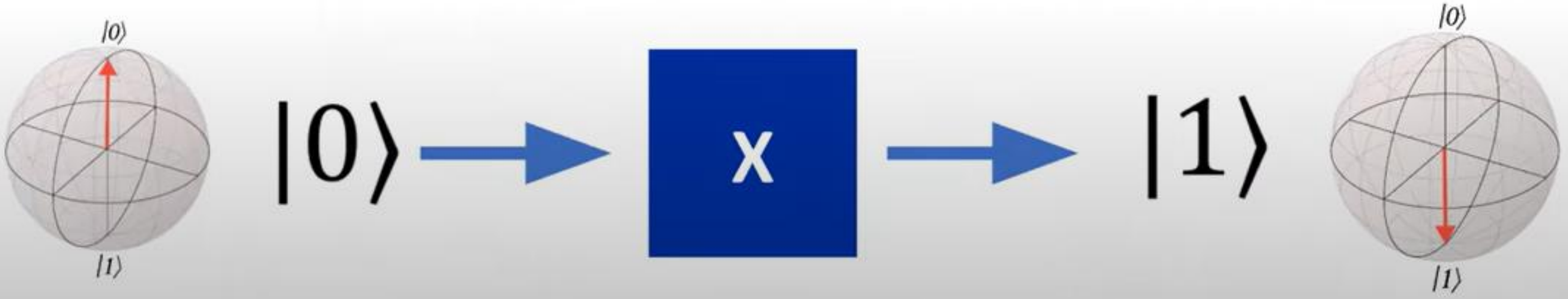
By now, we have learned what the circuit below means and how to simulate it using qiskit.

...but how can we know in practice what really happens?

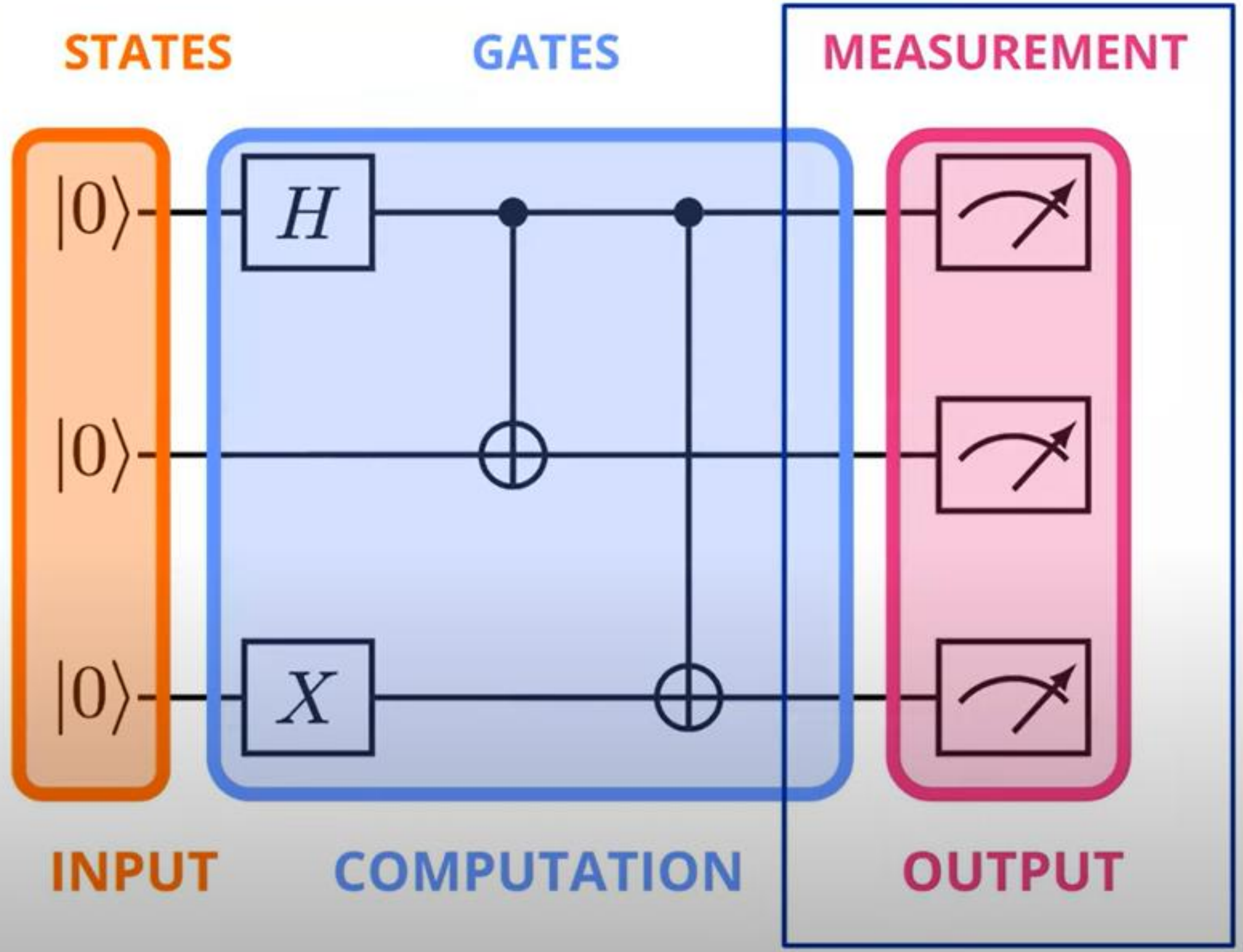


Measurement = extracting information

Measurement extracts information from systems.



Measurement in circuits



The final step is **measurement.**

This step is critical! Without it, we would just run gates all day long without actually knowing the value of the outcomes.

Quantum Measurements

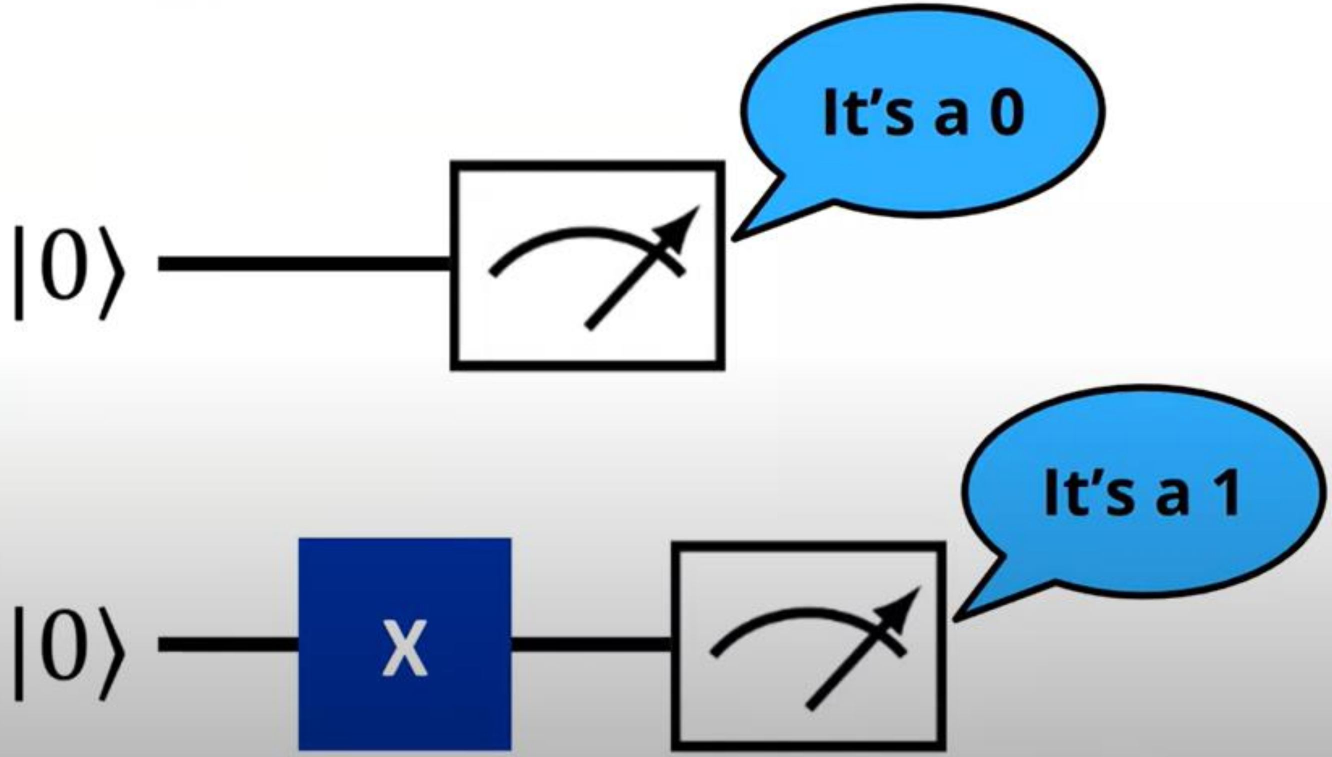
Quantum measurement is unique in two key ways:

1. **The outcome of a quantum measurement is often random.**

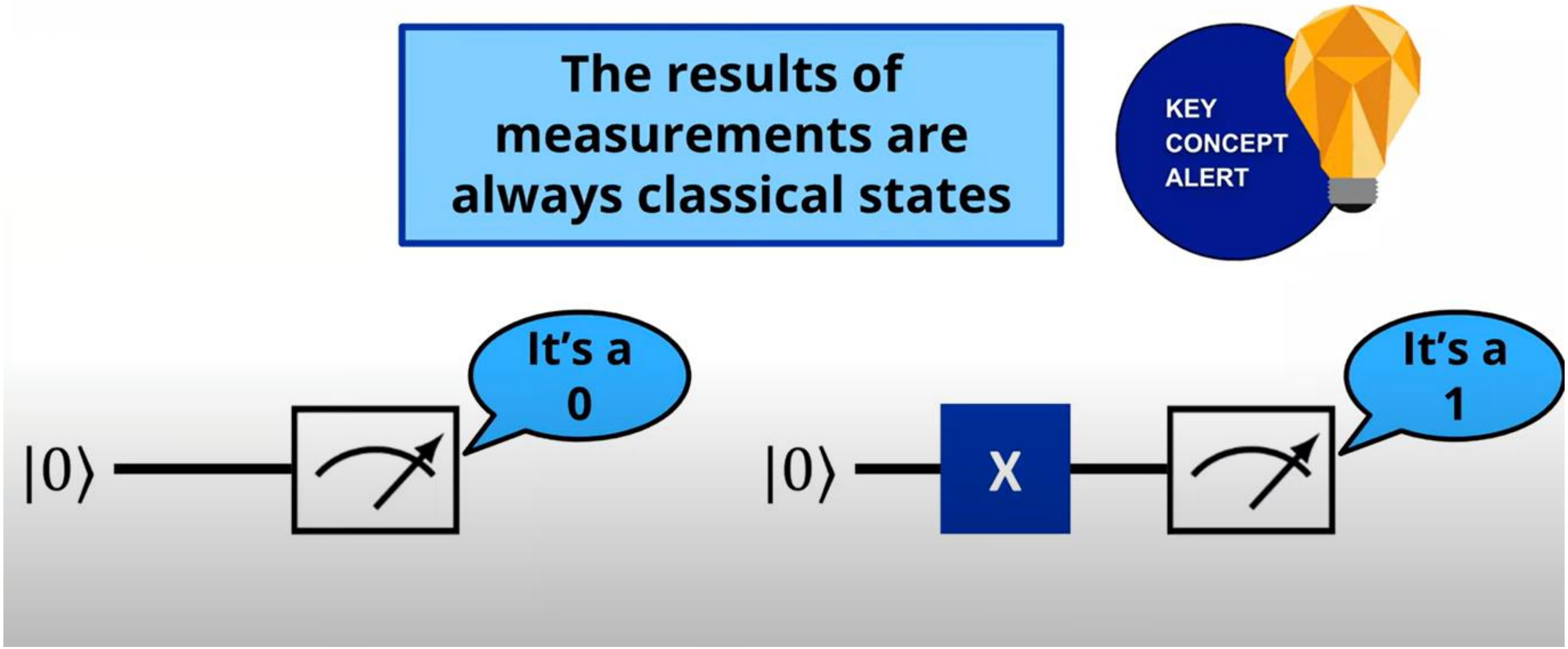
2. **By observing (measuring) a quantum state, we can change it.**

Measurement gives classical states

When we measure a qubit, we get out either 0 or 1.



Measurement gives classical states



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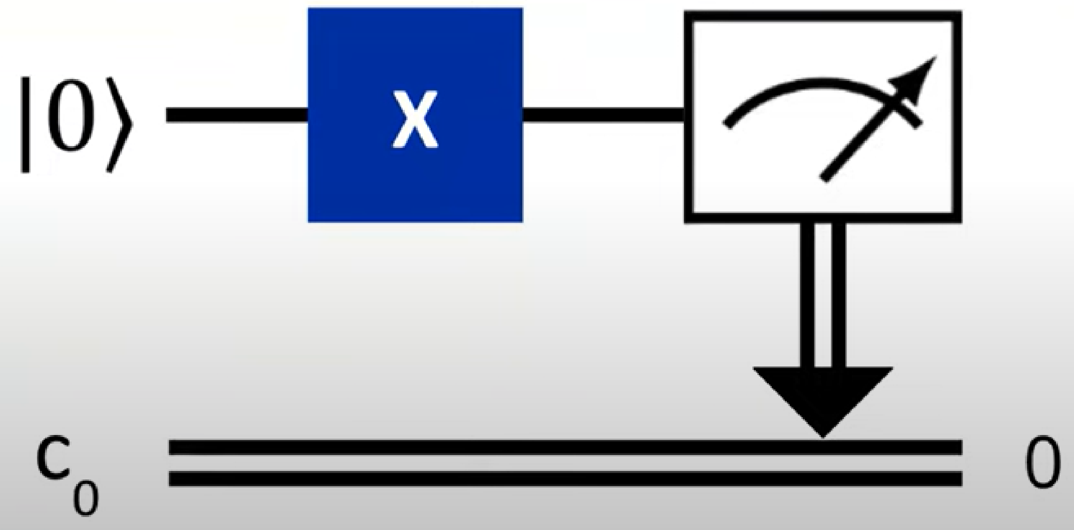
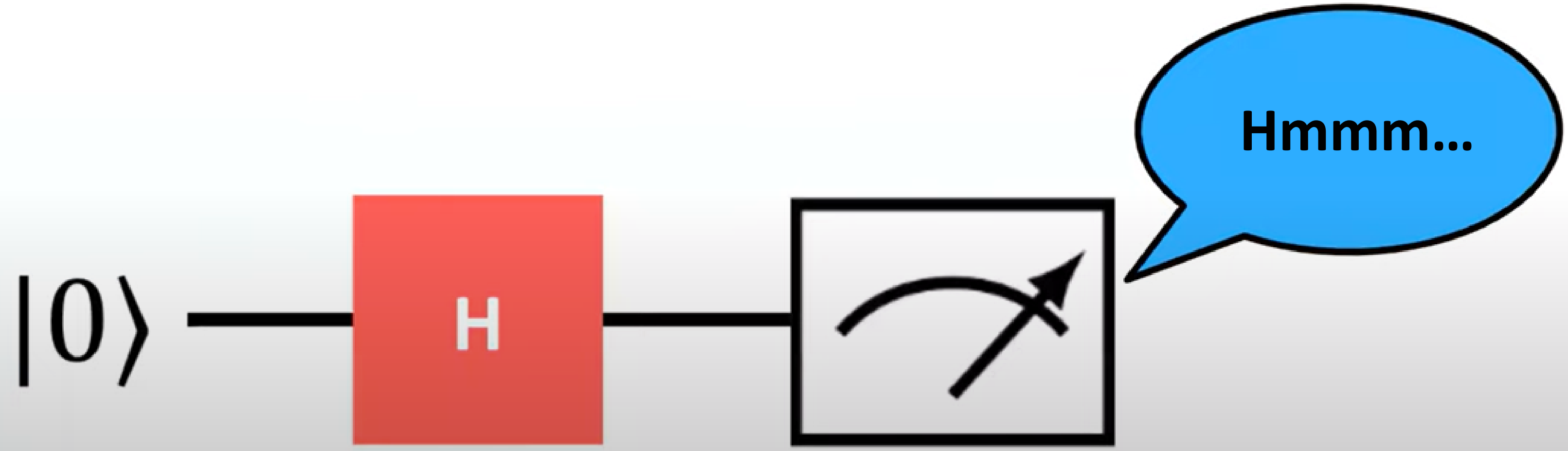
Measurement gives classical states

Measurement gives us a classical state, **so we need to store the result is a classical bit**.



