CS 4495 Computer Vision

Linear Filtering 2: Templates, Edges

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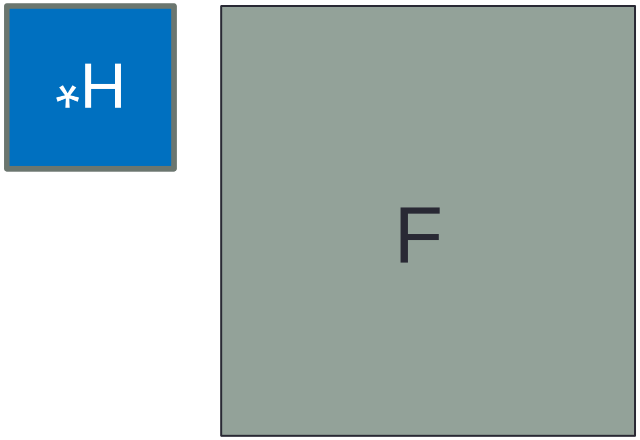
Last time: Convolution

- Convolution:
  - Flip the filter in both dimensions (right to left, bottom to top)
  - Then apply cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

$$G = H * F$$

Notation for convolution operator



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Convolution vs. correlation

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i-u, j-v]$$

$$G = H \star F$$

(Cross-)correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

$$G = H \otimes F$$

- When H is symmetric, no difference. We tend to use the terms interchangeably.
- Convolution with an impulse (centered at 0,0) is the identity

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Filters for features

- Previously, thinking of filtering as a way to remove or reduce **noise**
- Now, consider how filters will allow us to abstract higher-level "**features**".
    - Map raw pixels to an intermediate representation that will be used for subsequent processing
    - Goal: reduce amount of data, discard redundancy, preserve what's useful

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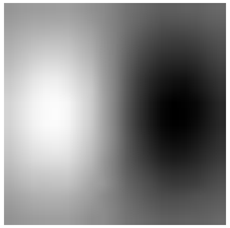
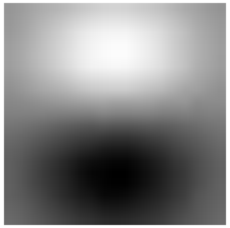
Template matching

- Filters as **templates**:

  Note that filters look like the effects they are intended to find --- “matched filters”





- Use (*normalized*) cross-correlation score to find a given pattern (template) in the image.
  - Normalization needed to control for relative brightness. More in problem sets.

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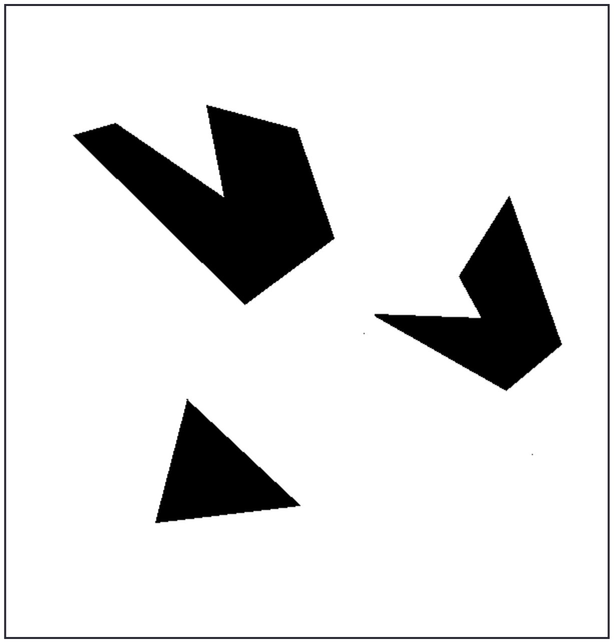
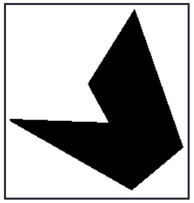
Template matching



Scene



Template (mask)

A toy example

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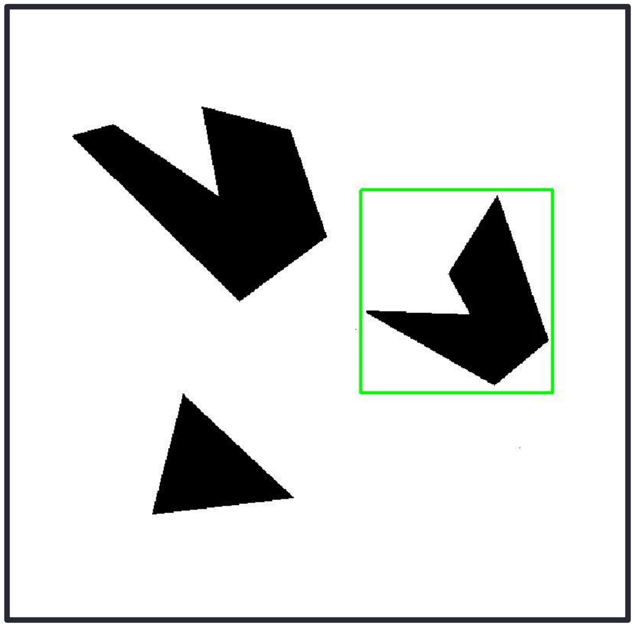
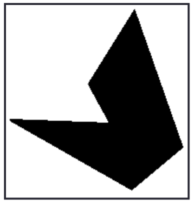
Template matching



Detected template



Template

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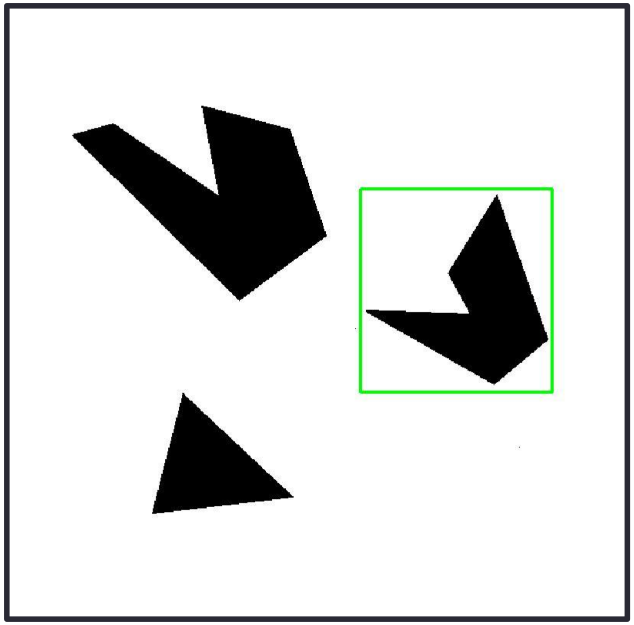
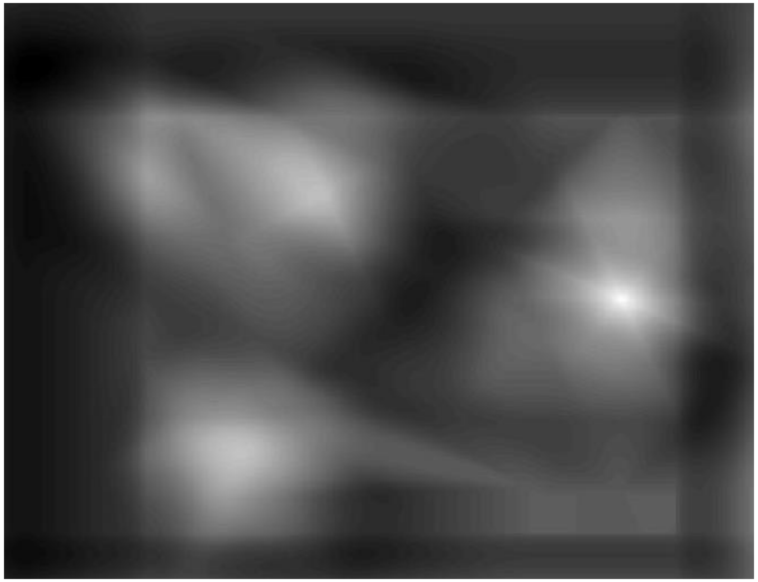
Template matching



Detected template



Correlation map

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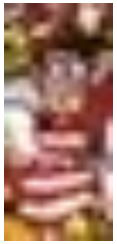
Where's Waldo?

Scene



Template

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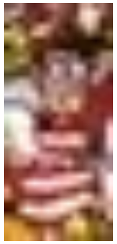
Where's Waldo?

Detected template



Template

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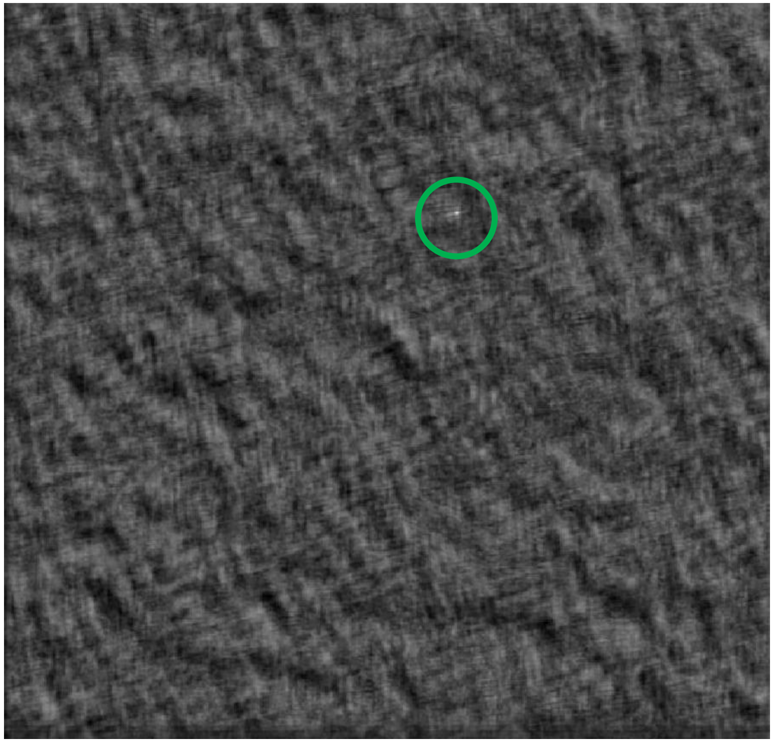
Template demo…

- In directory C:\Bobick\matlab\CS4495\Filter
- echodemo waldotemplate

Where's Waldo?

