A new approach to map building using relative position estimates

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ABSTRACT

A new method for automatic map building and vehicle localization is described. The method utilizes relative measurements of distance and angle between features of the environment. The noise in successive relative measurements is uncorrelated, hence a significant saving in memory and computation is achieved over other techniques, such as the Augmented State Kalman Filter.

Keywords: Autonomous vehicles, Map building, Navigation, Unknown environment, Relative measurements

1. INTRODUCTION

This paper examines the problem of automatically constructing a map of an unknown environment from a vehicle whose position is simultaneously localized. This problem has been addressed previously by several researchers,[1,2,4–6] many using the Kalman filter.[3]

In section 2 the efficiency problem associated with the Kalman filter in this context is briefly described. Section 3 introduces a new algorithm, called the Relative Filter, which does not suffer the efficiency problem of the Kalman filter. In Section 4 experimental results are presented showing the performance of the Relative filter in a real map building scenario. Conclusions are presented in Section 5.

2. THE KALMAN FILTER

The vehicle has the simultaneous task of estimating its own position continuously and building the map. These two tasks cannot be performed independently; the estimate $\hat{\mathbf{x}}$ of the vehicle position $\mathbf{x}$ is needed to generate an estimate $\hat{\mathbf{p}}_i$ of the position $\mathbf{p}_i$ of a feature i that has been observed, see Figure 1. Subsequent observation of feature i are used to update $\hat{\mathbf{p}}_i$ and $\hat{\mathbf{x}}$. This has the effect that the errors $\tilde{\mathbf{x}} = \mathbf{x} - \hat{\mathbf{x}}$ and $\tilde{\mathbf{p}}_i = \mathbf{p}_i - \hat{\mathbf{p}}_i$ are correlated to each other. Indeed, correlations also arise between $\tilde{\mathbf{p}}_i$ and $\tilde{\mathbf{p}}_j$, in other words all estimates have errors that are correlated to each other.

$$E\begin{bmatrix} \tilde{\mathbf{x}}\tilde{\mathbf{x}} & \tilde{\mathbf{x}}\tilde{\mathbf{p}}_i \\ \tilde{\mathbf{p}}_i\tilde{\mathbf{x}} & \tilde{\mathbf{p}}_i\tilde{\mathbf{p}}_i \end{bmatrix} \neq \mathbf{0},\ \textit{for all } i \tag{1}$$

$$E\begin{bmatrix} \tilde{\mathbf{p}}_i\tilde{\mathbf{p}}_i & \tilde{\mathbf{p}}_i\tilde{\mathbf{p}}_j \\ \tilde{\mathbf{p}}_j\tilde{\mathbf{p}}_i & \tilde{\mathbf{p}}_j\tilde{\mathbf{p}}_j \end{bmatrix} \neq \mathbf{0},\ \textit{for all } i,j \tag{2}$$

The correlations must be maintained explicitly in the form of the covariance matrix $\mathbf{P}$. Therefore, the estimate of the vehicle position is augmented with the estimate of the feature positions to produce an augmented state estimate, hence the name 'Augmented Sate Kalman Filter' (ASKF). If some of the correlations are neglected, for example a reduced order covariance matrix is employed, the estimates become inconsistent. Figure 2 is an example of this.

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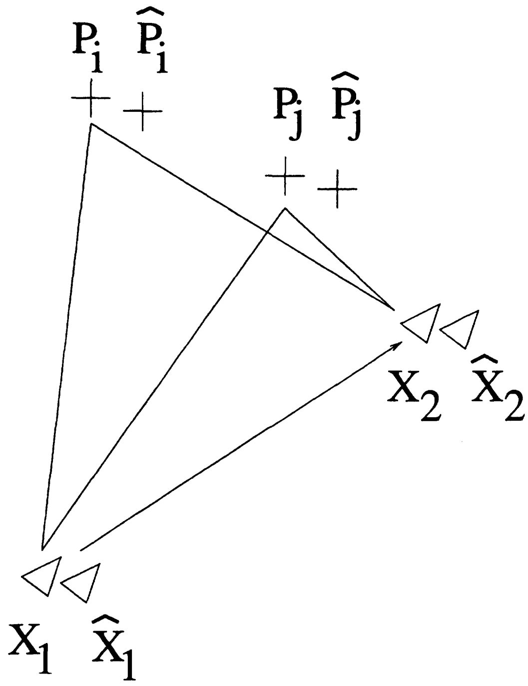

Figure 1. The source of the correlations: the estimate $\hat{\mathbf{x}}$ of the vehicle position $\mathbf{x}$ is needed to generate an estimate $\hat{\mathbf{p}}_i$ of the position $\mathbf{p}_i$ of a feature i that has been observed. Correlations arise between the errors of all feature estimates $\hat{\mathbf{p}}_i$ and $\hat{\mathbf{p}}_j$, and between the errors of any feature estimate $\hat{\mathbf{p}}_i$ and the vehicle estimate $\hat{\mathbf{x}}$. The error of a vehicle position estimate $\hat{\mathbf{x}}_2$ is also correlated to the previous vehicle position estimate $\hat{\mathbf{x}}_1$.