Measurement gives classical states

So, what will we measure in this circuit...?



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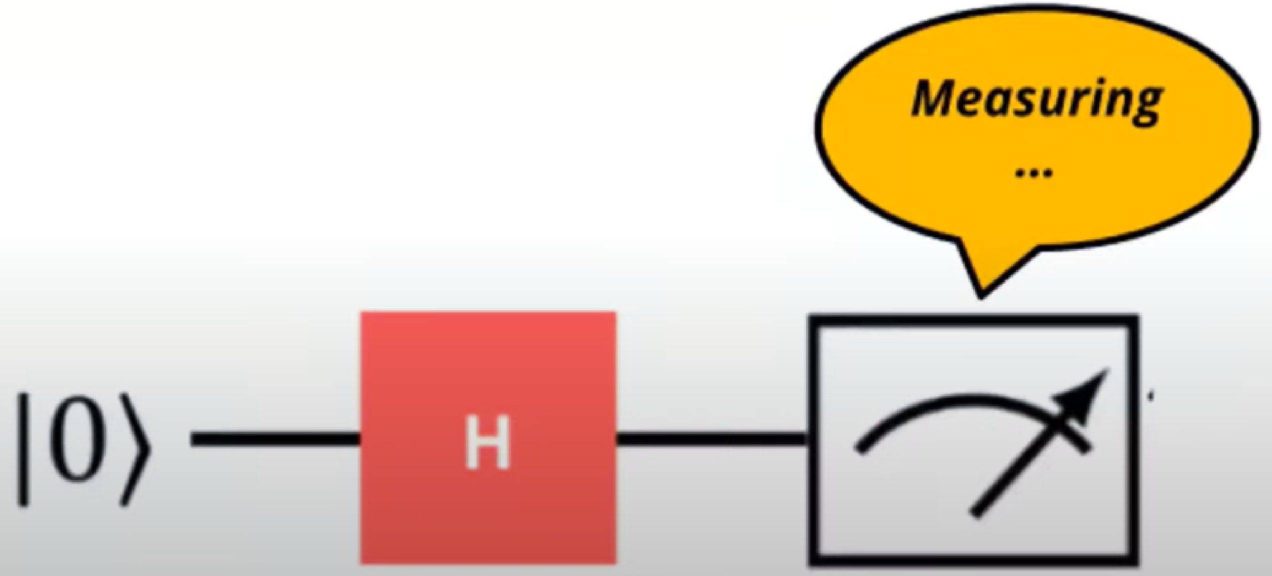
When we measure the + or - states, there is a 50% chance of getting 0 versus 1.

We have *no way* to predict anything beyond that. The outcome is *truly random*.

Random Outcomes

For example, when we measure the outcome of this circuit, the result will be **random**!



Random Outcomes

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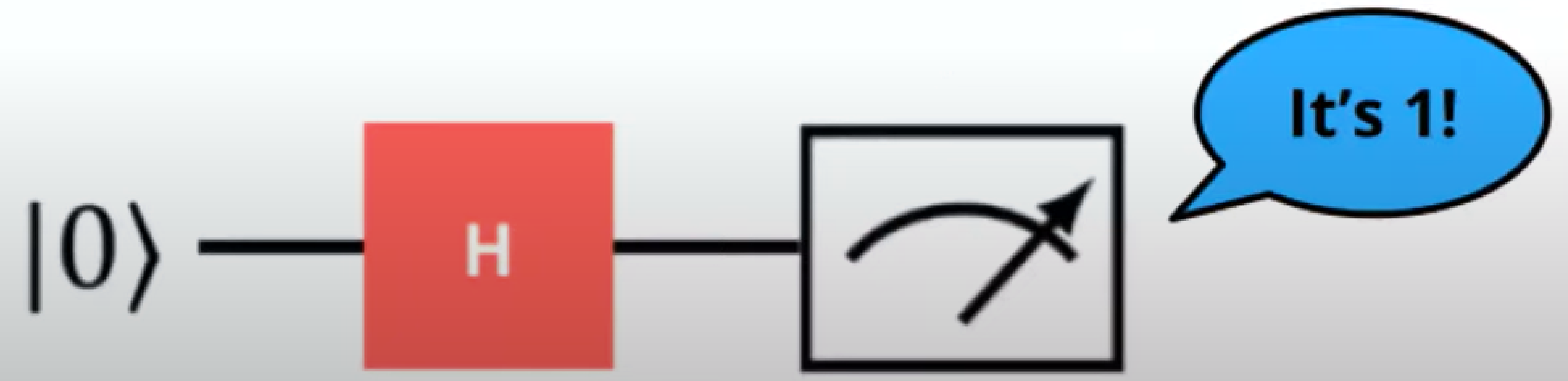

Random Outcomes

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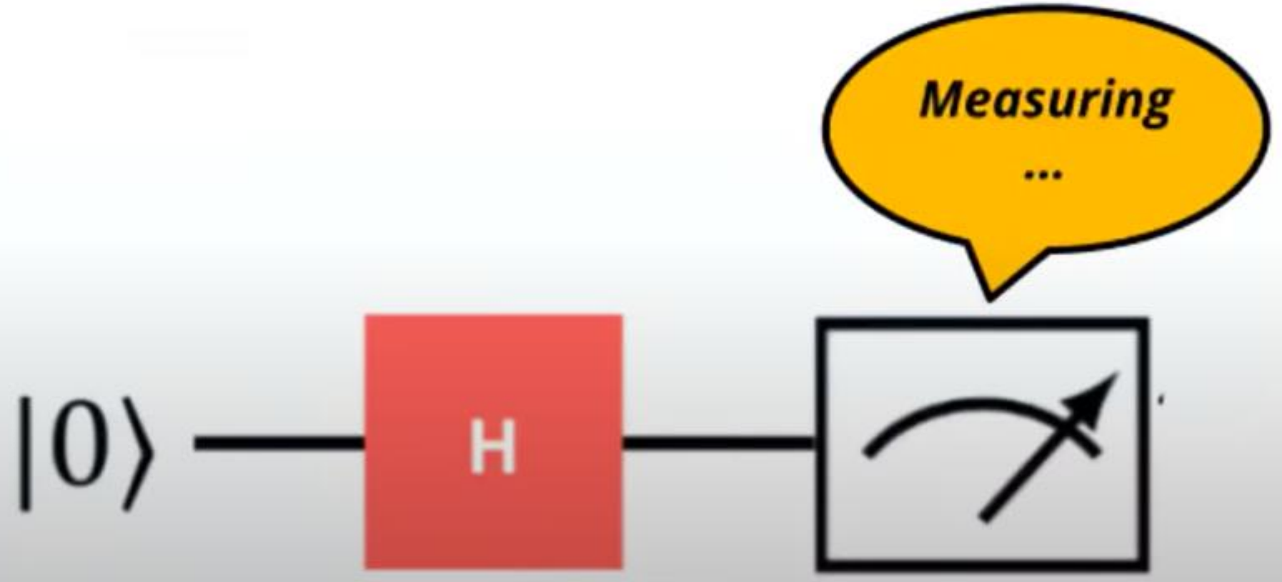
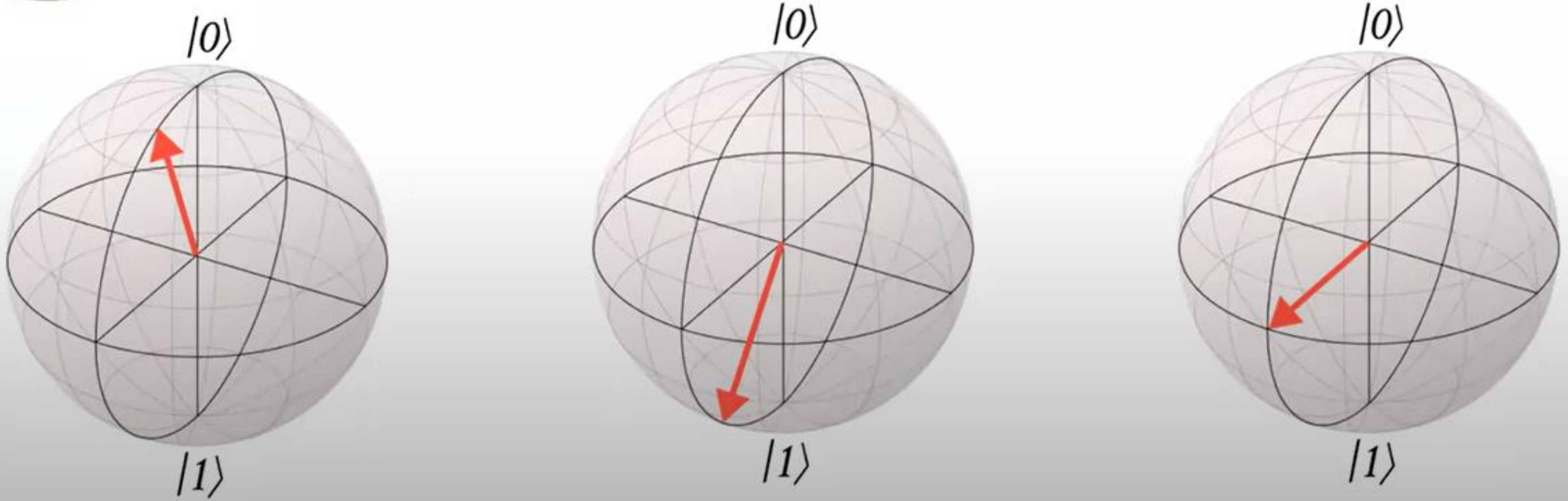


Random Outcomes

For example, when we measure the outcome of this circuit, the result will be **random**!

Random Outcomes

Measurement is random for *any* superposition states!

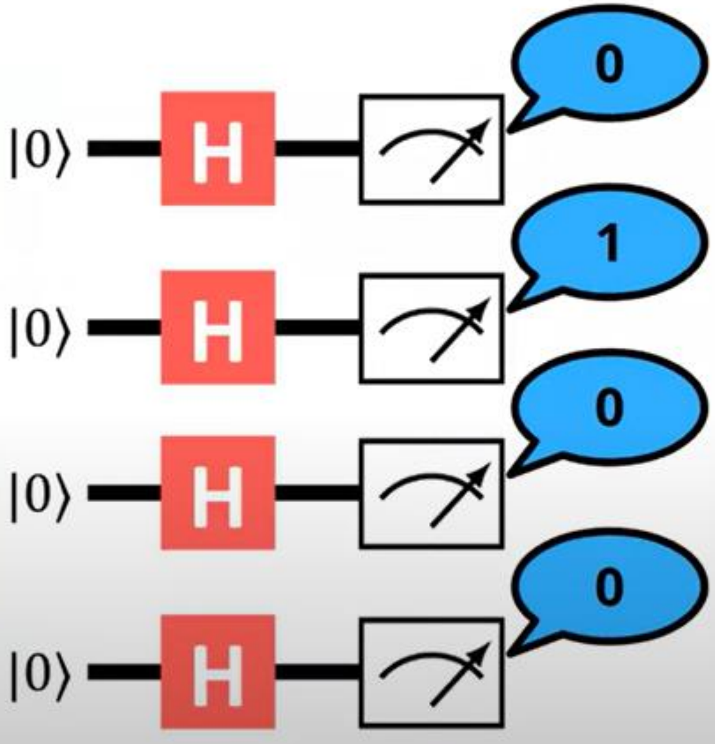


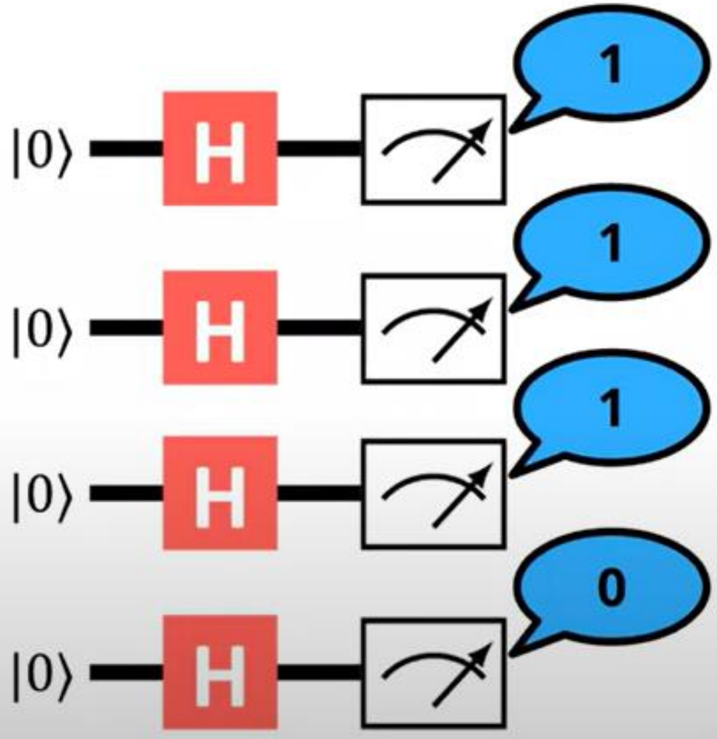
Random Outcomes

But we could measure a 0 from a circuit with no H gate at all.

So how can we know that our H gate is working as intended?

It's impossible to know with only one measurement! So, we need to **run this circuit and make measurements many times.** We then look at the **pattern of outcomes**.

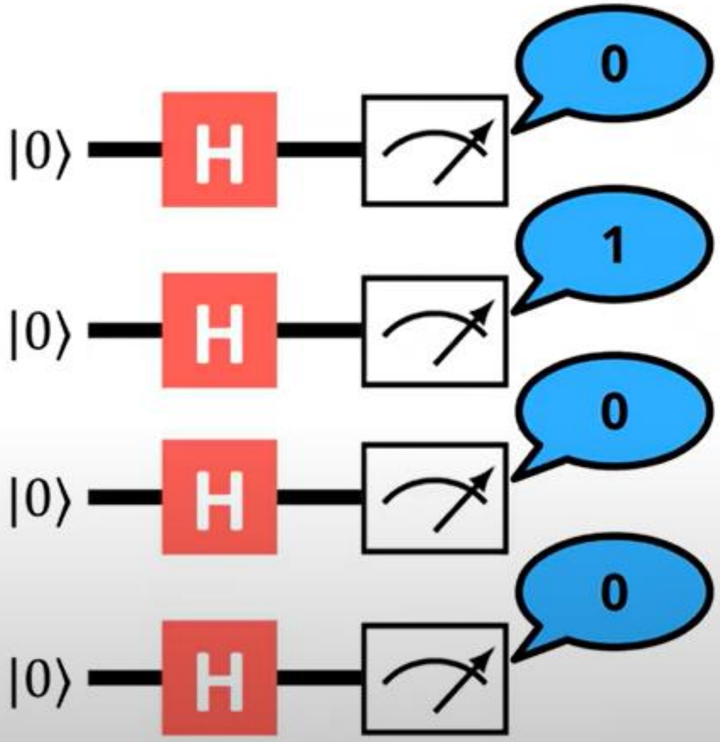




4 0s and 4 1s:

It's 50/50!

