Modeling Odometry and Uncertainty Propagation for a Bi-steerable Car

Jorge Hermosillo Cedric Pradalier Sepanta Sekhavat

INRIA Rhone-Alpes
655 av. de l'Europe, Montbonnot, 38334 St. Ismier Cedex, France
{First-name.Last-name}@inrialpes.fr

Abstract— **This paper addresses the problem of modeling the odometry (dead-reckoning) for a Bi-Steerable car (BS-car) and its uncertainty propagation model. The attractive feature of BS-robots is their enhanced maneuverability compared to car-like robots. At first, we take into account the steering ability of a BS-car in order to obtain the odometry model for vehicles showing this type of kinematics. Subsequently, we obtain a generic model for the uncertainty propagation of the odometry model by using a Bayesian probability approach. We show how it is possible to derive from this generic model a specific error propagation model for a particular system, under clearly stated hypothesis. We evaluate qualitatively and quantitatively the performance of the odometry model by experimenting with our BS-platform, the Cycab robot.**

Keywords— **Odometry, Bi-steerable cars, Bayesian uncertainty model.**

I. Introduction

THIS paper addresses the problem of modeling the odometry (dead-reckoning) for a Bi-Steerable car (BS-car) and its uncertainty propagation model. To our knowledge, there is no work on modeling the dead-reckoning for robots with such kinematics. Our goal is to establish at first the odometry model for a BS-car and subsequently, a generic error propagation model by using a Bayesian probabilistic approach. The advantage of the latter is that it is possible to derive from it a specific error propagation model for a particular system. Indeed, this is possible when explicit uncertainty factors and *a priori* knowledge about the system are considered [1]. In order to evaluate qualitatively and quantitatively the performance of our odometry model, we implemented it in our experimental BS-platform, the Cycab Robot, and calibrated its accuracy using a high-precision laser range finder.

A. Motivation and Related Work

As part of a new kind of Intelligent Transportation System, BS-cars are a new experimental platform that has gained momentum in some laboratories around the world[1]. One of the most attractive features of BS-robots is their capability to deflect its front wheels by an angle ϕ while inducing a deflection of its rear wheels as a function of ϕ (i.e. $\phi_{rear} = f(\phi_{front})$). This feature enhances maneuverability in cumbersome environments [2]. Our BS-platform, the Cycab robot, is intended to be part of a new public transportation system specifically designed for pedestrian zones (e.g. airport terminals, commercial zones, etc.).

Endowed with autonomy capacities, the Cycab must be able to localize itself relying on its proprioceptive and exteroceptive sensors. Hence, characterizing the accuracy of the odometry is crucial for two main reasons:

- Odometry is at the heart of uncertainty estimations when data fusion is performed with exteroceptive sensors;

- A reasonably accurate odometer may alleviate momentarily from computations (possibly heavy) for absolute localisation; this is particularly true when for some reason absolute position readings are unavailable (e.g. occlusion of landmarks), in which case it would be of great interest to trust as long as possible on the odometry itself.

Modeling odometry for a BS-robot sets challenges that are not encountered in mobile robots for which many work has been done: [3],[4],[5],[6]. However, a common agreement in robotics community is to embrace estimation theory techniques to deal with uncertainty. In the majority of cases Gaussian hypothesis are done at sensor and system levels so that Kalman Filter techniques are commonly employed to calibrate systematic errors and propagate the uncertainty. A more theoretical work on the underlying phenomena in systematic error propagation can be found in [7] where the analysis focus on the dynamics of the perturbed system. In this paper, we aim at a generic model for dealing with the uncertainty of the odometry model by using a Bayesian formalism[1], which can be readily particularized for a specific robot.

B. Organization of the Paper

This paper is organized as follows: We begin section II by recalling the principles of differential heading odometry and the kinematics of a BS-robot. The odometry model for a BS-car is then obtained. In section III we discuss the uncertainty model in the framework of a Bayesian probability description. In section IV we assess qualitatively and quantitatively the odometry model using our experimental platform. In doing this, we assume Gaussian hypothesis at the sensor and system levels. We discuss the calibration results using an Extended Kalman Filter -EKF-. We close the paper with some concluding remarks in section V.