Detected template



Correlation map

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Template matching

Scene

Template

What if the template is not identical to some subimage in the scene?

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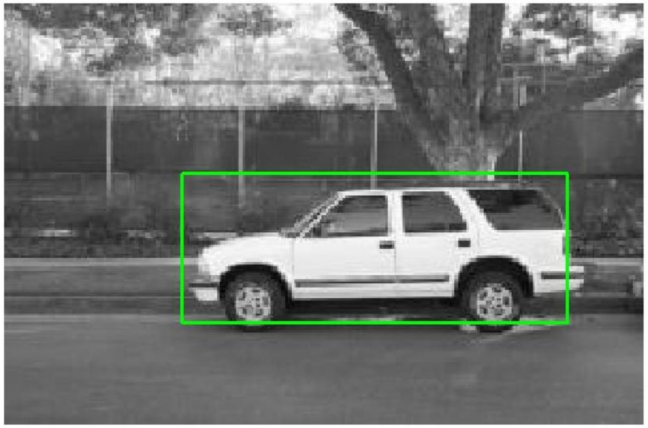
Template matching



Detected template

Template

Match can be meaningful, if scale, orientation, and general appearance is right.

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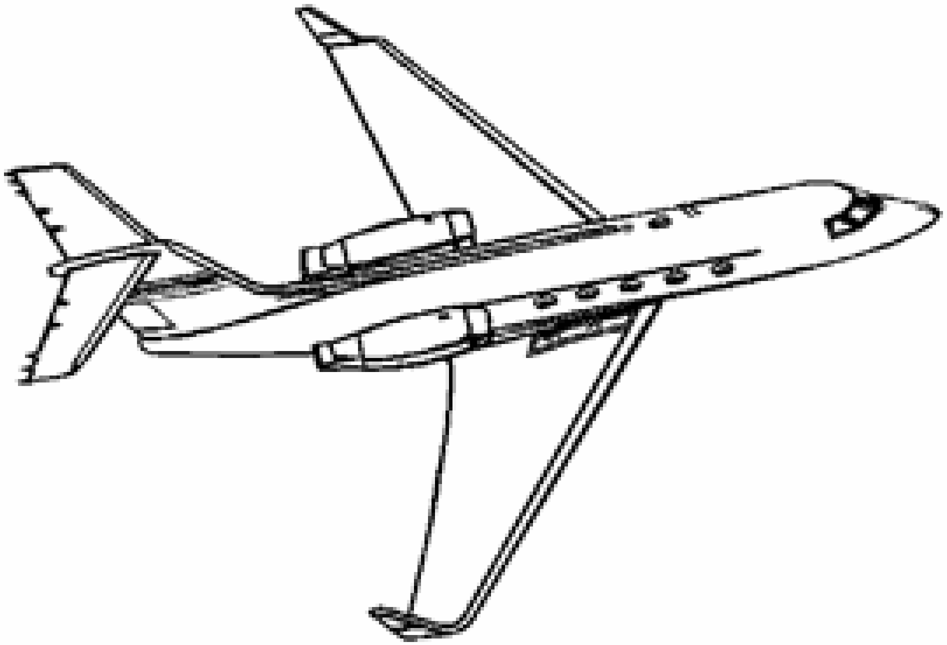
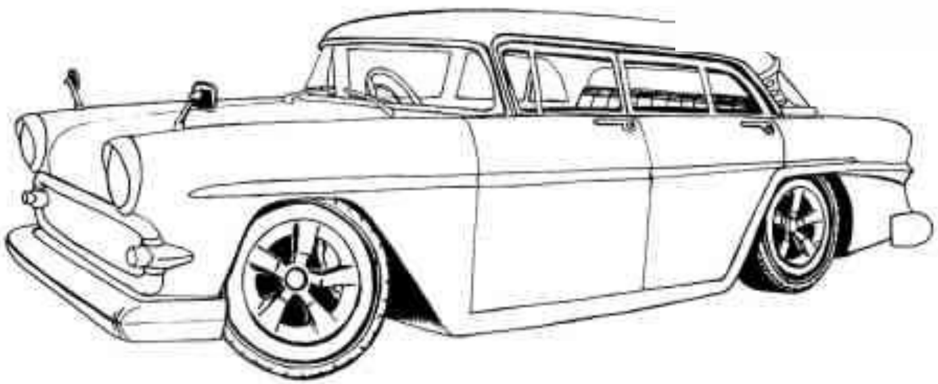
Generic features…

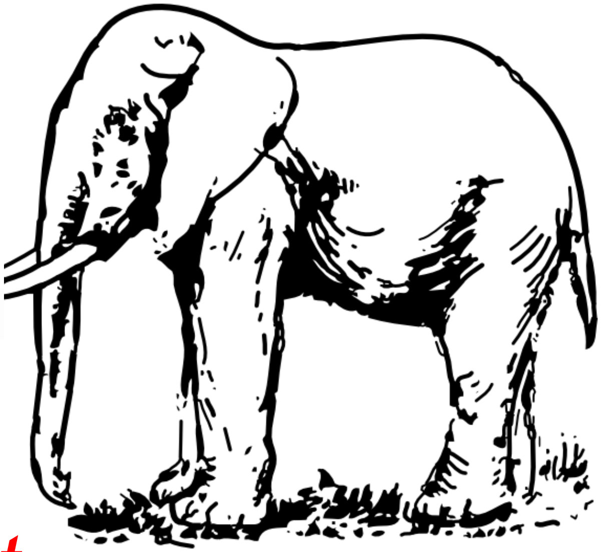
- When looking for a specific object or pattern, the **features** can be defined for that pattern – we will do this later in the course for specific object recognition.

- But for **generic** images, what would be good features? What are the parts or properties of the image that encode its “meaning” for human (or other biological) observers?

- Some examples of greatly reduced images…

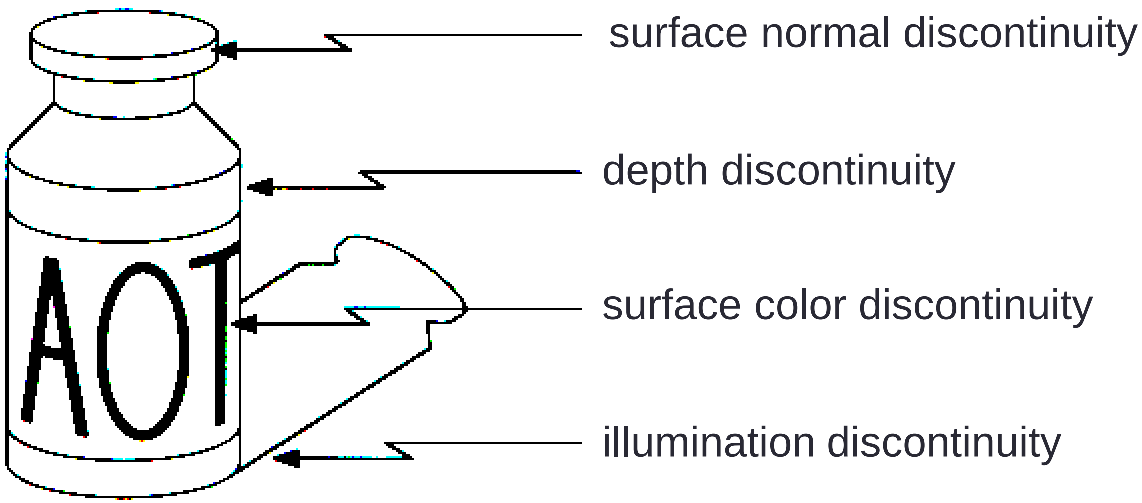






Edges seem to be important…

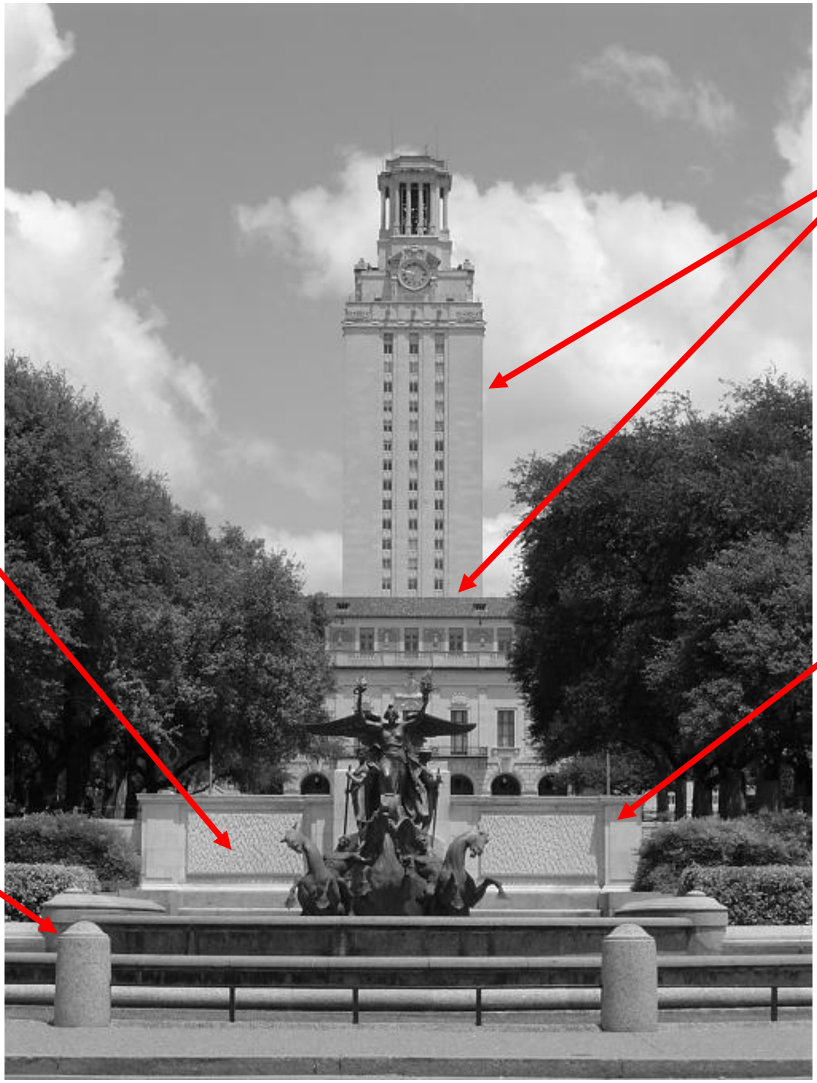
Origin of Edges



- Edges are caused by a variety of factors
- Information theory view: edges encode change, change is what is hard to predict, therefore edges efficiently encode an image