Running and measuring the same quantum circuit many times



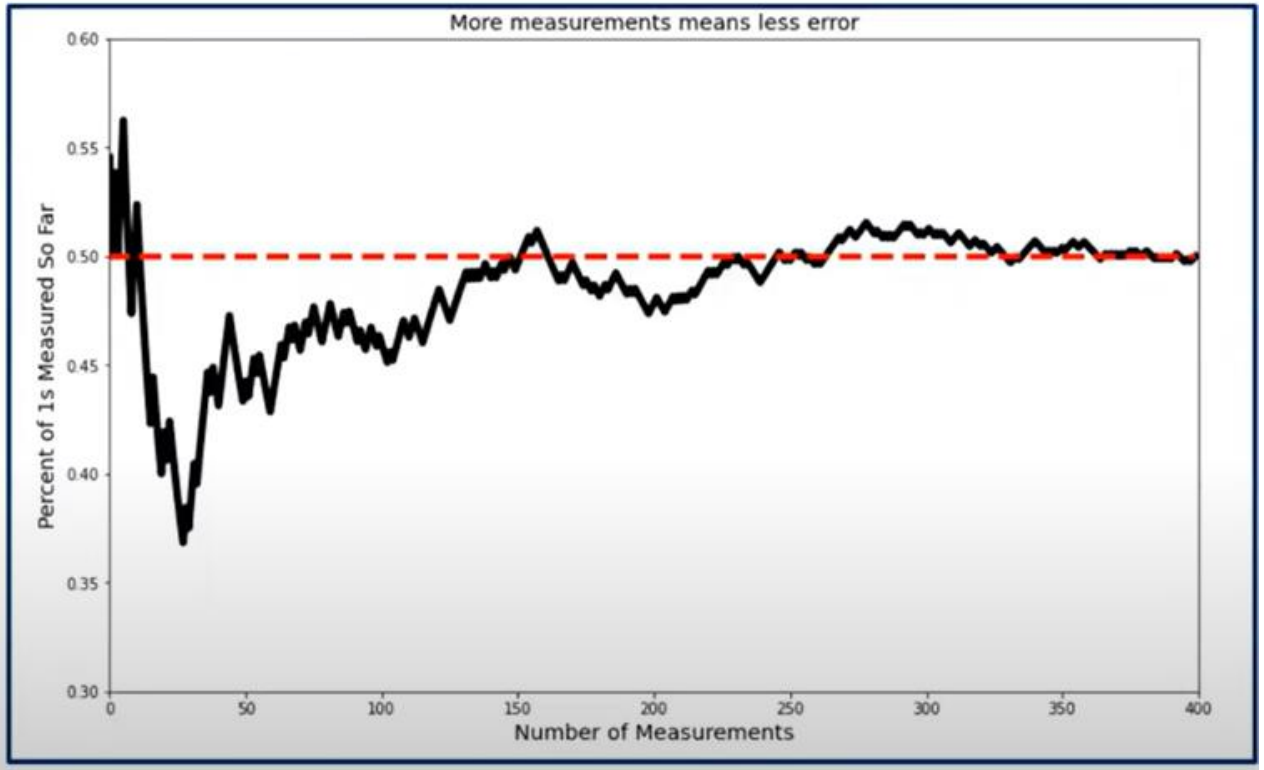
is just like determining the likelihood of heads vs. tails by flipping a coin many times

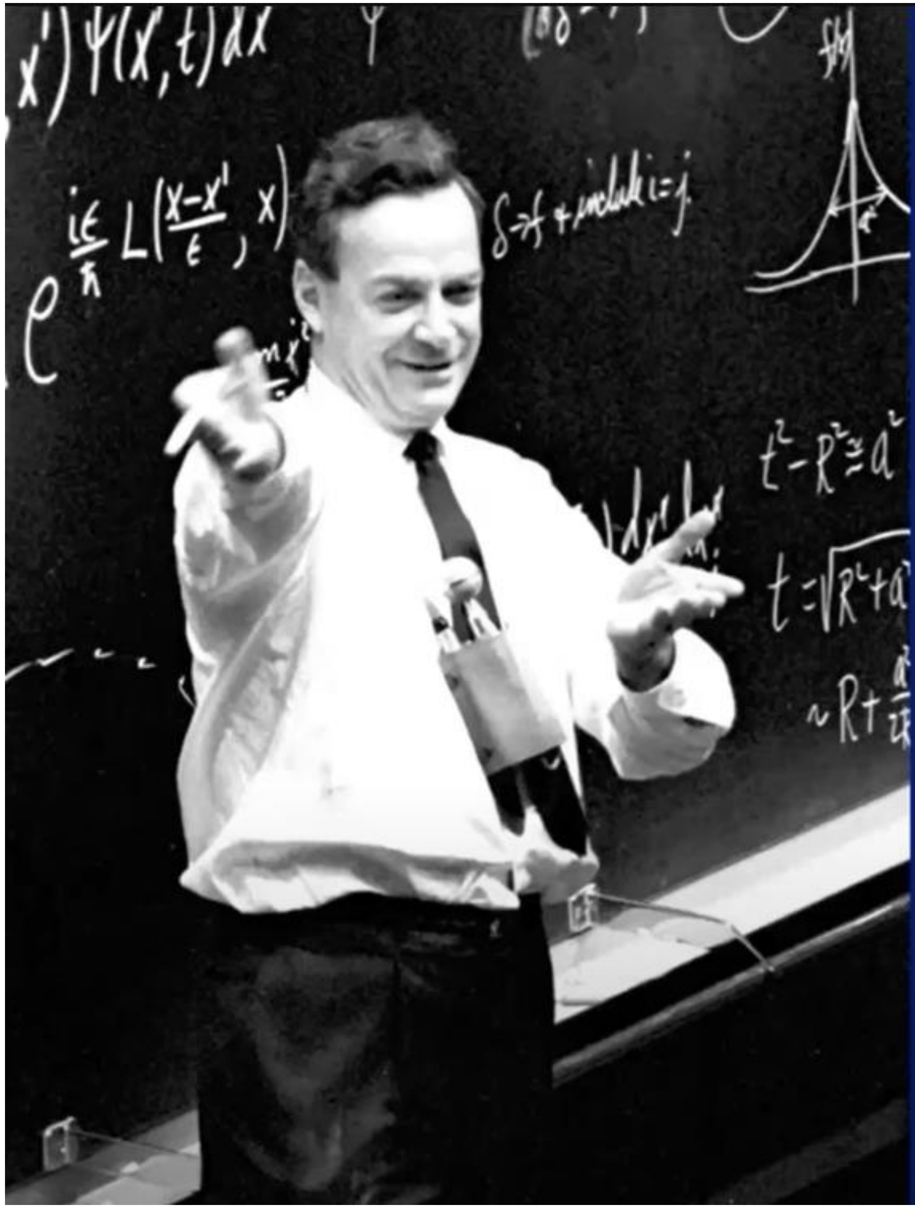
More is better

So, given this inherent randomness, we have to **perform many measurements** to try to arrive at the right answer.

We can use these measurements to **derive the probability that a qubit will behave one way versus another**.

The **more measurements we make, the more confident we can be in our results**.





"In its efforts to learn as much as possible about nature, modern physics has found that certain things can never be "known" with certainty. Much of our knowledge must always remain uncertain. **The most we can know is in terms of probabilities.**"

- Richard Feynman, American Theoretical Physicist

Quantum Measurements

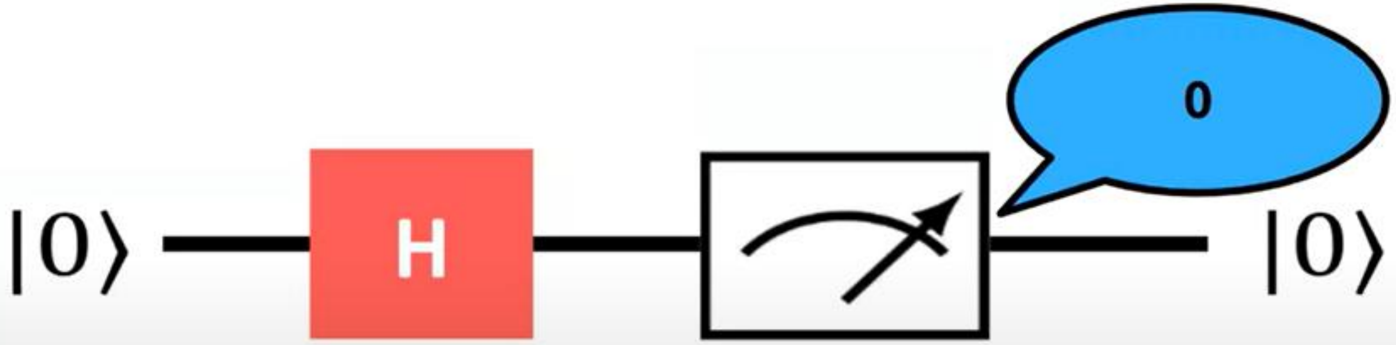
Quantum measurement is unique in two key ways:

1. **The outcome of a quantum measurement is often random.**

2. **By observing (measuring) a quantum state, we can change it.**

Measurements change states!

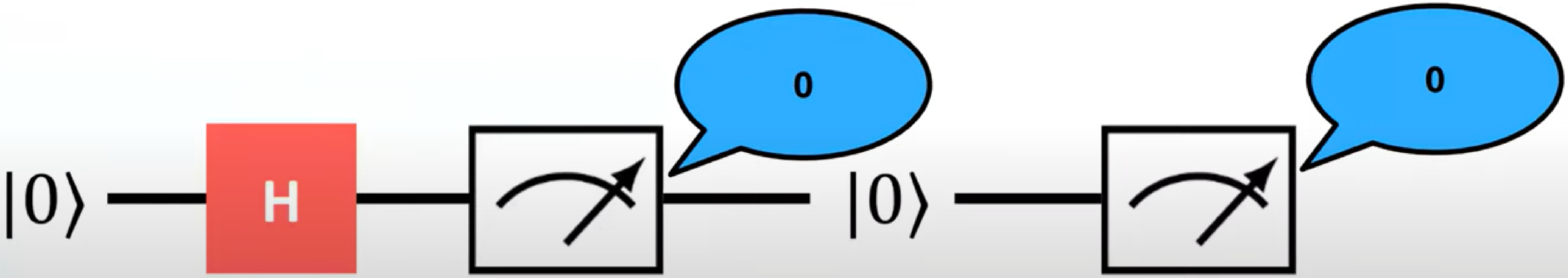
Furthermore, if we happen to measure 0 in the circuit below, the qubit will end up in the |0> state instead of the |+> state.



Measurements change states

Measurements change states!

If we measure again, it will be 0 again.

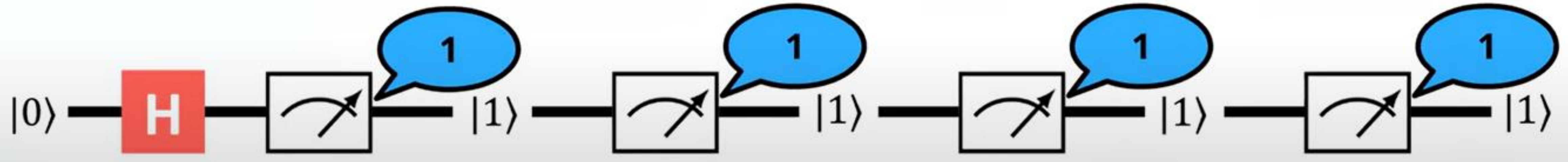


Measurements change states!

We will *always* measure 0 now, no matter how many times we measure it.

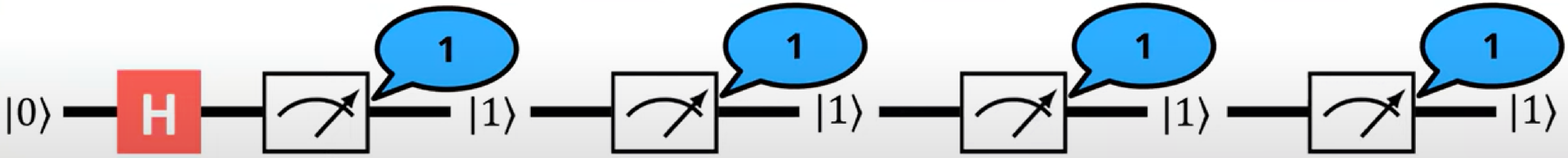
Measurements change states!

And if we happened to measure 1 at first,
then we would *always* measure 1 afterwards.



Measurements change states!

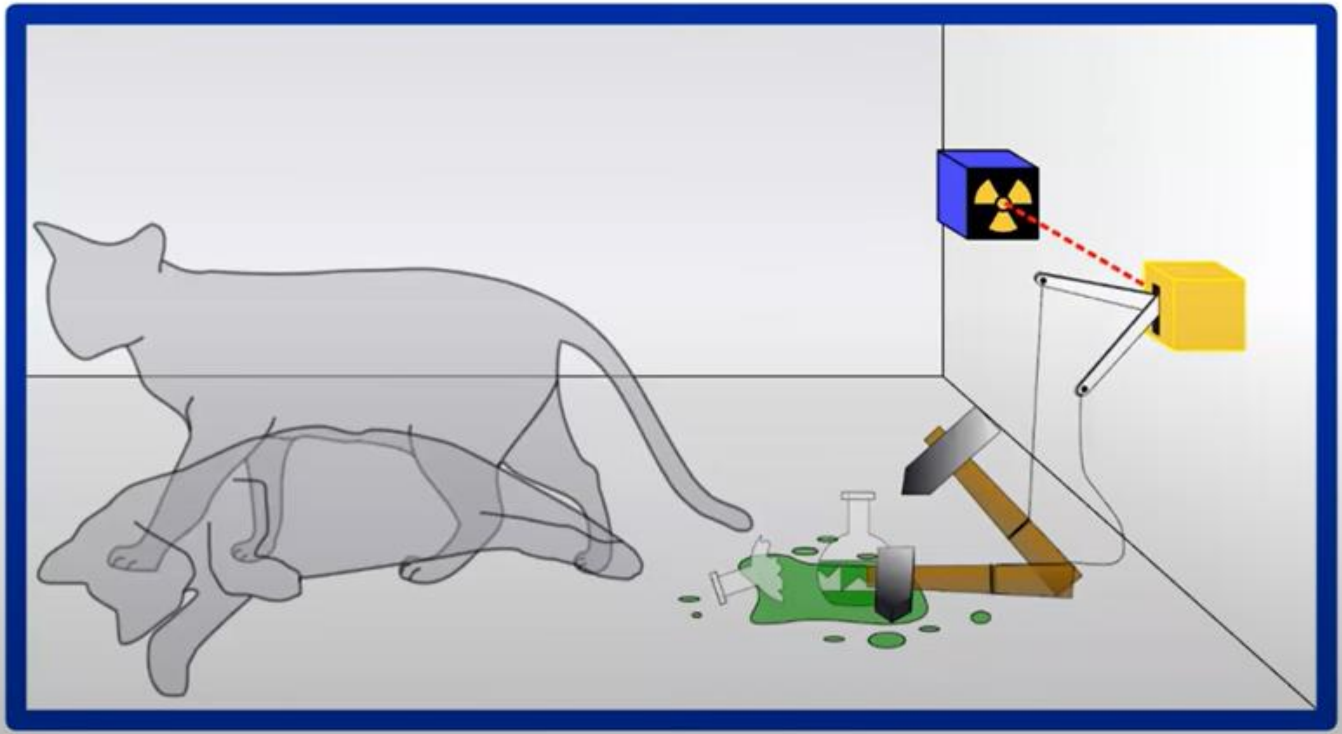
We really meant it when we said that measurements *extract information*! Measurements take that information away from the qubit, often destroying or **collapsing superpositions**.