[1]e.g. the "πCar" prototype of IEF ("Institut d'Electronique Fondamentale" of Paris-Sud University.)and the Cycab robot at INRIA in France and NTU in Singapore

II. Odometry Model for a BS-Car

A. Differential Heading Odometry

In obtaining the odometry model for a BS-car we assume that the robot is equipped with encoders in the four wheels. Thus, we are interested in a differential heading odometry model. Hereafter the following hypothesis are made: (H1. the elementary path described by the wheels between two sampling times ($i-1$ and i) is an arc of circle (see Fig.1); this assumes that the steering command of the robot is "frozen" between sampling periods; and (H2. the sampling rate of the state and controls is very small compared to their rate of change; this ensures that linearizing errors and curvature discontinuities at elementary path ends are small. Therefore, we recall here established results regarding differential heading odometry mainly coming from [3] and revisited by [6].

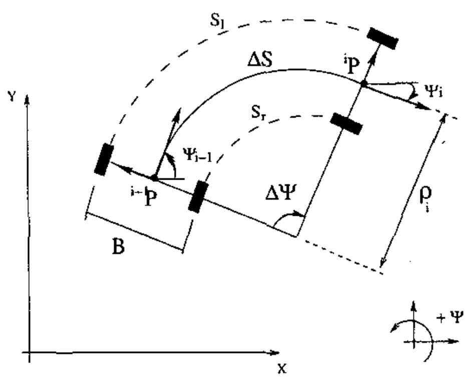


Fig. 1. The differential heading odometry model in the general case

Consider the geometric construction of Fig.1. The position of point P at sampling instant i (${}^{i}\mathcal{P}$) with respect to the reference frame on point P at sampling instant $i-1$ (${}^{i-1}\mathcal{P}$) is given by the following system of equations as a function of the turning radius ρ_i and the angular displacement $\Delta\Psi$.

$$\begin{cases} {}^{i-1}x_i = \rho_{i-1}\sin(\Delta\Psi) \\ {}^{i-1}y_i = \rho_{i-1}(1-\cos(\Delta\Psi)) \\ {}^{i-1}\Psi_i = \Delta\Psi \end{cases}$$

In world coordinates we have:

$$\begin{cases} x_i &= x_{i-1} + \rho_{i-1}(\sin(\Psi_{i-1}) - \sin(\Psi_{i-1}+\Delta\Psi)) \\ y_i &= y_{i-1} + \rho_{i-1}(\cos(\Psi_{i-1}+\Delta\Psi) - \cos(\Psi_{i-1})) \\ \Psi_i &= \Psi_{i-1} + \Delta\Psi \end{cases}$$

This system corresponds to [6], since for differential heading odometry we have $\Delta\Psi = \frac{S_r - S_l}{B}$, $\Delta S = \frac{S_r + S_l}{2}$; where S_r and S_l are the traveled distance by the right and left wheels respectively and B is the wheels base. By letting $\rho_i = \Delta S/\Delta\Psi$ and using trigonometric identities, the system becomes the one reported in [3]:

$$\begin{cases} x_i &= x_{i-1} + 2\frac{\Delta S}{\Delta\Psi}\sin(\frac{\Delta\Psi}{2})\cos(\Psi_{i-1} + \frac{\Delta\Psi}{2}) \\ y_i &= y_{i-1} + 2\frac{\Delta S}{\Delta\Psi}\sin(\frac{\Delta\Psi}{2})\sin(\Psi_{i-1} + \frac{\Delta\Psi}{2}) \\ \Psi_i &= \Psi_{i-1} + \Delta\Psi \end{cases} \qquad (1)$$

This is the general model for differential heading odometry under the hypothesis H1 and H2.

B. Kinematic Aspects of a BS-car

The kinematics of a BS-robot (and in particular of the Cycab) have been studied in previous work [8],[2]. The main characteristic which makes a BS-car a highly maneuverable vehicle is that it can steer the rear wheels as a function of the deflection of the front wheels ($\phi_{rear} = f(\phi_{front})$), see Fig.2.