In a real image

Reflectance change: appearance information, texture

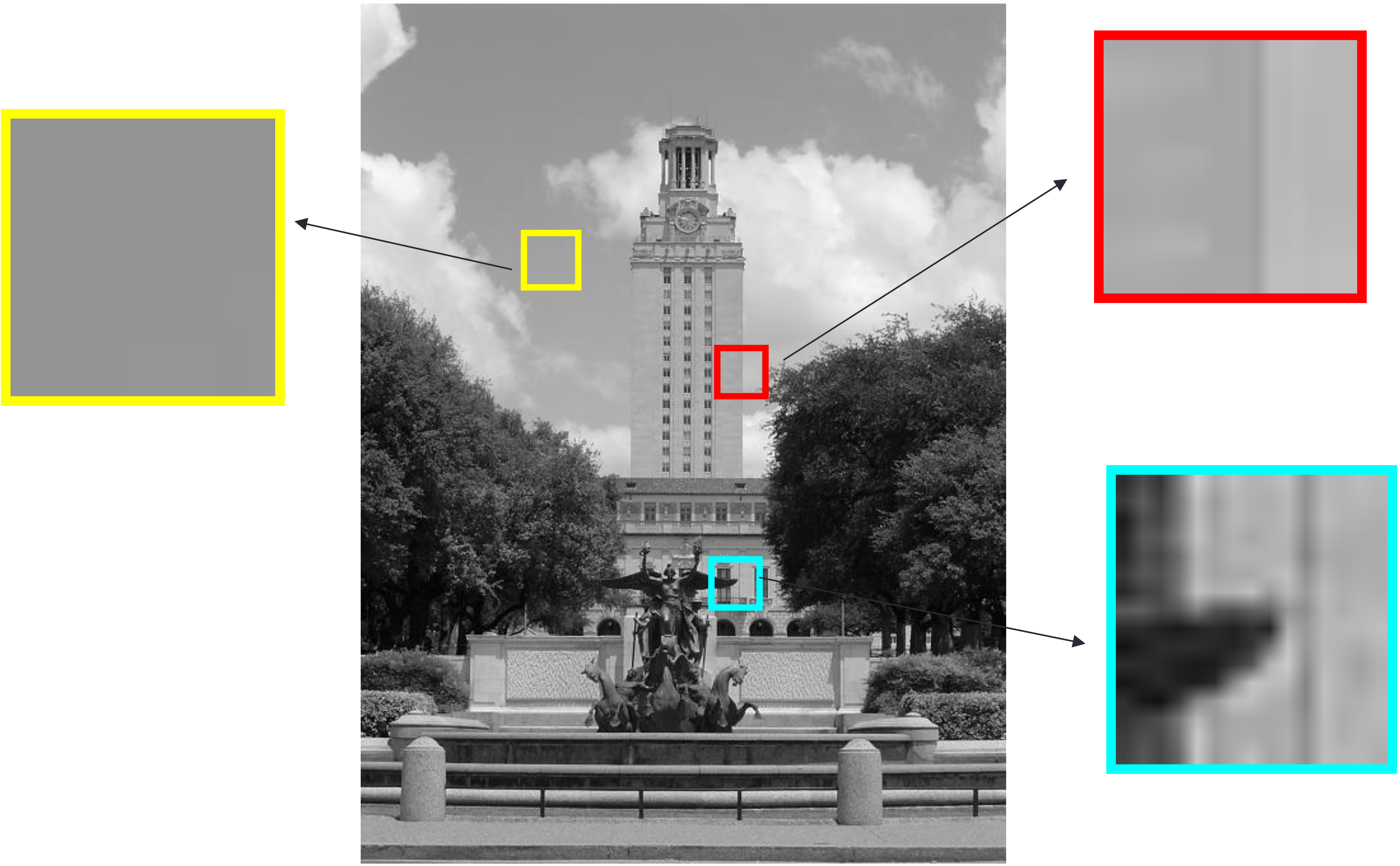


Depth discontinuity: object boundary

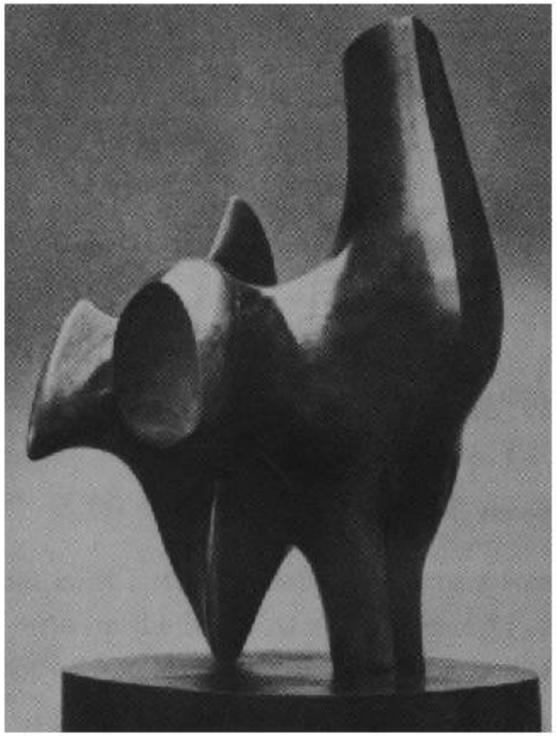
Cast shadows

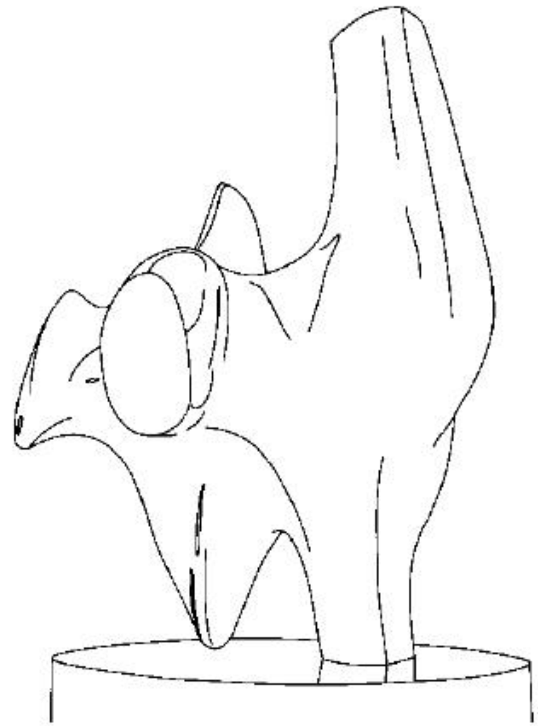
Change in surface orientation: shape

Contrast and invariance



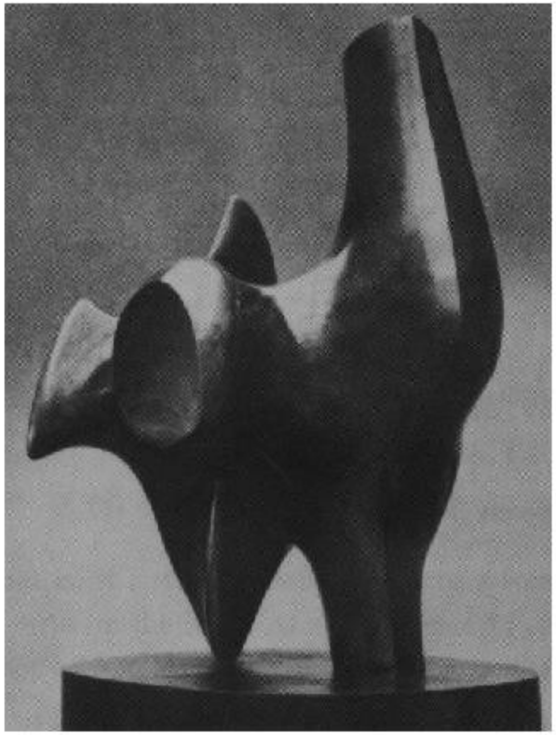
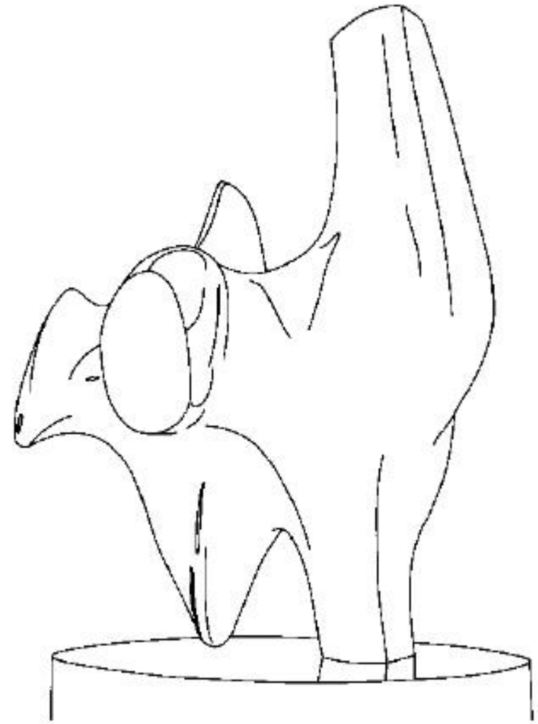
Edge detection