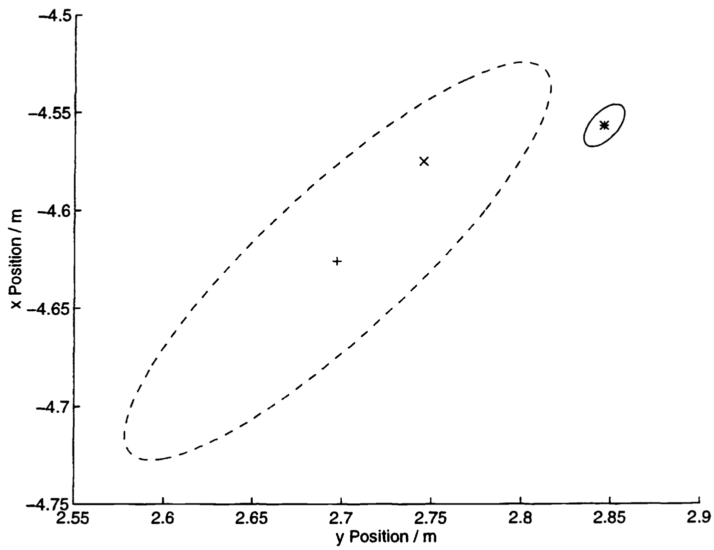

Figure 2. Effect of neglecting the correlations: the solid line shows the 1σ bound of the position estimate ($\star$) when a reduced order covariance matrix is used, and the dashed line is the 1σ bound of the position estimate ($+$) when a full covariance matrix is used. The true position is marked with a 'x'.

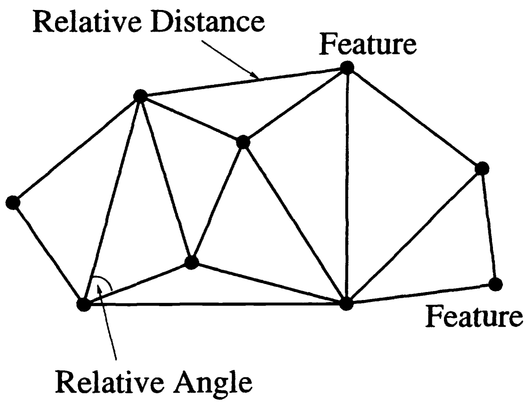

Figure 3. A relative map

However, for a scenario of N features, this results in a matrix of dimension that scales as N^2, each term of which has to be updated with each observation. This computational burden and storage requirement prohibits the application of the ASKF to real time applications of map building.

An alternative solution is required that can be used to automatically construct a map of the environment, but can be performed in real time. The Relative filter is such a solution.

3. THE RELATIVE FILTER

In this section the Relative Filter is examined. The section starts with an overview of the data structure, followed by an explanation of how relative distances and angles are computed. A brief discussion on avoiding correlations follows and the section concludes with an outline of the Relative filter algorithm.

3.1. Structure of the Relative Map

Figure 3 is an example of a Relative Map. The map consists of estimates of the relative distance between features (the nodes of the map) and their neighbors. This information may on its own be insufficient to construct an unambiguous map. To eliminate the ambiguity angles are introduced. Given a feature and two of its neighbors, an estimate of the angle subtended by the two neighboring features is also maintained by the map.

A particular feature F_i in a Relative map will typically be linked to a few other features. A link L_{ij} of the feature F_i to the feature F_j consists of an estimate (and its variance) of the distance between F_i and F_j. Associated with a link L_{ij} is a pointer *p_link*$_{ij}$ that points to F_j (in the map) and a pointer *p_link*$_{ji}$ that points to F_i (in the map). These pointers allow the features F_i and F_j to identify each other, see Figure 4.

The Relative Filter also maintains a circular buffer. This buffer stores a history of observations and a pointer with each observation which points to the feature in the map with which it has been associated (e.g. pointer *p_obs*$_k$ for the observation of F_k). Each feature can have only one observation of itself in the buffer. If a new observation is identified with a particular feature, the previous observation of that feature is removed from the buffer. A feature F has a pointer *p_node* of its own which points to the observation of F found in the buffer, or has a $NULL$ pointer if no observation is currently available (see *p_node*$_k$ for F_k in Figure 4).

For each link L_{ij} the feature F_i has a pointer *p_link*$_{obs}$ that points to the last observation of F_j that was used to update L_{ij}, or is a $NULL$ pointer if that observation of F_j is no longer available. This pointer is used to avoid correlations when updating relative distance estimates.