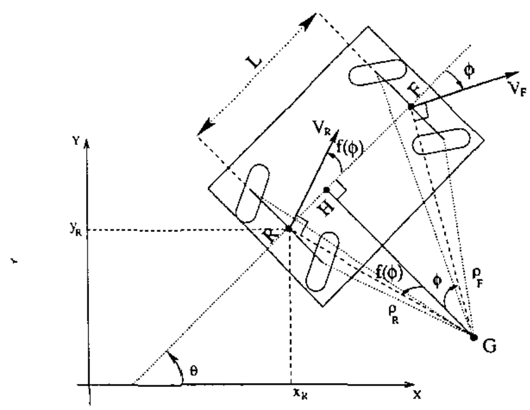


Fig. 2. Kinematics of a BS-car

Conventionally, the configuration of the robot is taken as the vector $(x, y, \theta)^T$ where (x, y) are either the coordinates of the point F or R (middle points of an axle) and θ is the heading of the robot (i.e. the orientation of the body). For modeling purposes, the steering control ϕ (alt. $f(\phi)$) is assumed to be applied to an imaginary wheel placed at point F (alt. R) so that the velocity vector associated to that point V_f (alt. V_r) is collinear to the orientation of this wheel.

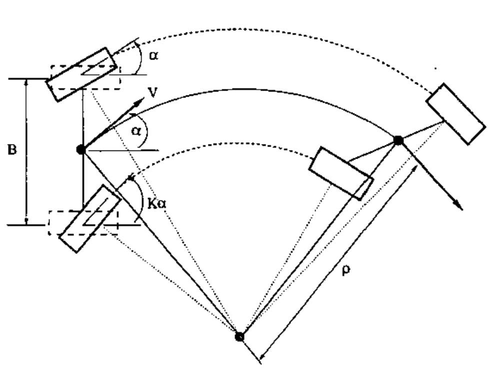
In the sequel, we are only concerned with the motion of one pair of wheels. Since for odometry purposes both axles are the same, we will assume a general steering command α likely to be applied to either pair of wheels.

C. Odometry Model for a BS-Car

At this point geometric modeling choices must be made. Indeed, it is well known that the mechanical design of a steering axle implies that each wheel must have different attitude for describing an exact circle as shown in Fig.3.a. In modeling this phenomena, we pivot the steering axle round the middle point. Thus, the pivot link leads to a "shortening" of the steering axle as represented in Fig.3.b.

Referring back to equation (1), the position and orientation of the robot can be determined from ΔS and $\Delta\Psi$. Therefore, we are interested in the relation between θ and Ψ since the former is the actual orientation of the vehicle. Moreover, we are interested in the effect of the steering ability of the BS-car on the robot's motion.

¿From the kinematics model of a BS-robot we have: $\Psi = \alpha + \theta$. This relation suggests that changes in angle Ψ are



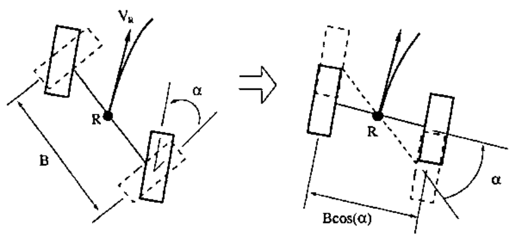


Fig. 3. a) Mechanical steering capability of a BS-car; b) its geometric model for odometry purposes

composed of changes in the steering angle and changes in the orientation of the vehicle. Now, under hypothesis H1, α is constant between two samplings. Hence $\Delta\alpha = \alpha_i - \alpha_{i-1} = 0$ and then $\Delta\Psi = \Delta\theta$.