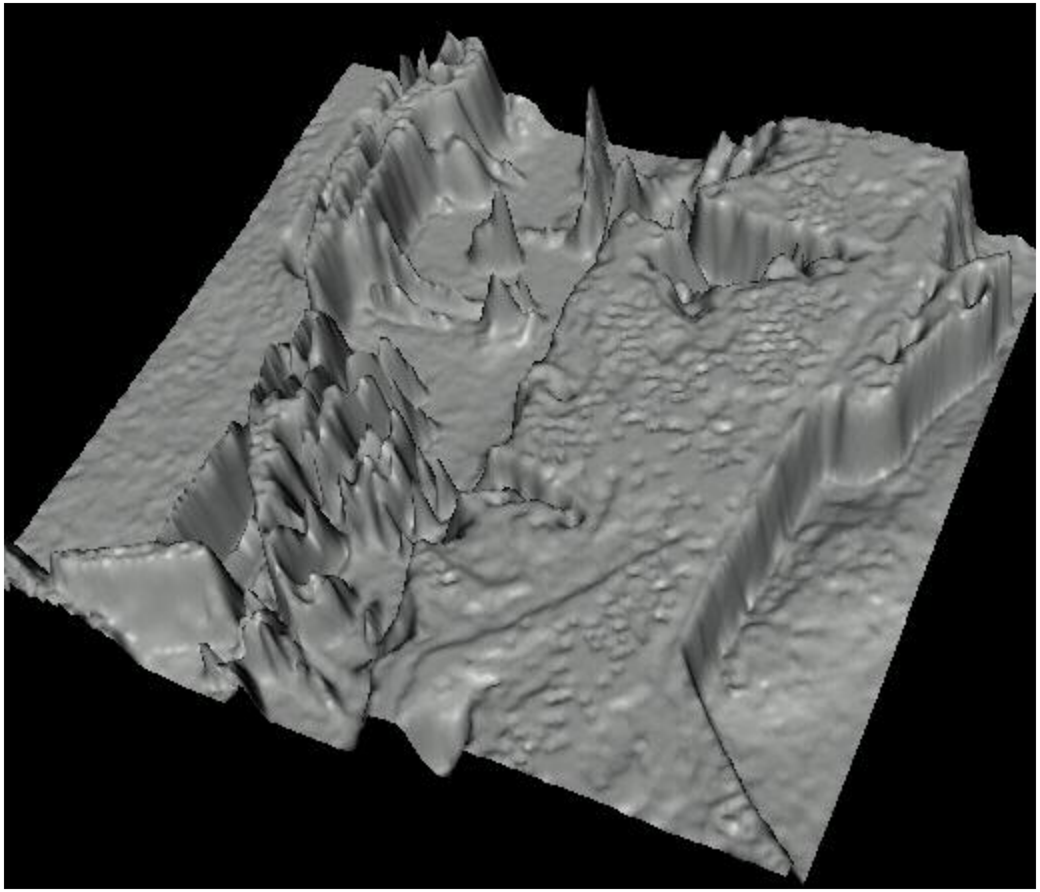
- Convert a 2D image into a set of curves
  - Extracts salient features of the scene
  - More compact than pixels

Edge detection

- How can you tell that a pixel is on an edge?

Images as functions…



- Edges look like steep cliffs

Edge Detection

Basic idea: look for a neighborhood with strong signs of change.

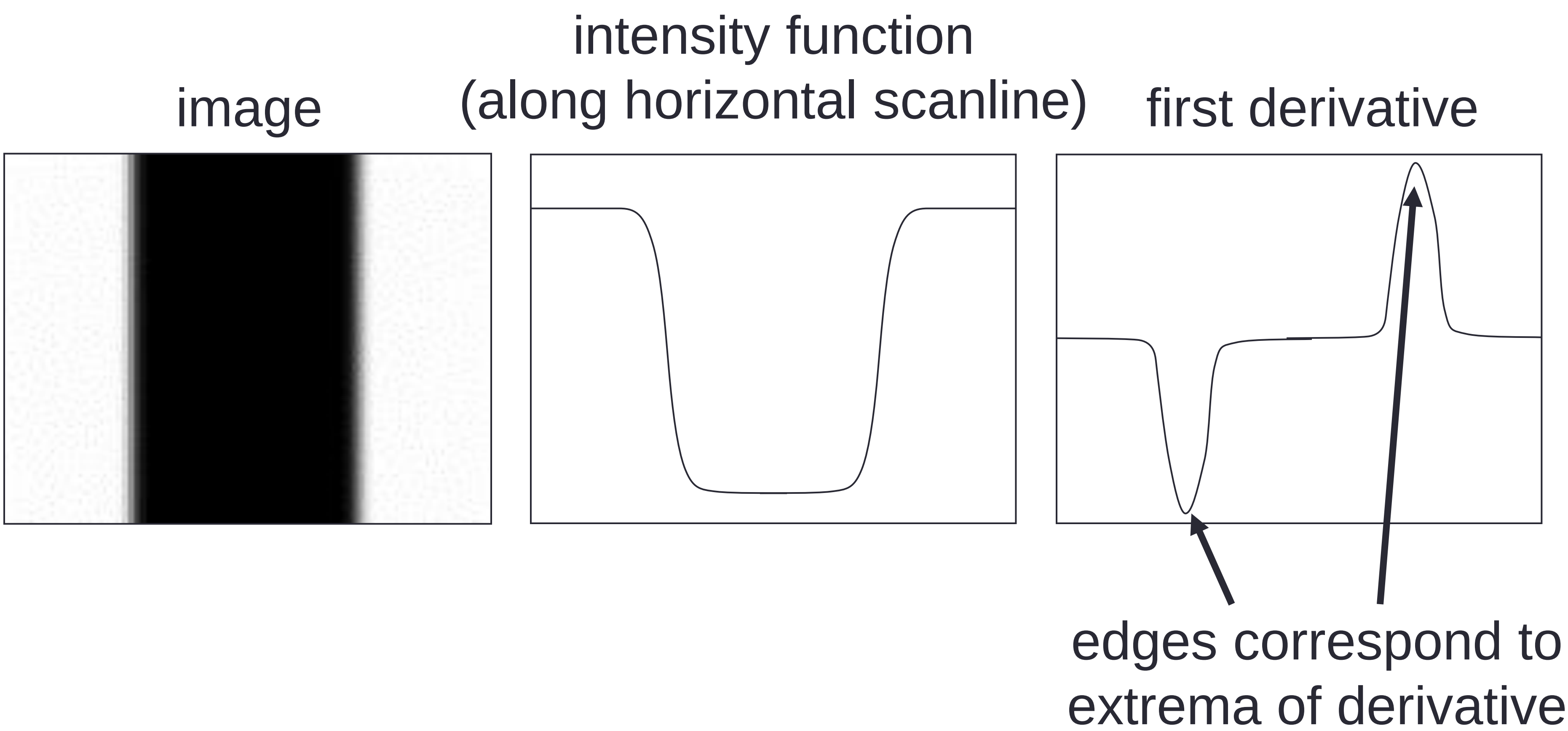
Problems:

- neighborhood size
- how to detect change

81	82	26	24
82	33	25	25
81	82	26	24

Derivatives and edges

An edge is a place of rapid change in the image intensity function.



Source: L. Lazebnik

Differential Operators

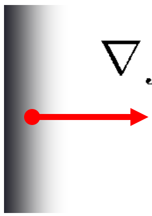
- Differential operators – here we mean some operation that when applied to the image returns some derivatives.
- We will model these “operators” as masks/kernels which when applied to the image yields a new function that is the image ***gradient function***.
- We will then threshold the this ***gradient function*** to select the edge pixels.
- Which brings us to the question:

What’s a gradient?

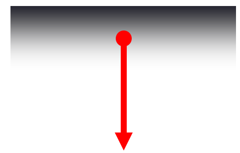
Image gradient

The gradient of an image:

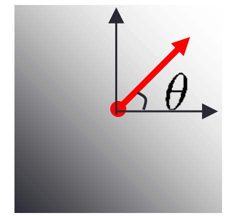
$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The gradient points in the direction of most rapid increase in intensity

The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- how does this relate to the direction of the edge?

The *edge strength* is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Discrete gradient

For 2D function, f(x,y), the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