Measurements change states!

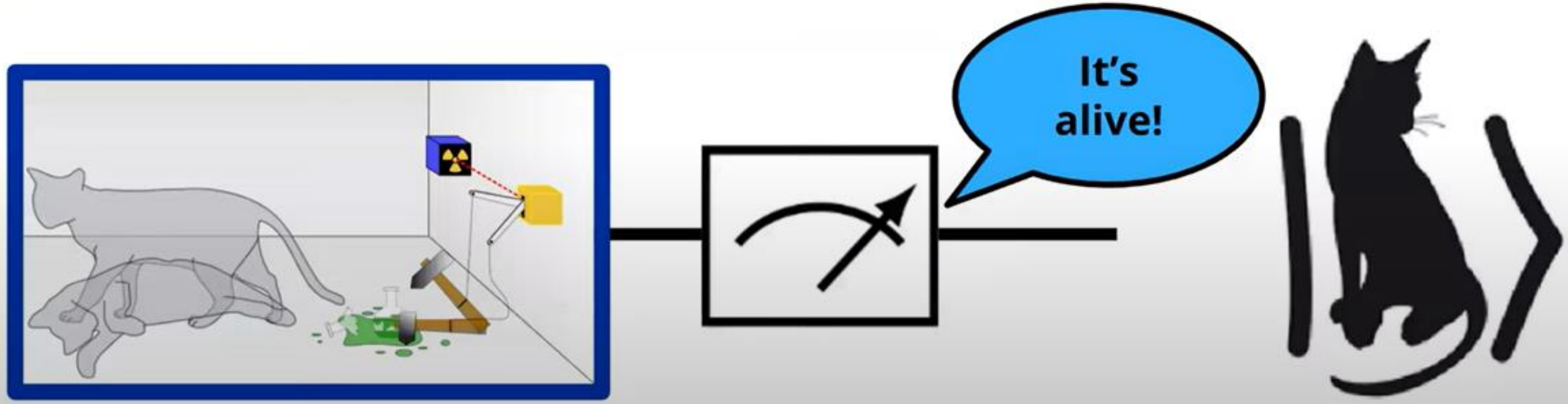
Remember Schrodinger's cat?

The cat is in a superposition of being dead and alive.



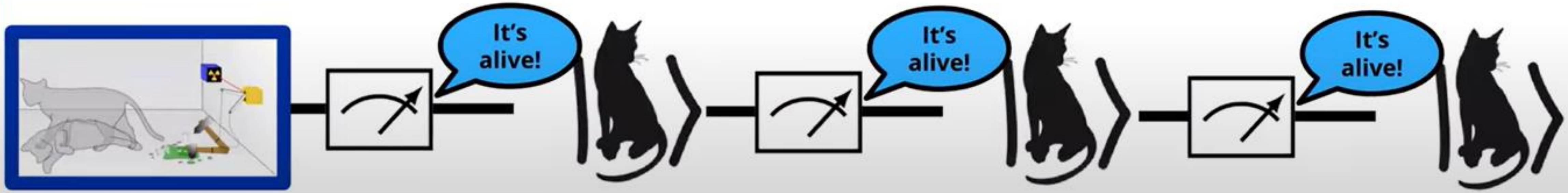
Measurements change states!

When we open the box to see what happened, we are measuring the cat's state. This causes the cat's state to collapse into either alive or dead.



Measurements change states!

Now we just have a living cat. Assuming nothing else happens, **there is no reason for the experiment's outcome to change if we measure sometime later**!



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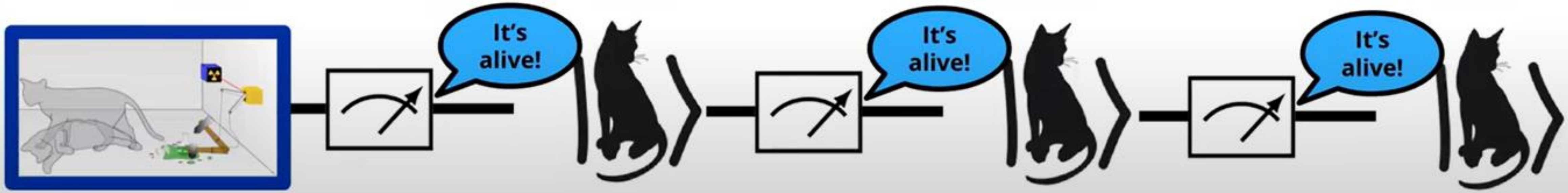
Measurements change states!

The first measurement changed the cat's state from a superposition to the alive state. So, every time afterwards, the only possible measurement was alive.

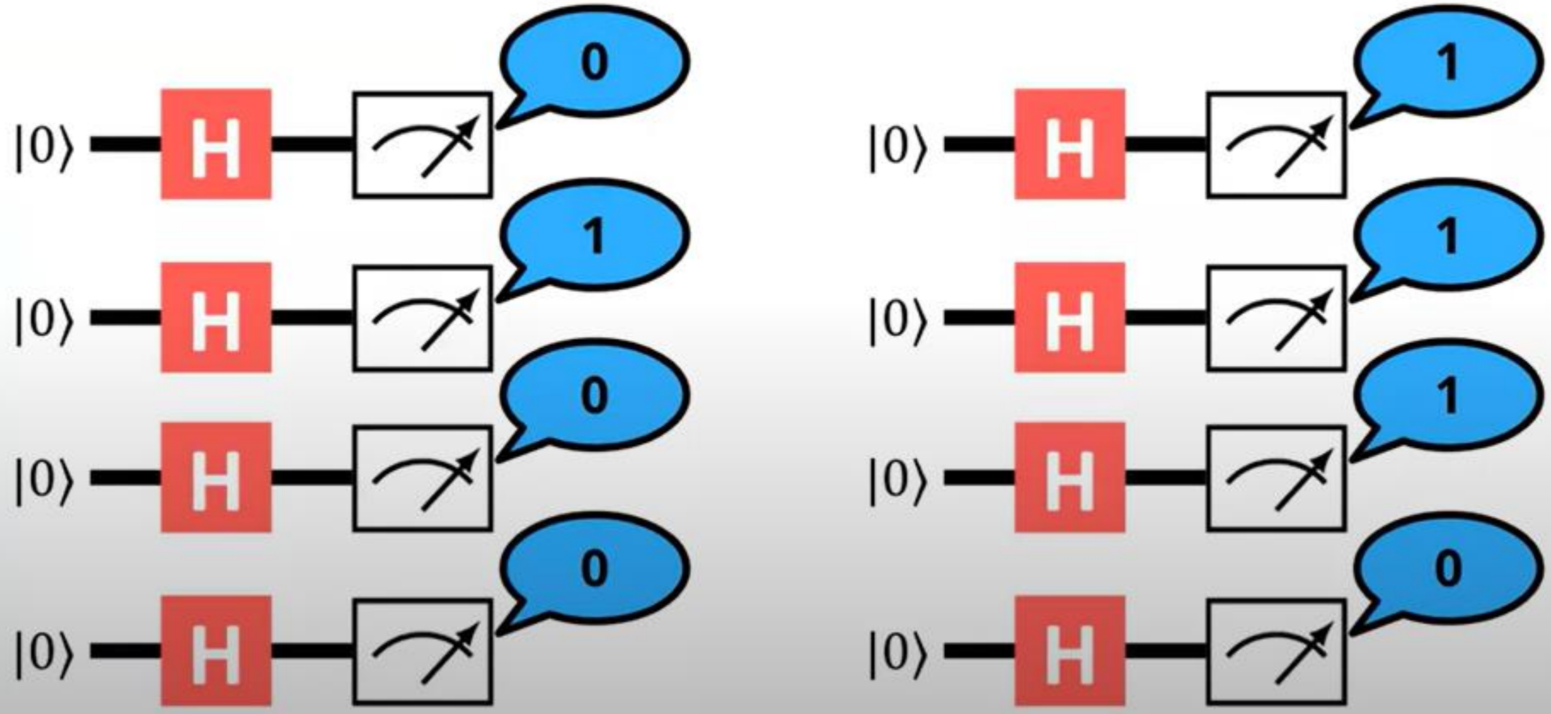


The Paradox of Quantum Measurement

- We need to make measurements to know what our quantum system is doing.
- But the moment we make a measurement, we've destroyed the quantum properties that made the system so special!

Random Outcomes

To get the many outcomes necessary to understand our quantum system, we need to completely rerun it every time!




This inherent
randomness is
not all bad.

Application: Harnessing Quantum Randomness

Researchers have built a device that can derive **truly random sequences of numbers from photons,** a quantum particle.

JANUARY 22, 2021 REPORT

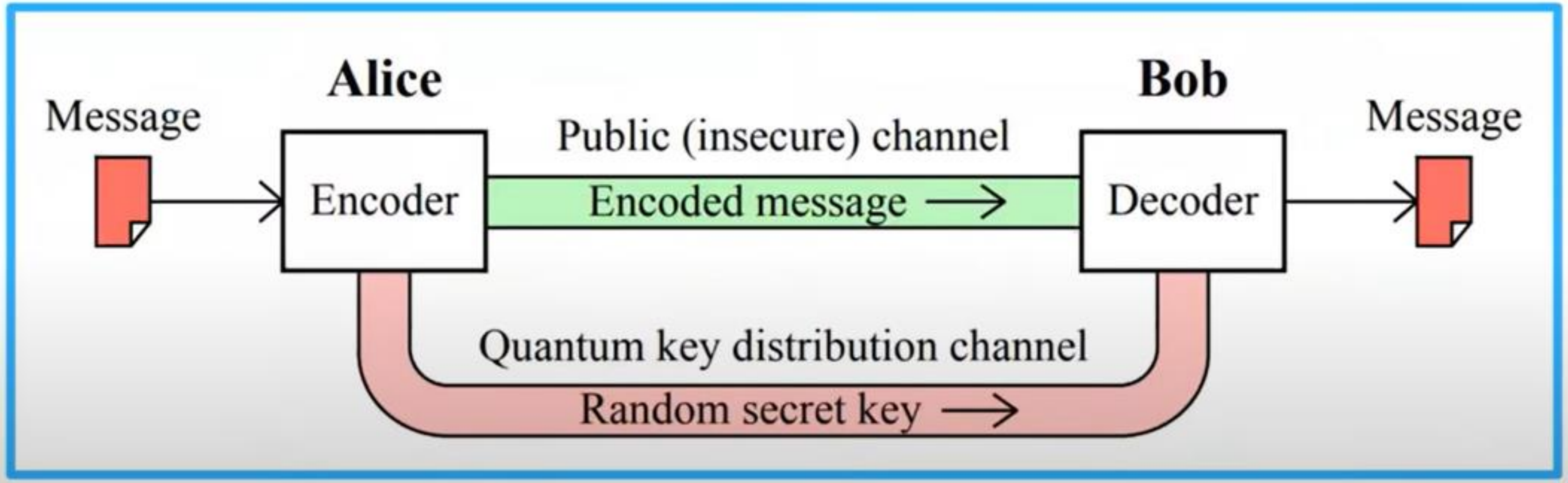
Using the unpredictable nature of quantum mechanics to generate truly random numbers

by Bob Yirka , Phys.org

This is a crucial advancement because random number generation is the foundation of cryptography and cybersecurity.

Application: Quantum Cryptography

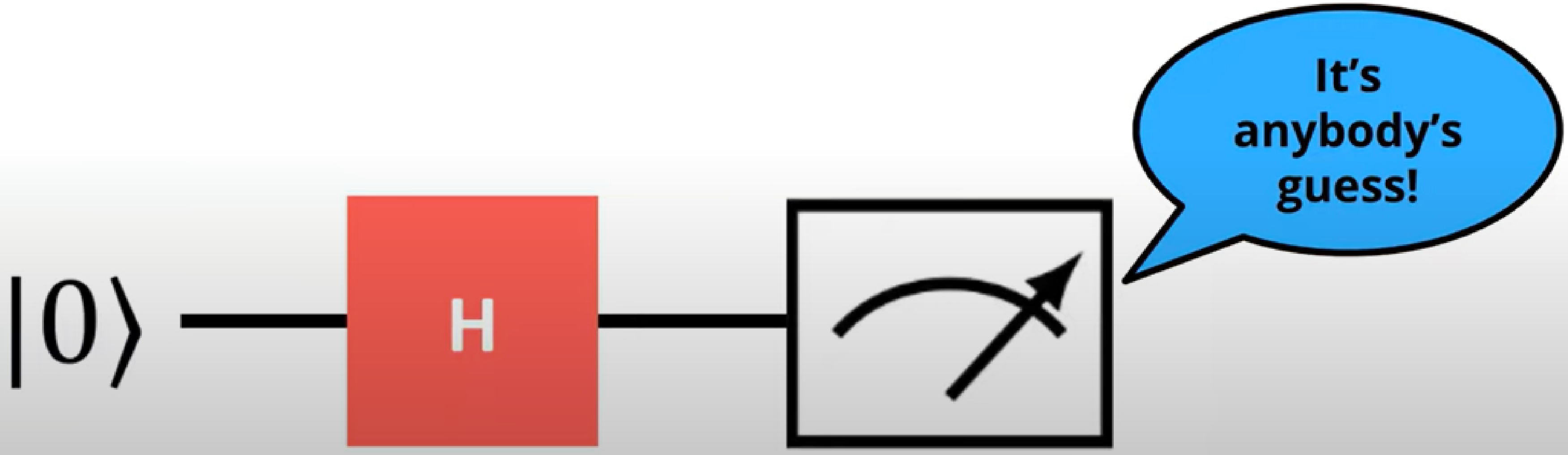
We can use the properties of quantum measurement to implement secure cryptography schemes! Learning how measurements work this week will help us learn these quantum cryptography schemes in a few weeks.



A useful protocol we will learn is Quantum Key Distribution.

Why 50/50?

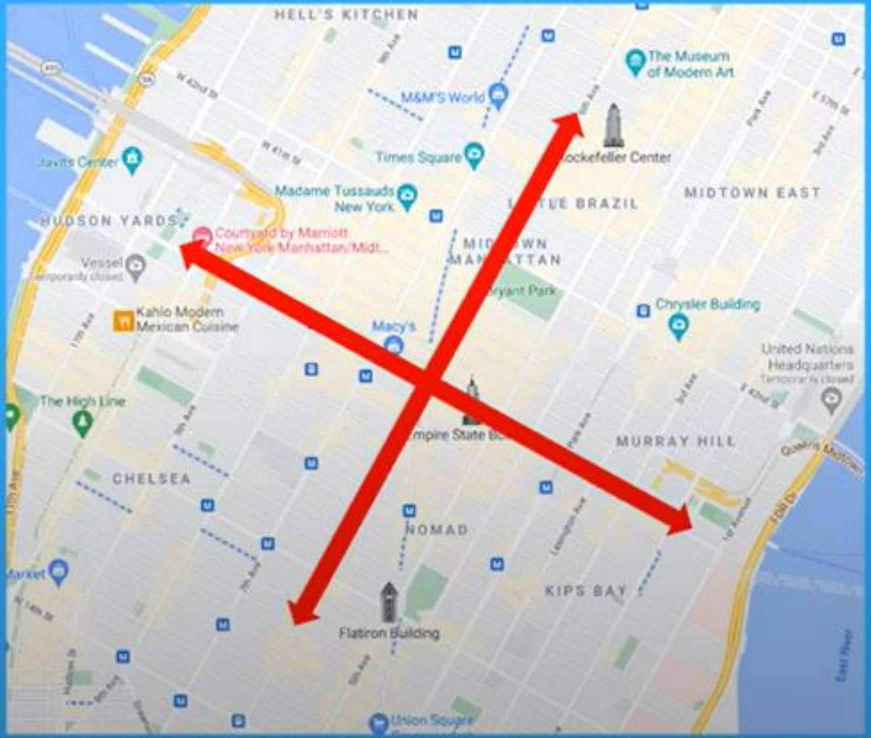
So far we have just told you that there's a 50% of measuring 0 or 1 in the + and - states. But is there any way we could actually predict this?