Thus, if we let:

$$\Delta S = \frac{S_r + S_l}{2} = \frac{R_r\delta_r + R_l\delta_l}{2}$$

$$\Delta\theta = \frac{S_r - S_l}{B\cos(\alpha_i)} = \frac{R_r\delta_r - R_l\delta_l}{B\cos(\alpha_i)},$$

where R_r (resp. R_l) is the radius of the right (resp. left) wheel and δ_r (resp. δ_l) is the angular displacement of the right (resp. left) wheel, then the substitution of these expressions into system (1), and the application of the Taylor's first order approximation of the $\sin(\Delta\Psi/2)$ term (this is a good approximation under hypothesis H2) yield the following general odometry model for a BS-car:

$$\left\{ \begin{array}{lcl} x_i & = & x_{i-1} + \Delta S\cos(\theta_{i-1} + \alpha_i + \frac{1}{2}\Delta\theta) \\ y_i & = & y_{i-1} + \Delta S\sin(\theta_{i-1} + \alpha_i + \frac{1}{2}\Delta\theta) \\ \theta_i & = & \theta_{i-1} + \Delta\theta \end{array} \right. \quad (2)$$

The model is consistent since in the absence of motion, the orientation of the vehicle (θ) remains unaltered even in the presence of a steering command. However, consider now the absolute value of the angular argument of the sinusoids in the x and y coordinate functions. We can appreciate that as soon as the vehicle starts moving, the steering command will induce a faster change in coordinate y and a slower change in coordinate x implying a turning maneuver.

Now, as we are modeling a real system, system (2) is inevitably incomplete. Incompleteness leads to uncertainty[9]. Therefore we are obliged to use estimation tools from probability theory in order to propagate it. The following section aims at establishing an error propagation model by accounting for uncertainties of different character.

III. The Uncertainty Model

A. Dealing with Uncertainty

In order to assess the uncertainty related to system (2), it is important to analyze the nature of the uncertainty of each term in the model. For example, at the sensor level, sensor signals are commonly contaminated with white Gaussian noise (e.g. thermal noise). In the best case, this uncertainty can be propagated to upper levels of the architecture so they can be accounted for. In practice this is rather unusual. In most cases, we have a pre-treatment phase which sends us steady levels concerning sensed variables but for which accuracy and precision knowledge is lost. Furthermore, systematic errors concern mainly the robot's parameters (e.g. the deviations of the wheels' radii from the nominal radius)[5]. But in establishing an accurate error model for these, we would have to make uncertainty assumptions according to the knowledge we have about the real parameters, whose random behavior might not be necessarily Gaussian in nature (e.g. deviations could be uniformly distributed).

In obtaining the uncertainty model of the odometry model(2), we will identify three possible groups of variables which might show a random character (possibly different). Thus, let $X = \{x, y, \theta\}$ be the random state vector, $U = \{\delta_r, \delta_l, \alpha\}$. be the random sensor vector, and $A = \{R_r, R_l, B\}$ be the random robot's parameters vector. Hence, system (2) expresses the dynamics of the state vector as a function of these random vectors; that is:

$$X_i = f(X_{i-1}, U_i, A)$$

In order to establish the uncertainty of (2) in a generic fashion we will use a Bayesian probabilistic formalism ([1]) to model the uncertainty propagation of our system.

The reasons for this are threefold: Firstly, we can clearly state independence assumptions and reflect them into the uncertainty model. Secondly, this formalism enables to use numerical inference tools ([9]) to estimate X_i by using *a priori* knowledge on variables of our system whose probability density might not be Gaussian in nature. Last but not least, this formalism enables natural expansions for data fusion purposes aiming global localisation systems (e.g. see [10]).

B. A Generic Uncertainty Model for Odometry

Bayes rules help us to establish the probability density of present states X_i by knowing past values of the state, current sensor levels and the robot's parameters. This distribution is given by the joint probability density of X_i, X_{i-1}, U_i, A as:

$$p(X_iX_{i-1}U_iA) = \\ p(U_i)\,p(X_{i-1}|U_i)\,p(A|X_{i-1}U_i)\,p(X_i|X_{i-1}U_iA) \quad (3)$$

This is a generic uncertainty model for (2) which directly derives from the Bayes rules.

As a matter a fact, this model can be simplified or even particularized under certain assumptions. The interesting point here is that the model (3) contains all the information about uncertainty interdependencies. At this point, it is possible to formulate in an explicit way conditional independence hypothesis so as to simplify the model.