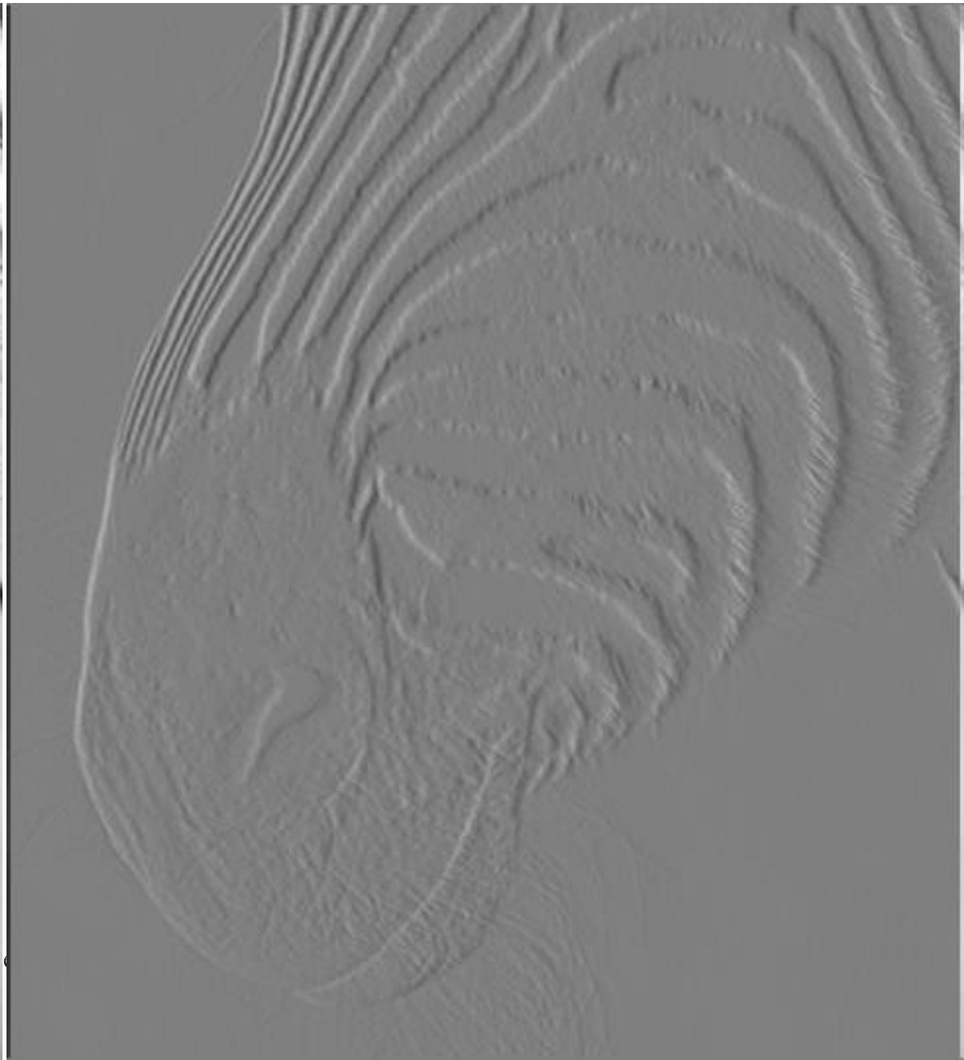
For discrete data, we can approximate using *finite* differences:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

$$\approx f(x+1, y) - f(x,y)$$

"right derivative"
But is it???

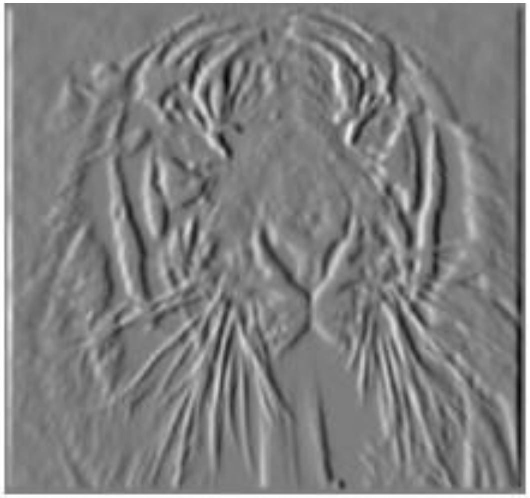
Finite differences

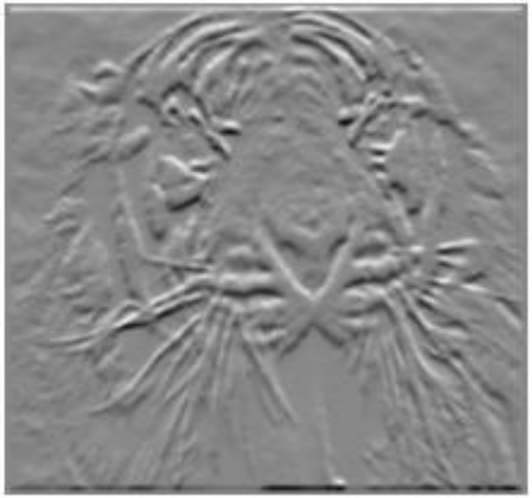


Partial derivatives of an image

$$\frac{\partial f(x,y)}{\partial x}$$

$$\frac{\partial f(x,y)}{\partial y}$$





-1	1

-1
1

? or

1
-1

Which shows changes with respect to x?

(showing correlation filters)

Differentiation and convolution

- For 2D function, f(x,y), the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

- For discrete data, we can approximate using finite differences:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

- To implement above as convolution, what would be the associated filter?

The discrete gradient

- We want an "operator" (mask/kernel) that we can apply to the image that implements:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

How would you implement this as a cross-correlation?

(not flipped)

0	0
-1	+1
0	0

H

Not symmetric around image point; which is "middle" pixel?

0	0	0
-1/2	0	+1/2
0	0	0

H

Average of "left" and "right" derivative. See?

Example: Sobel operator

$$\frac{1}{8}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad \frac{1}{8}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$s_x \qquad\qquad s_y$$

On a pixel of the image I

- Let g_x be the response to mask S_x (sometimes * 1/8)
- Let g_y be the response to mask S_y

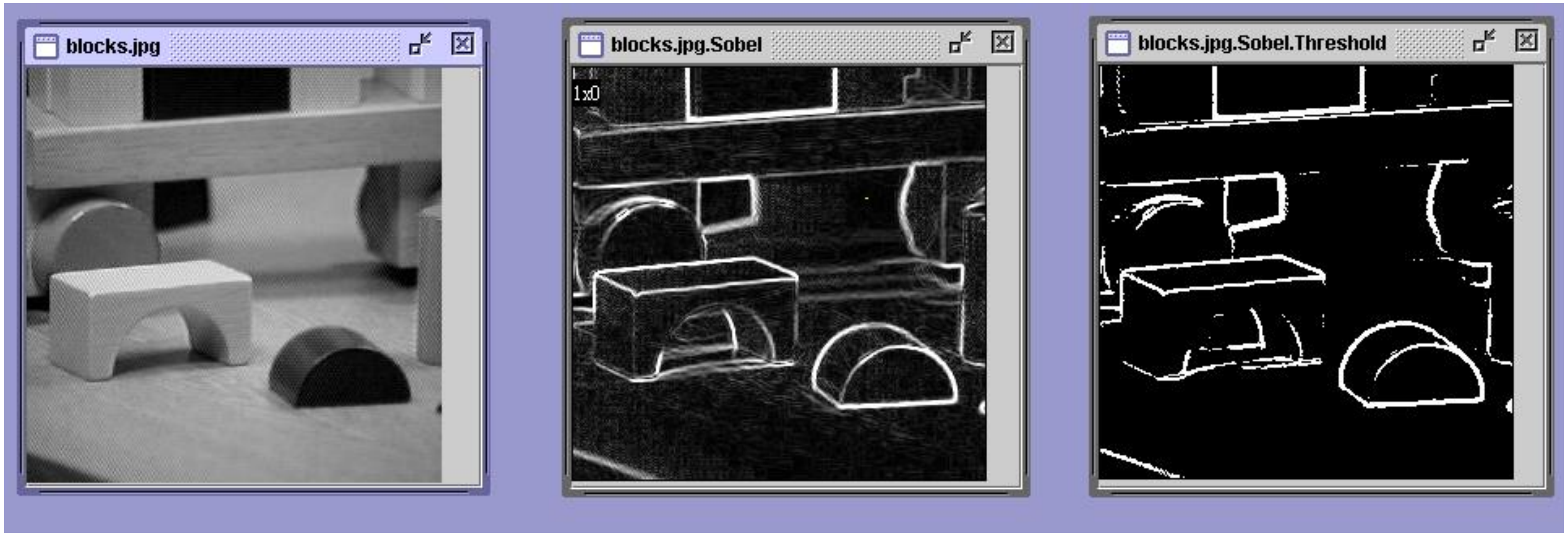
What is the gradient?

(Sobel) Gradient is $\nabla I = [g_x \;\; g_y]^T$

$g = (g_x^2 + g_y^2)^{1/2}$ is the gradient magnitude.

$\theta = \text{atan2}(g_y, g_x)$ is the gradient direction.

Sobel Operator on Blocks Image



original image

gradient magnitude

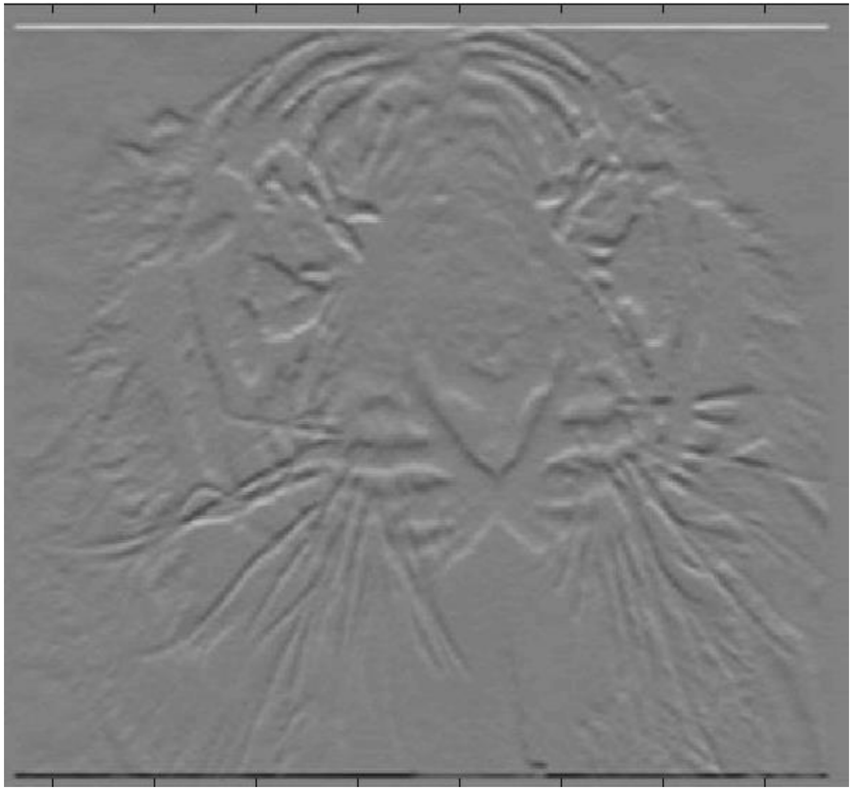
thresholded gradient magnitude

Some Well-Known Masks for Computing Gradients

	Sx	Sy
• Sobel:	$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$
• Prewitt:	$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$
• Roberts	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Matlab does edges

