Summer School on SLAM
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Data Association in SLAM

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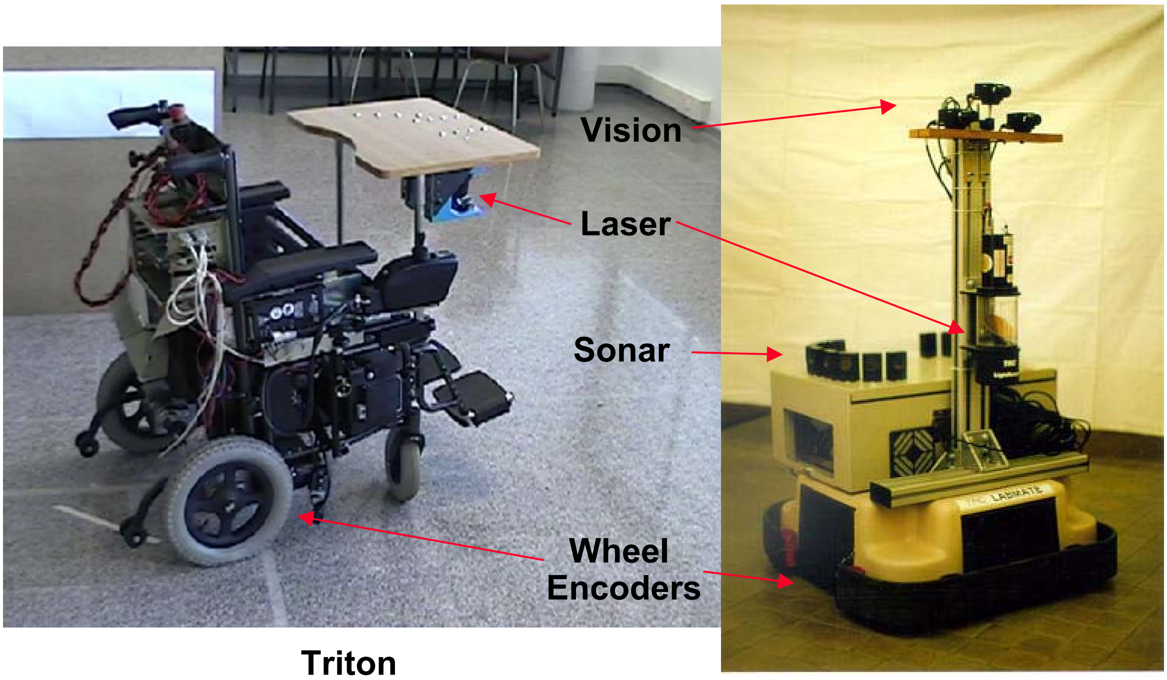
UZ – Robotics, Vision and Real Time Group

10 Faculty members

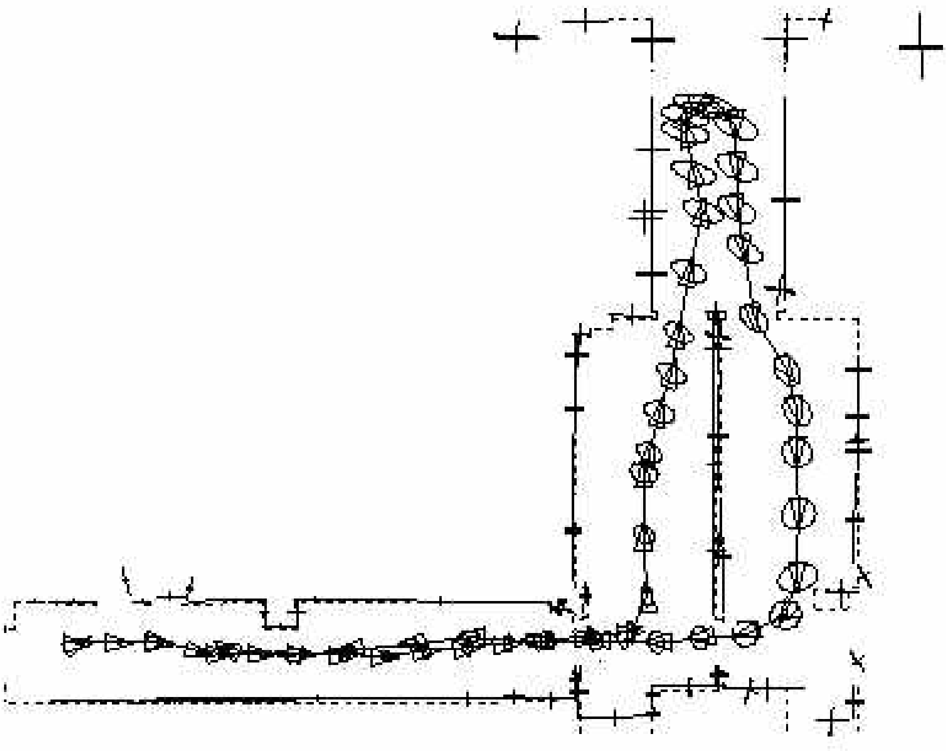
~10 PhD and Master students



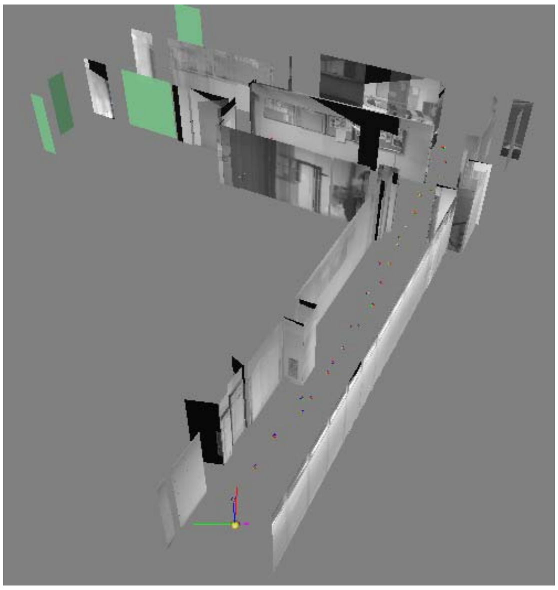
Triton

Otilio

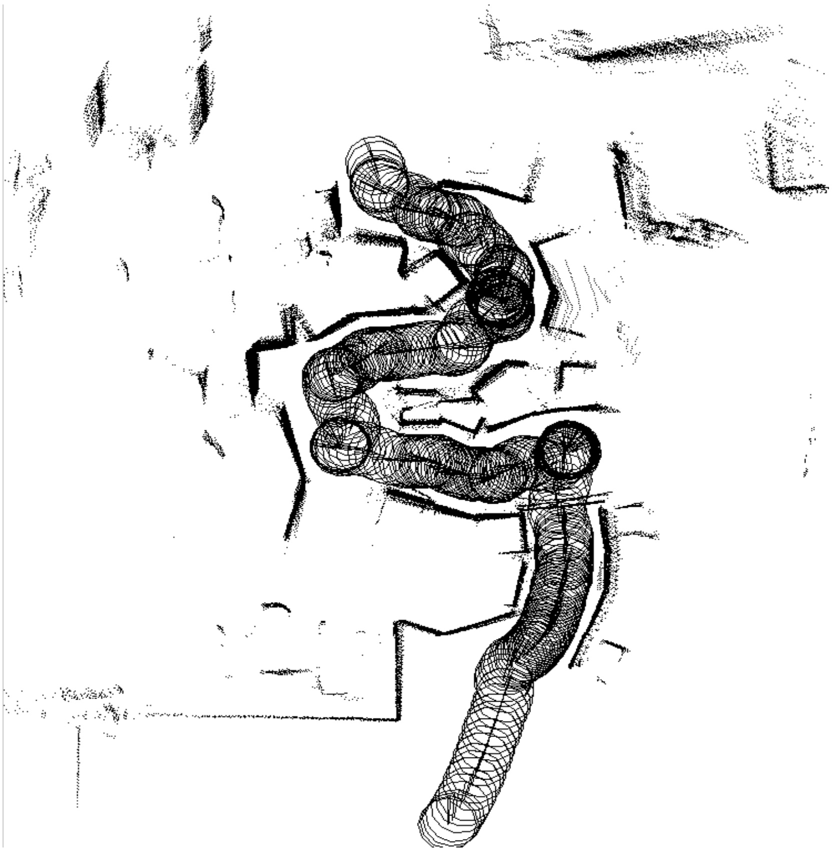
UZ: SLAM



UZ: Computer Vision and 3D Model Building



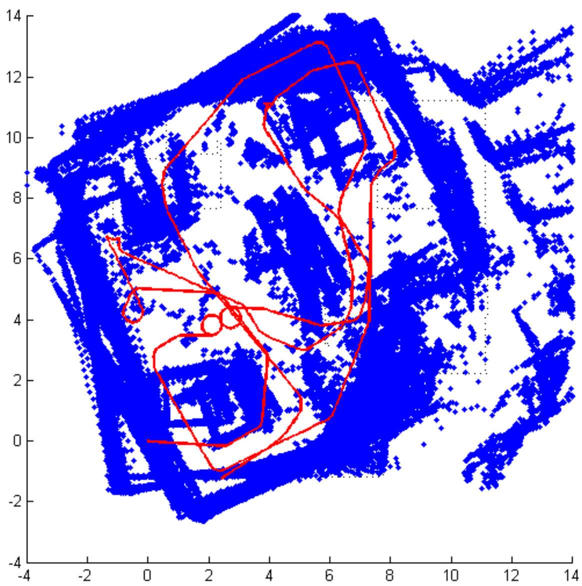
UZ: Robot Navigation

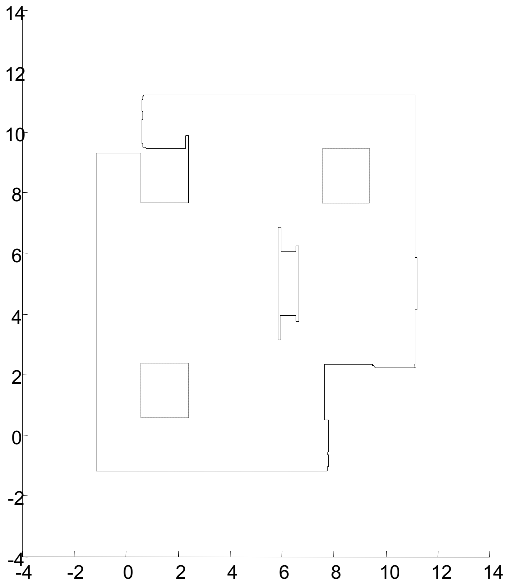


Data Association in SLAM

1. Introduction
2. Feature extraction: Laser, Sonar,…
3. Data association in continuous SLAM
   - Nearest Neighbor .vs. Joint Compatibility
   - SLAM with Laser and Sonar
   - Map joining
   - The loop closing problem
4. Robot relocation and map matching
   - Geometric Constraints
   - A linear time algorithm
   - Application to multi-robot mapping
5. Conclusion

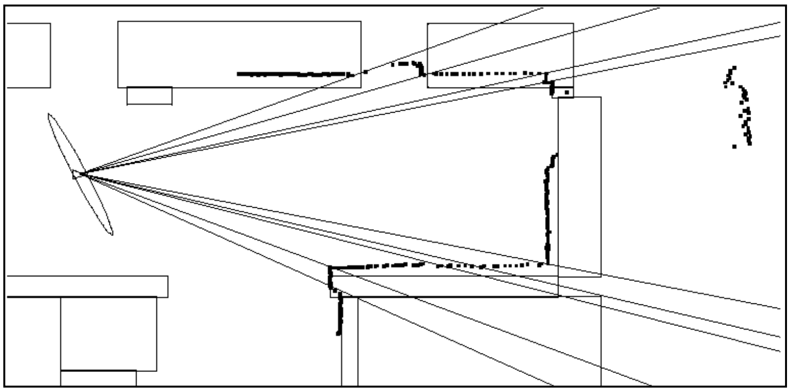
The SLAM problem





Data Association

- Given an environment map
- And a set of sensor observations
- Associate observations with map elements



Importance of Data Association

- Measurement $\mathbf{y}$ is used to improve estimation of $\mathbf{x}$:

EKF update:

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k-1} + \mathbf{K}_k\left(-\mathbf{h}_{k-1}\right)$$
$$\mathbf{P}_k = \mathbf{P}_{k-1} - \mathbf{K}_k\mathbf{H}_{k-1}\mathbf{P}_{k-1}$$
$$\mathbf{K}_k = \mathbf{P}_{k-1}\,\mathbf{H}_{k-1}^T\left(\mathbf{H}_{ij}\,\mathbf{P}\;\mathbf{H}_{ij}^T + \mathbf{G}_{ij}\,\mathbf{S}\;\mathbf{G}_{ij}^T\right)^{-1}$$

- If the association of E_i with feature F_j is…..

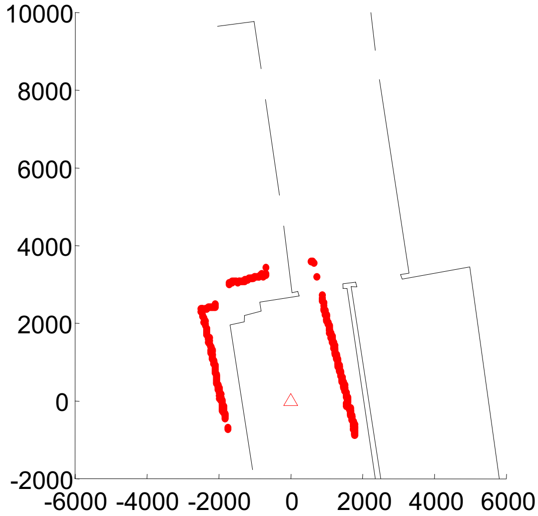
	correct:	spurious:
error:	$\mathbf{x}-\hat{\mathbf{x}}$ ↓	$\mathbf{x}-\hat{\mathbf{x}}$ ↑
covariance:	$\mathbf{P}$ ↓	$\mathbf{P}$ ↓
	Consistency	Divergence!