Conditional Independence Hypothesis: Equation (3) can be simplified by establishing conditional independence hypothesis between X, U, A on the basis of the knowledge we have about them. An obvious simplification is the fact that the parameters A are uncorrelated to X and U all time. A second simplification stems from the assumption that U_i and X_{i-1} are statistically independent; that is: current values from the encoders are uncorrelated to the previous position of the robot. Notice however that this could eventually be a strong modeling choice under other circumstances[2]. Furthermore, it is possible to assume that the probability density of U_i is invariant along time (i.e. $p(U_i) = p(U_{i-1})$), allowing us to calibrate its effects once for all[3]. The point we want to stress here is that equation (3) aware us from modeling choices (possibly having sound physical meaning) that may have a non negligible impact on the model's accuracy.

Hence, under these hypothesis, the joint probability distribution (3) can be expressed as:

$$p(X_iX_{i-1}U_iA) = \\ p(X_{i-1})\,p(U)\,p(A)\,p(X_i|X_{i-1}UA) \quad (4)$$

Therefore, in order to have an estimate of the configuration reached ($\hat{X}_i$), we require to know the probability density of the state vector $p(X_i)$ which is given by:

$$p(X_i) = \int_{\mathcal{O}} p(X_iX_{i-1}UA) = \\ \int_{\mathcal{O}} p(X_{i-1})\,p(U)\,p(A)\,p(X_i|X_{i-1}UA) \quad (5)$$

where $\mathcal{O} = \{X_{i-1}, U, A\}$. To solve this equation the following probability distributions must be known:

$$p(X_i|X_{i-1}UA), p(X_{i-1}), p(U), p(A)$$

Here, it is possible to elucidate the physical meaning of this. In fact, equation (5) involves systematic errors coming from $p(A)$ and $p(U)$, representing the uncertainty factors linked to the robot's characteristics, and non-systematic errors coming from $p(X_{i-1})$, containing the cumulated uncertainty, and from $p(X_i|X_{i-1}UA)$ which is a family of possible probability distributions according to the values taken by X_{i-1}, U and A. Therefore, equation (5) is the uncertainty propagation model of the odometry model (2) under the hypothesis stated above. Clearly, calibration of odometry means here characterizing $p(A)$ and $p(U)$ as discussed in the following section.

[2]e.g. think about the propagation of the uncertainty within the system's architecture for path planning purposes

[3]i.e. at system start-up or seldom during long periods of use if automated calibration is available.

IV. Odometry Performance

For sake of simplicity, it seems a common agreement in robotics community to consider system (2) as being contaminated with Gaussian noise. Indeed, this allows to use Kalman Filter techniques to characterize systematic errors and to propagate the uncertainty in a computationally convenient way. Also, it is a common practice to assess the error stemming from U and A together (e.g. the heading error gain from [5]). We will adopt this approach in a similar way. Hence, we will assume a system-noise vector S susceptible of being calibrated as will be discussed below.

In the experiments reported here we used a Cycab whose rear-to-front steering relation is $\phi_{rear} = -\phi_{front}$. Thus in model (2) we replaced α by $-\phi$.

A. Uncertainty under Gaussian Hypothesis

If we assume that errors in system (2) are of Gaussian nature, equation (5) may be reduced to:

$$X_i \sim \mathcal{N}(f(X_{i-1}, S); \Sigma_{(X_i,S)}) \quad (6)$$

That is, X_i is normally distributed with first moment system (2) and second moment:

$$\Sigma_{(X_i,S)} = \Sigma_{X_{i-1}} + J_{(X_{i-1},S)}f \cdot \Sigma_{(X_{i-1},S)} \cdot J^T_{(X_{i-1},S)}f \quad (7)$$

where $J_{(X_{i-1},S)}$ is the Jacobian of f with respect to the vector $\{X_{i-1}, S\}$. Under the independence hypothesis stated in sectionIII-B, the odometric error (uncertainty of the system) can be alternatively represented by the following covariance matrix:

$$\Sigma_{(X_i,S)} = \Sigma_{X_{i-1}} + J_{X_{i-1}}f \cdot \Sigma_{(X_{i-1},S)} \cdot J^T_{X_{i-1}}f + \\ J_S f \cdot \Sigma_{(X_{i-1},S)} \cdot J^T_S f \quad (8)$$

For evaluating the performance of our model, we implemented (2) in our experimental platform, the Cycab robot, to compute the rear-wheel odometry. In the sequel, we assume Gaussian hypothesis.