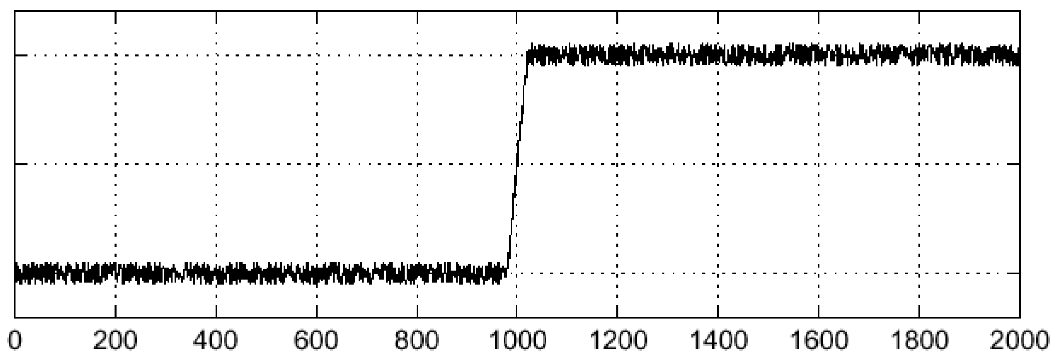
```
>> My = fspecial('sobel');
>> outim = imfilter(double(im), My);
>> imagesc(outim);
>> colormap gray;
```



But…

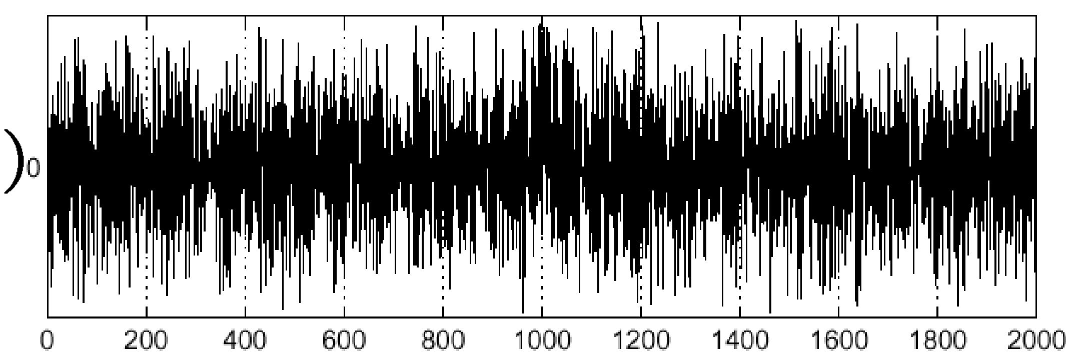
- Consider a single row or column of the image
  - Plotting intensity as a function of *x*

$$f(x)$$



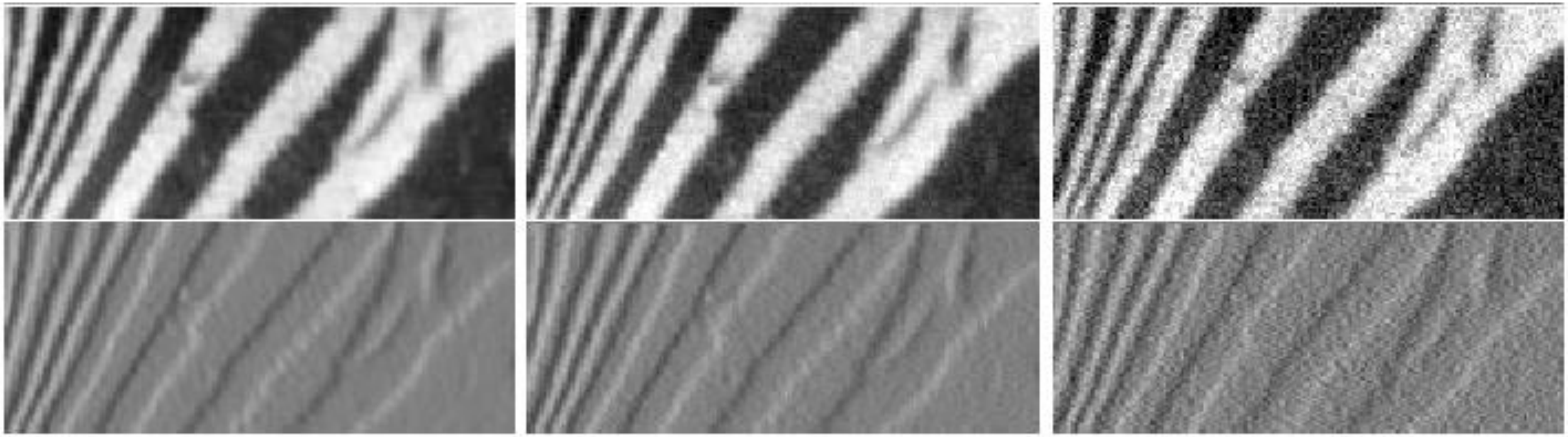
  - Apply derivative operator….

$$\frac{d}{dx}f(x)$$



Uh, where's the edge?

Finite differences responding to noise



Increasing noise ->
(this is zero mean additive gaussian noise)

D. Forsyth

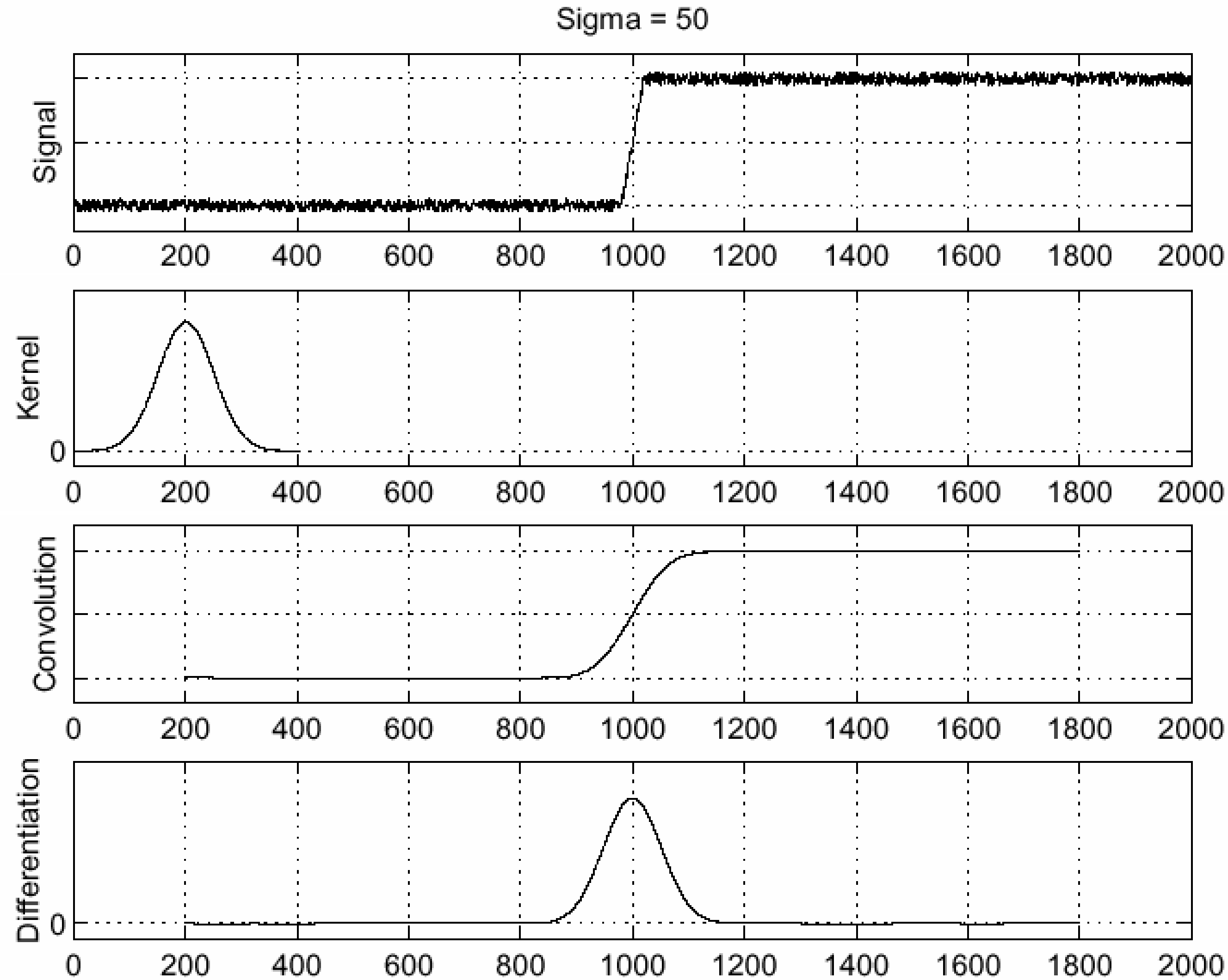
Solution: smooth first

f

h

$h * f$

$\frac{\partial}{\partial x}(h * f)$



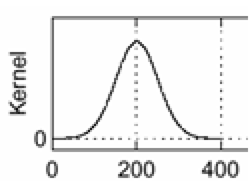
Where is the edge? Look for peaks in $\frac{\partial}{\partial x}(h * f)$

Derivative theorem of convolution

- This saves us one operation: $\frac{\partial}{\partial x}(h * f) = (\frac{\partial}{\partial x}h) * f$