Difficulties: clutter

- Influence of the ***type***, ***density, precision*** and ***robustness*** of features considered:

Laser scanner:

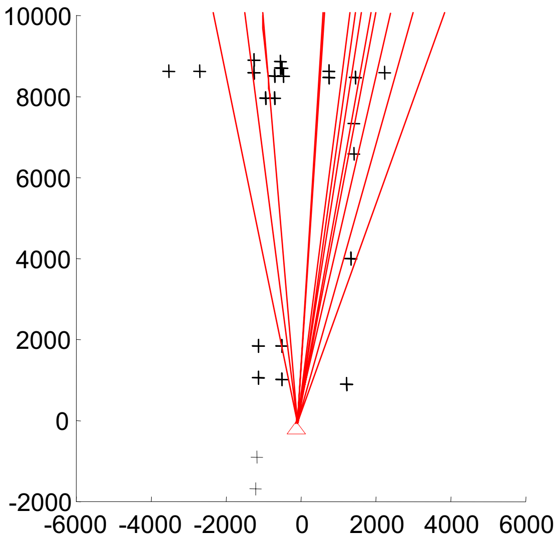
- *Small amount of features (n)*
- *Small amount of measurements (m)*
- Low spuriousness



Low clutter

Difficulties: clutter

Vertical Edge Monocular vision:

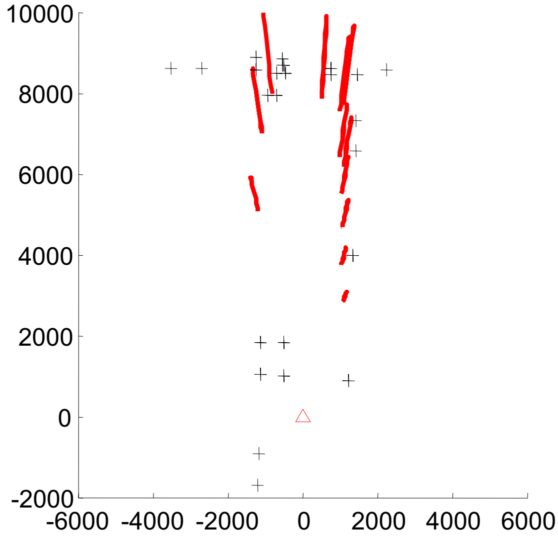


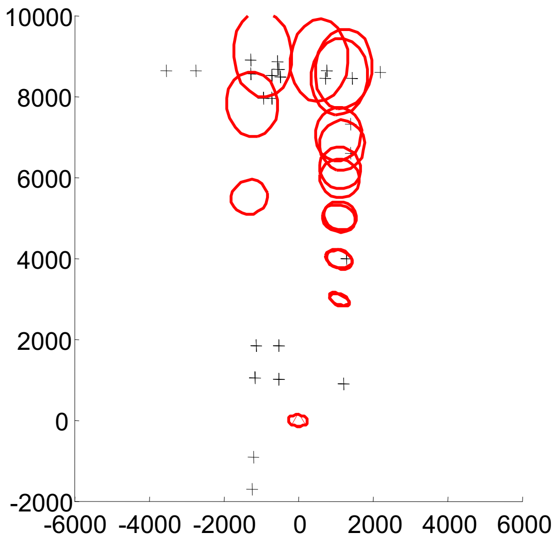
- Many features (*n* large)
- Many measurements (*m* large)
- no depth information
- higher spuriousness

High clutter

Difficulties: imprecision

- Both the sensor and the vehicle introduce imprecision

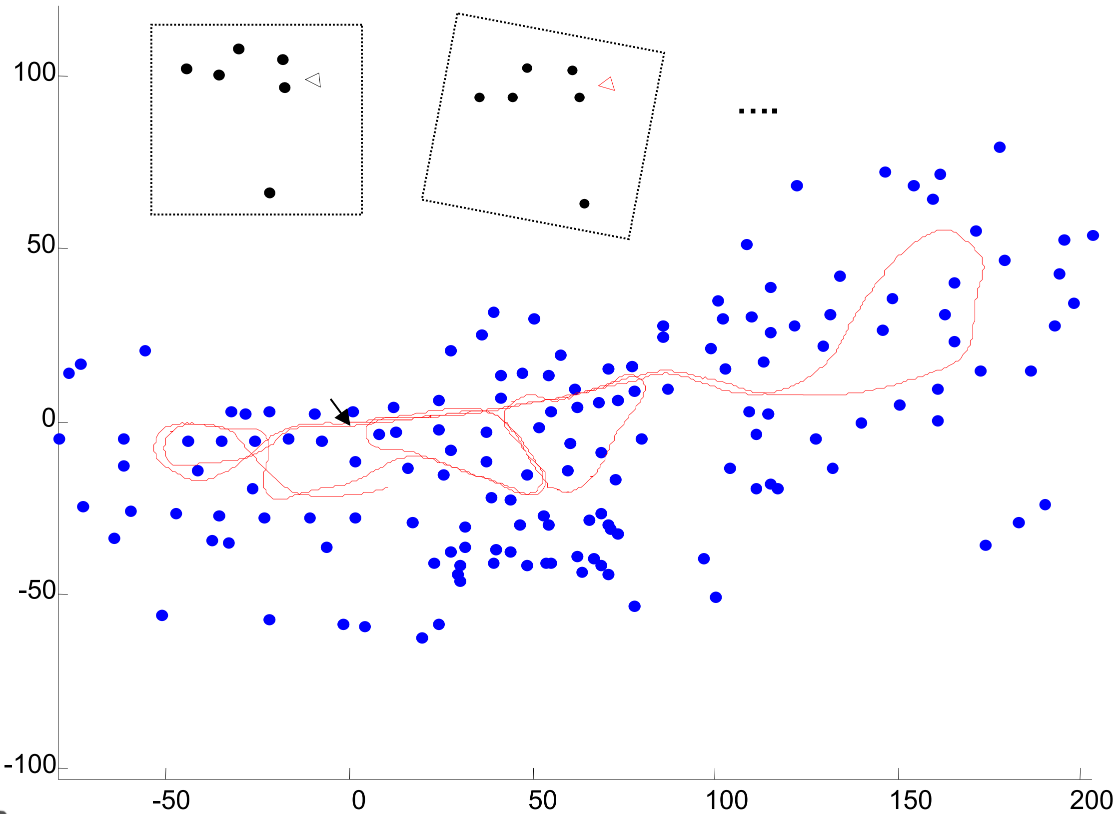




Vertical Edge Trinocular vision:
variable depth precision
good angular precision

Robot imprecision:
introduces CORRELATED error

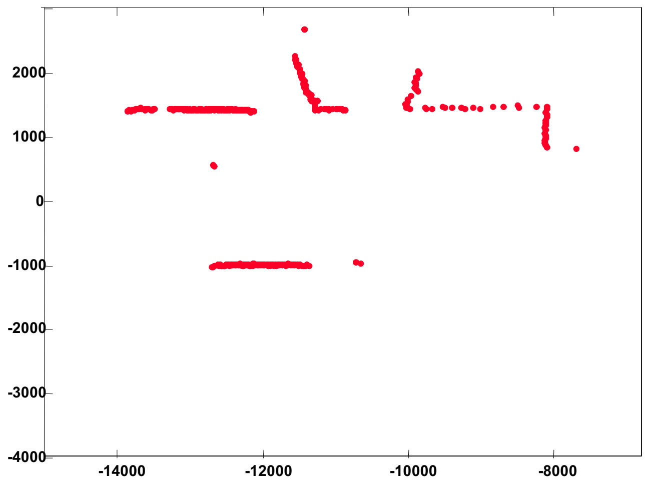
Search in configuration space



Approaches to Data Association

- Search in configuration space: find robot location with maximal data to map overlapping
  - Can be done with raw data
  - Or with features
    - » Speed of convergence?
- Search in correspondence space: find a consistent correspondence hypothesis and compute robot location
  1. Extract features from data
     - » If sparse data, move and build a local map
  2. Feature-based map (points, lines, trees, …)
  3. Search for data feature to map feature correspondences
     - » Exponential number of solutions?

2. Feature extraction: Laser

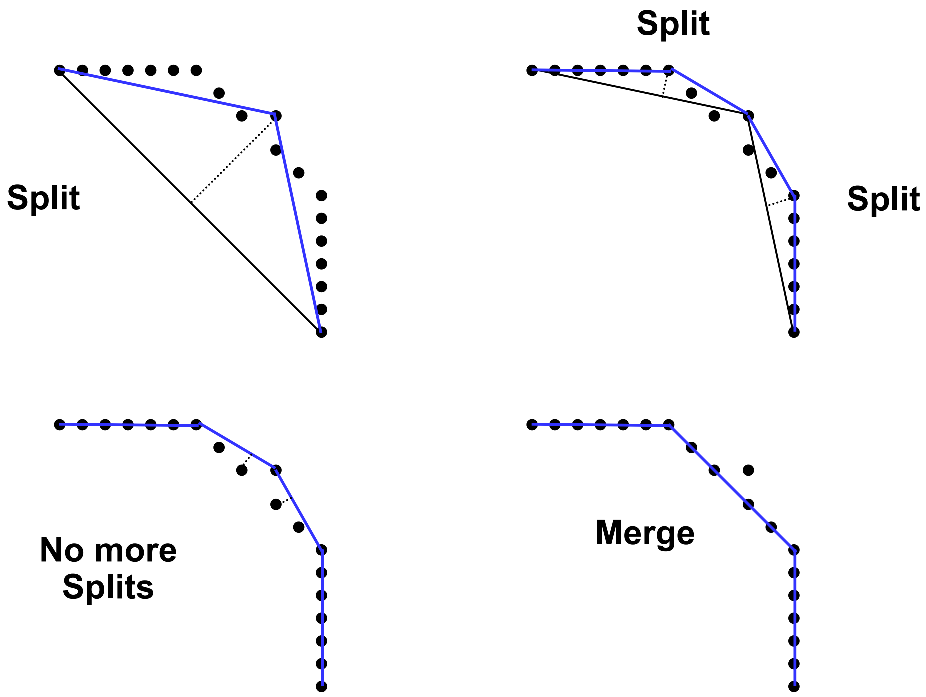


- Obtain line segments from a laser scan:
  - Segmentation
  - Line estimation

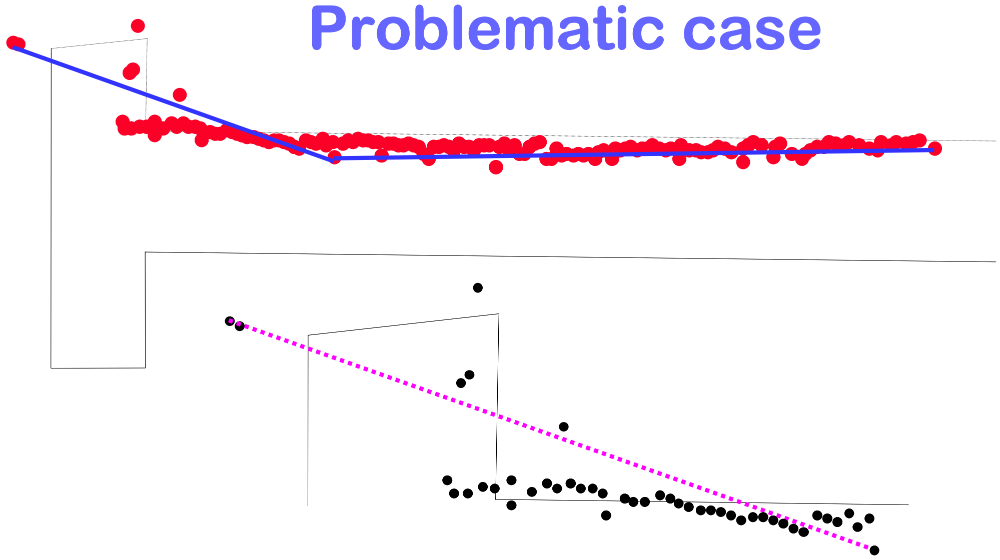
Split and merge:

1. **Recursive Split:**
   1. Obtain the line passing by the two extreme points
   2. Obtain the point more distant to the line
   3. If distance > error_max, split and repeat with the left and right sub-scan
2. **Merge:**
   1. If two consecutive segments are close enough, obtain the common line and the more distant point
   2. If distance <= error_max, merge both segments
3. **Prune short** segments
4. **Estimate line equation**

Split and Merge



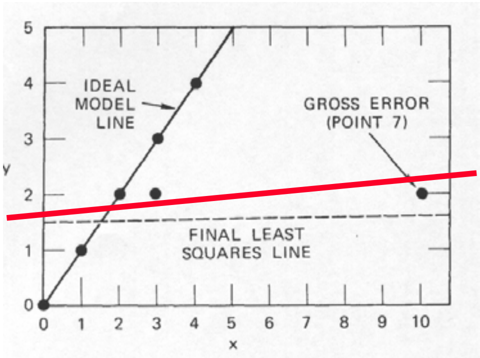
Problematic case



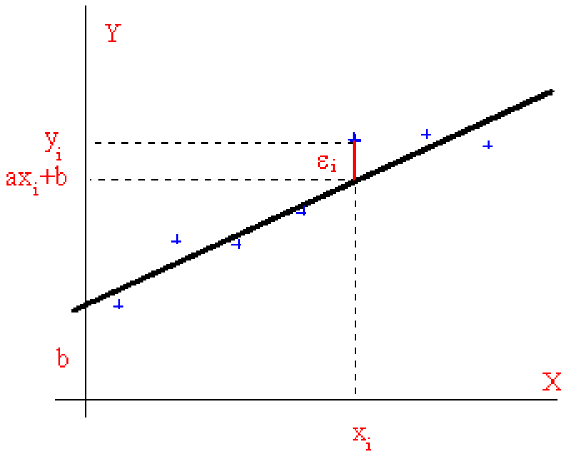
- **Split and merge:** uses only extreme points
- **Other options:**
  - TLS: Total Least squares (not much better)
  - RANSAC: RANdom SAmpling Consensus

RANSAC (Fischler y Bolles, 1981)

- TLS: A single "leverage point" produces a big estimation error
- Heuristics such as remove more discrepant points may also fail



POINT	x	y
1	0	0
2	1	1
3	2	2
4	3	2
5	3	3
6	4	4
7	10	2



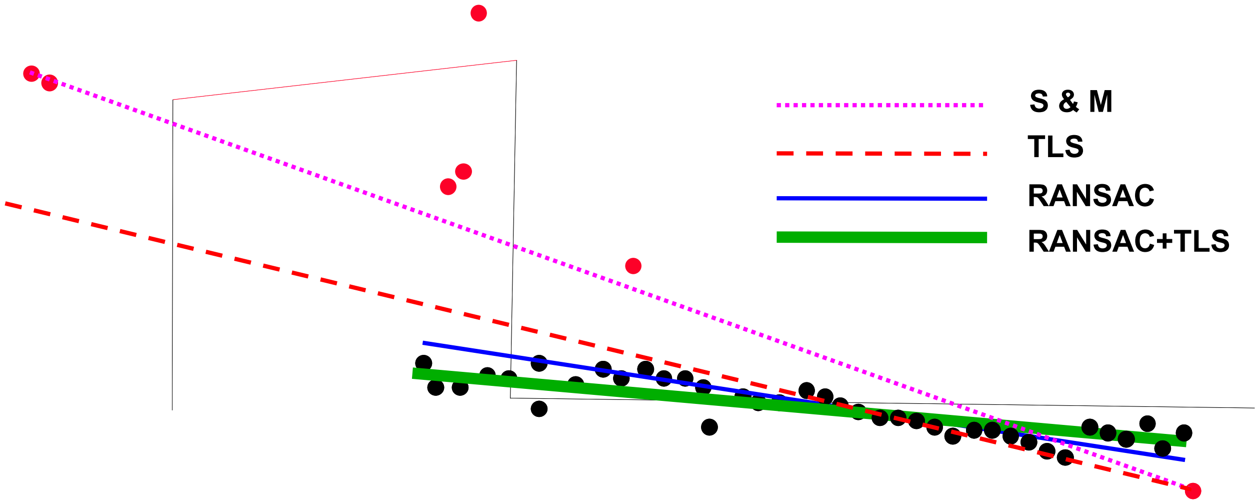
SUCCESSIVE LEAST SQUARES APPROXIMATIONS		
ITERATION	DATA SET	FITTING LINE
1	1, 2, 3, 4, 5, 6, 7	1.48 + .16x
2	1, 2, 3, 4, 5, 7	1.25 + .13x
3	1, 2, 3, 4, 7	0.96 + .14x
4	2, 3, 4, 7	1.51 + .06x