B. Qualitative Results

In order to assess the qualitative behavior of our model, we compared it against a global localisation system based on an accurate laser range-finder (from the company Sick). The localisation system performs a Kalman Filter using odometry predictions and laser corrections.

The experiment consists in tracking artificial landmarks using the laser range-finder while filtering the odometry with this information. At a certain moment we stop filtering and leave the odometry to its own. Fig.4 shows the qualitative results.

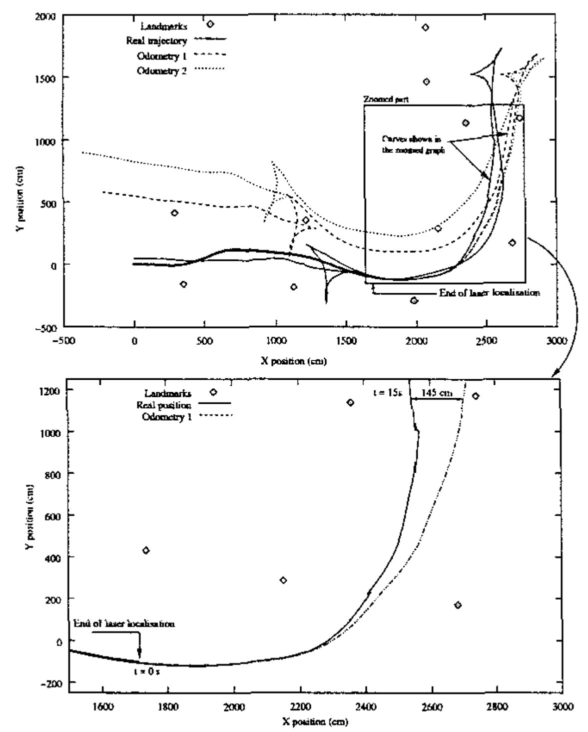


Fig. 4. Qualitative behavior of the odometry model for a BS-car: a) **Odometry 1** trace: The performance of model (2) compared against the data given by the laser range-finder ("real trajectory"); **Odometry 2** trace: performance when we introduce an error by omitting the shortening effect of the steering axle when the wheels turn (i.e. we use B instead of $B\cos(\phi)$ in model (2). b) a zoomed view of the deviation of the odometry as computed by model (2). It is expected that this error can be reduced by calibrating the odometry in the real robot.

Refer to Fig4. This figure shows a good qualitative behavior of the odometry model even without calibration. We can appreciate that the odometry follows the laser range-finder with good accuracy (below 10cm.) along an x-distance of 8m. and a y-distance of 2m. The performance decays as we continue to move in the y direction to reach a maximum deviation of 1.5m along a y-distance of 12m and during a total time of 15 seconds. Interesting to appreciate is the effect of the modeling of the steering of the wheels (see trace labeled *Odometry 1*). This trace remains closer to the "real" trajectory of the robot (measured with the laser) than the trace labeled *Odometry 2* which corresponds to a model where the "shortening" of the axle was omitted.

C. Numerical Calibration of Systematic Errors

Systematic errors stem from many sources: "hidden" variables of the real system (since model (2) is by definition incomplete); deviations from nominal parameters of the robot (e.g. wheels' radii deviations), encoder gains bias (i.e. the calibration process of the encoder induces systematic errors), etc.. We chose a simplified parametric representation of the systematic errors of our system in a similar manner to [11].

We model systematic errors of system(2) as error gains associated to each wheel (k_r, k_l) and to the wheel base (k_b) of the real robot. For each wheel we have calibrated the gain of the encoder (ζ) in meters per pulse (m./pulse). In this way, the relations giving the geometric information of an elementary motion of the robot are parametrized with three systematic error gains as follows:

$$\Delta S = \frac{k_r \cdot \zeta_r E_r + k_l \cdot \zeta_l E_l}{2}$$
$$\Delta\theta = \frac{k_r \cdot \zeta_r E_r - k_l \cdot \zeta_l E_l}{k_b \cdot B\cos(\alpha_i)}, \tag{9}$$

where E_r, E_l are the encoder readings in pulses.