The relative angle A_{ijk} formed by three features F_i, F_j and F_k (with F_j forming the center) is stored with feature F_j. Associated with A_{ijk} are two pointers *p_angle*$_i$ and *p_angle*$_k$ which point to the last observation of F_i and F_k that were used to update A_{ijk}. Either *p_angle*$_i$ or *p_angle*$_k$ may be a $NULL$ pointer if the particular observation is no longer in the buffer. The pointers *p_angle*$_i$ and *p_angle*$_k$ are used to avoid correlations when updating relative angle estimates.

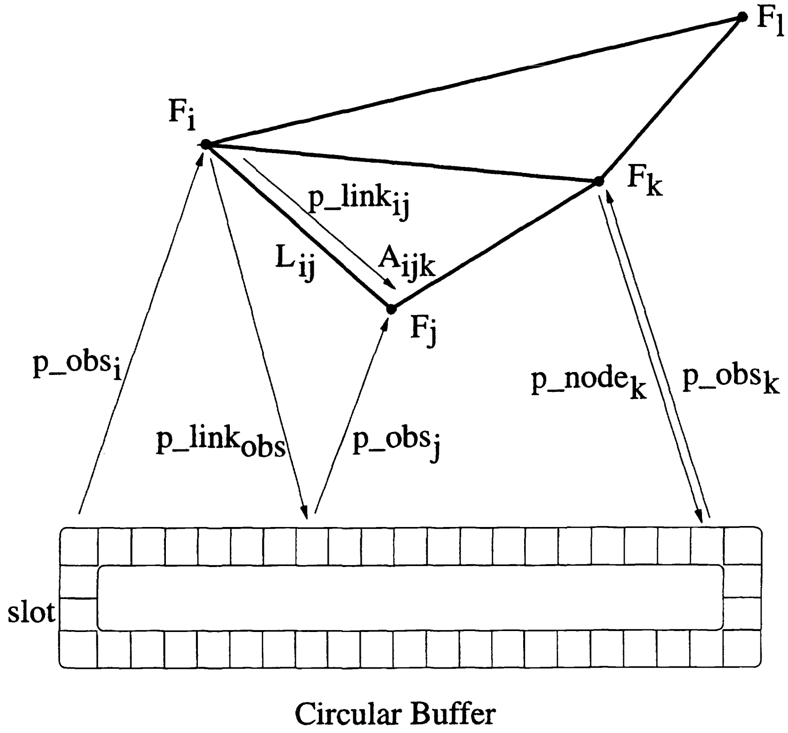

Figure 4. A relative map and its buffer

3.2. Calculating relative distances and angles

Given simultaneous observations of two distinct features, it is possible to calculate a measurement of the relative distance between the features without knowing the position of the observer. Similarly, given simultaneous observations of three distinct features, it is possible to calculate a measurement of the angle formed by the three features without knowing the position of the observer.

If the observer has moved between the observations, the only additional information required to construct the measurements is the relative motion between the observations. Again, knowledge of the absolute position of the observer (now different at each observation) is not required.

Denoting the cartesian coordinates of the first and second feature with respect to the first vehicle position as (x_1, y_1) and (x_2, y_2) respectively, the following expressions are obtained (see Figure 5):

$$x_1 = r_1 \cos(\delta\theta_1) \quad (3)$$
$$y_1 = r_1 \sin(\delta\theta_1) \quad (4)$$
$$x_2 = \delta x + r_2 \cos(\phi + \delta\theta_2) \quad (5)$$
$$y_2 = \delta y + r_2 \sin(\phi + \delta\theta_2) \quad (6)$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (7)$$

Where (r, θ) is an observation of a feature provided by the sensor and $(\delta x, \delta y, \delta\phi)$ is the change in position and orientation of the vehicle between the two observations. The desired relative distance between the two features is d. An estimate of $(\delta x, \delta y, \delta\phi)$ can be obtained by using the control inputs to predict the vehicle motion between the two observations.

Denoting the cartesian coordinates of the first, second and third feature as (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively, the following expressions are obtained. The desired relative angle subtended at the First Feature by the Third