COMPUTATION OF RESIDUALS				
POINT	ITERATION 1 RESIDUALS	ITERATION 2 RESIDUALS	ITERATION 3 RESIDUALS	ITERATION 4 RESIDUALS
1	-1.48	-1.25	-.96*	—
2	-0.64	-0.38	-.10	-.57
3	-0.20	0.49	.76	.37
4	0.05	0.36	.63	.31
5	1.05	1.36*	—	—
6	1.89*	—	—	—
7	-1.06	-0.57	-.33	-.11

RANSAC

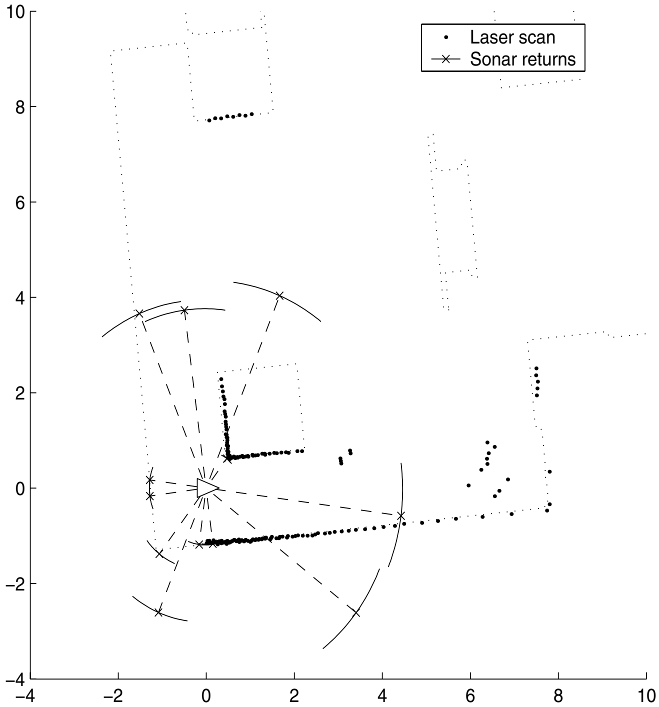
- Given a model that requires n data points to compute a solution and a set of data points P, with #(P) > n :
  - Randomly select a subset S1 of n data points and compute the model M1
  - Determine the consensus set S1* of points is P compatible with M1 (within some error tolerance)
  - If #(S1*) > t, use S1* to compute (maybe using least squares) a new model M1*
  - If #(S1*) < t, randomly select another subset S2 and repeat
  - If, after some predetermined number of trials there is no consensus set with t points, return with failure

RANSAC

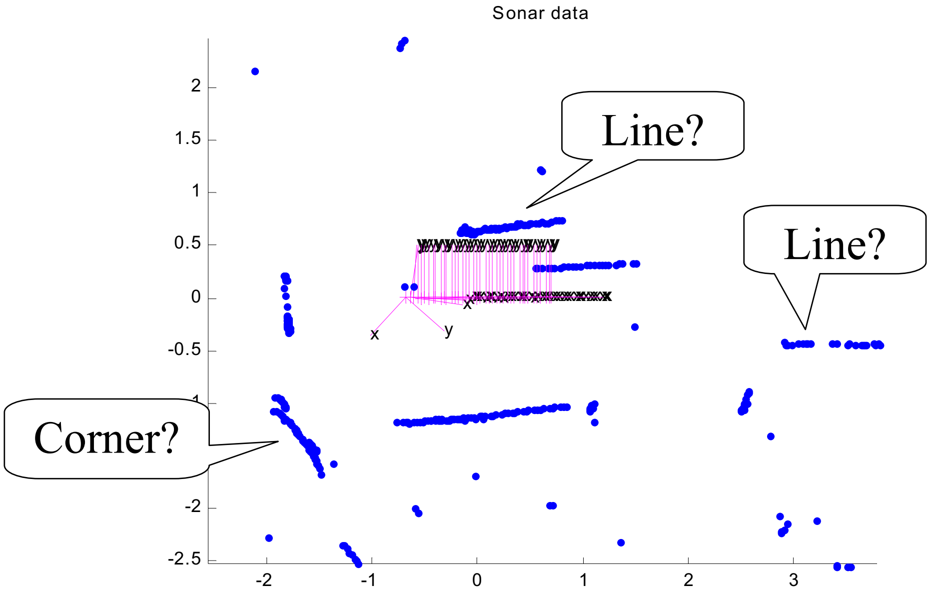


Feature extraction: Sonar

- Very sparse and noisy data

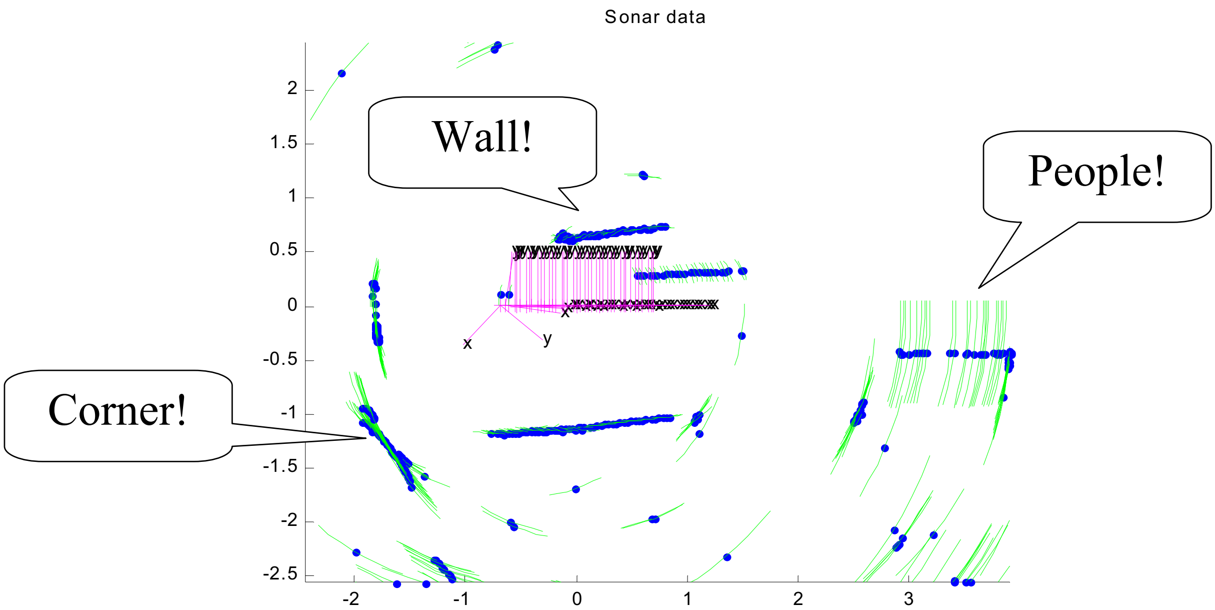


Sonar data, several steps



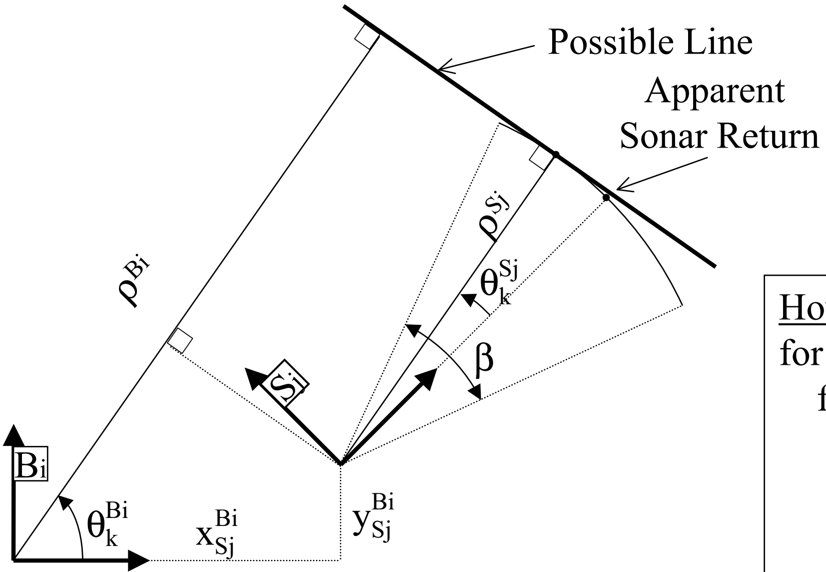
- You need to move the robot
- You need to delay decisions

Sonar data, several steps



- You need a good sensor model

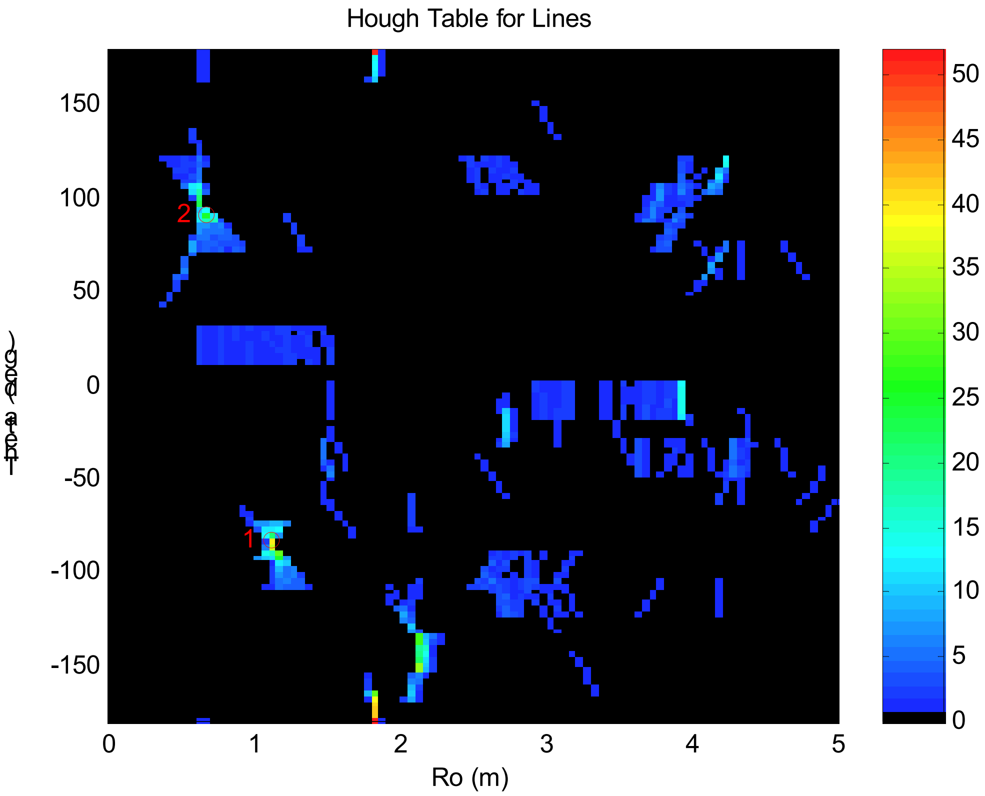
Sonar Model for Lines



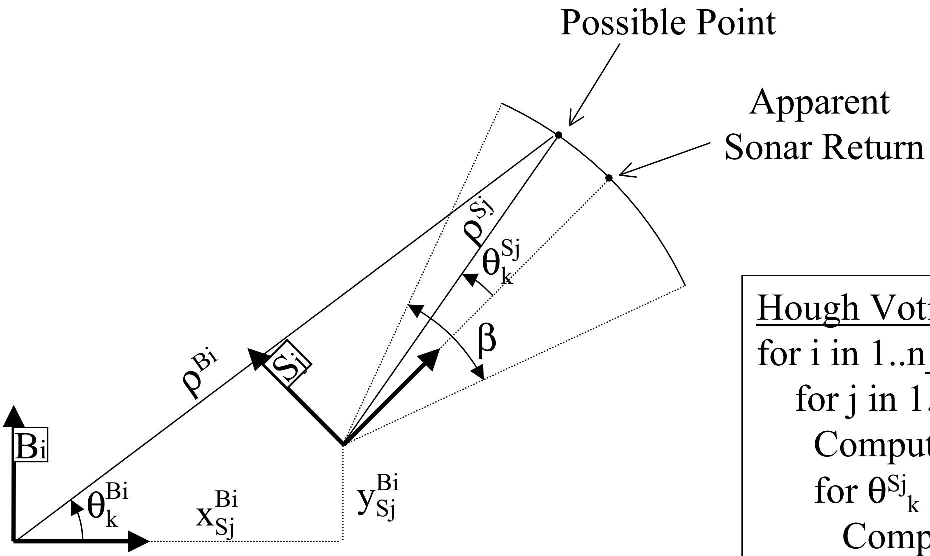
<u>Hough Voting</u>

for i in 1..n_positions
  for j in 1..n_sensors
    Compute $\mathbf{x}^{Bi}_{Sj}$
    for θ^{Sj}_k in $-\beta/2..\beta/2$ step δ
      Compute θ^{Bj}_k ρ^{Bj}_k
      Vote(θ^{Bj}_k, ρ^{Bj}_k)
    end
  end
end