Inspired from [5] and [11], we decided to express one of the wheel error gains (e.g. k_l) as a function of the heading error gain k_d, estimated during a straight-line motion using the following predictor:

$$\begin{cases} S[i+1] = S[i] + k_d[i] \cdot \dfrac{\zeta_r E_r[i] + \zeta_l E_l[i]}{2} \\ k_d[i+1] = k_d[i], \end{cases}$$

Henceforth it is more convenient to estimate the right error gain k_r and the wheel base error gain k_b through a new heading gain defined as $\zeta_a = k_d \cdot (\zeta_r E_r + \zeta_l E_l)$, since a new predictor can be constructed using (9):

$$\begin{cases} \Delta S[i+1] = \frac{k_r[i]\cdot\zeta_r E_r[i] + (\zeta_a[i] - k_r[i]\cdot\zeta_l)E_l[i]}{2} \\ \Delta\theta[i+1] = \frac{k_r[i]\cdot\zeta_r E_r[i] - (\zeta_a[i] - k_l[i]\cdot\zeta_l)E_l[i]}{k_b[i]\cdot B\cos(\alpha[i])} \\ k_r[i+1] = k_r[i] \\ k_b[i+1] = k_b[i], \end{cases} \tag{10}$$

We used a Kalman Filter[12] (sub-optimal EKF version) in order to calibrate the systematic errors of our system. When substituting the expressions (10) into system (2), our augmented state vector becomes $X = \{x, y, \theta, k_r, k_b\}$, our system-noise vector is $S = \{E_l, E_r, \alpha\}$ (the sources of noise) and our measurement vector (laser correcting readings) is $Z = \{x, y, \theta\}$. The linearized error covariance matrix ($\mathbf{P}_\mathbf{k}^{(-)}$[12]) is readily obtained from equation (8). Results are shown in figures Fig.5 through Fig.8. It is possible to appreciate the improved quality of the odometry predictions after the substitution of the calibrated error gains into system (10).

V. Conclusion

In this work, we presented an odometry model for a Bi-steerable Car and its generic uncertainty propagation

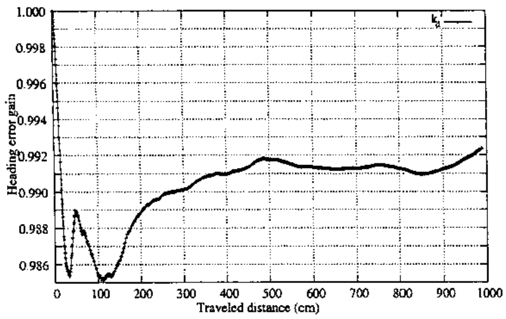


Fig. 5. estimation of k_d during the execution of a straight-line motion

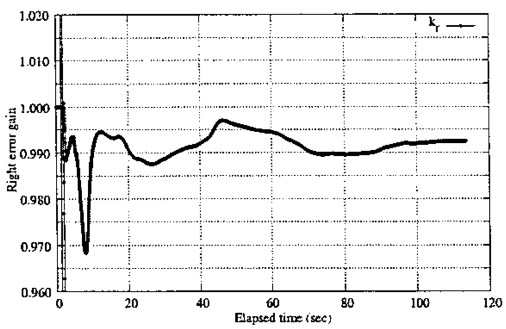


Fig. 6. estimation of k_r during motion depicted in Fig.4

model. To this end, we modeled at first the effect of the steering capability of a BS-car and introduced it into a differential heading odometry model. Subsequently, we used a Bayesian probabilistic approach in order to establish a generic uncertainty model for odometry. We showed that it is possible to simplify or particularize this model by stating conditional independence hypothesis and by introducing *a priori* knowledge about the particular system. Under the assumption of these hypothesis and Gaussian noise contamination at system and sensor stages, we chose a simplified parametric representation of the systematic errors of our model by introducing error gains associated to the encoders. We calibrated these error gains using a Kalman Filter approach. The qualitative behavior of the odometry model was satisfactory even without calibration but improved substantially after calibration. As part of the future work we will learn the odometry model using bayesian inference techniques[9].