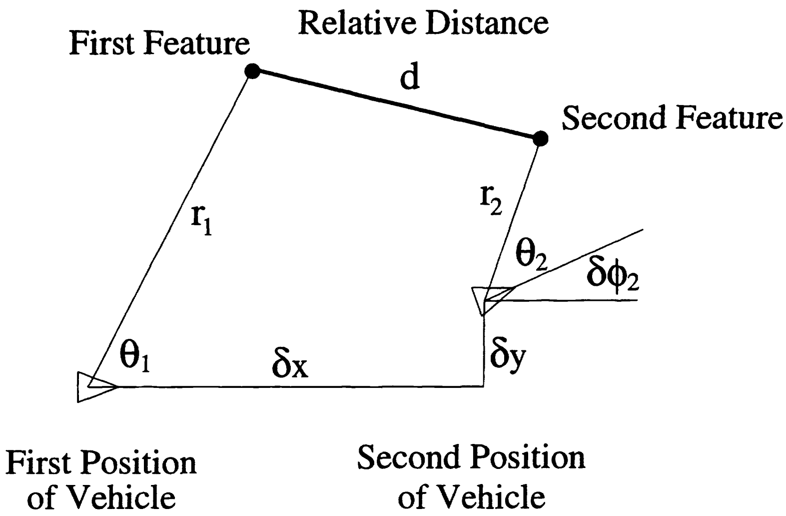

Figure 5. Calculating the relative distance

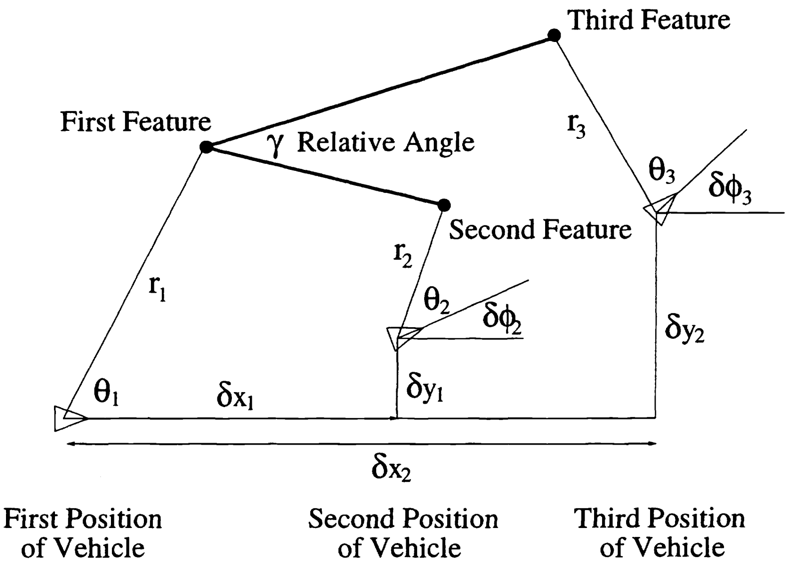

Figure 6. Calculating the relative angle

and Second Features is γ, see Figure 6. Care should be taken when evaluating $\tan^{-1}(\cdot)$ to ensure that the correct quadrant is obtained.

$$\begin{aligned}
x_1 &= r_1 \cos(\delta\theta_1) && (8)\\
y_1 &= r_1 \sin(\delta\theta_1) && (9)\\
x_2 &= \delta x_1 + r_2 \cos(\phi_2 + \delta\theta_2) && (10)\\
y_2 &= \delta y_1 + r_2 \sin(\phi_2 + \delta\theta_2) && (11)\\
x_3 &= \delta x_2 + r_3 \cos(\phi_3 + \delta\theta_3) && (12)\\
y_3 &= \delta y_2 + r_3 \sin(\phi_3 + \delta\theta_3) && (13)\\
\gamma &= \tan^{-1}(\frac{y_3 - y_1}{x_3 - x_1}) - \tan^{-1}(\frac{y_2 - y_1}{x_2 - x_1}) && (14)
\end{aligned}$$

The notation is as before but has been extended to three observations. The above expressions may not represent the best numerical solution, but have been chosen for their clarity.

All the information necessary for the map building process can be stored in the circular buffer. The control inputs of the vehicle are logged continuously in the buffer and any observation is logged with the current control signal. This allows a measurement of the relative distance or the relative angle between all observed features currently in the buffer to be calculated.

3.3. Avoiding correlations

When estimates are fused it is necessary to account for correlations in the error of the estimates, otherwise the new estimate obtained will be inconsistent as described in Section 2. The alternative to keeping track of correlations is to avoid them all together, in which case a simple update equation can be used:

$$\begin{aligned}
x_3 &= \frac{x_1\sigma_2 + x_2\sigma_1}{\sigma_1 + \sigma_2} && (15)\\
\sigma_3 &= \frac{\sigma_1\sigma_2}{\sigma_1 + \sigma_2} && (16)
\end{aligned}$$

Where x_3 (with σ_3) is the update, and x_1 (with σ_1) and x_2 (with σ_2) are the current estimate and the new measurement interchangeably.

Given two observations (O_i and O_j) were used to calculate a measurement d_{ij} of the relative distance between two features F_i and F_j, two more observations (O'_i and O'_j) are needed if a new measurement d'_{ij} is desired such that the errors in d'_{ij} and d_{ij} are uncorrelated. For the same reason three new observations, one of each feature, are needed in order to form a new measurement of a relative angle that can be fused with the current estimate according to Equations 15 and 16.

A measurement of a relative distance requires two observations and also requires an estimate of the relative motion of the vehicle between the observations. The relative motion estimate, formed by using a sequence of control inputs stored in the buffer, is another potential source of undesired correlations. The error of the estimate of the relative motion ΔM_{12} between two observations O_1 and O_2 will be correlated to the error of the estimate of the relative motion ΔM_{34} between two other observations O_3 and O_4 if the process noise is correlated or there is a common set of control inputs * that are used to form ΔM_{12} and ΔM_{34}.