Hough Transform for Lines



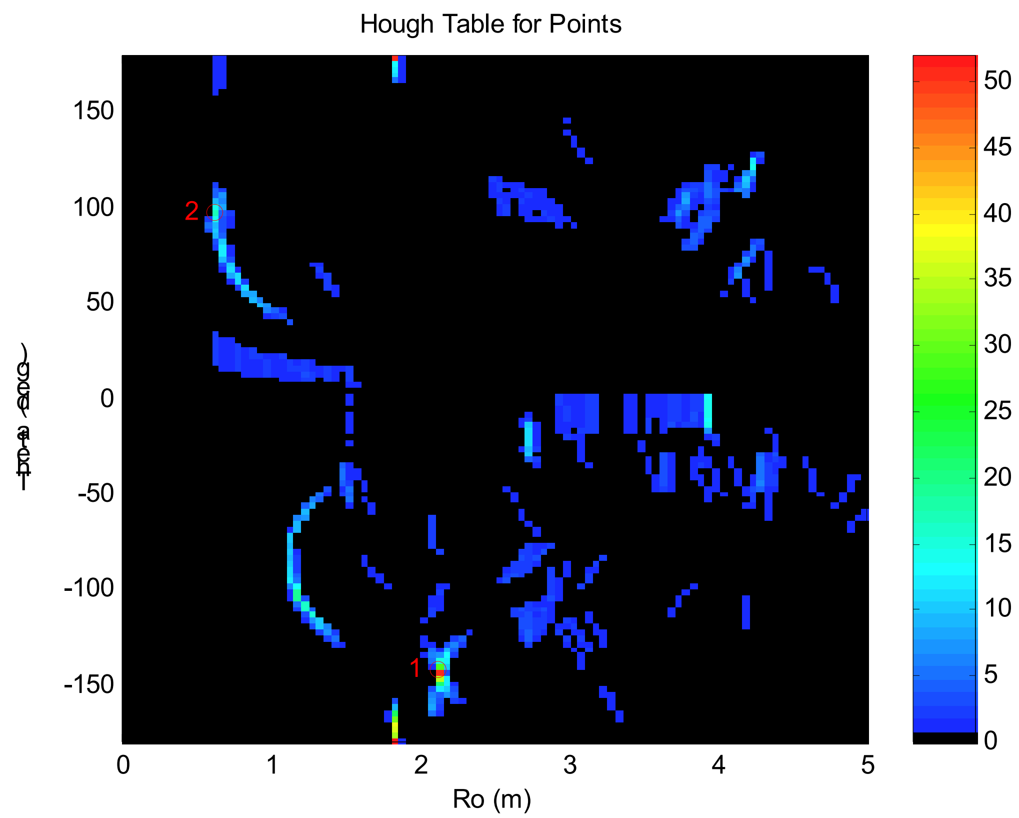
Sonar Model for Points



<u>Hough Voting</u>

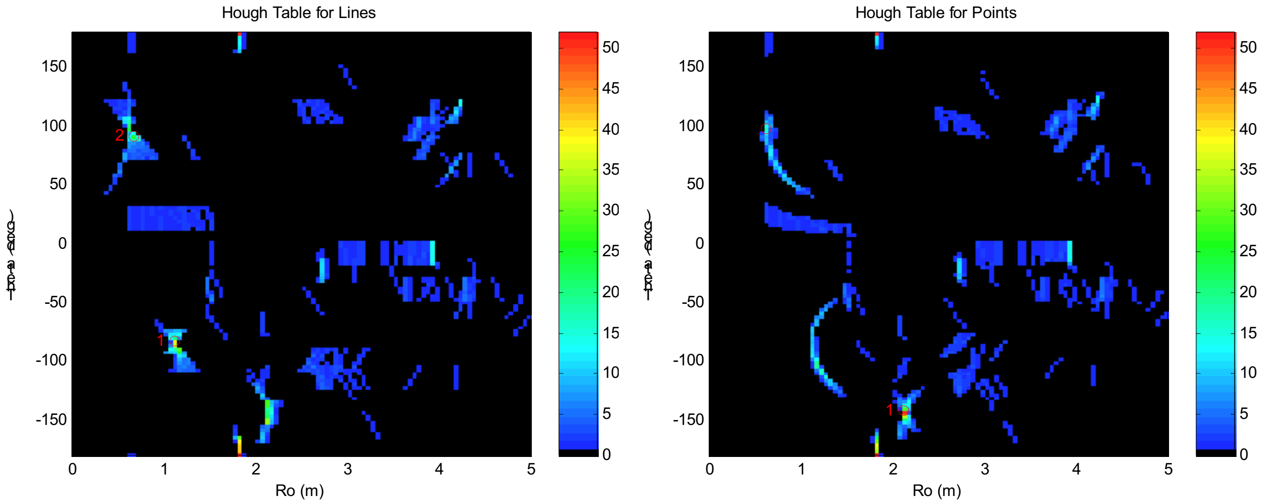
for i in 1..n_positions
    for j in 1..n_sensors
        Compute $\mathbf{x}_{Sj}^{Bi}$
        for θ_k^{Sj} in $-\beta/2..\beta/2$ step δ
            Compute θ_k^{Bj} ρ_k^{Bj}
            Vote(θ_k^{Bj}, ρ_k^{Bj})
        end
    end
end

Hough Transform for Points

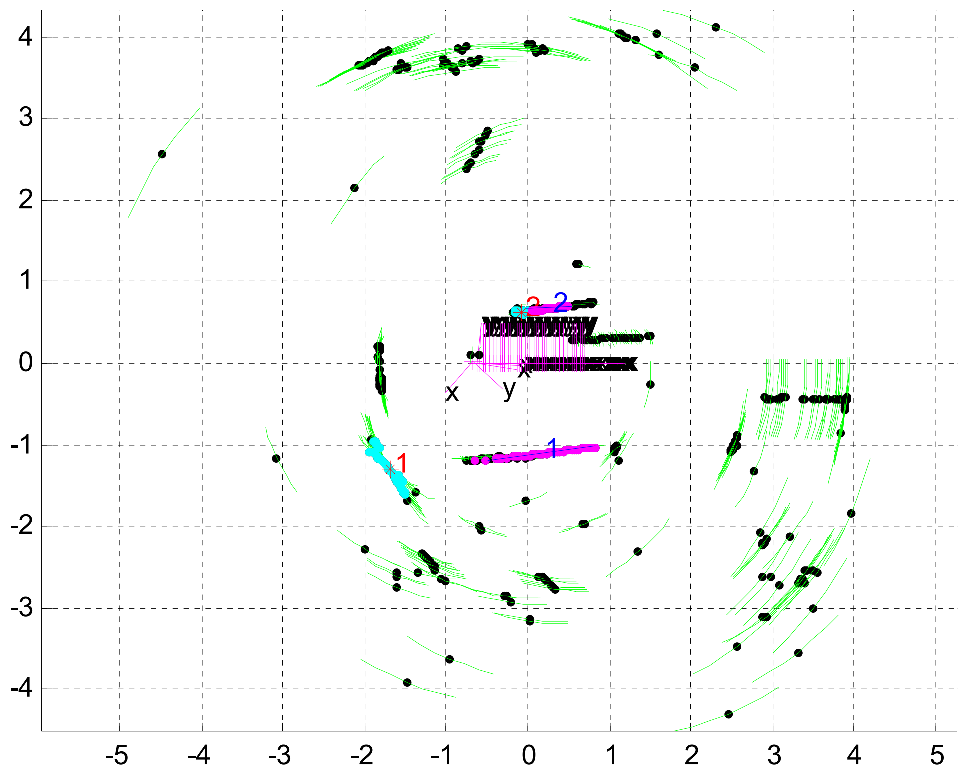


Looking for Local Maxima

- Winner-takes-all strategy



Perceptual Grouping with Hough



4. Data Association

- n map features:

$$\mathcal{F} = \{F_1 \dots F_n\}$$

- m sensor measurements:

$$\mathcal{E} = \{E_1 \dots E_m\}$$

- **Goal:** obtain a hypothesis that associates each observation E_i with a feature F_{ji}

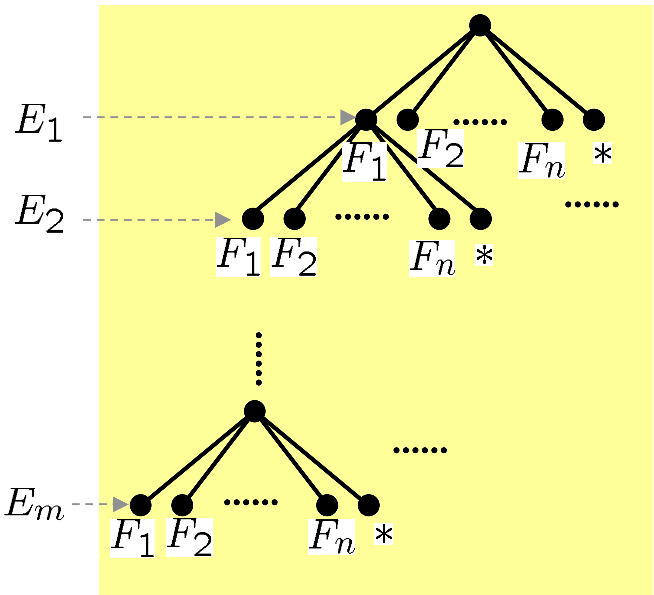
$$\mathcal{H}_m = [j_1 \dots j_i \dots j_m]$$

$$E_i \qquad F_{j_i}$$

- Non matched observations:

$$j_i = 0$$

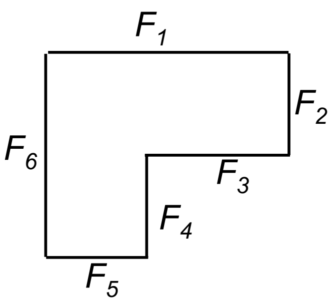
Interpretation tree (Grimson et al. 87):



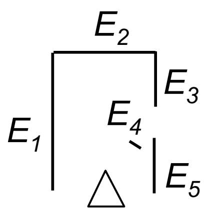
$(n+1)^m$ possible hypotheses

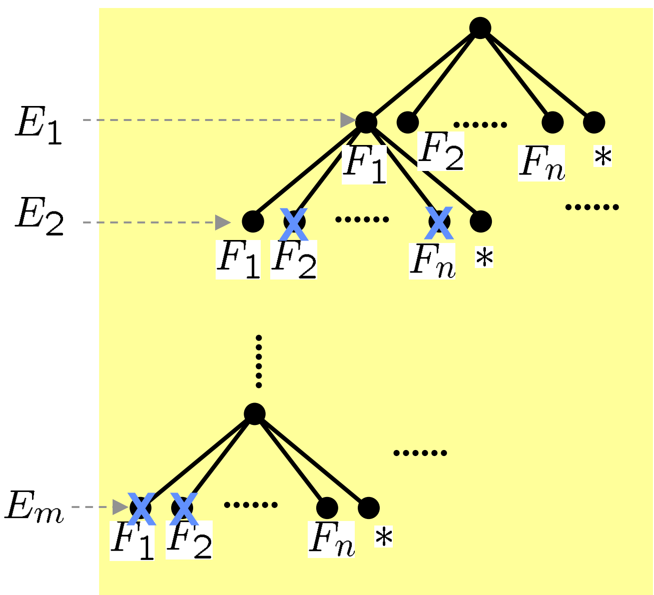
Use constraints to prune the tree

- Map:



- Observations:





- Constraints:
  - Feature location (needs an estimation of robot location)
  - Geometric relations: angles, distances,… (location independent)

Individual Compatibility

- Measurement equation for observation Ei and feature Fj

$$\mathbf{z}_i = \mathbf{h}_{ij}(\mathbf{x}_{\mathcal{F}}^B) + \mathbf{w}_i \qquad E[\mathbf{w}_i \mathbf{w}_i^T] = \mathbf{R}_i$$

$$\mathbf{z}_i \simeq \mathbf{h}_{ij}(\hat{\mathbf{x}}_{\mathcal{F}}^B) + \mathbf{H}_{ij}(\mathbf{x}_{\mathcal{F}}^B - \hat{\mathbf{x}}_{\mathcal{F}}^B) \qquad \mathbf{H}_{ij} = \left.\frac{\partial \mathbf{h}_{ij}}{\partial \mathbf{x}_{\mathcal{F}}^B}\right|_{(\hat{\mathbf{x}}_{\mathcal{F}}^B)}$$

- Ei and Fj are compatible if:

$$D_{ij}^2 = (\mathbf{z}_i - \mathbf{h}_{ij}(\hat{\mathbf{x}}_{\mathcal{F}}^B))^T C_{ij}^{-1} (\mathbf{z}_i - \mathbf{h}_{ij}(\hat{\mathbf{x}}_{\mathcal{F}}^B)) < \chi_{d,\alpha}^2$$

$$C_{ij} = \mathbf{H}_{ij} \mathbf{P}_{\mathcal{F}}^B \mathbf{H}_{ij}^T + \mathbf{R}_i$$

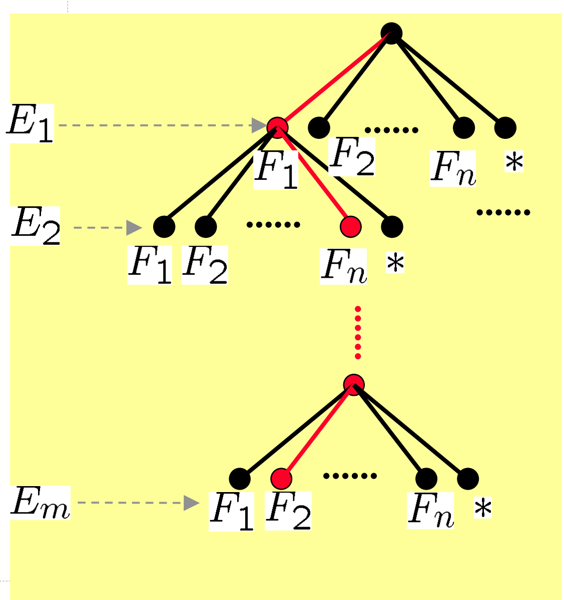
d = length(z_i)

- Nearest Neighbour (NN) rule:
  - associate Ei with the feature Fj having smaller Mahalanobis distance Dij

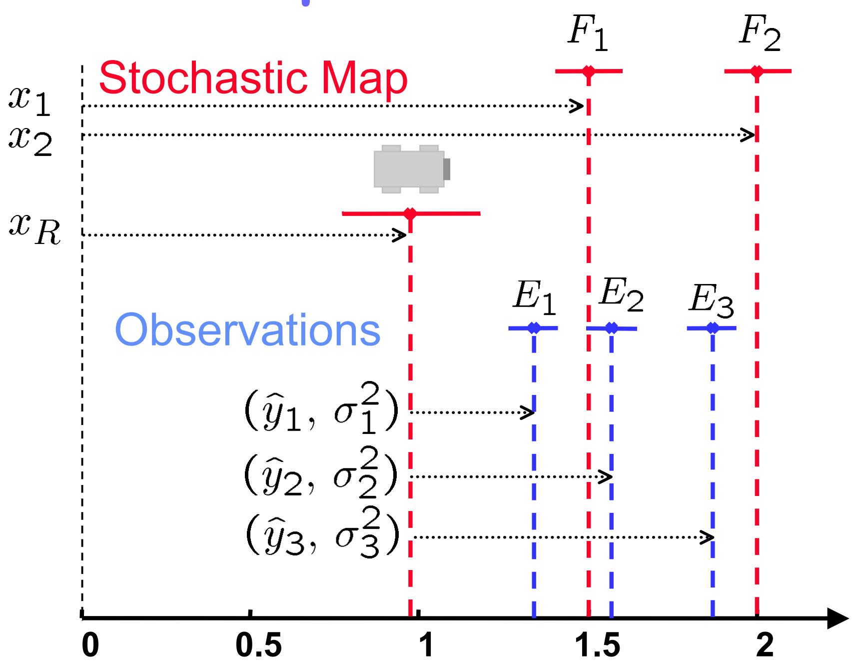
Nearest Neighbor

```
NN:
for i = 1 to m -- feature E_i
   D2min = Mahalanobis2(E_i, F_1)
   nearest = 1
   for j = 2 to n
      Dij2 = Mahalanobis2(E_i, F_j)
      if Dij2 < D2min then
         nearest = j
         D2min = Dij2
      fi
   rof
   if D2min <= Chi2(d_i,alpha) then
      H(i) = nearest
   else
      H(i) = 0
   fi
rof
```

Greedy algorithm: O(mn)



Example: MonoRob



Nearest Neighbor: $\mathcal{H} = [0,\ 1,\ 2]$

True Association: $\mathcal{H} = [1,\ 0,\ 2]$

Joint Compatitibily

- Given a hypothesis $\mathcal{H} = [j_1,\ j_2, \cdots, j_s]$
- Joint measurement equation

$$
\begin{aligned}
\mathbf{z}_{\mathcal{H}} &= \mathbf{h}_{\mathcal{H}}(\mathbf{x}_{\mathcal{F}}^{B}) + \mathbf{w}_{\mathcal{H}} \\
\mathbf{h}_{\mathcal{H}} &= \begin{bmatrix} \mathbf{h}_{1j_1} \\ \mathbf{h}_{2j_2} \\ \vdots \\ \mathbf{h}_{sj_s} \end{bmatrix}
\end{aligned}
$$