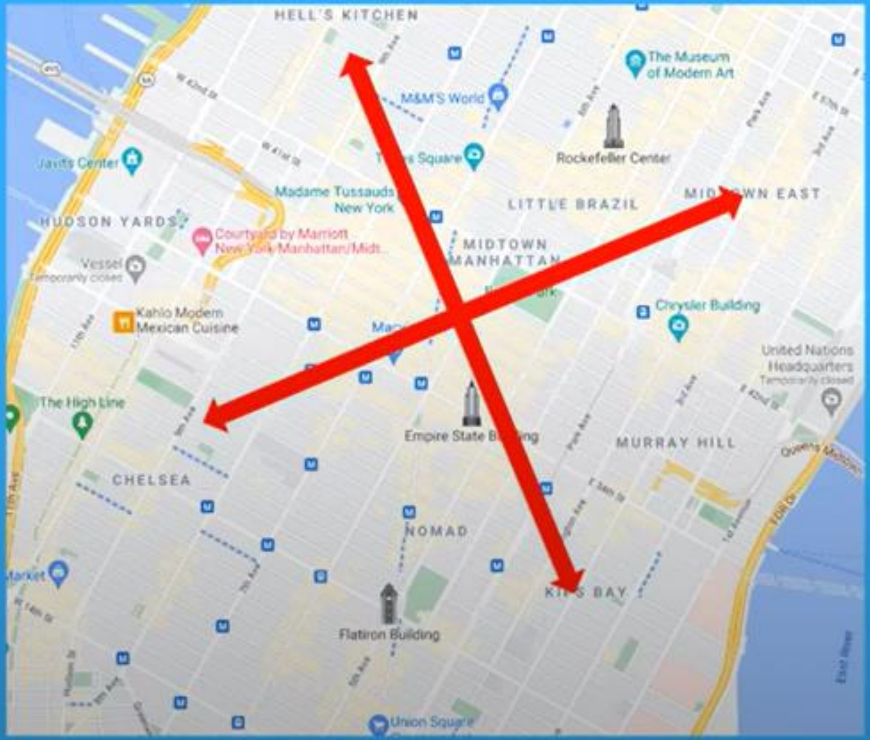



Review
Measurement
Bases
Bases

Basis

A basis is the specific point of view or frame of reference you are using to look at a state. It is another word for coordinate system.

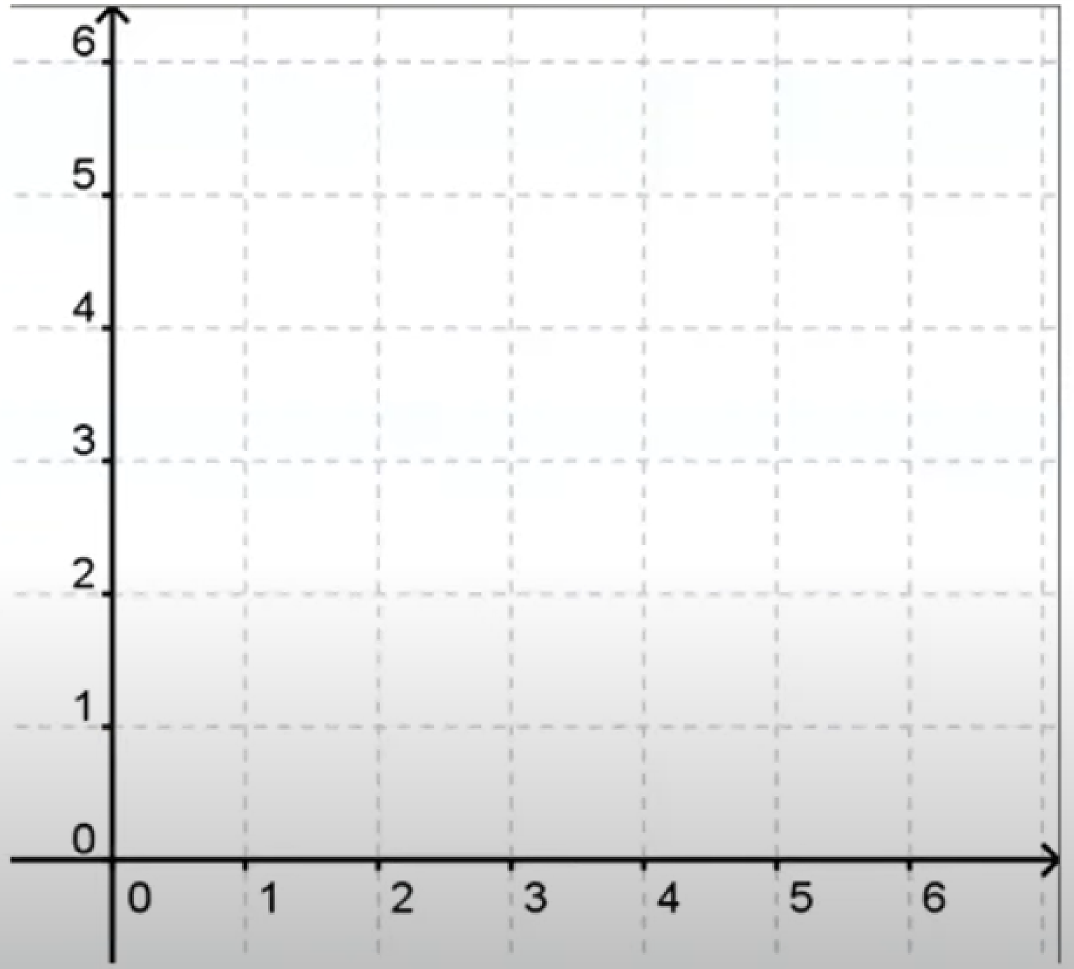




Bases are Coordinate Systems

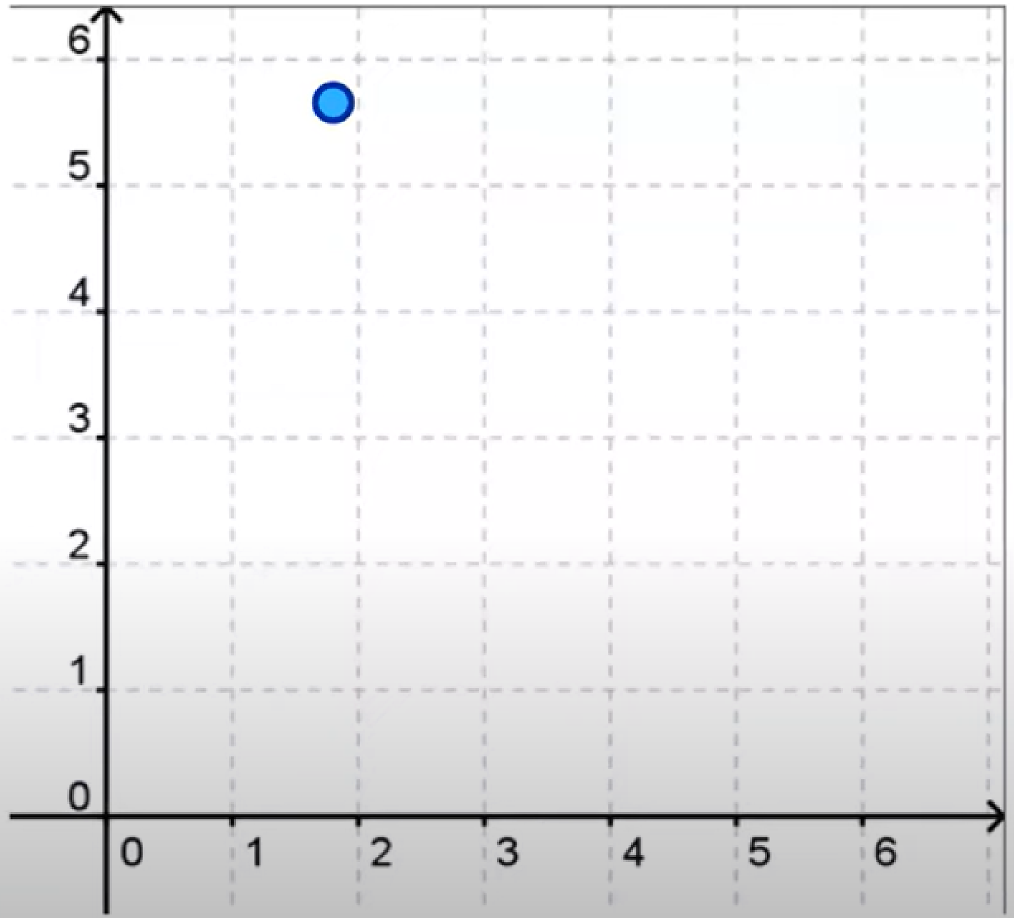
Each basis is its own coordinate system.

For instance, the coordinate system on the right is one choice of basis.



Bases are Coordinate Systems

Where is the point in this basis?



Bases are Coordinate Systems

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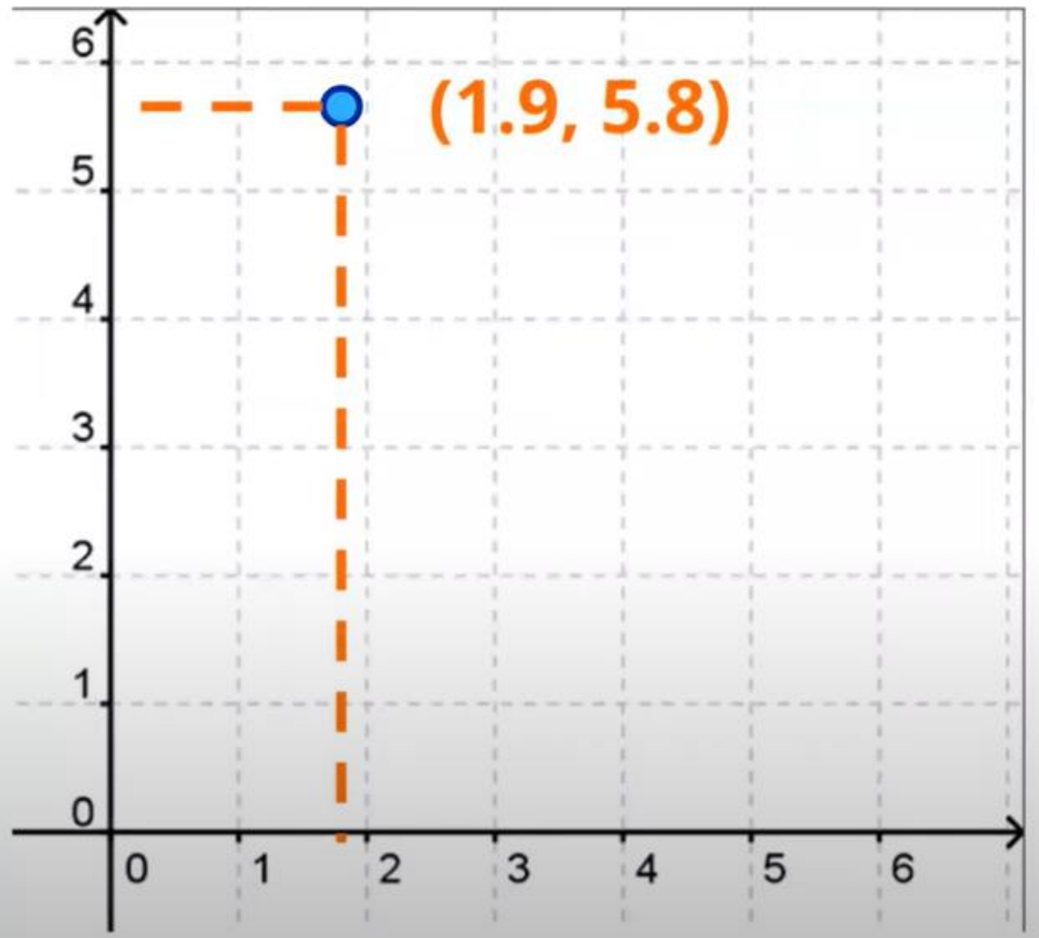
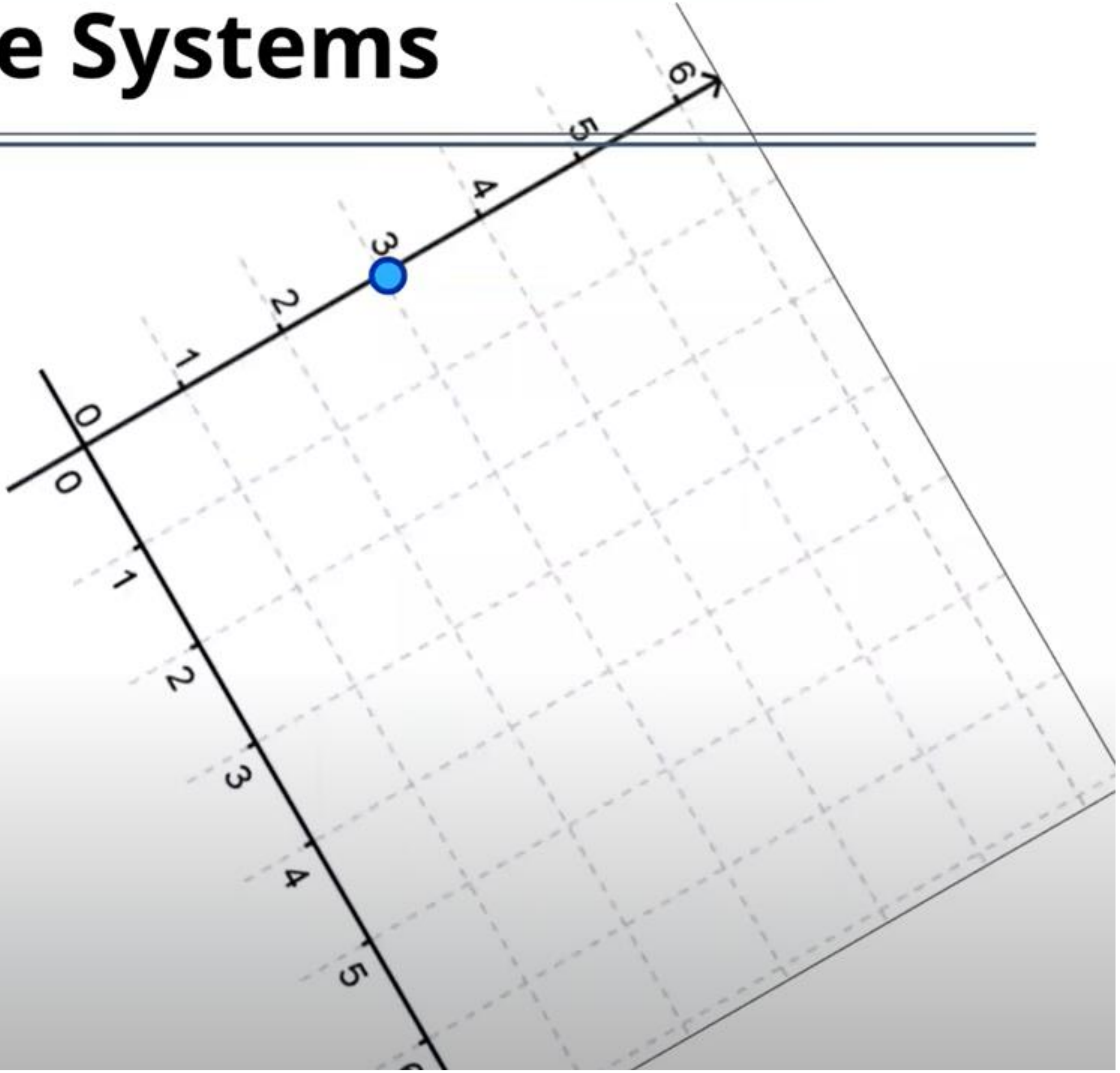