Acknowledgements

This work was partially supported by the Inria LaRA *(La Route Automatisée - 1998-2002)* program on urban public transport and partially by CONACYT *Consejo Nacional de Ciencia y Tecnologia (Mexico)*.

REFERENCES

[1] K. Mekhnacha, E. Mazer, and P. Bessière, "A robotic CAD system using a Bayesian framework," in *Proc. of the IEEE-RSJ Int. Conf. on Intelligent Robots and Systems*, Takamatsu (JP), Nov. 2000.

[2] S. Sekhavat and J. Hermosillo, "The cycab robot: a differentially flat system," in *Proc. of the IEEE-RSJ Int. Conf. on Intelligent Robots and Systems*, Takamatsu (JP), Nov. 2000.

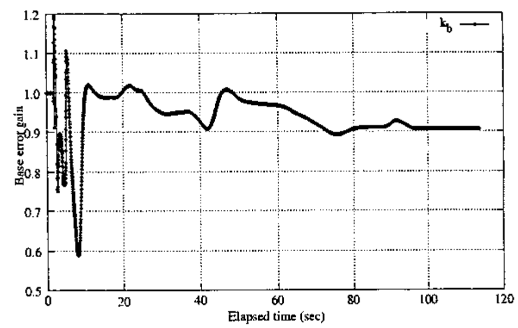


Fig. 7. estimation of k_b during motion depicted in Fig.4

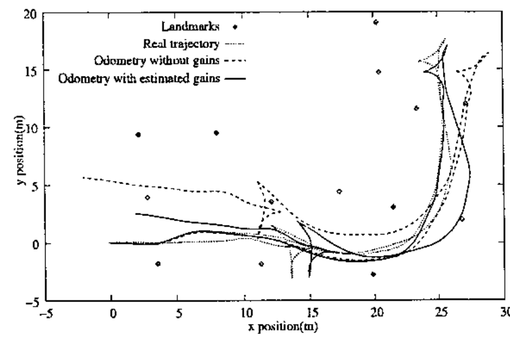


Fig. 8. Qualitative behavior of the odometry model for a BS-car after calibration: It is interesting to appreciate that the predictions of the odometry model (2) after calibrating k_r, k_l and k_b correspond better to the information sent by the laser range finder.

[3] C. Ming Wang, "Location estimation and uncertainty analysis for mobile robots," in *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Philadelphia, PA (US), Apr. 1988.

[4] Frédéric Chenavier and James L. Crowley, "Position estimation for a mobile robot using vision and odometry," in *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Nice (FR), May 1992.

[5] Johann Borenstein and Liqiang Feng, "Correction os systematic odometry errors in mobile robots," in *Proc. of the IEEE-RSJ Int. Conf. on Intelligent Robots and Systems*, Pittsburgh, PA (US), Oct. 1995.

[6] Kok Seng Chong and Lindsay Kleeman, "Accurate odometry and error modelling for a mobile robot," in *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Albuquerque, NM (US), Apr. 1997.

[7] Alonzo Kelly, "General solution for linearized systematic error propagation in vehicle odometry," in *Proc. of the IEEE-RSJ Int. Conf. on Intelligent Robots and Systems*, Hawaii, HI (US), Oct.-Nov. 2001.

[8] S. Sekhavat, J. Hermosillo, and P. Rouchon, "Motion planning for a bi-steerable car," in *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Seoul (KR), May 2001, pp. 3294–3299.

[9] O. Lebeltel, J. Diard, P. Bessière, and E. Mazer, "A Bayesian framework for robotic programming," in *Proc. of the Int. Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering*, Paris (FR), July 2000.

[10] Sebastian Thrun, Dieter Fox, Wolfram Burgard, and Frank Dellaert, "Robust monte carlo localization for mobile robots," *Artificial Intelligence*, vol. 128, no. 1, pp. 99–142, Jan. 2001.

[11] M. Bak, *Control of Systems with Constraints*, Ph.D. thesis, Technical University of Denmark, 2000.

[12] A. Gelb, J.F. Kasper, R.A Nash, C.F. Price, and A.A Sutherland, *Applied Optimal Estimation*, The M.I.T. PRESS, 1996.