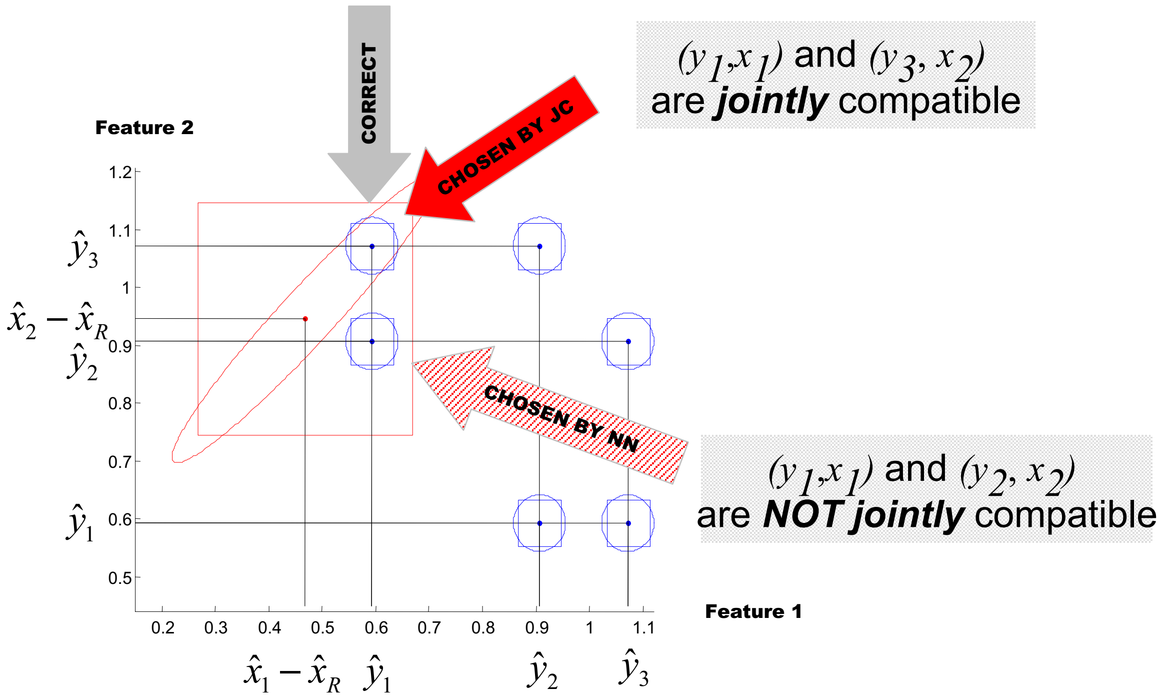
- The joint hypothesis is compatible if:

$$
\begin{aligned}
D_{\mathcal{H}}^2 &= (\mathbf{z}_{\mathcal{H}} - \mathbf{h}_{\mathcal{H}}(\hat{\mathbf{x}}_{\mathcal{F}}^{B}))^T C_{\mathcal{H}}^{-1} (\mathbf{z}_{\mathcal{H}} - \mathbf{h}_{\mathcal{H}}(\hat{\mathbf{x}}_{\mathcal{F}}^{B})) < \chi_{d,\alpha}^2 \\
C_{\mathcal{H}} &= \mathbf{H}_{\mathcal{H}} \mathbf{P}_{\mathcal{F}}^{B} \mathbf{H}_{\mathcal{H}}^T + \mathbf{R}_{\mathcal{H}}
\end{aligned}
$$

d = length(z)

Individual .vs. Joint Compatibility

- Joint Compatibility assures consistency:

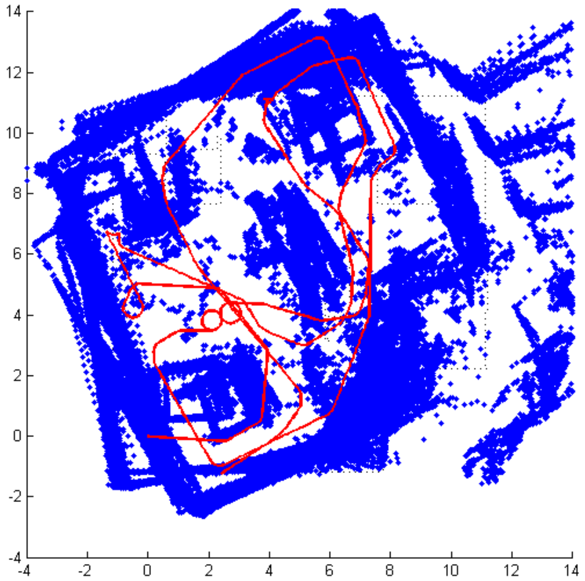


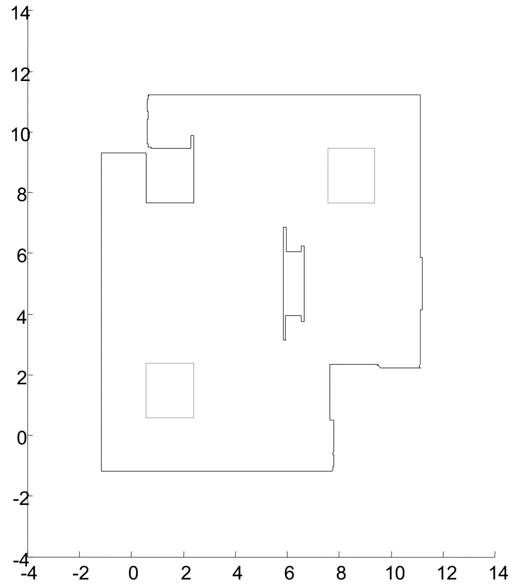
Joint Compatibility Branch and Bound

- Find the largest hypothesis with jointly consistent pairings

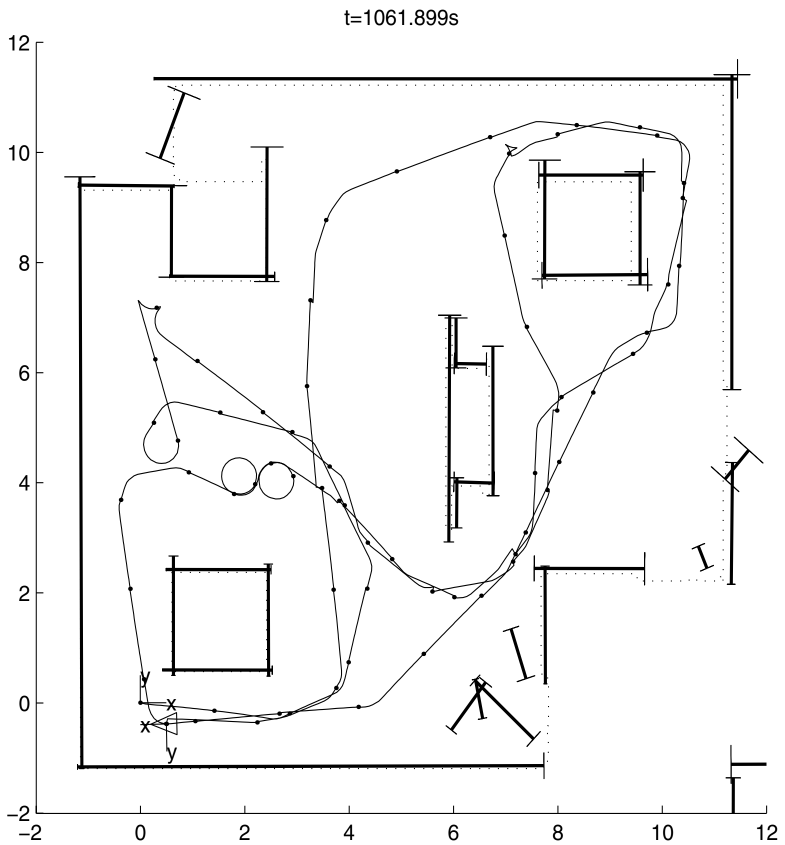
```
JCBB(H, i): -- find pairings for feature E_i

if i > m -- leaf node?
   if pairings(H) > pairings(Best) -- did better?
      Best = H
   fi
else
   for j = 1 to n
       if individual_compatibility(E_i, F_j) and then
          joint_compatibility(H, E_i, F_j)
          JCBB([H j], i + 1) -- pairing (E_i, F_j) accepted
       fi
   rof
   if pairings(H) + m - i >= pairings(Best) -- can do better?
      JCBB([H 0], i + 1) -- star node: E_i not paired
   fi
fi
```

SLAM with laser segments

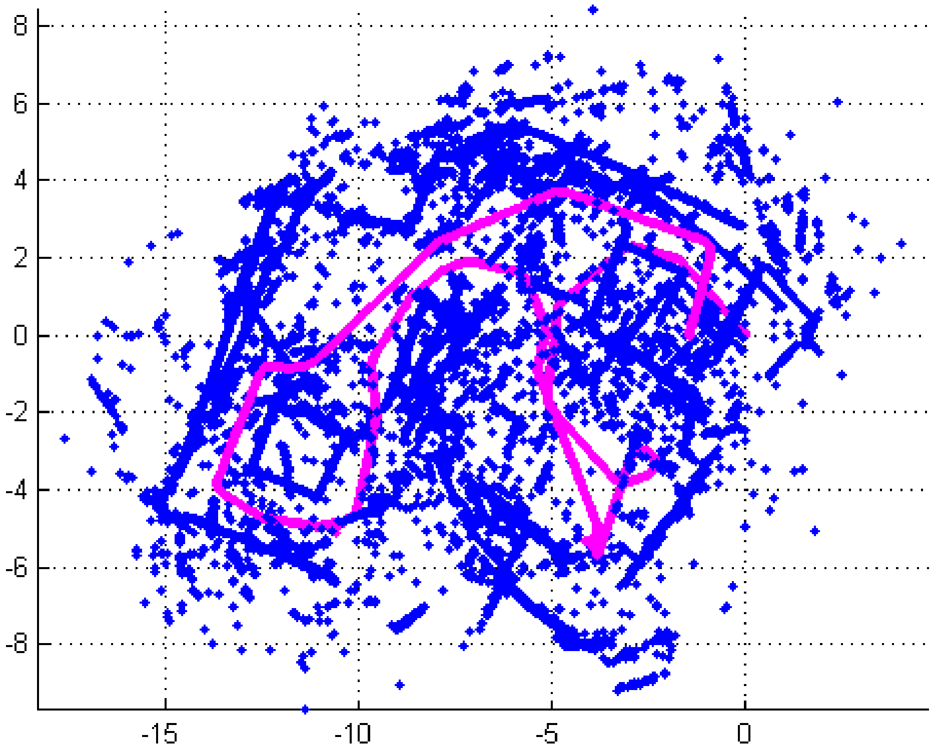




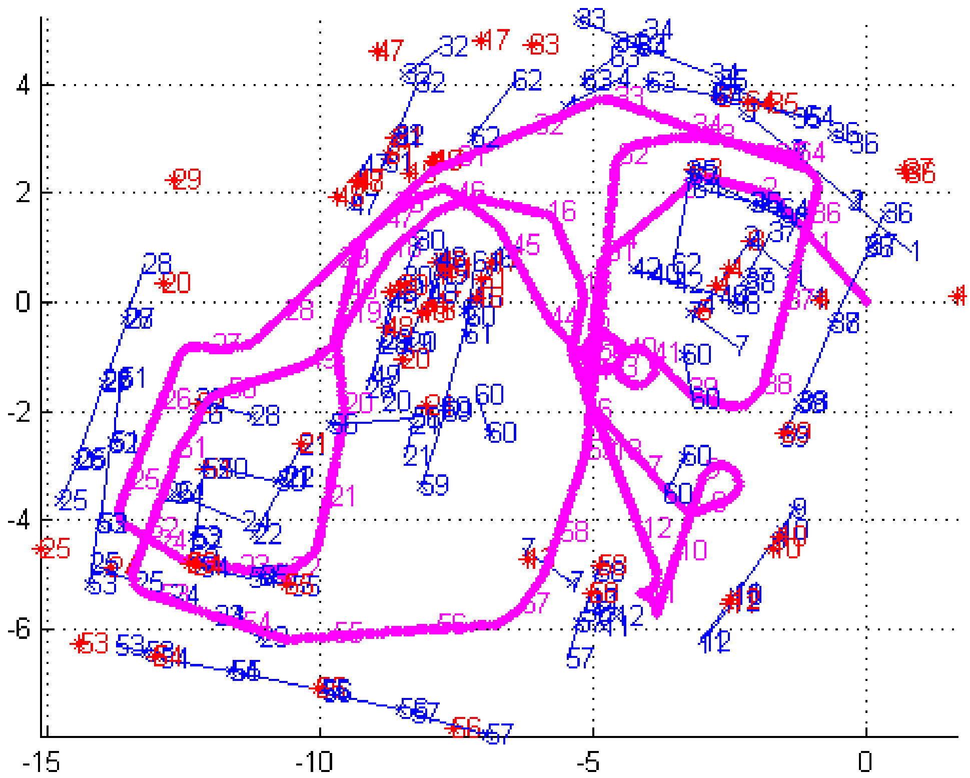
SLAM with laser segments



SLAM with Sonar



Perceptual Grouping with Hough: Lines and Points



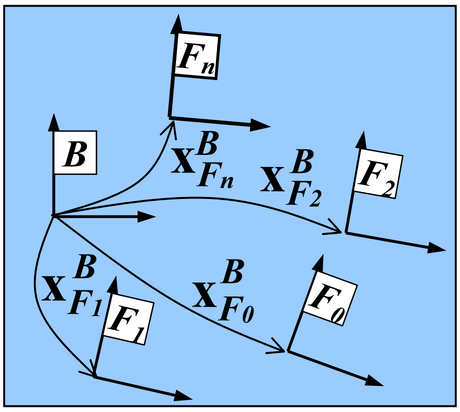
Map Joining: local maps

- Environment information related to a set of elements:

$$\mathcal{F} = \{B,\, F_0,\, F_1,\, \ldots,\, F_n\}$$

F_0 = Robot

- represented by a map: $\mathcal{M}_{\mathcal{F}}^{B} = \left(\hat{\mathbf{x}}_{\mathcal{F}}^{B}, \mathbf{P}_{\mathcal{F}}^{B}\right)$



$$\hat{\mathbf{x}}_{\mathcal{F}}^{B} = \begin{bmatrix} \hat{\mathbf{x}}_{F_0}^{B} \\ \vdots \\ \hat{\mathbf{x}}_{F_n}^{B} \end{bmatrix}$$

$$\mathbf{P}_{\mathcal{F}}^{B} = \begin{bmatrix} \mathbf{P}_{F_0F_0}^{B} & \cdots & \mathbf{P}_{F_0F_n}^{B} \\ \vdots & \ddots & \vdots \\ \mathbf{P}_{F_nF_0}^{B} & \cdots & \mathbf{P}_{F_nF_n}^{B} \end{bmatrix}$$

Local map building

- Periodically, the robot starts a new map, relative to its current location:

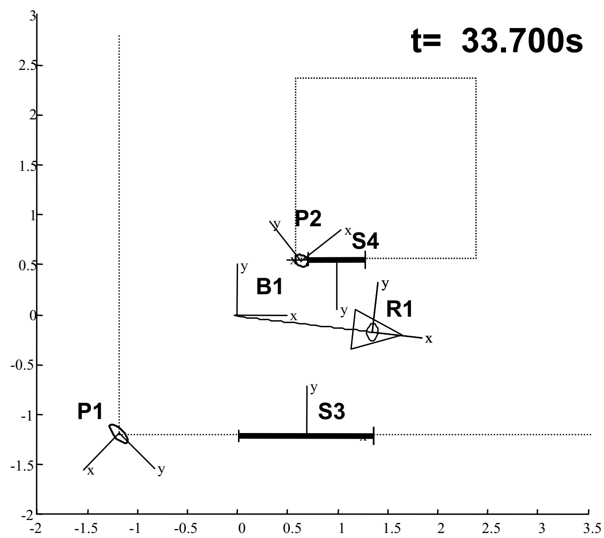
$$\hat{\mathbf{x}}_{R_0}^{B} = 0$$
$$\mathbf{P}_{R_0}^{B} = 0$$

- EKF approximates the conditional mean:

$$\boxed{\hat{\mathbf{x}}_{\mathcal{F}_1}^{B_1} \simeq E\left[\mathbf{x}_{\mathcal{F}_1}^{B_1} \mid D^{1\ldots k_1},\ \mathcal{H}^{1\ldots k_1}\right]}$$

- Given measurements:

$$D^{1\ldots k_1} = \left\{\mathbf{u}_1\, \mathbf{z}_1 \ldots \mathbf{u}_{k_1}\, \mathbf{z}_{k_1}\right\}$$
$$\mathbf{u}_k = \hat{\mathbf{x}}_{R_k}^{R_{k-1}}$$



Local map building

- Second map: $D^{k_1+1\ldots k_2} = \left\{\mathbf{u}_{k_1+1}\,\mathbf{z}_{k_1+1}\ldots\mathbf{u}_{k_2}\,\mathbf{z}_{k_2}\right\}$

$$\boxed{\hat{\mathbf{x}}^{B_2}_{\mathcal{F}_2} \simeq E\left[\mathbf{x}^{B_2}_{\mathcal{F}_2} \mid D^{k_1+1\ldots k_2},\ \mathcal{H}^{k_1+1\ldots k_2}\right]}$$

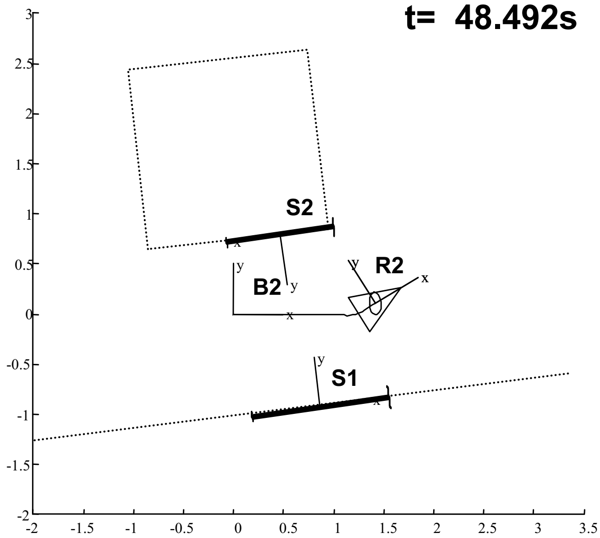
- No information is shared:

$$D^{1\ldots k_1} \cap D^{k_1+1\ldots k_2} = \emptyset$$

Maps are uncorrelated

- Common reference:

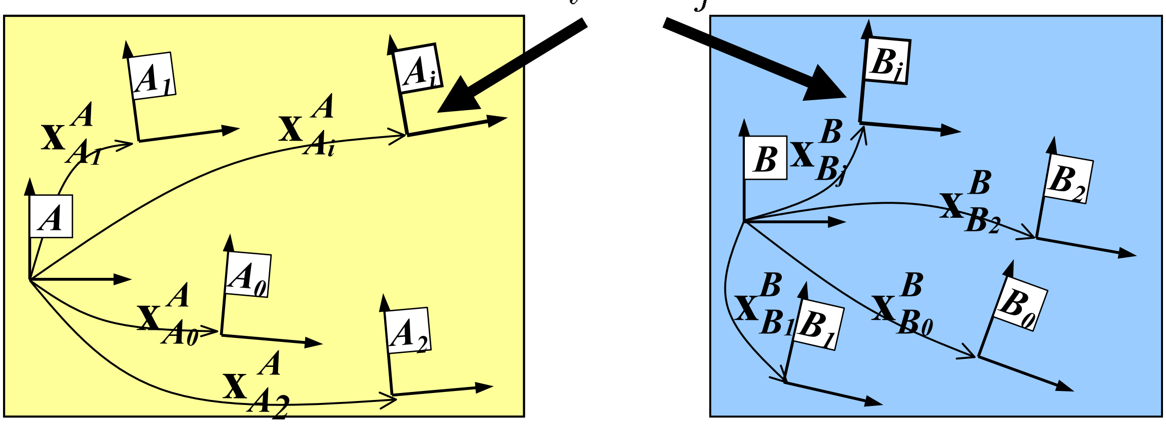
$$B_2 = R_1$$



Map Joining

- Given:
  1. Two statistically independent stochastic maps
  2. A common reference

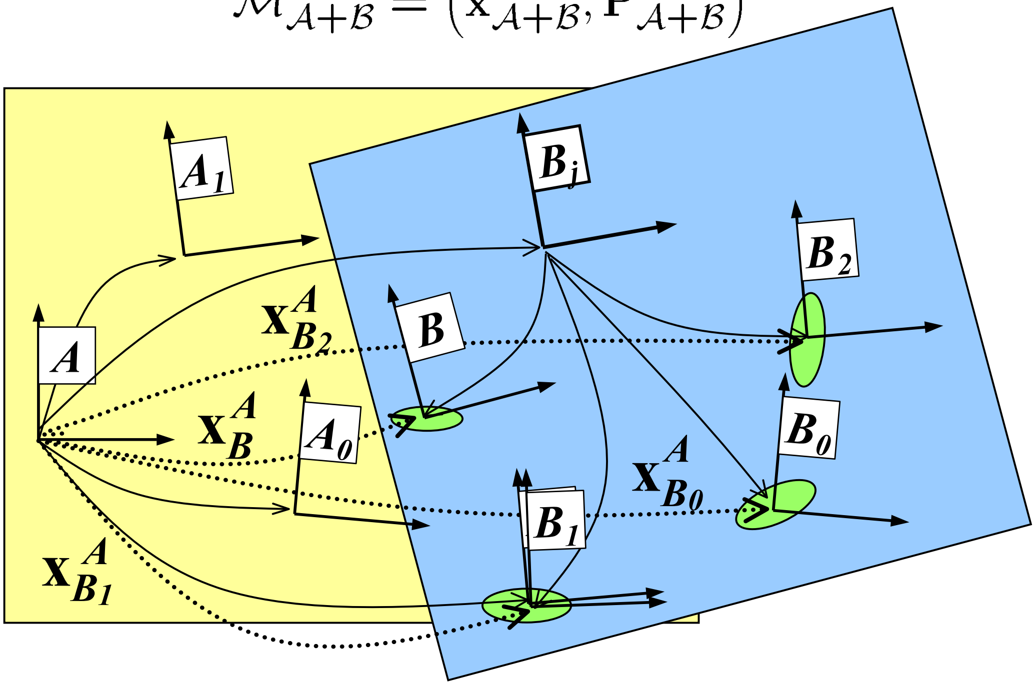
$\mathcal{M}^A_{\mathcal{A}} = \left(\hat{\mathbf{x}}^A_{\mathcal{A}}, \mathbf{P}^A_{\mathcal{A}}\right)$ $\quad A_i = B_j \quad$ $\mathcal{M}^B_{\mathcal{B}} = \left(\hat{\mathbf{x}}^B_{\mathcal{B}}, \mathbf{P}^B_{\mathcal{B}}\right)$



Map Joining

- Conveys the information of the two maps into a single **fully consistent** stochastic map:

$$\mathcal{M}^{A}_{\mathcal{A}+\mathcal{B}} = \left(\hat{\mathbf{x}}^{A}_{\mathcal{A}+\mathcal{B}}, \mathbf{P}^{A}_{\mathcal{A}+\mathcal{B}}\right)$$



Change the base of map B to B_j

- New state vector:

$$\hat{\mathbf{x}}^{B_j}_{\mathcal{B}} = \begin{bmatrix} \hat{\mathbf{x}}^{B_j}_{B_0} \\ \vdots \\ \hat{\mathbf{x}}^{B_j}_{B} \\ \vdots \\ \hat{\mathbf{x}}^{B_j}_{B_m} \end{bmatrix} = \begin{bmatrix} \ominus\hat{\mathbf{x}}^{B}_{B_j} \oplus \hat{\mathbf{x}}^{B}_{B_0} \\ \vdots \\ \ominus\hat{\mathbf{x}}^{B}_{B_j} \\ \vdots \\ \ominus\hat{\mathbf{x}}^{B}_{B_j} \oplus \hat{\mathbf{x}}^{B}_{B_m} \end{bmatrix}$$

- New covariance matrix: $\mathbf{P}^{B_j}_{\mathcal{B}} = \mathbf{J}^{B_j}_{B}\, \mathbf{P}^{B}_{\mathcal{B}}\, {\mathbf{J}^{B_j}_{B}}^T$

$$\mathbf{J}^{B_j}_{B} = \frac{\partial \hat{\mathbf{x}}^{B_j}_{\mathcal{B}}}{\partial \hat{\mathbf{x}}^{B}_{\mathcal{B}}} = \begin{bmatrix} \mathbf{J}_{00} & \cdots & \mathbf{J}_{0j} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots & & \vdots \\ \mathbf{0} & \cdots & \mathbf{J}_{jj} & \cdots & \mathbf{0} \\ \vdots & & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{J}_{mj} & \cdots & \mathbf{J}_{mm} \end{bmatrix}$$

$$\begin{aligned} \mathbf{J}_{jj} &= \mathbf{J}_{\ominus}\left\{\hat{\mathbf{x}}^{B}_{B_j}\right\} \\ \mathbf{J}_{ij} &= \mathbf{J}_{1\oplus}\left\{\ominus\hat{\mathbf{x}}^{B}_{B_j}, \hat{\mathbf{x}}^{B}_{B_i}\right\} \mathbf{J}_{\ominus}\left\{\hat{\mathbf{x}}^{B}_{B_j}\right\} && i = 0..m,\ i \neq j \\ \mathbf{J}_{ii} &= \mathbf{J}_{2\oplus}\left\{\ominus\hat{\mathbf{x}}^{B}_{B_j}, \hat{\mathbf{x}}^{B}_{B_i}\right\} && i = 0..m,\ i \neq j \end{aligned}$$

Map Joining

- New state vector: $$\hat{\mathbf{x}}^{A}_{\mathcal{A}+\mathcal{B}} = \begin{bmatrix} \hat{\mathbf{x}}^{A}_{\mathcal{A}} \\ \hat{\mathbf{x}}^{A}_{\mathcal{B}} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{x}}^{A}_{\mathcal{A}} \\ \hat{\mathbf{x}}^{A}_{A_i} \oplus \hat{\mathbf{x}}^{B_j}_{B_0} \\ \vdots \\ \hat{\mathbf{x}}^{A}_{A_i} \oplus \hat{\mathbf{x}}^{B_j}_{B_m} \end{bmatrix}$$

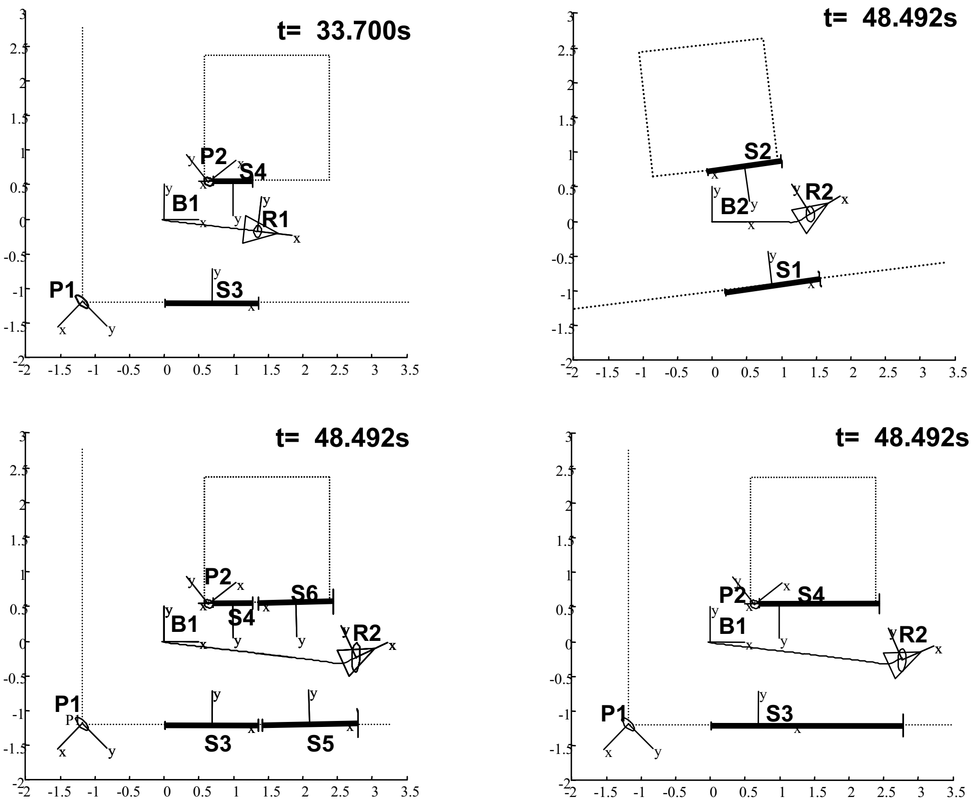
- New covariance matrix:

$$\begin{aligned} \mathbf{P}^{A}_{\mathcal{A}+\mathcal{B}} &= \mathbf{J}^{\mathcal{A}+\mathcal{B}}_{\mathcal{A}} \mathbf{P}^{A}_{\mathcal{A}} \mathbf{J}^{\mathcal{A}+\mathcal{B}\,T}_{\mathcal{A}} + \mathbf{J}^{\mathcal{A}+\mathcal{B}}_{\mathcal{B}} \mathbf{P}^{B_j}_{\mathcal{B}} \mathbf{J}^{\mathcal{A}+\mathcal{B}\,T}_{\mathcal{B}} \\ &= \begin{bmatrix} \mathbf{P}^{A}_{\mathcal{A}} & \mathbf{P}^{A}_{\mathcal{A}} \mathbf{J}_1^T \\ \mathbf{J}_1 \mathbf{P}^{A}_{\mathcal{A}} & \mathbf{J}_1 \mathbf{P}^{A}_{\mathcal{A}} \mathbf{J}_1^T \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_2 \mathbf{P}^{B_j}_{\mathcal{B}} \mathbf{J}_2^T \end{bmatrix} \end{aligned}$$