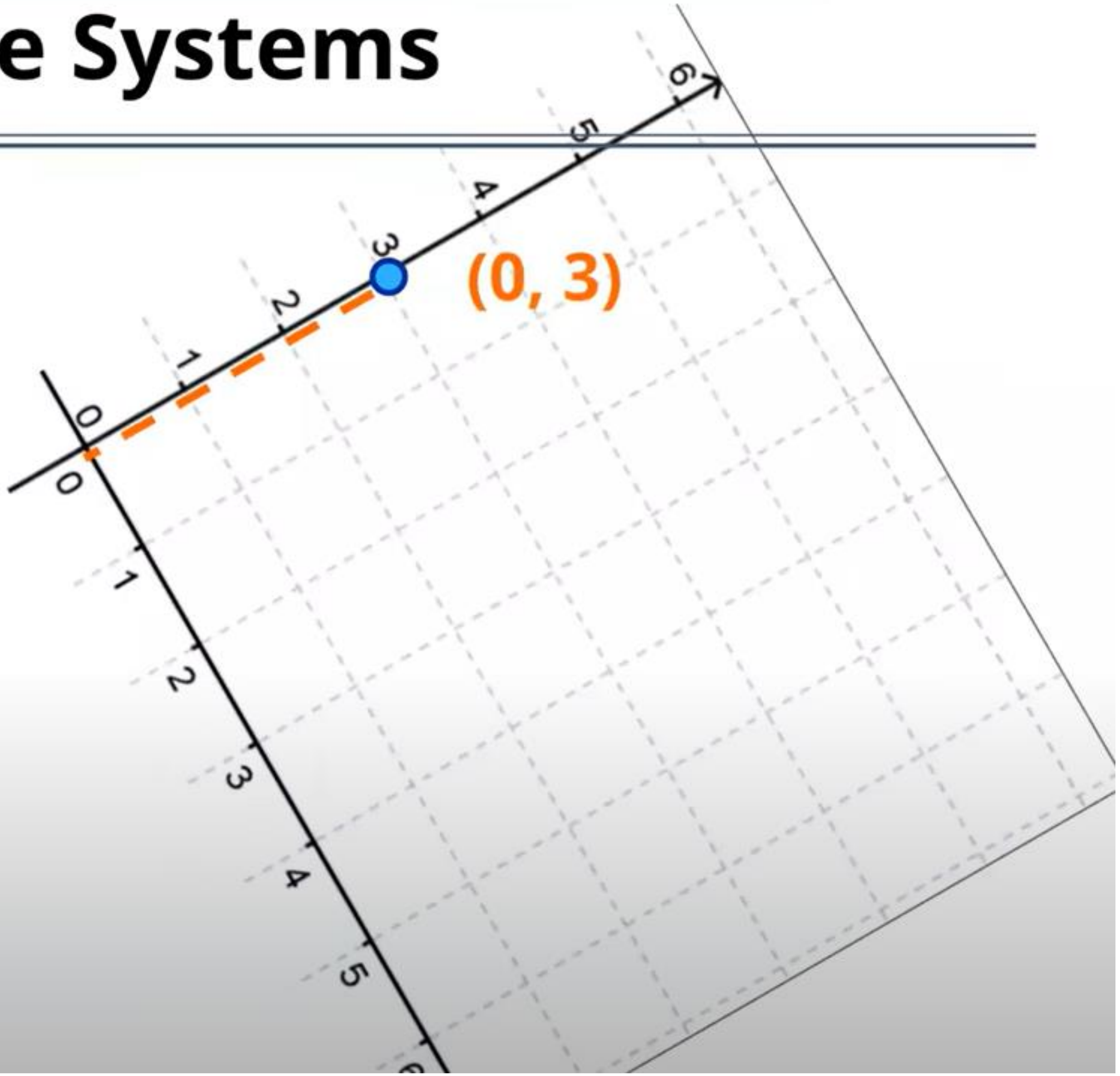
Bases are Coordinate Systems

How about this basis?



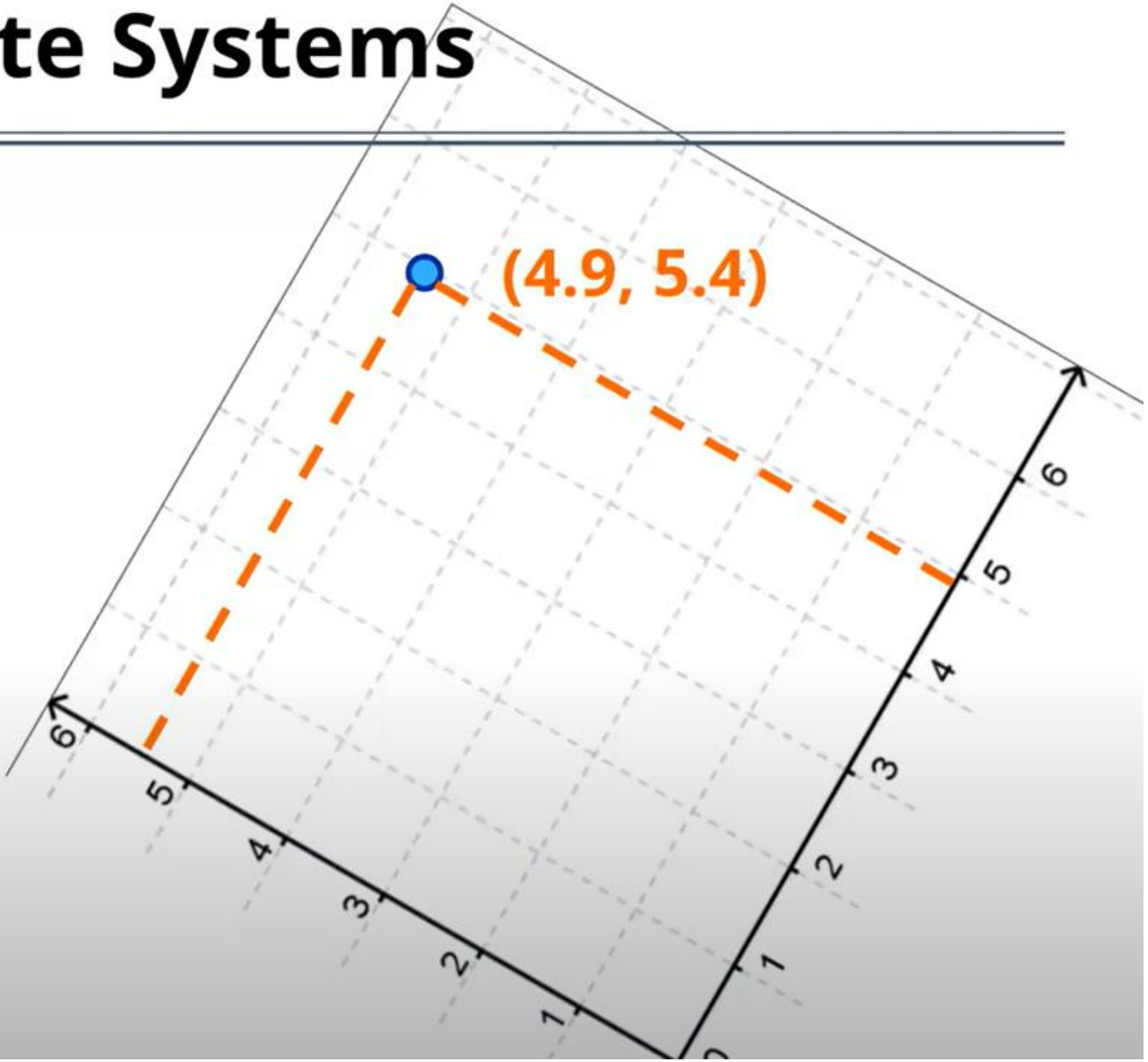
Bases are Coordinate Systems

How about this basis?



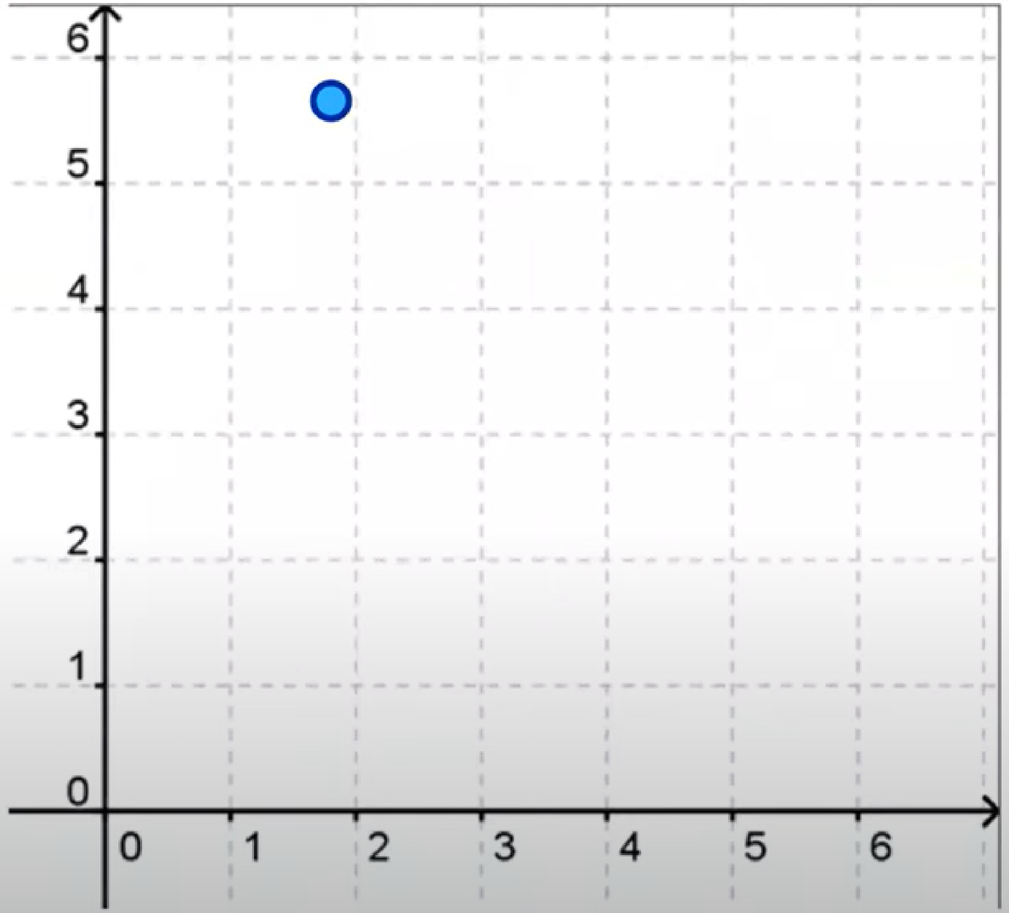
Bases are Coordinate Systems

And in this basis?



Bases are Coordinate Systems

Notice that only the coordinates move while **the point stays in the same place**!



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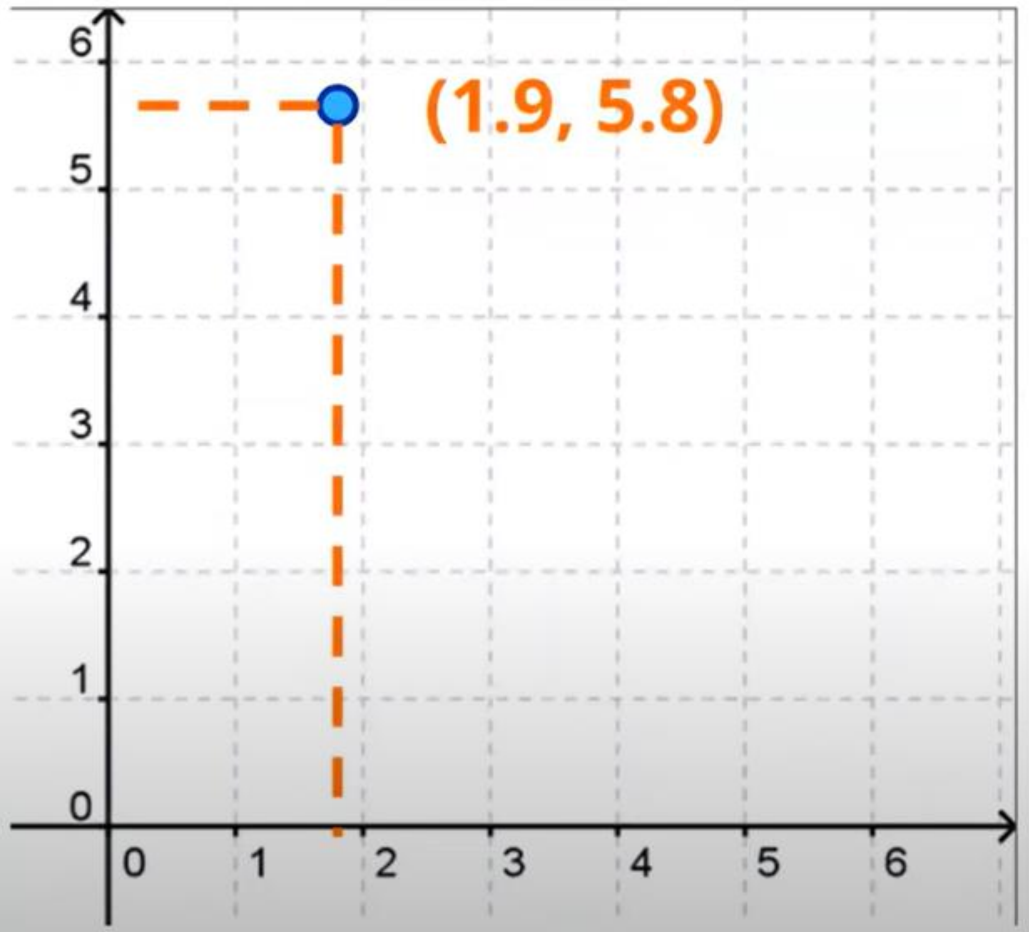