The process noise is assumed to be uncorrelated (the same assumption is made when using a Kalman filter), therefore it is only necessary to avoid the use of a common set of control inputs. Since observations are entering chronologically into the buffer, insisting on a new set of observations for constructing a new measurement of the relative distance assures that there will be no common set of control inputs used in the construction of ΔM_{12} and ΔM_{34}.

*The process noise is assumed to affect the control inputs.

Similar reasoning leads to the conclusion that three new observations O_i, O_j and O_k of features the F_i, F_j and F_k respectively are necessary to permit the construction of a measurement of A_{ijk} that will have an error which is uncorrelated to the current estimate of A_{ijk}. The actual construction of a new measurement of A_{ijk} may involve using two estimates of relative motion ΔM_{ji} and ΔM_{jk} which have correlated errors, depending on the order in which O_i, O_j and O_k were made. This correlation, however, can be determined and does not change the fact that the error in the new measurement of A_{ijk} is uncorrelated to the error in the current estimate of A_{ijk}. Therefore, the new measurement and the current estimate can be fused according to Equations 15 and 16.

3.4. Operation of the Relative Filter

The operation is most easily understood by considering a typical filtering cycle.

Start:

An observation O_{new} is available of the feature F_{new}. The new observation is gated with features already in the map. If gating is successful, the observation is used to update the map, otherwise it is used to add a new feature to the map.

Gate:

It is desired to test whether F_{new} is equivalent to a known feature, to be called F_{test}. A successful F_{test} has to pass a relative distance test followed by a relative angle test.

Relative Distance Test

- An observation O_1 is selected from the buffer, and the feature associated with it shall be called F_1.
- The relative distance d between F_1 and F_{new} is calculated using O_{new} and O_1.
- d is tested against all the links of F_1.
- If none of the links gate successfully, a new F_1 is chosen and the process is repeated until a link gates successfully or no more observations are available in the buffer (in which case F_{new} could not be associated with any feature and will be added to the map).
- If a link gates successfully, the node F_{test} at the end of the link is potentially equivalent to F_{new}, and a relative angle test is performed on F_{test}.

Relative Angle Test

- The links of F_1 not leading to F_{test} are examined to see if an observation O_2 of one of them is still in the buffer.
- The node F_2 at the end of such a link must form the same angle with F_1 and F_{test} as it does with F_1 and F_{new}.
- If the angle test fails, F_{test} cannot be considered equivalent to F_{new}, a new F_1 is selected and the distance test repeated.
- If the angle test is successful, F_{new} can be associated with F_{test} and an update can be made. F_{test} will now be called F_j.

Update:

A new observation of a feature F_j in the map has been identified. The observation is used to calculate a new measurement of the relative distances and angles that involve F_j.

- The links of F_j are examined. The pointer p_link_{obs} of link L_{ji} is checked to identify the last observation of N_i that was used to update L_{ji}. If this is different to the most recent observation of N_i, a new measurement of the relative distance of L_{ji} can be formed and fused with the current estimate, because a new observation of F_j *and* of F_i is available. All pointers are updated to reflect the use of the new observations. This process is repeated for all links of F_j.

- The angles formed at F_j are examined. For each angle A_{ijk} (j fixed) the pointers p_angle_i and p_angle_k are checked to identify the last observation of F_i and F_k that were used to update A_{ijk}. If both are different to the most recent observation of F_i and F_k, a new measurement of the relative angle A_{ijk} can be formed using O_{new} and the most recent observations of F_i and F_k. The new measurement can be fused with the current estimate of A_{ijk}. All pointers are updated to reflect the use of the new observations. The process is repeated for all angle estimates at F_j.

Adding A New Feature:
The new observation O_{new} failed to gate with any feature in the map. F_{new} will become a new feature in the map, and shall be called F_k.

- F_k is joined to those of its nearest neighbors of which there is still an observation in the buffer. For a neighboring feature F_j, a measurement of the relative distance is formed and entered as the distance estimate for the new link L_{jk}, and the necessary pointers are initialized.
- Previous links L_{ji} of F_j are examined. If an observation of F_i is available in the buffer, measurements of A_{ijk} can be constructed and entered as the angle estimate of A_{ijk}, and the necessary pointers are initialized.
- Angle measurements with F_k at the center are also constructed after more than one link has been made with F_k.

4. EXPERIMENTAL RESULTS

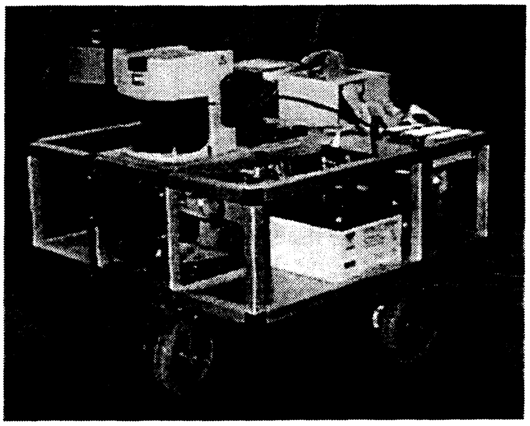

Figure 7. The experimental vehicle equipped with a laser range finder

In this section a real map building scenario is examined. A vehicle (see Figure 7), equipped with a laser range finder, explores a corridor and generates a map of the railings of a staircase and some additional poles placed in the environment.