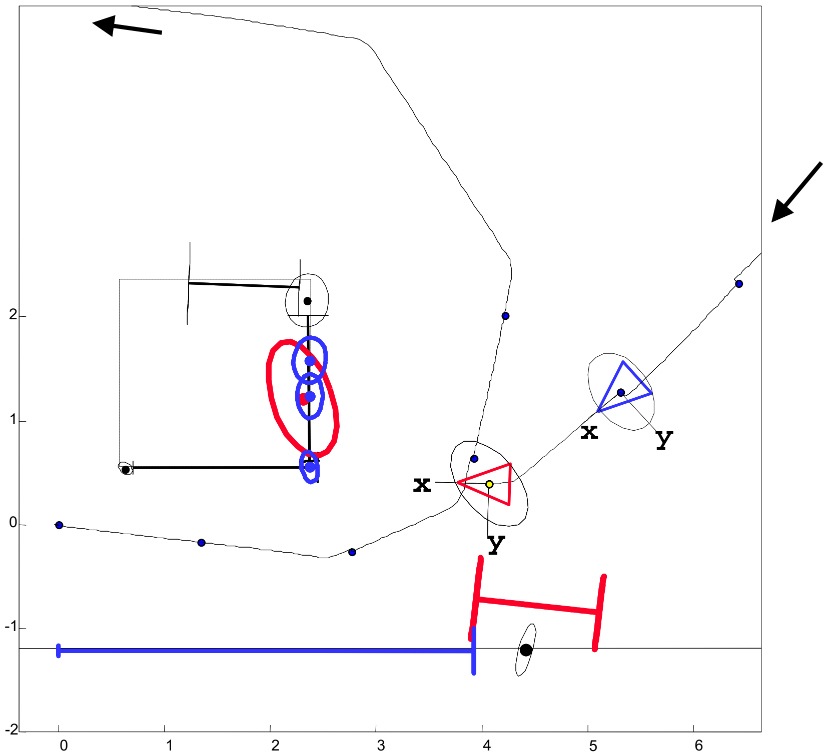
$$\mathbf{J}^{\mathcal{A}+\mathcal{B}}_{\mathcal{A}} = \frac{\partial \hat{\mathbf{x}}^{A}_{\mathcal{A}+\mathcal{B}}}{\partial \hat{\mathbf{x}}^{A}_{\mathcal{A}}} = \begin{bmatrix} \mathbf{I} \\ \mathbf{J}_1 \end{bmatrix} \qquad \mathbf{J}_1 = \begin{bmatrix} \mathbf{0} & \cdots & \mathbf{J}_{1\oplus}\left\{\hat{\mathbf{x}}^{A}_{A_i}, \hat{\mathbf{x}}^{B_j}_{B_0}\right\} & \cdots & \mathbf{0} \\ \vdots & & \vdots & & \vdots \\ \mathbf{0} & \cdots & \mathbf{J}_{1\oplus}\left\{\hat{\mathbf{x}}^{A}_{A_i}, \hat{\mathbf{x}}^{B_j}_{B_m}\right\} & \cdots & \mathbf{0} \end{bmatrix}$$

$$\mathbf{J}^{\mathcal{A}+\mathcal{B}}_{\mathcal{B}} = \frac{\partial \hat{\mathbf{x}}^{A}_{\mathcal{A}+\mathcal{B}}}{\partial \hat{\mathbf{x}}^{B_j}_{\mathcal{B}}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{J}_2 \end{bmatrix} \qquad \mathbf{J}_2 = \begin{bmatrix} \mathbf{J}_{2\oplus}\left\{\hat{\mathbf{x}}^{A}_{A_i}, \hat{\mathbf{x}}^{B_j}_{B_0}\right\} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{J}_{2\oplus}\left\{\hat{\mathbf{x}}^{A}_{A_i}, \hat{\mathbf{x}}^{B_j}_{B_m}\right\} \end{bmatrix}$$

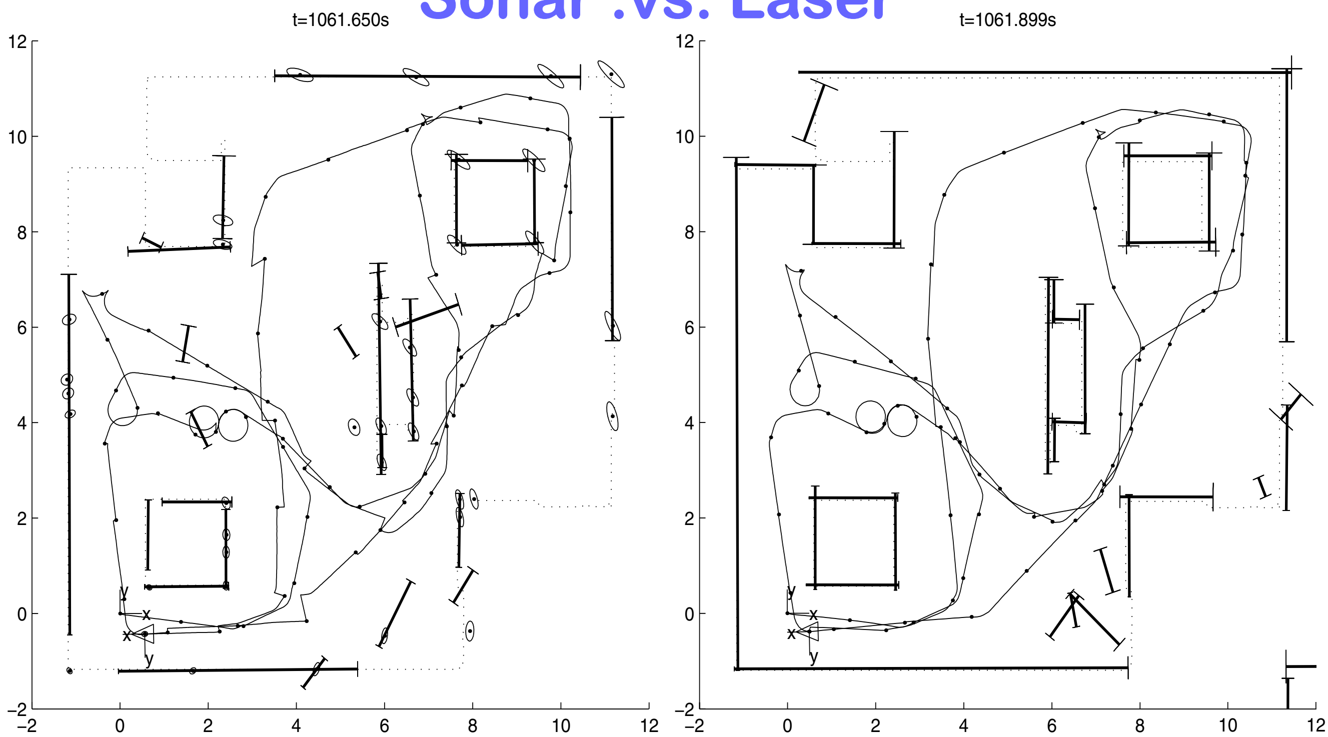
Local map sequencing



Loop closing: Joint Compatibility!



Sonar .vs. Laser

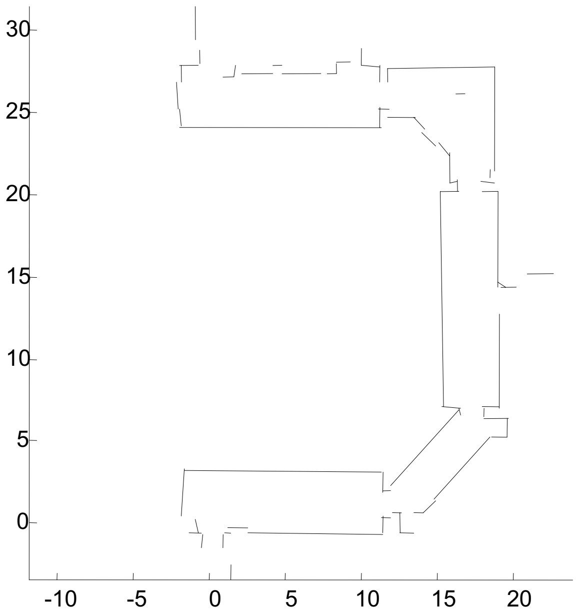


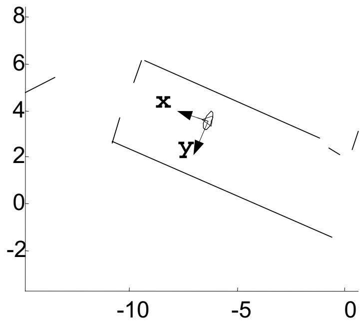
- Very robust map building with sonar
- Local maps: 60x faster than standard SLAM

4. Robot Relocation

Previously built map (A)

Local map (B)





Where is the robot?

No robot estimation: use geometric constraints

- Location independent constraints
  - Unary: depend on a single matching (size, color,…)
    - » Tree diameter
    - » Wall length: the observed length should be less than the length in the map (if map is complete)
  - Binary: depend on two matchings (angles, distances)
    - » Distances between points
    - » Angles between lines
    - » "Distances" between segments
  - N-ary: more complex, little pruning

Binary Constraints for Points

- Distance between two map points and the two matching observed points should coincide:

$$||d_{obs} - d_{map}|| \leq \epsilon$$

  – If using a probabilistic model, 95% error bound:

$$\epsilon = 2\sqrt{\sigma_{obs}^2 - \sigma_{map}^2}$$

  – Or use an empiric value

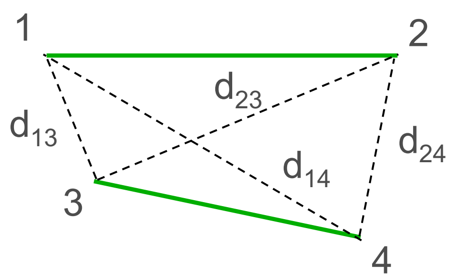
Binary Constraints for Segments

- Angle constraint:

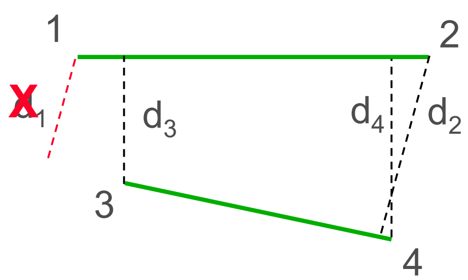
$$||\alpha_{obs} - \alpha_{map}|| \leq \epsilon$$

- Distance constraint (verifies overlapping)
  – Let [d, D] be the interval of possible distances between any two points of two segments:

$$[d_{obs} - \epsilon,\ D_{obs} + \epsilon] \cap [d_{map} - \epsilon,\ D_{map} + \epsilon] \neq \emptyset$$



$D = \max(d_{13}, d_{23}, d_{14}, d_{24})$

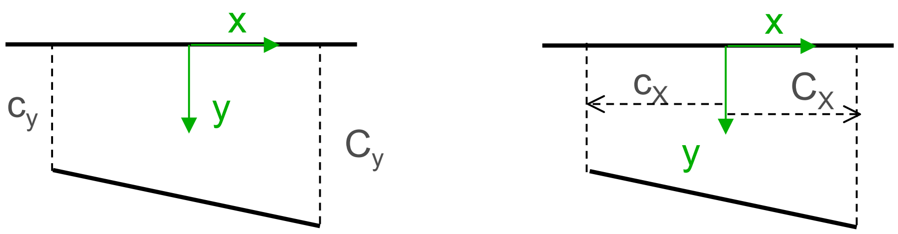


$d = \min(d_{13}, d_{23}, d_{14}, d_{24}, d_1, d_2, d_3, d_4)$

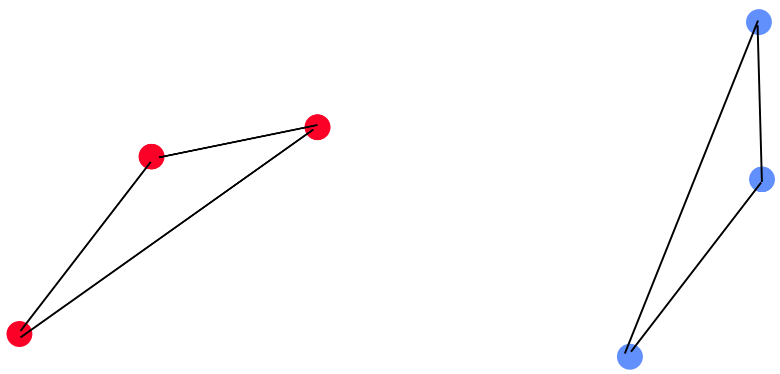
Binary Constraints for Segments

- Component constraint:
  - Let [c, C] be the interval of possible longitudinal or lateral component any point of one segment relative to the center of the other segment:

$$[c_{obs} - \epsilon,\, C_{obs} + \epsilon] \cap [c_{map} - \epsilon,\, C_{map} + \epsilon] \neq \emptyset$$



Location independent constraints are not a sufficient condition

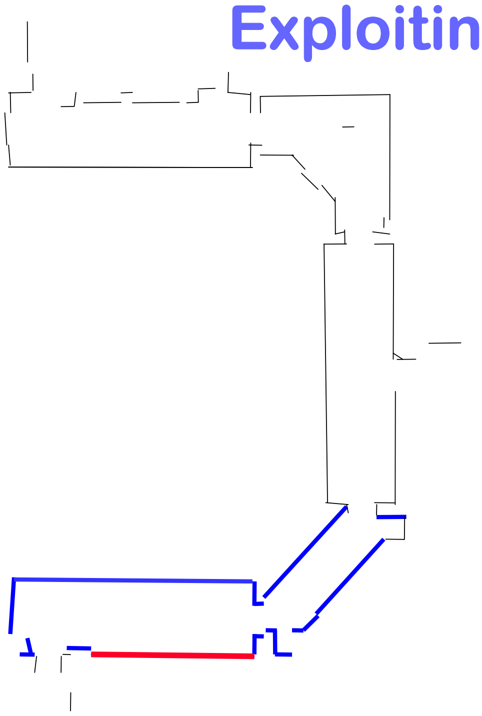


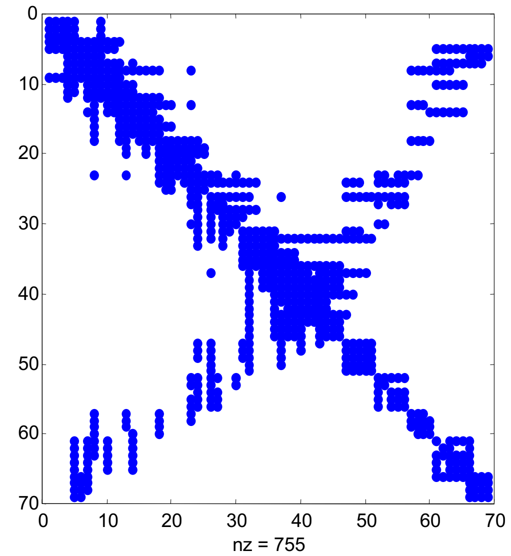
- You need to verify global consistency

Geometric Constraints Branch and Bound

```
GCBB(H, i): -- find pairings for feature E_i

if i > m -- leaf node?
   if pairings(H) > pairings(Best) and then -- did better?
      is_consistent(H) then -- globaly consistent?
      Best = H
   fi
else
   for j = 1 to n
       if unary(E_i, F_j)and then
          binary(H, E_i, F_j) -- test new with all pairings in H
          GCBB([H j], i + 1) -- pairing (E_i, F_j) accepted
       fi
   rof
   if pairings(H) + m - i > pairings(Best) -- can do better?
      GCBB([H 0], i + 1) -- star node: E_i not paired
   fi
fi
```