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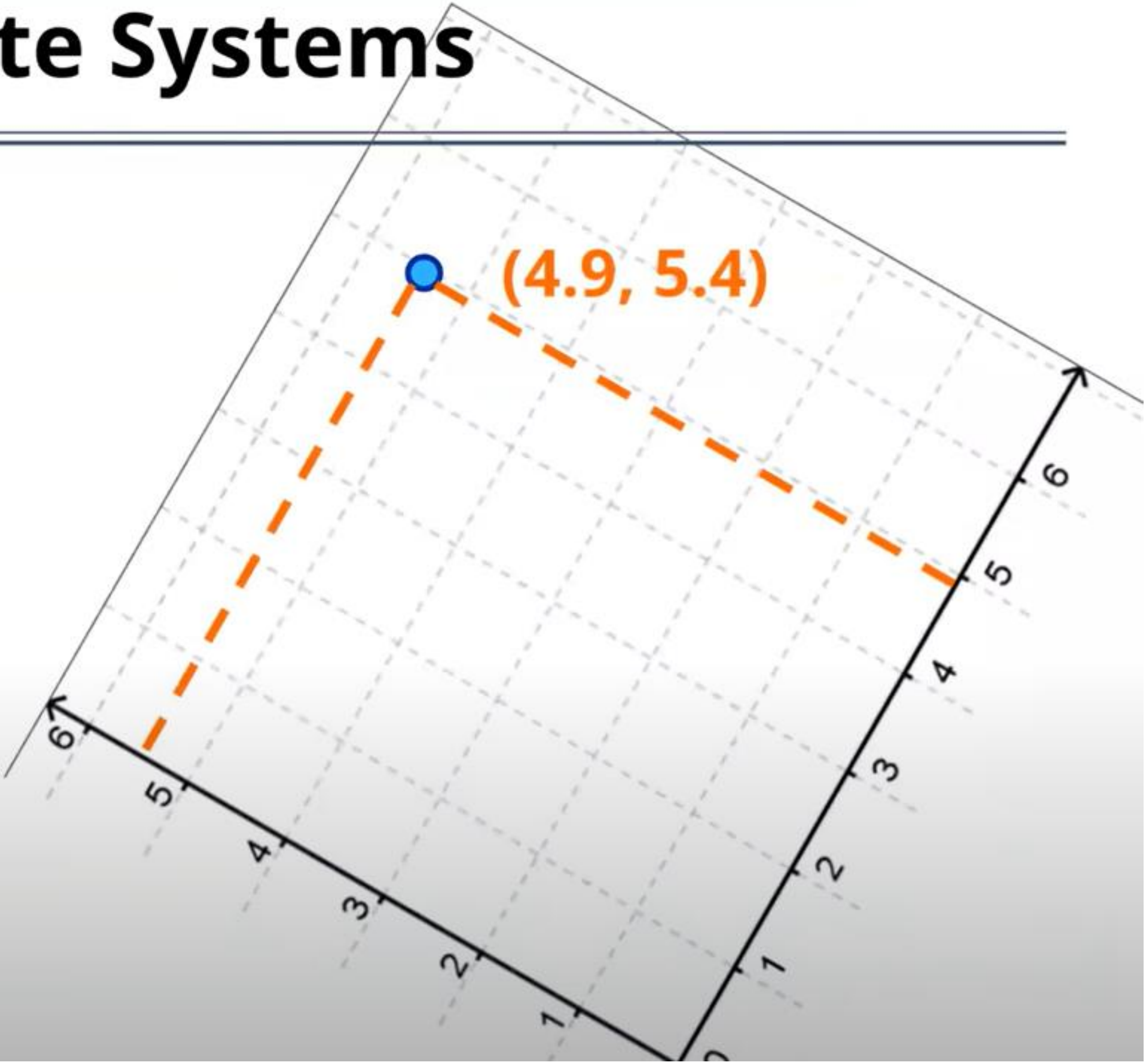

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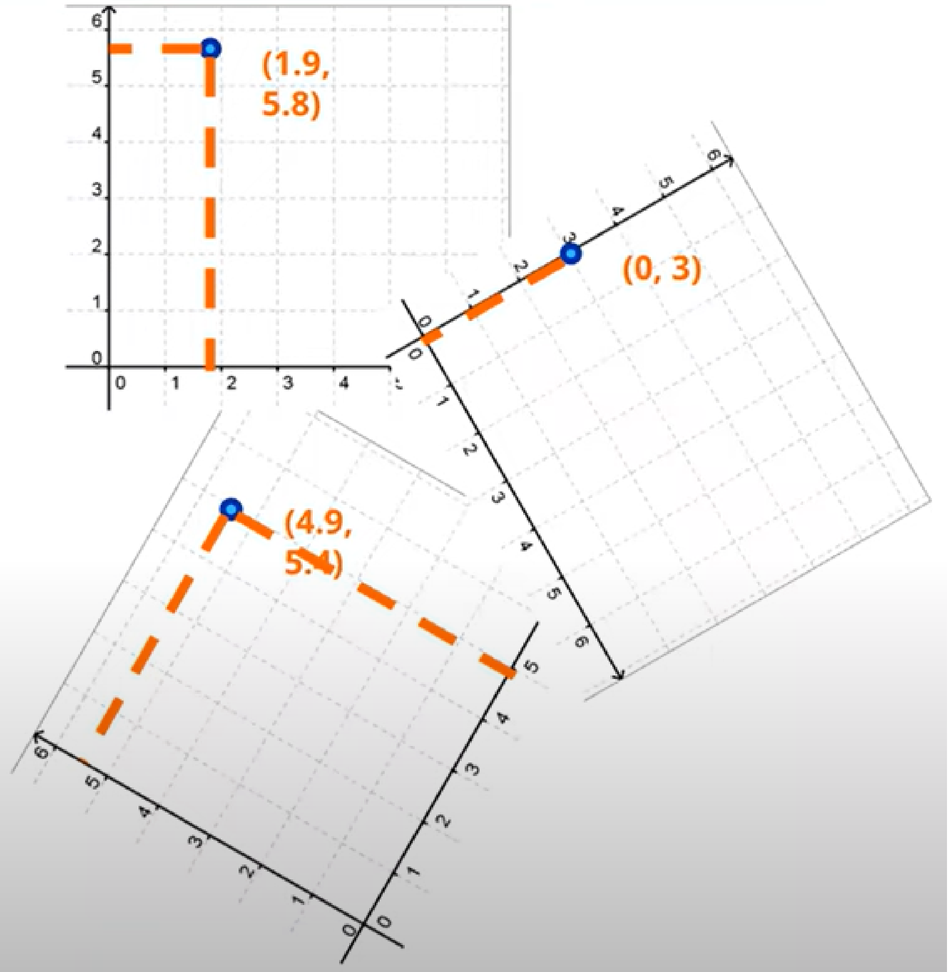
Bases are Coordinate Systems

Measuring the location of a point in different bases will give different results even though the point doesn't move.

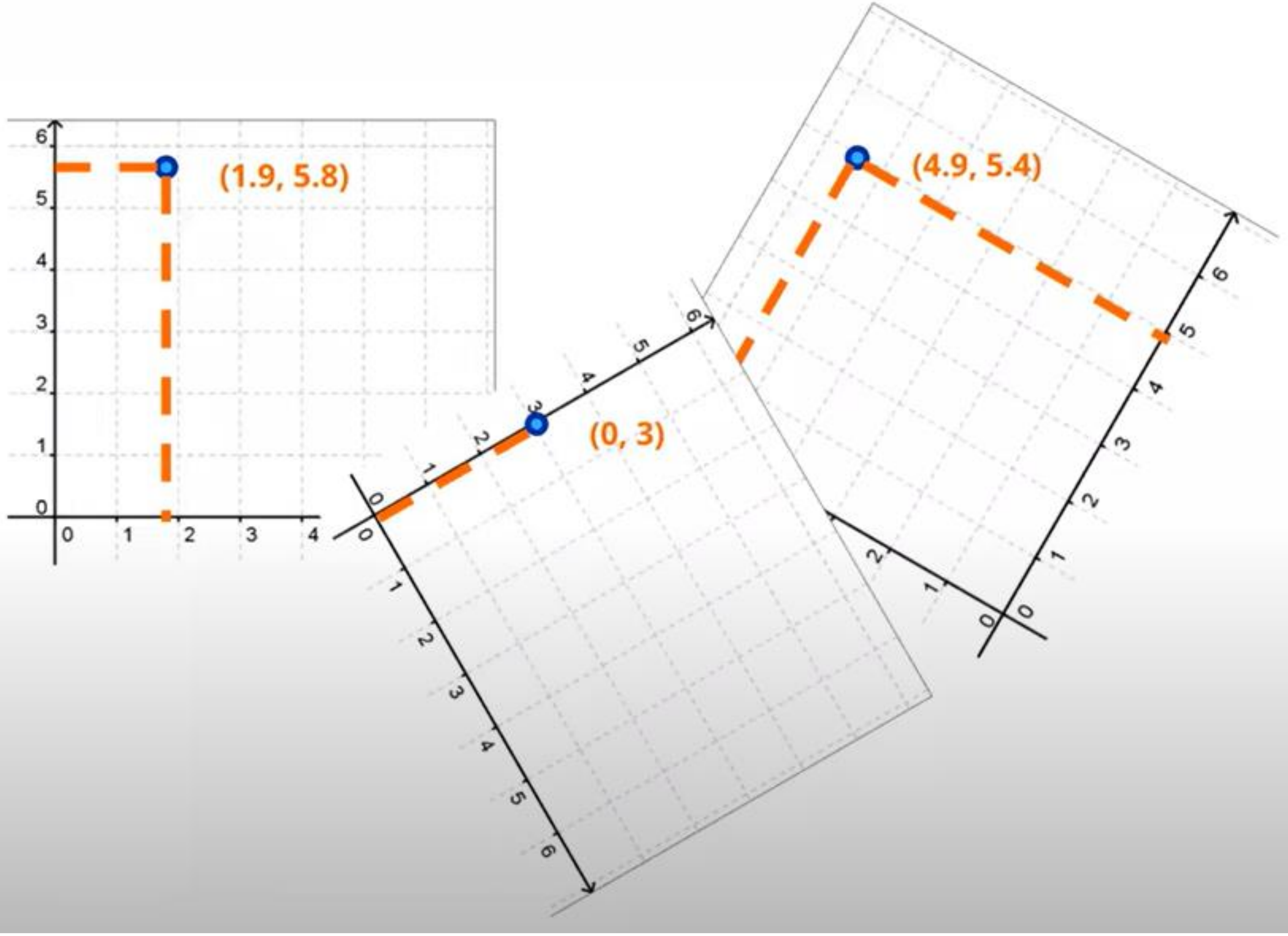
Each basis is equally valid so long as we can describe all possible points using it.



Bases are Coordinate Systems

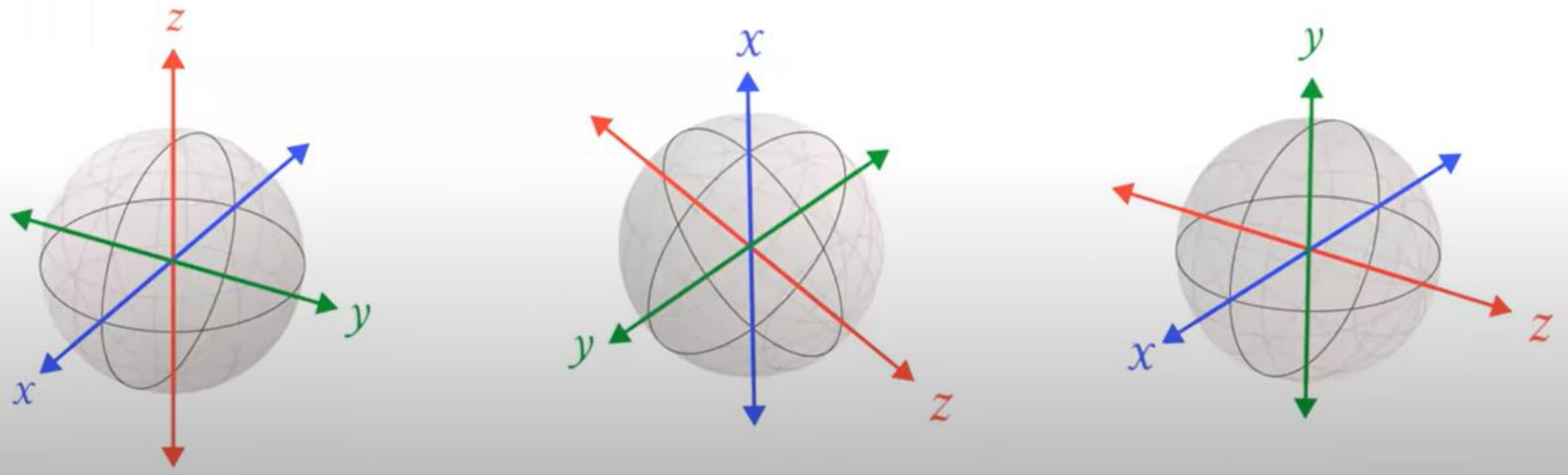
There is no such thing as a “correct” basis.

Each one has its advantages and disadvantages.



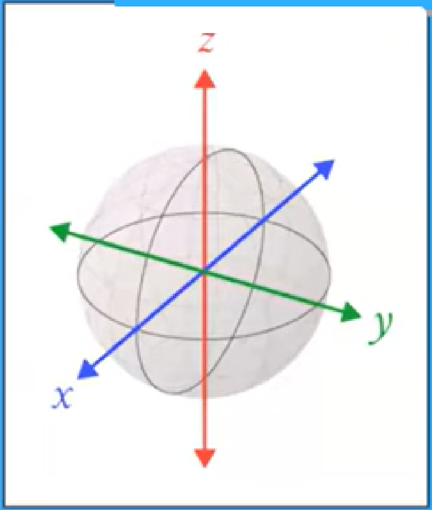
Qubit Bases

Qubits can be described and measured in different bases too! Instead of rotating a graph like we just did, we rotate the whole Bloch sphere!



Z Basis

The z basis describes states with the z axis pointed up. This is commonly called the *computational basis* and it is the standard basis to talk about qubits in.



Basis States

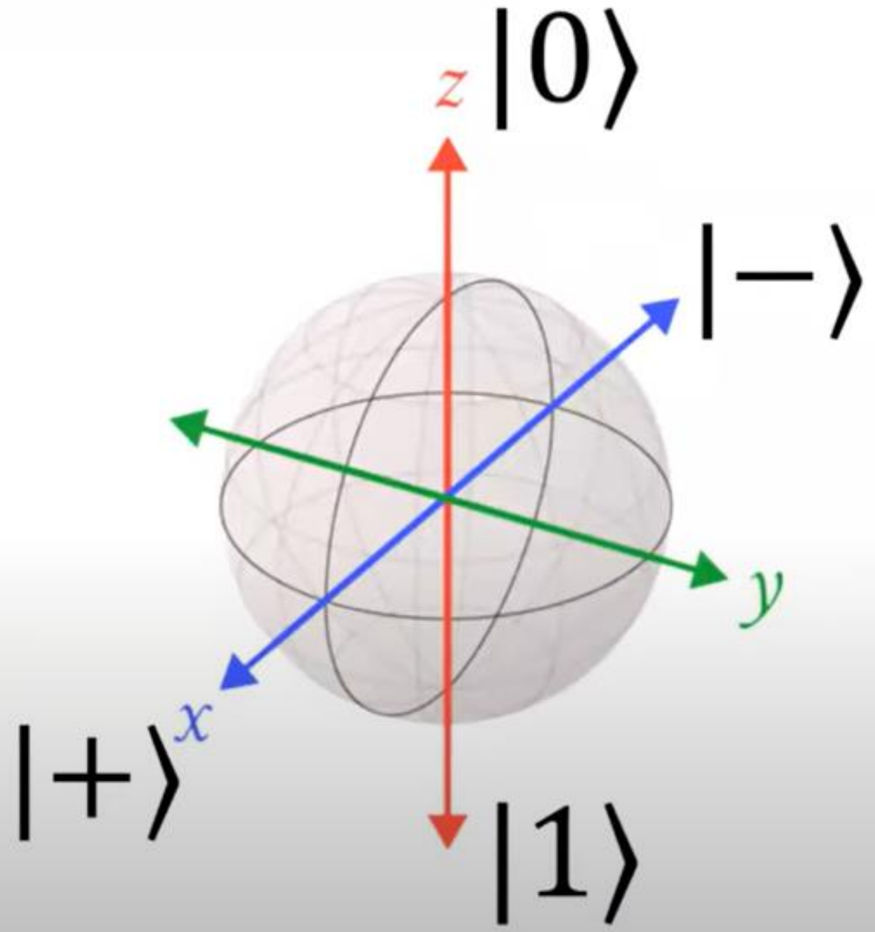
The states corresponding to the up and down directions in a given basis are called the basis states.