The first set of plots (Figures 8-17) show the evolution of the map. No estimate of the vehicle position is necessary to generate the map, as described in Section 3. The second set of plots (Figures 18 and 19) show the consistency of the map. For a particular distance estimate (Figure 18) and a particular angle estimate (Figure 19), the error of the estimate was plotted with the respective 1σ bound. In both plots the error is well inside the 1σ bound showing the consistency of the Relative filter. Similar consistency was found for all estimates of distance and angle.

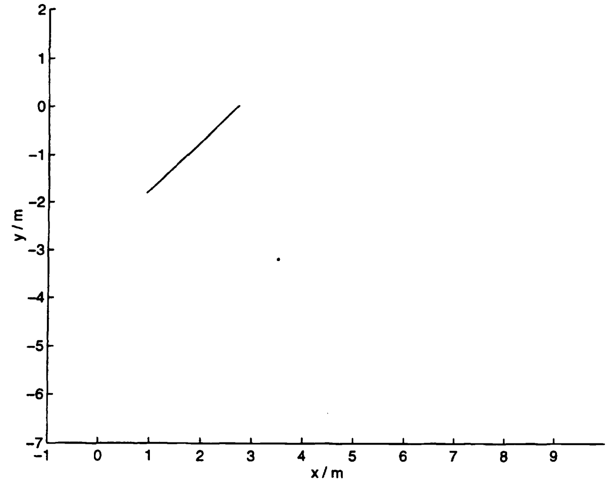

Figure 8: Timestep 0

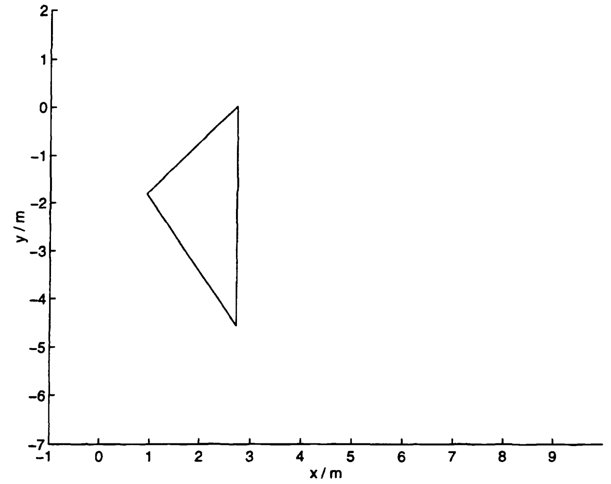

Figure 9: Timestep 45

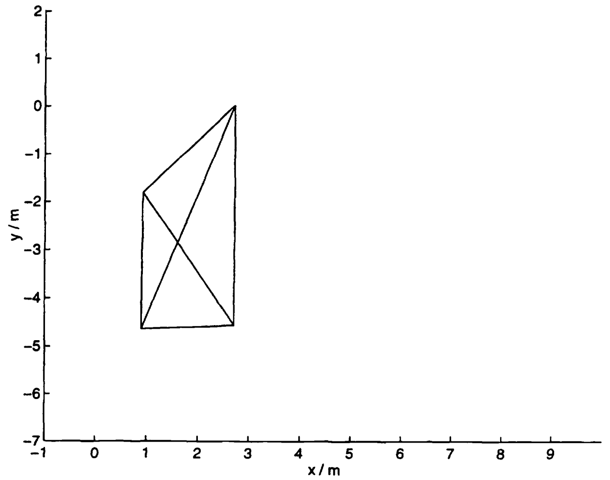

Figure 10: Timestep 60

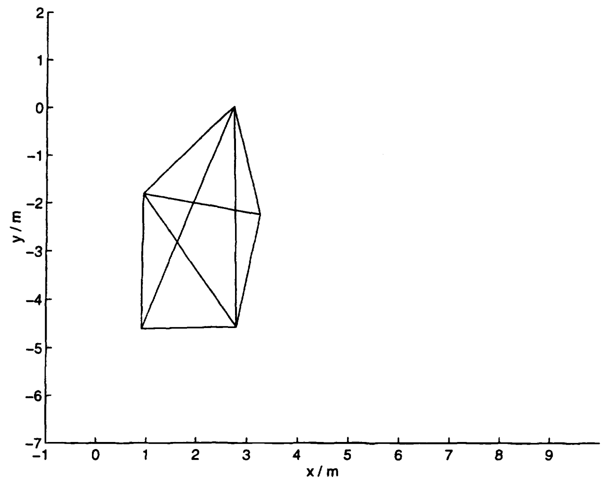

Figure 11: Timestep 75

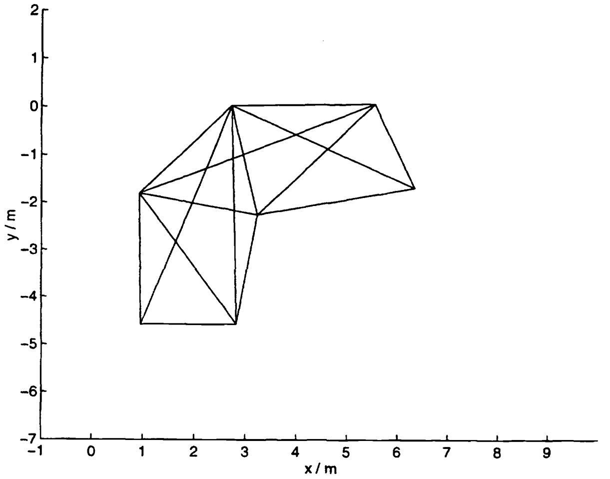

Figure 12: Timestep 273

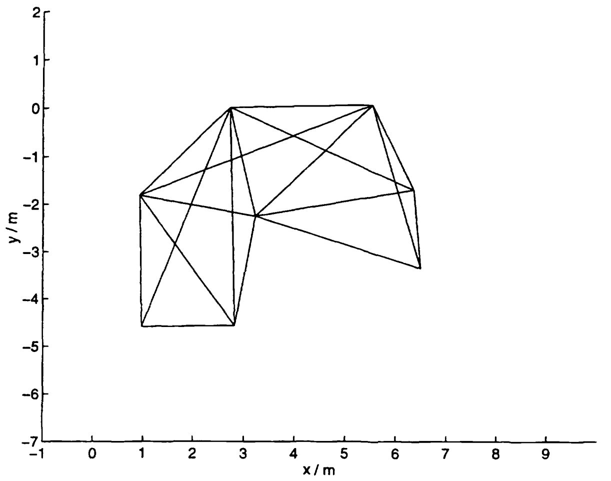

Figure 13: Timestep 409

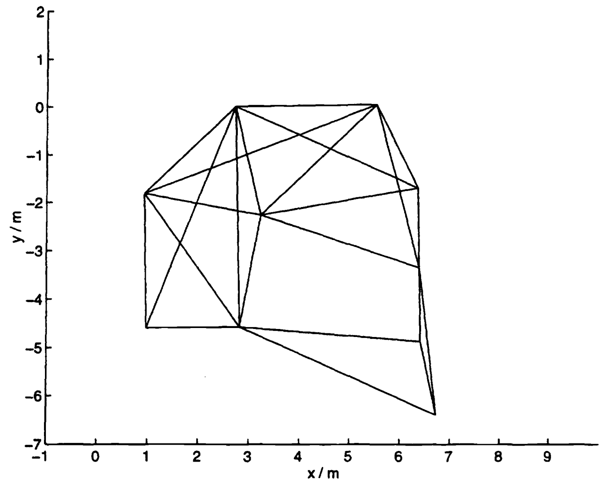

Figure 14: Timestep 986

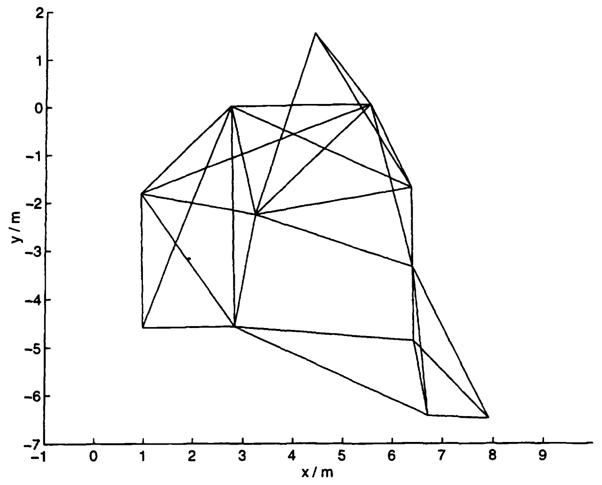

Figure 15: Timestep 1274

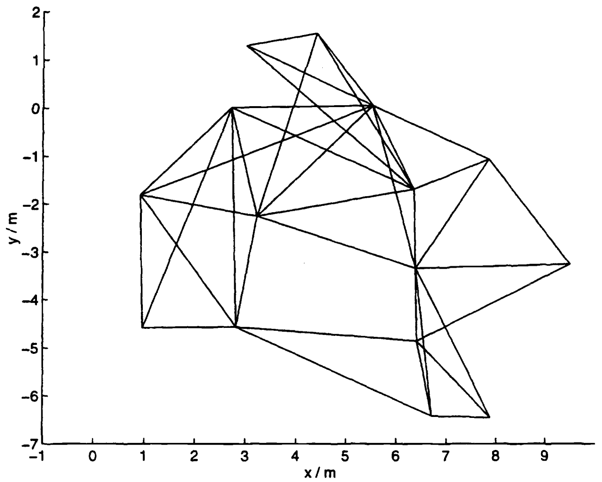

Figure 16: Timestep 1456

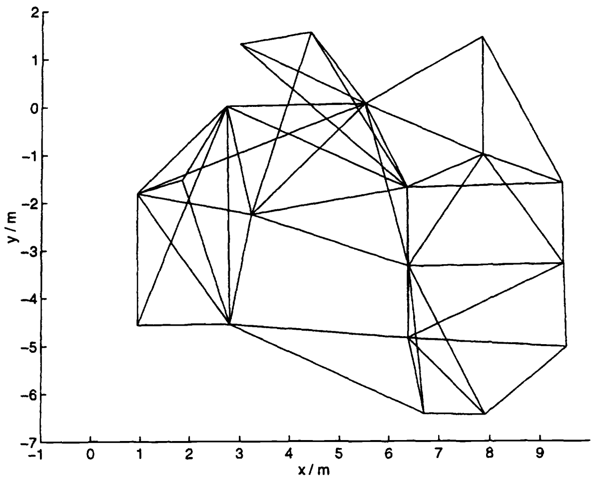

Figure 17: Timestep 3686