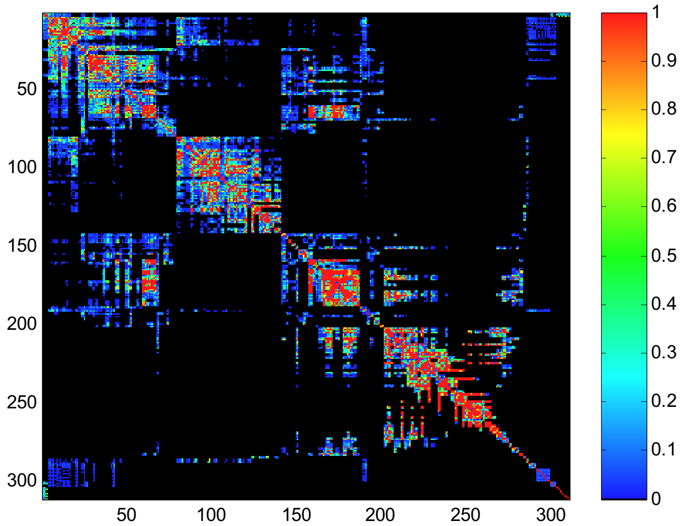

Exploiting locality

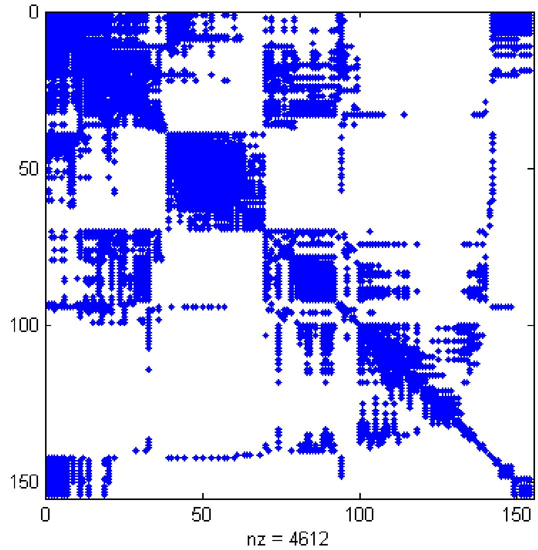




- Limit search in the global map to subsets of covisible features
- Locality makes GCBB **linear** with the global map size

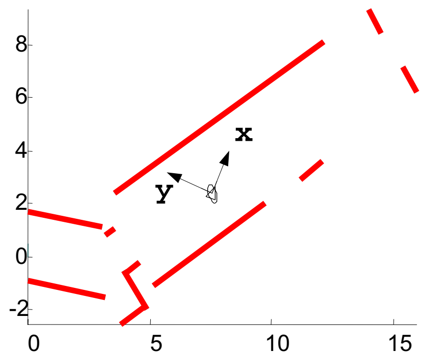
Information Matrix .vs. Covisibility

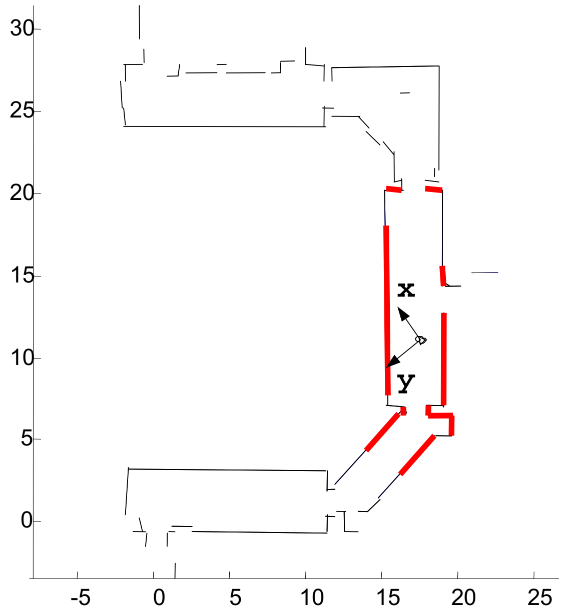




- Sydney park dataset (Nebot et al.)

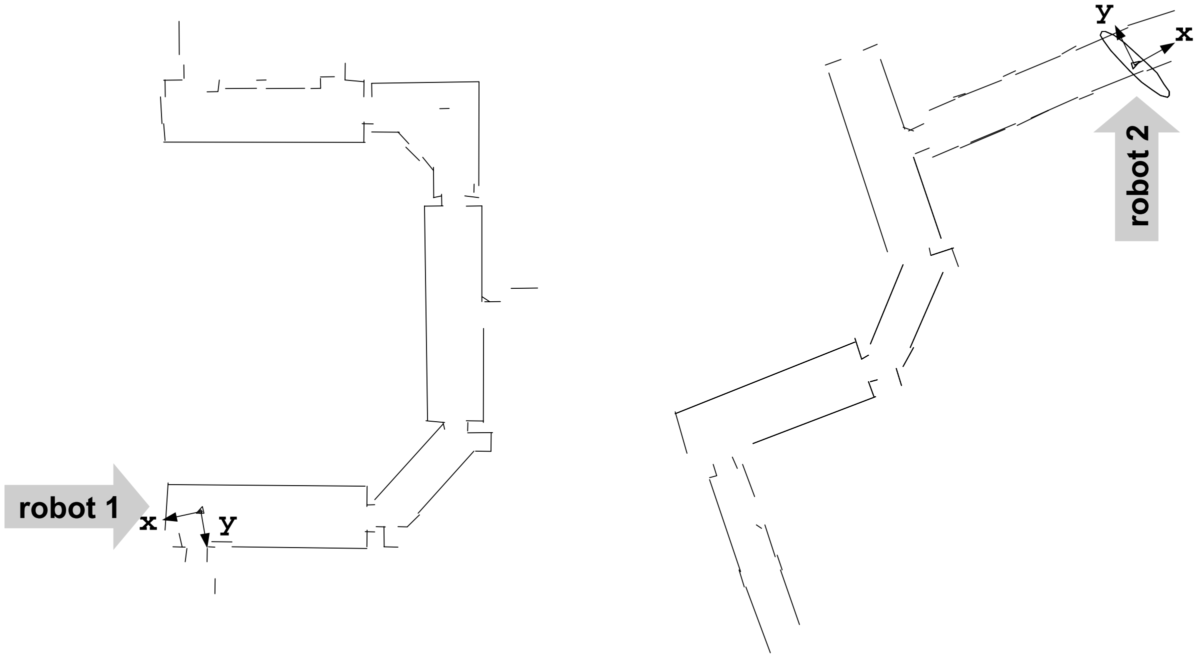
Robot relocation example





100% success rate for n=30 local maps

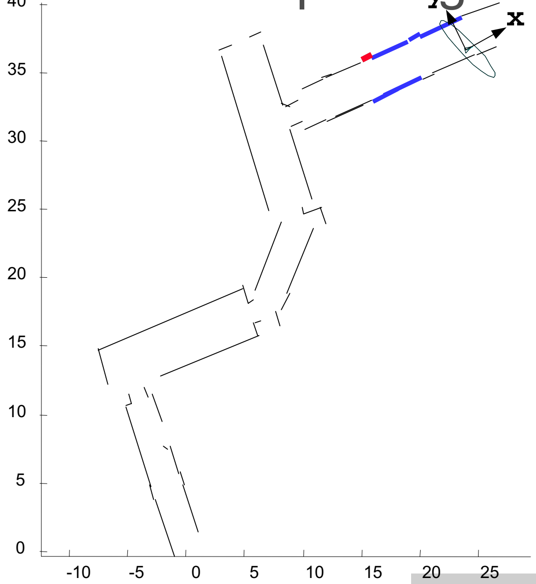
Multirobot map building

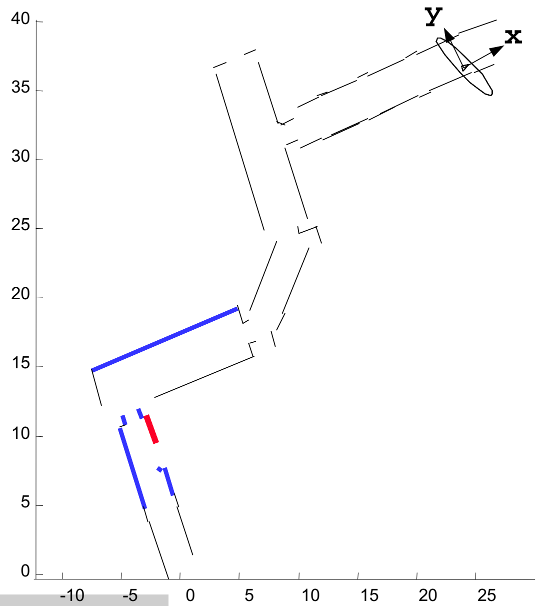


Match, join and fuse to one full stochastic map

Multirobot map building

- Randomly select a **feature** in one map
- Try to associate its **covisible features** in the other map using GCBB

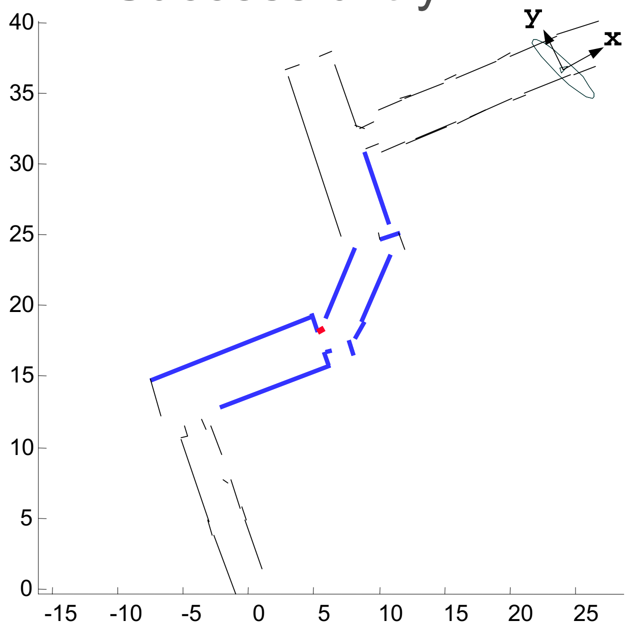


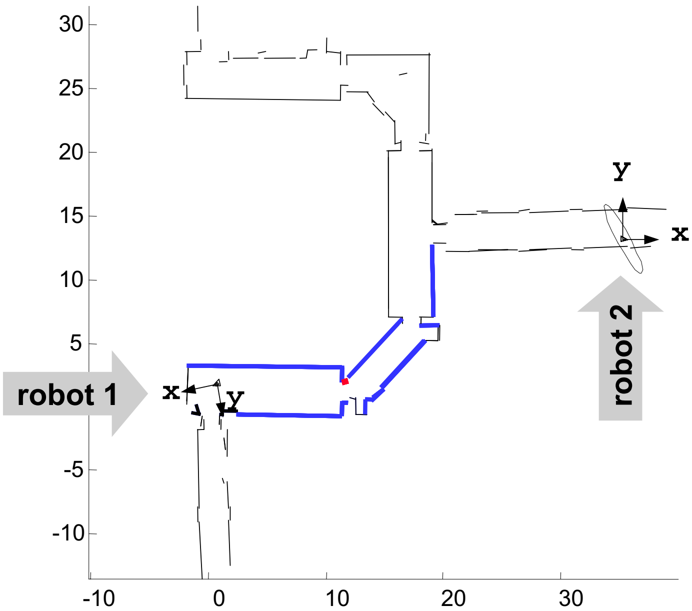


Unsuccessful tries

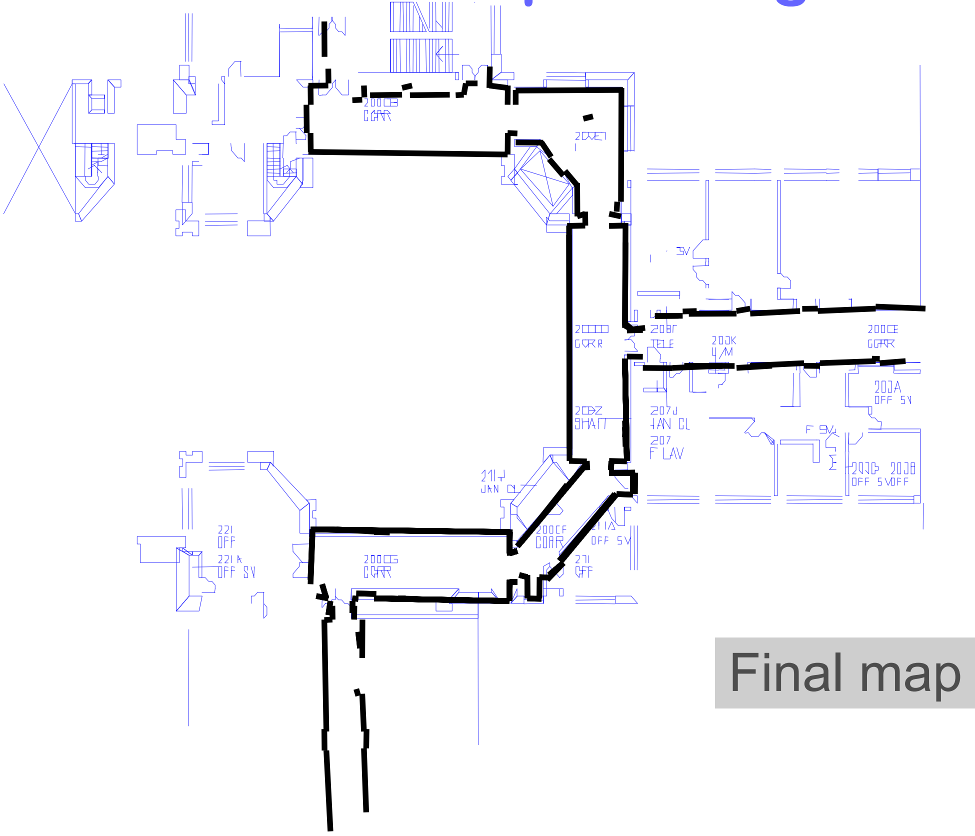
Multirobot map building

- Successful try
- Final map





Multirobot map building



5. Open Issues in SLAM

- ~~Data association~~
- **Big environments**
  - Big loop closing
  - Computational efficiency (constant time?)
  - New estimation techniques
- **Outdoor & complex environments**
  - Feature extraction and modeling
  - Data association
- **SLAM with vision**
- **3D SLAM**

Recommended Readings

- Grimson, W. E. L. : "Object Recognition by Computer: The Role of Geometric Constraints", The MIT Press, Cambridge, Mass., 1990
- José A. Castellanos and Juan D. Tardós, Mobile Robot Localization and Map Building: A Multisensor Fusion Approach, Kluwer Ac. Pub., Boston, 1999
- J. Neira and J.D. Tardós: “Data Association in Stochastic Mapping Using the Joint Compatibility Test”, IEEE Trans. Robotics and Automation, vol. 17, no. 6, pp. 890-897, Dec 2001.
- P.M. Newman, J.J. Leonard, J. Neira and J.D. Tardós: “Explore and Return: Experimental Validation of Real Time Concurrent Mapping and Localization”. IEEE Int. Conf. Robotics and Automation, May, 2002.
- J.D. Tardós, J. Neira, P.M. Newman and J.J. Leonard: “Robust Mapping and Localization in Indoor Environments using Sonar Data”, Int. J. Robotics Research, 2002 (to appear)