problem_set_6
October 20, 2021

1 Problem Set 6

1.0.1 Learning Objective:

• Formulate linear optimization models to inform a business decision.

1.0.2 Overview:

This problem set assesses your ability to analyze a given business descision by creating a concrete formulation of a linear optimization model, as discussed in the lectures for Week 8. If you know some LaTeX, you can typeset within notebook cells. Otherwise you can create a separate word or pdf file to turn in. It is not ideal but you may handwrite and scan.

1.0.3 Grading

There are three possible scores you can get from submitting this assignment on time (submitting a blank file or one without any apparent effort does not count). Note that the rubric is designed to incentivize you to go for 100% mastery of the material, as the little details matter a lot in business analytics.

Grade	Description
5 out of 5	Perfect submission with no significant errors.
4 out of 5	Near perfect submission with one or more significant errors.
2 out of 5	Apparent effort but far from perfect.

1.1 Q1. Production Planning

(Adapted from DMD Exercise 7.2) The Magnetron Company manufactures and markets microwave ovens. Currently, the company produces two models: full-size and compact. Production is limited by the amount of labor available in the general assembly and electronic assembly departments, as well as by the demand for each model. Each full-size oven requires 2 hours of general assembly and 2 hours of electronic assembly, whereas each compact oven requires 1 hour of general assembly and 3 hours of electronic assembly. In the current production period, there are 500 hours of general assembly labor available and 800 hours of electronic assembly labor available.

In addition, the company estimates that it can sell at most 220 full-size ovens and 180 compact ovens in the current production period. The earnings contribution per oven is 120 dollars for a full-size oven and 130 dollars for a compact oven. The company would like to find an earnings-maximizing production plan for the current production period.

a) Succintly describe the decision, objective and constraints in English.
   Decision variables:
   Number of full-size ovens produced
   Number of compact ovens
   Objective:
   Maximize the profit for selling two types of ovens
   Constraints:
   General assembly labor
   Electronic assembly
   Demand for each type of oven

b) Translate the above English description into a concrete formulation of a linear optimization problem. **Decision variables:**
   f: Number of full-size ovens produced (Integer)
   c: Number of compact ovens (Integer)
   Objective:

$$\textbf{Maximize: } 120f + 130c$$

   Constraints:

(General assembly labor)	$2f + c \leq 500$
(Electronic assembly labor)	$2f + 3c \leq 800$
(Demand for full-size ovens)	$f \leq 220$
(Demand for compact ovens)	$c \leq 180$
(Non-negativity)	$f, c \geq 0$

1.2 Q2. Portfolio Planning

(Adapted from DMD Exercise 7.12) An investor would like to construct an optimal portfolio consisting of five possible funds. (A portfolio consists of a certain amount of money in each fund.) The five funds and their respective fund categories, risk levels, and percentage annual returns are shown below.

Fund	Category	Risk Level	Percentage Annual Return
A	Money Market	1	4.50%
B	Money Market	2	5.62 %
C	Bond	2	6.80%
D	Bond	3	10.15%
E	Aggressive Growth	5	20.60%

The risk level of risky. The investor would like to maximize the total annual return (i.e., the total monetary amount earned) of the portfolio subject to the following restrictions:

1. The average risk level of the entire investment should not exceed 2.5. (The average here is weighted by the amount of money in each fund. For example, if the entire investment consists of 7500 in C and 1000 in D, then the average risk level is (7500×2+1000×3)/(7500+1000) ≈ 2.12.)
2. At least 30% of the money invetsed should be placed in money market funds.
3. At most 2,000 dollars should be invested in the aggressive growth fund.
4. The total amount of initial investment should be between 5,000 and 10,000 dollars (inclusive).

a) Succintly describe the decision, objective and constraints in English.

Decision variables:

Amount of investment in each fund

Objective:

Maximize total annual return

Constraints:

Average risk level
Ratio of fund A and B
Amount of money in Fund E
Total initial investment

b) Translate the above English description into a concrete formulation of a linear optimization problem.

Decision variables:

a: Amount of investment in fund A (Continuous)
b: Amount of investment in fund B (Continuous)
c: Amount of investment in fund C (Continuous)
d: Amount of investment in fund D (Continuous)
e: Amount of investment in fund E (Continuous)

Objective:

Maximize: $0.045a + 0.0562b + 0.068c + 0.1015d + 0.206e$

Constraints:

(Average risk level) $\frac{a+2b+2c+3d+5e}{a+b+c+d+e} \leq 2.5$

(Ratio of fund A and B) $\frac{a+b}{a+b+c+d+e} \geq 0.3$

(Amount of money in Fund E) $e \leq 2000$

(Total initial investment) $5000 \leq a + b + c + d + e \leq 10000$

(Non-negativity) $a, b, c, d, e \geq 0$

1.3 Q3. Assignment of Consultants to Projects

There are two projects and four consultants: Alice, Bob, Charles, and Daphne. Each consultant can be assigned to at most one project, and each project requires at least two consultants. As a manager, you evaluated the relative fitness of the four consultants for each project on a scale of 1 to 5, with 5 being the best fit and 1 being the worst.

	Project 1	Project 2
Alice	5	2
Bob	3	2
Charles	4	5
Daphne	3	1

Furthermore, Alice, Bob and Daphne are senior consultants and each project requires at least one senior on the team.

Formulate a linear optimization problem to maximize the total fitness of the consultants to their assigned project, subject to all the business constraints.

a) Succintly describe the decision, objective and constraints in English.

Decision variables:

Whether each consultants do project 1 or not

Objective:

Maximize total fitness

Constraints:

Required consultants in each project

Required senior consultants in each project

b) Translate the above English description into a concrete formulation of a linear optimization problem.

Decision variables:

a: Whether Alice do project 1 (Binary)

b: Whether Bob do project 1 (Binary)

c: Whether Charles do project 1 (Binary)

d: Whether Daphne do project 1 (Binary)

Objective:

$$\textbf{Maximize: } 5a + (1-a)2 + 3b + (1-b)2 + 4c + (1-c)5 + 3d + (1-d)$$

Constraints:

(Consultants) $a + b + c + d = 2$

(Senior consultants) $1 \leq a + b + d \leq 2$