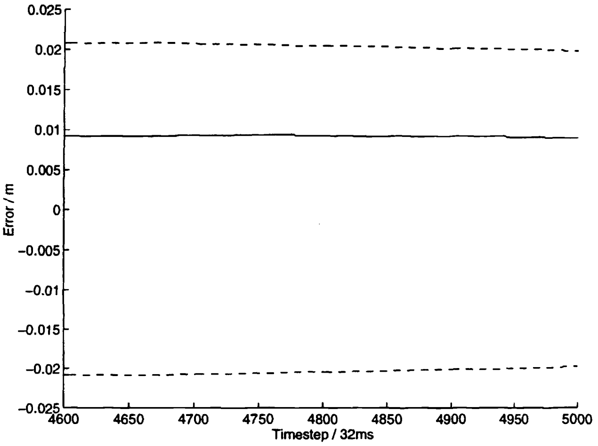

Figure 18: Error of a particular distance estimate and the associated 1σ bound

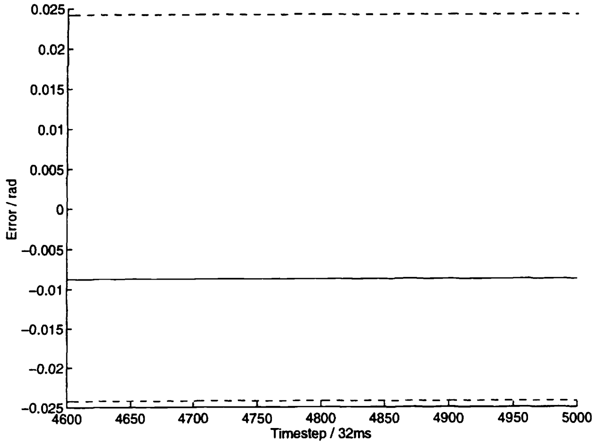

Figure 19: Error of a particular angle estimate and the associated 1σ bound

5. CONCLUSIONS

The results confirm that the Relative filter is a solution to the Automatic Map Building Problem. The Relative filter does not use absolute position estimates, and thereby avoids the correlation problem. This allows it to operate with a memory requirement that scales only as N, whereas the memory requirement of the ASKF scales as N^2. The update of the Relative filter involves only the currently observed features, resulting in a computation burden that does not increase with N. In contrast, the ASKF must update each term in the covariance matrix, hence the computation burden scales at least as N^2.

A long period of no observations may cause a problem, because the buffer may contain no observations, leading to a halt in the map building process. At that point, a new set of observations would have to initiate a search to match the new observations to a part of the map. However, the problem does not arise in a reasonably feature-rich environment.

Figure 16 (Timestep 1456) shows two features, located at positions (2.7,0.0) and (3.0,1.1), that are not connected but are quite close. The reason for this is that there was no observation of the older feature, at position (2.7,0.0), when the other feature was initialized. The current implementation of the algorithm does not add additional links between features that are already part of the map.

A further advantage of a relative map over an absolute position map is that the map quality does not deteriorate with the distance traveled by the vehicle. For a large absolute map the variance on position estimates of features far from the vehicle starting point would be considerable. This could lead to a gating problem where the vehicle could not decide with which feature to associate a new observation. This problem does not arise with a relative position approach to map building.

The processes using the map, such as path planning and mission control, have to operate in a relative framework. This may be a disadvantage, but the authors feel that the benefit of being able to construct a dynamic map in real time will prove the relative approach superior to the absolute approach.

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