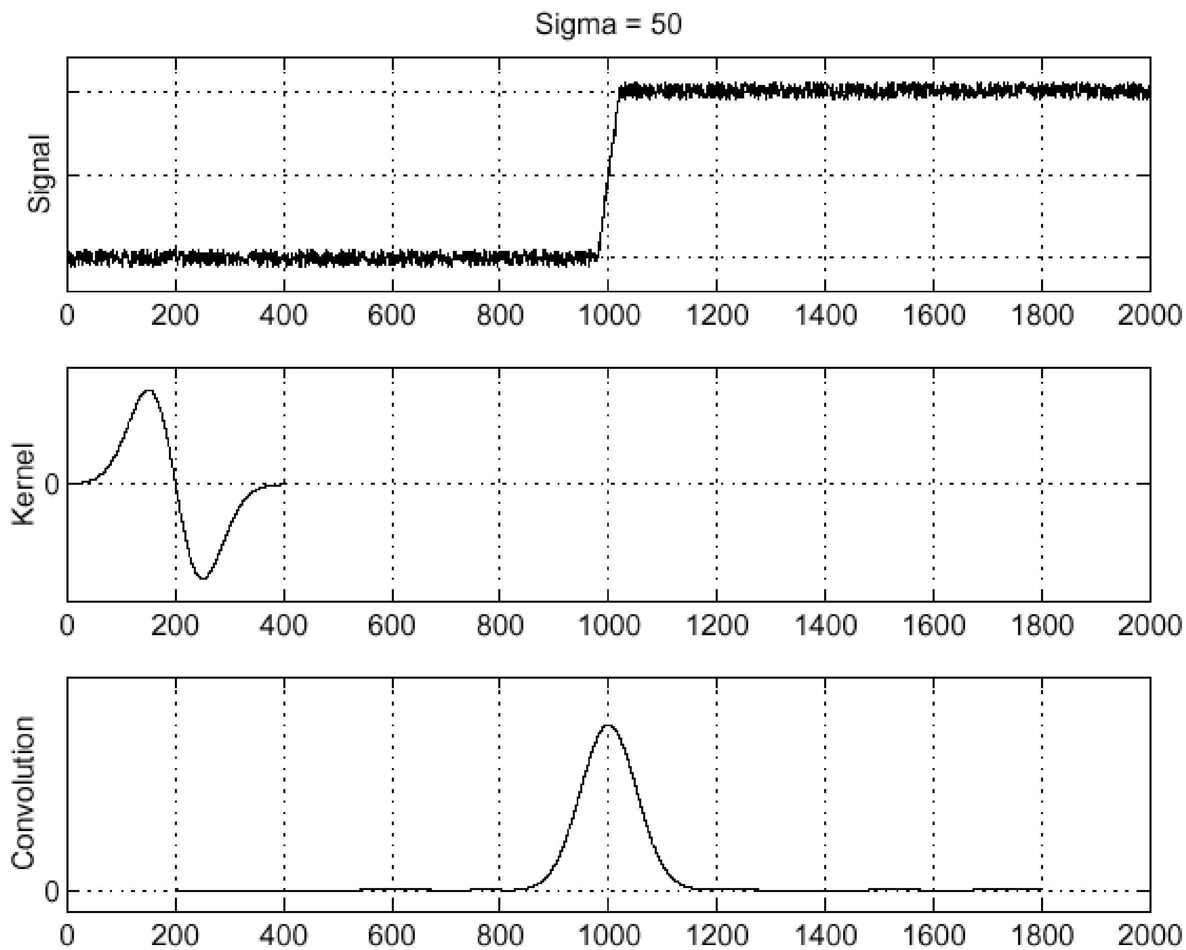
f

h



$\frac{\partial}{\partial x}h$

$(\frac{\partial}{\partial x}h) * f$



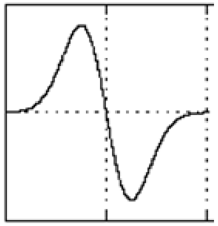
How can we find (local) maxima of a function?

2nd derivative of Gaussian

- Consider $\frac{\partial^2}{\partial x^2}(h * f)$

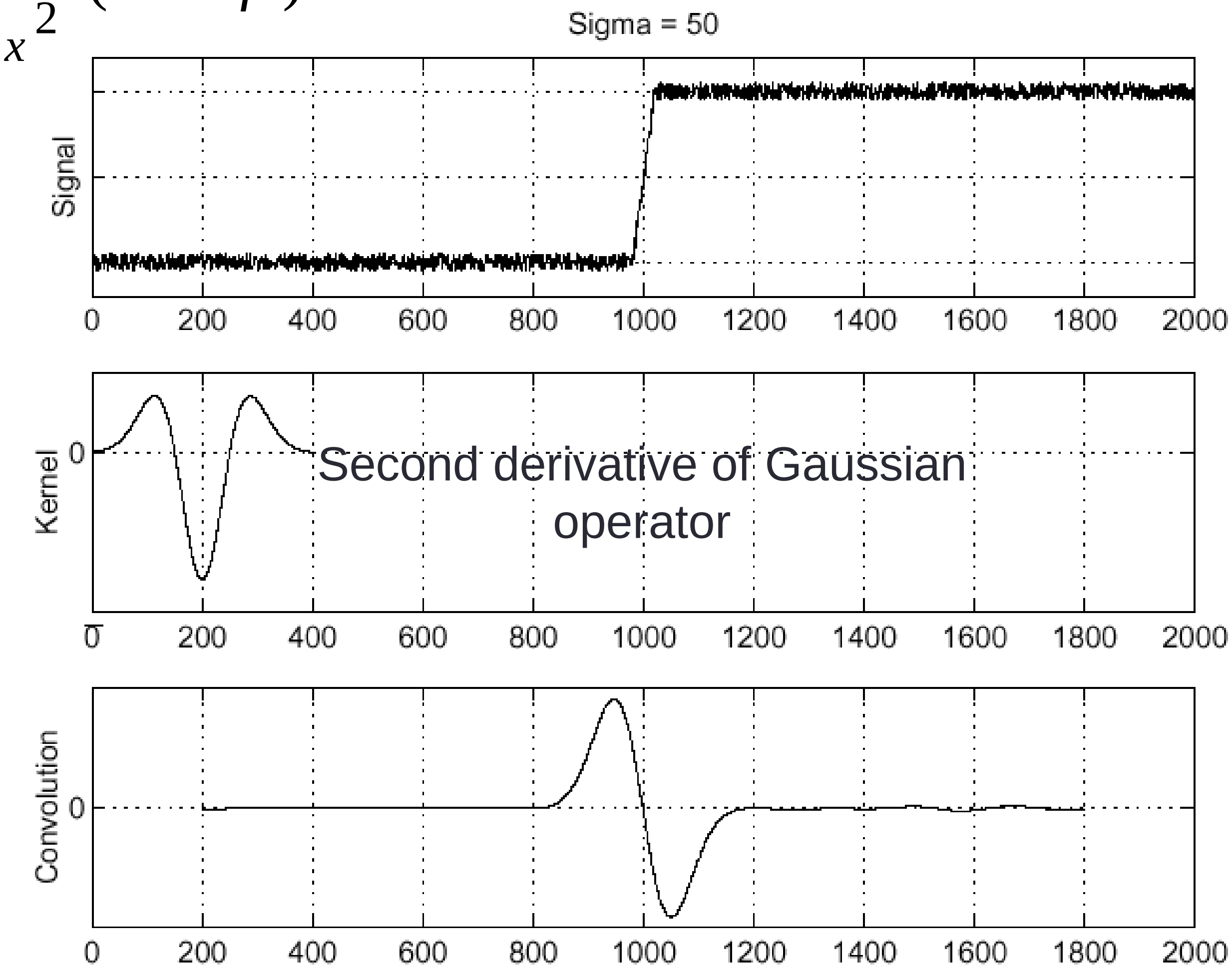
f

$\frac{\partial}{\partial x} h$



$\frac{\partial^2}{\partial x^2} h$

$\frac{\partial^2}{\partial x^2}(h * f)$



Where is the edge? Zero-crossings of bottom graph

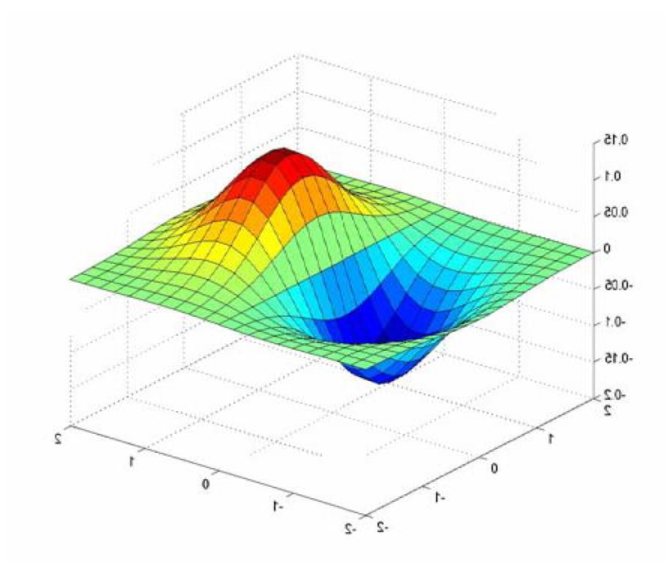
What about 2D?

