Lista 08

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1 - a)

$$\frac{n(n+1)(2n+1)}{6}, \; n \geq 1$$

1) Passo base : $1^2 = \frac{1(1+1)(2(1)+1)}{6} = 1$ ✓

2) Passo Indutiva :

$$P(k): 1^2 + 2^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

$$P(k+1): \underbrace{1^2 + 2^2 + \ldots + k^2}_{\frac{k(k+1)(2k+1)}{6}} + (k+1)^2 = \frac{(k+1)[(k+1)+1](2(k+1)+1)}{6}$$

$$\frac{k(k+1)(2k+1)}{6} + k+1^2 = \frac{(k+1)[(k+1)+1](2(k+1)+1)}{6}$$

$$\frac{(k^2+k)(2k+1) + 6(k^2+2k+1)}{\cancel{6}} = \frac{(k+1)(k+2)(2k+3)}{\cancel{6}}$$

$$(k^2+k)(2k+1) + 6k^2 + 12k + 6 = (k+1)(k+2)(2k+3)$$

$$2k^3 + k^2 + 2k^2 + k + 6k^2 + 12k + 6 = 2k^3 + 9k^2 + 13k + 6$$

$$2k^3 + 9k^2 + 13k + 6 = 2k^3 + 9k^2 + 13k + 6$$ ✓

Provado Por Indução

b) $\frac{n}{n+1}, \; n \geq 1$

1) Passo base : $\frac{1}{1.2} = \frac{1}{1+1} = \frac{1}{2}$ ✓

2) Passo Indutiva :

$$P(k) = \frac{1}{1.2} + \frac{1}{2.3} + \ldots + \frac{1}{k(k+1)} = \frac{k}{k+1}$$

$$P(k+1) = \underbrace{\frac{1}{1.2} + \frac{1}{2.3} + \dots + \frac{1}{k(k+1)}}_{\frac{k}{k+1}} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$$

$$\frac{k}{k+1} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2} \Rightarrow \frac{k\cancel{(k+2)} + 1}{(k+1)\cancel{(k+2)}} = \frac{k+1}{\cancel{k+2}} \Rightarrow$$

$$\frac{k^2 + 2k + 1}{k+1} = k+1 \Rightarrow \frac{(k+1)^2}{\cancel{(k+1)}} = k+1$$

$k+1 = k+1$ Provados por indução

e) $\frac{m+1}{2m}, m \geq 2$

1) Passo Base: $\left(1 - \frac{1}{2^2}\right) = \frac{2+1}{2(2)} = \frac{3}{4}$

2) Passo Indutivo:

$$P(k) = \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\cdots\left(1 - \frac{1}{k^2}\right) = \frac{k+1}{2k}$$

$$P(k+1) = \underbrace{\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\cdots\left(1 - \frac{1}{k^2}\right)}_{\frac{k+1}{2k}} \cdot \left(1 - \frac{1}{(k+1)^2}\right) = \frac{k+2}{2(k+1)}$$

$$\left(\frac{k+1}{2k}\right)\left(1 - \frac{1}{(k+1)^2}\right) = \frac{k+2}{2(k+1)} \Rightarrow \left(\frac{k+1}{2k}\right) \cdot \left(\frac{k+1}{2k(k+1)^2}\right) = \frac{k+2}{2(k+1)} \Rightarrow$$

$$\Rightarrow \frac{(k+1)^3 - k+1}{2k(k+1)^2} = \frac{k+2}{2(k+1)} \Rightarrow \frac{\cancel{(k+1)^2}\left[(k+1) - \frac{1}{k+1}\right]}{2k\cancel{(k+1)^2}} = \frac{k+2}{2(k+1)} \Rightarrow$$

$$\Rightarrow \frac{k+1 - \frac{1}{k+1}}{2k} = \frac{k+2}{2(k+1)} \Rightarrow \frac{(k+1)^2 - 1}{\cancel{k+1}} = \frac{\cancel{2}k(k+2)}{\cancel{2}\cancel{(k+1)}} \Rightarrow$$

$$\Rightarrow (k+2)^2 - 1 = k(k+2) \Rightarrow (k^2 + 2k + \cancel{1}) - \cancel{1} = k^2 + 2k \Rightarrow$$

$\Rightarrow k^2 + 2k = k^2 + 2k \rightarrow$ Provado por indução

9- a)

1) Passo Base: $n=1 \rightarrow \frac{7-2}{5} = q = 1$

$n=2 \rightarrow \frac{49-4}{5} = 9 = q$

2) Passo Indutivo:

$$7^k - 2^k = 5q_1 \Rightarrow 7^{k+1} - 2^{k+1} = 5q_2$$

$$7^{k+1} - 2^{k+1} = 7.7^k - 2.2^k = 7(5q_1 + 2k) - 2.2^k =$$

$$7.5q_1 + 7.2^k - 2.2^k = 35q_1 + 2^k(7-2) = \underbrace{5(7q_1 + 2^k)}_{q}$$

Provada Por Indução

b) $n^3 - 4n + 6 \div 3$

1) Passo base: $\frac{(1)^3 - 4(1) + 6}{3} = q = 1$

2) Passo Indutivo:

$$k^3 - 4k + 6 = 3q_1 \Rightarrow (k+1)^3 - 4(k+1) + 6 = 3q_2$$

$$k^3 + 3k^2 + 3k + 1 - 4k + 4 + 6 = 3q_2$$

$$\underbrace{(k^3 - 4k + 6)}_{3q_1} + 3k^2 + 3k + 1 + 4 = 3q_2$$

$$3k^2 + 3k + 5 = 3q_2 - 3q_1$$

$$3k^2 + 3k + 5 = \underbrace{3(q_2 - q_1)}_{q}$$

Provada Por Indução

c) $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}^n = \begin{bmatrix} 2^n & n.2^{n-1} \\ 0 & 2^n \end{bmatrix}, n \geq 1$

1) Passo base: $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}^1 = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

2) Passo Indutivo:

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}^k = \begin{bmatrix} 2^k & k.2^{k-1} \\ 0 & 2^k \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}^{k+1} = \begin{bmatrix} 2^{k+1} & (k+1).2^{(k)} \\ 0 & 2^{k+1} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}^k . \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2^k & k.2^{k-1} \\ 0 & 2^k \end{bmatrix} . \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2^{k+1} & (k+1).2^k \\ 0 & 2^{k+1} \end{bmatrix}$$

$$\begin{bmatrix} 2.2^k & 2^k + 2(k.2^{k-1}) \\ 0 & 2.2^k \end{bmatrix} = \begin{bmatrix} 2^{k+1} & (k+1)2^k \\ 0 & 2^{k+1} \end{bmatrix}$$

$$2.2^k = 2^{k+1};$$

$$2^k + 2(k.2^{k-1}) = (k+1)2^k$$

$$2^k + 2.k.2^{k-1} = (k+1)2^k$$

$$2^k + k.2^{k-1(+1)} = k.2^k + 2k$$

$$k.2^k + 2k = k.2^k + 2k \; C$$ Provado por Indução