$|0\rangle$

$|1\rangle$

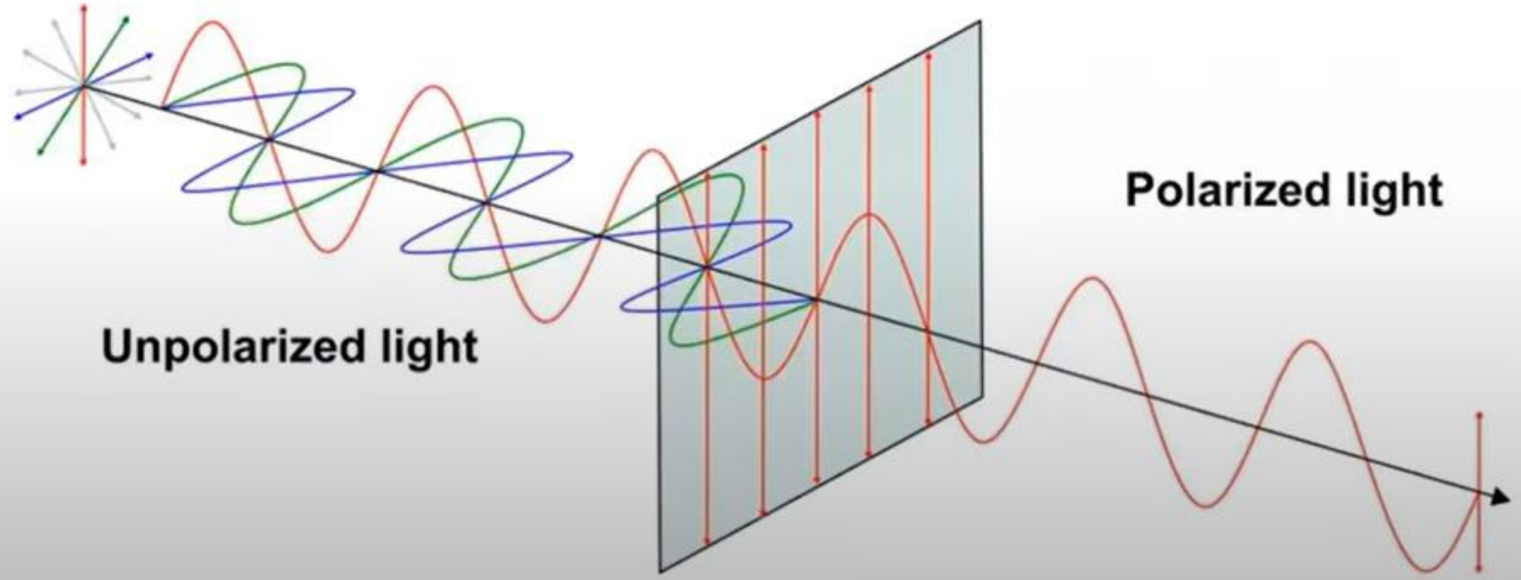
Computational Basis

We have already seen how to describe states in the computational basis:



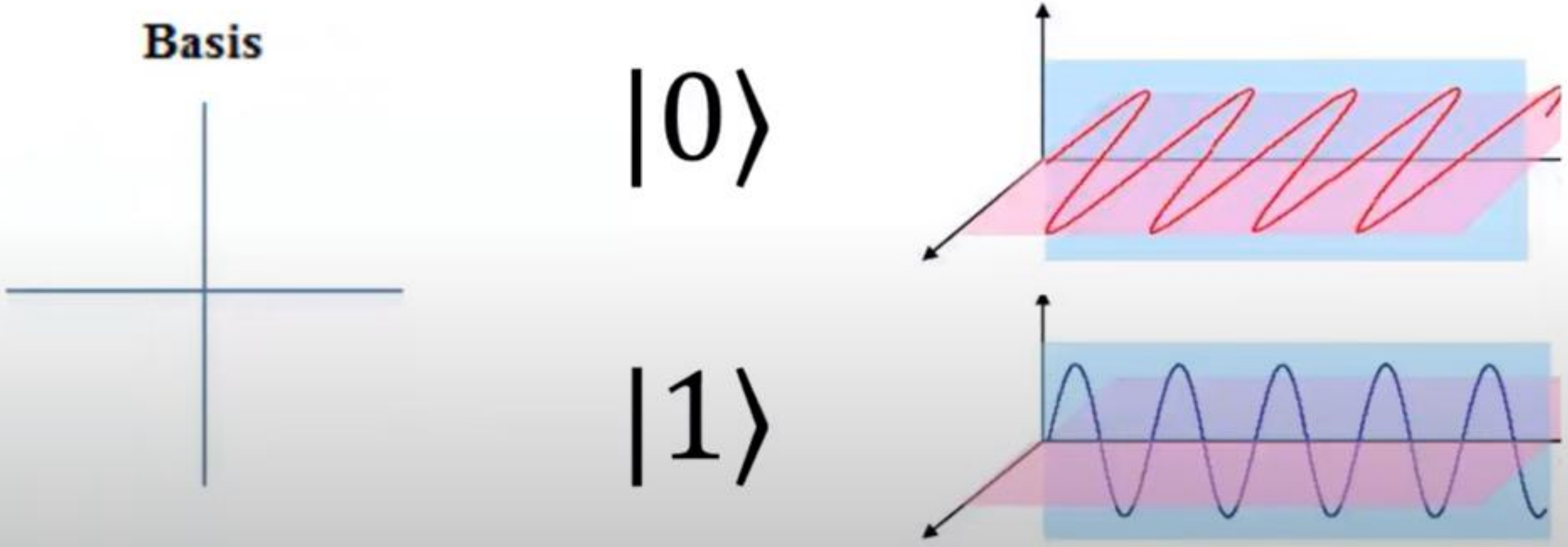
Basis example: photons

We say a light wave is “unpolarized” when it has parts in all directions or axes. Whereas “polarized” light only has a part in one direction or axis as shown below.



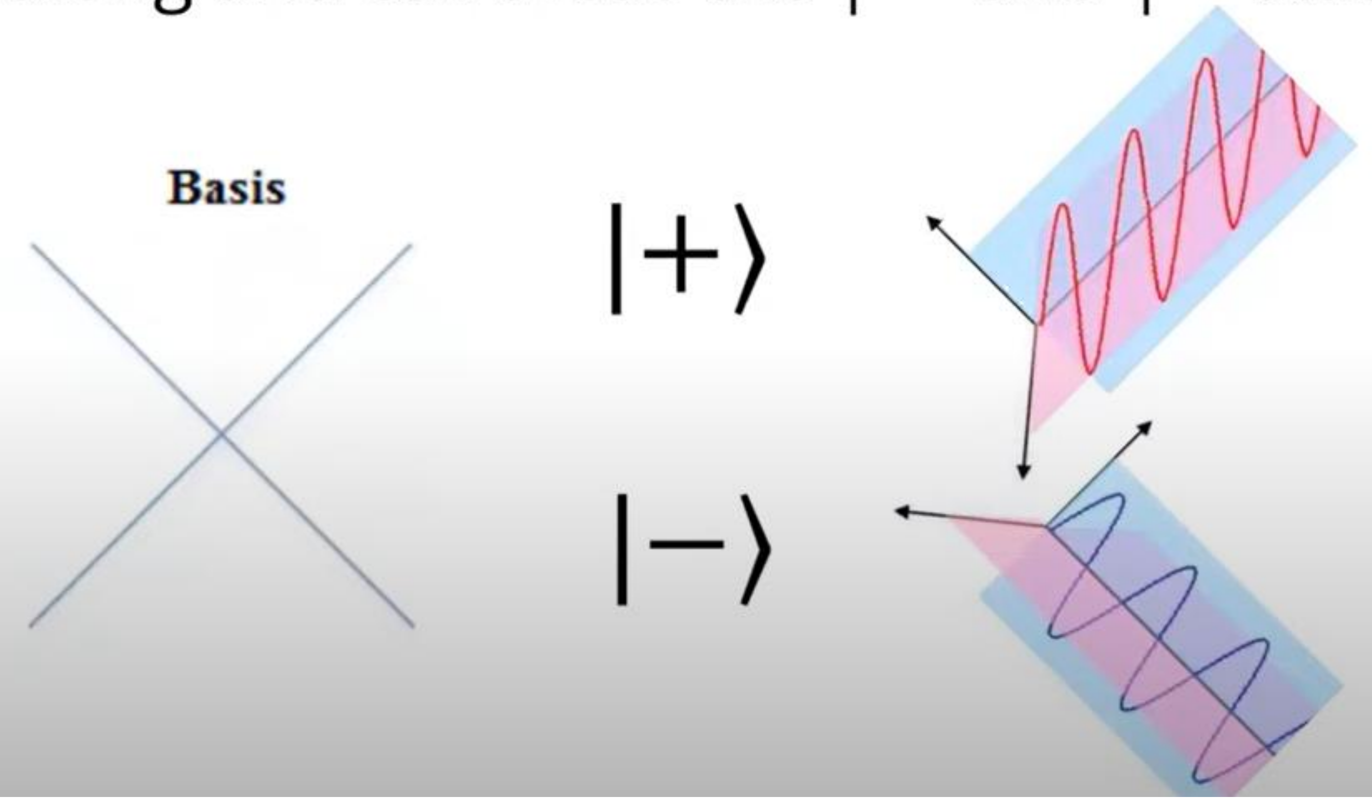
Basis example: photons

So, we can polarize a photon in one of two ways and call that way |0> and the other way |1> as shown below.



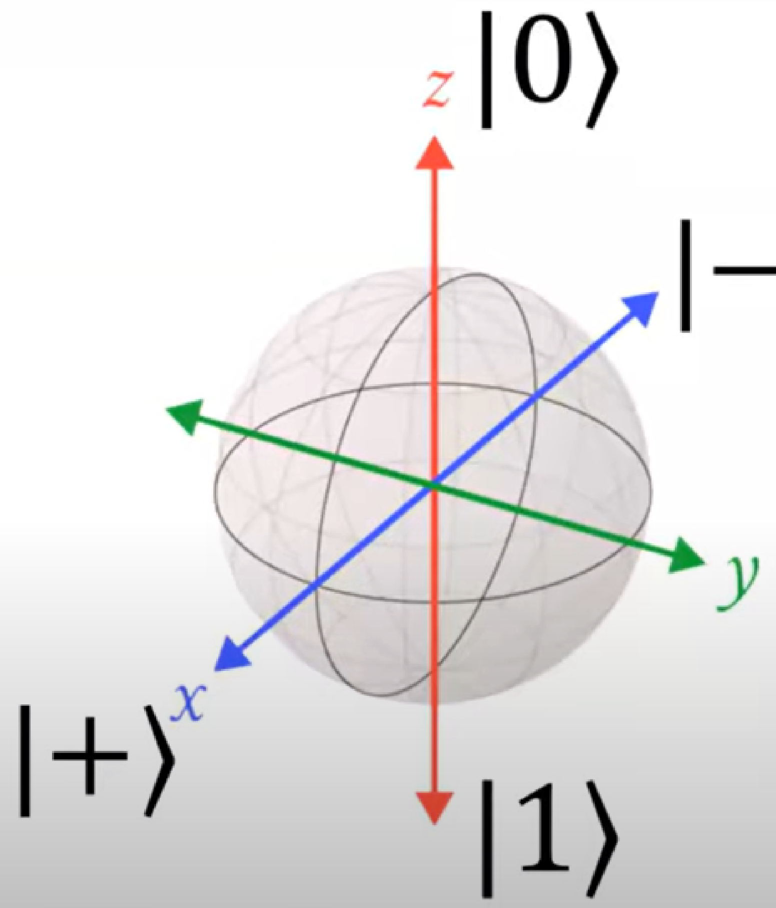
Basis example: photons

Or, we could choose a different set of axes to polarize the photon along and call these the |+> and |-> states.



Computational Basis

How can we describe the + and - states using the computational basis states: 0 and 1?



Computational Basis: + and - states

How can we describe the + and - states using the computational basis states: 0 and 1?

The only difference is a + or -. That's why we call these the + and – states!

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Computational Basis: + and - states

But what does this mean and how does this relate to a 50/50 chance of getting 0 or 1?

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

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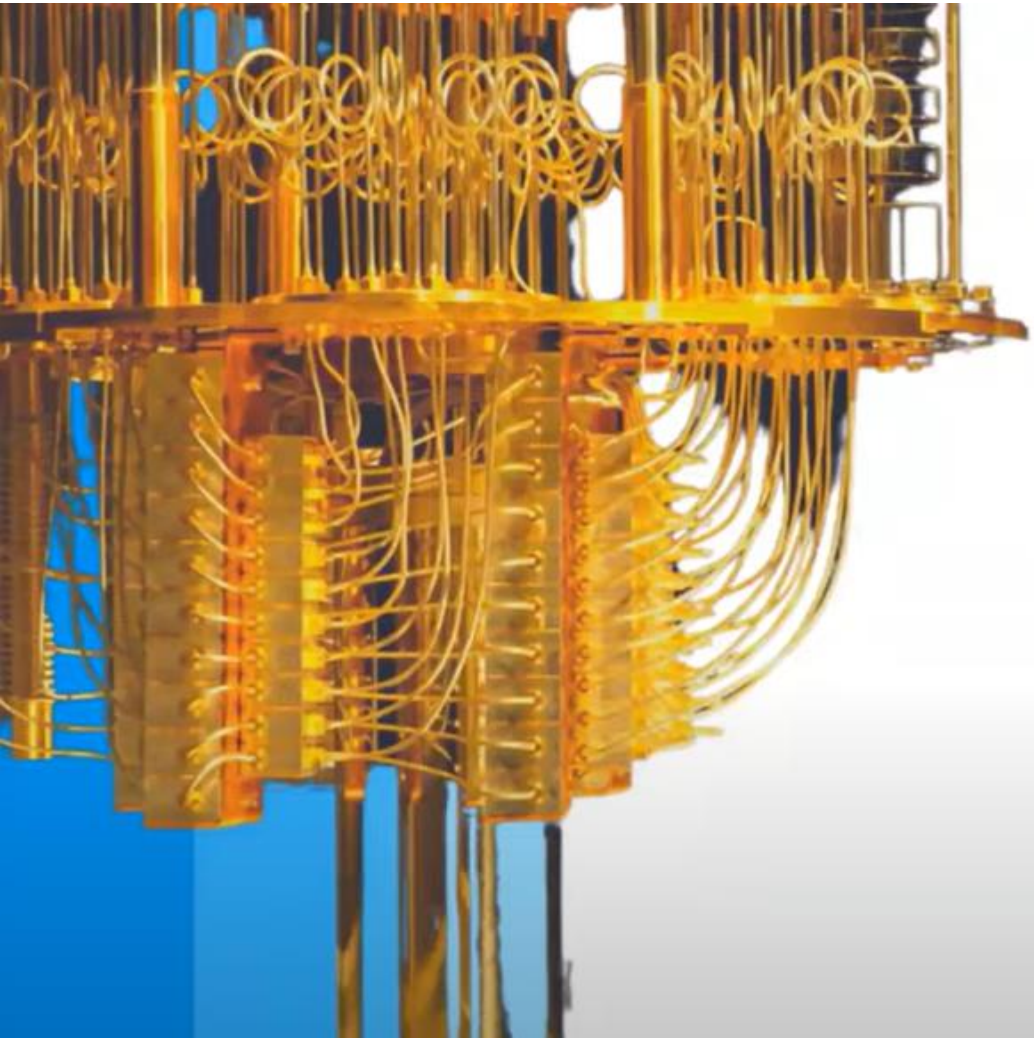
Computational Basis: + and - states

Notice that $(\frac{1}{\sqrt{2}})^2 = \frac{1}{2}$ ...

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$



Lesson Recap

Measurement

Here is what you need to know from today's lecture:

- **Measurement extracts information** and **changes states.**
- The **results of measurements** are always **classical states** and they are **random for any superposition states.**
- We need to run a circuit and **measure many times.**

Bases

Here is what you need to know from today's lecture:

- A **basis** is another word for **coordinate system.**
- The **z basis**, also called the **computational basis**, describes states with the **z axis as up**.