Derivative of Gaussian filter

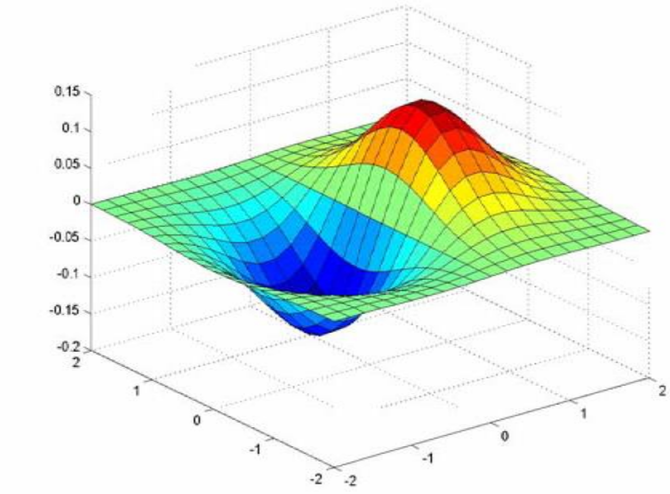
$$(I \otimes g) \otimes h = I \otimes (g \otimes h)$$

$$\begin{bmatrix} 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0219 & 0.0983 & 0.1621 & 0.0983 & 0.0219 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \end{bmatrix} \otimes \begin{bmatrix} 1 & -1 \end{bmatrix}$$

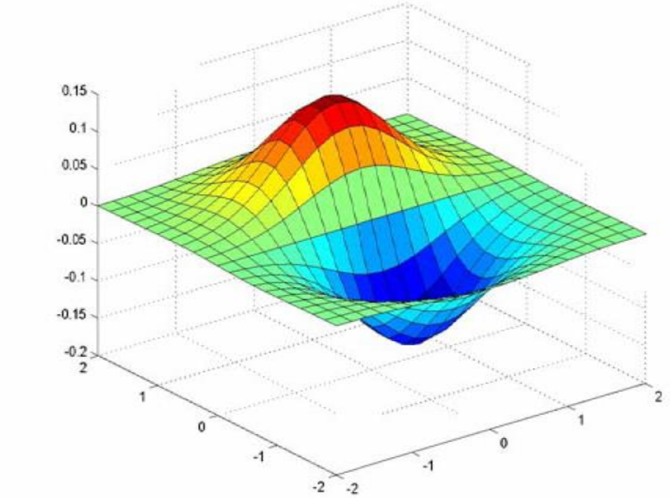
Why is this preferable?



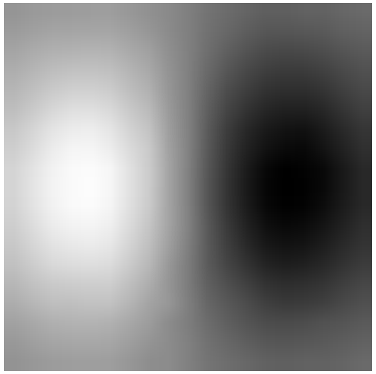
Derivative of Gaussian filters



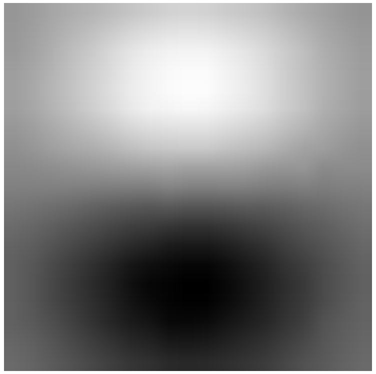
x-direction



y-direction



Is this for correlation or convolution?

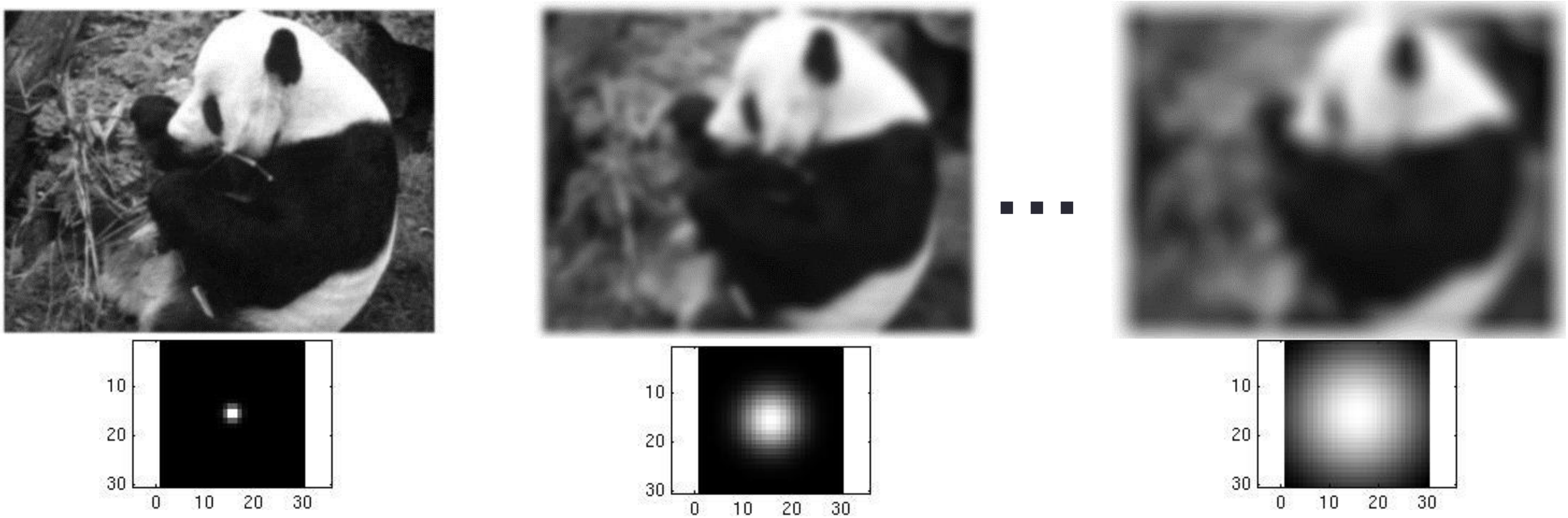


And for y it's always a problem!

Source: L. Lazebnik

Smoothing with a Gaussian

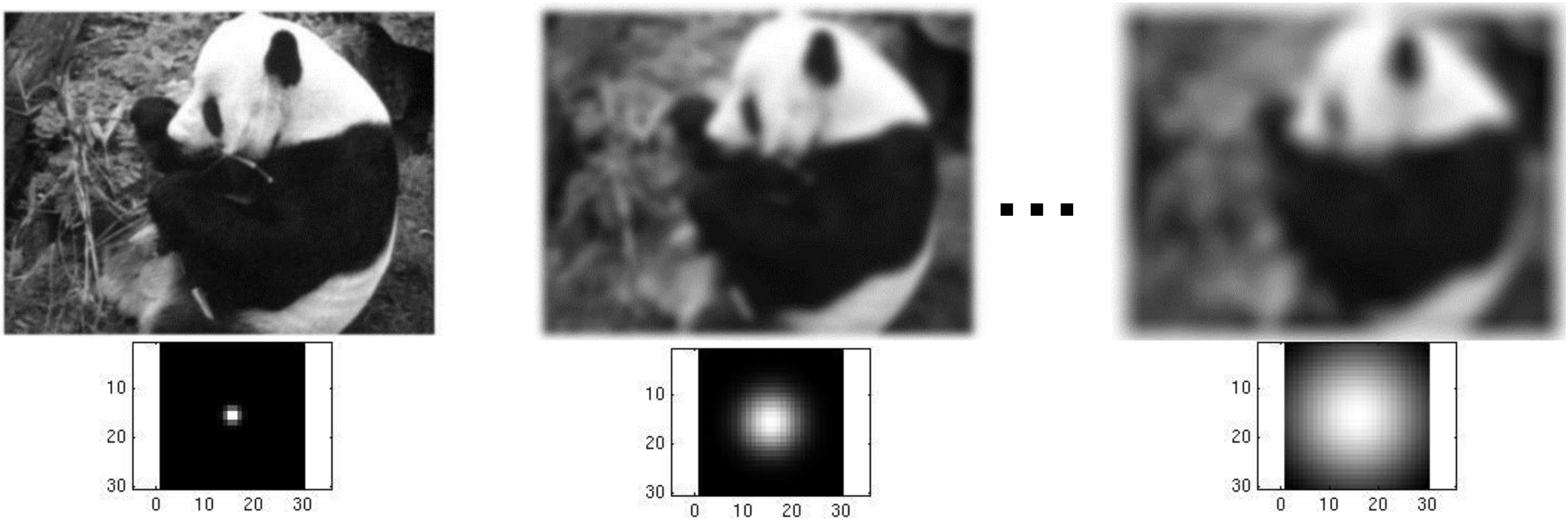
Parameter σ is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.



```
for sigma=1:3:10
  h = fspecial('gaussian`, fsize, sigma);
  out = imfilter(im, h);
  imshow(out);
  pause;
end
```

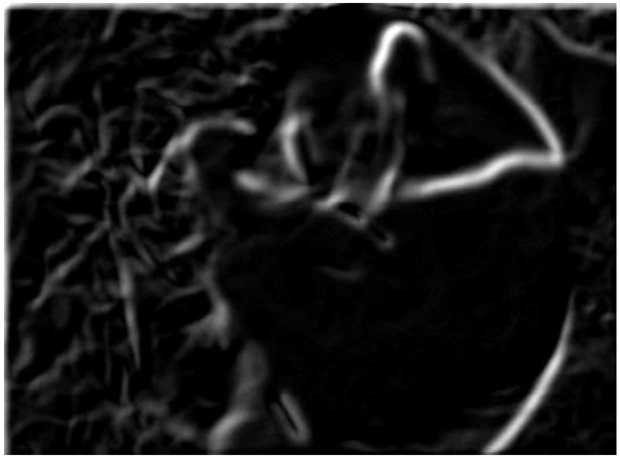
Smoothing with a Gaussian

Recall: parameter σ is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.



Effect of σ on derivatives

σ = 1 pixel



σ = 3 pixels

The apparent structures differ depending on Gaussian's scale parameter.

Larger values: larger scale edges detected
Smaller values: finer features detected

Gradients -> edges

- Primary edge detection steps:
- 1. Smoothing: suppress noise
- 2. Edge “enhancement”: filter for contrast
- 3. Edge localization
  - Determine which local maxima from filter output are actually edges vs. noise
    - Threshold, Thin



Canny edge detector

- Filter image with derivative of Gaussian
- Find magnitude and orientation of gradient
- **Non-maximum suppression**:
  - Thin multi-pixel wide “ridges” down to single pixel width
- Linking and thresholding (**hysteresis**):
  - Define two thresholds: low and high
  - Use the high threshold to start edge curves and the low threshold to continue them

- MATLAB: **`edge(image, ‘canny’);`**
- **`>>help edge`**

Source: D. Lowe, L. Fei-Fei

The Canny edge detector

original image (Lena)

The Canny edge detector

magnitude of the gradient

The Canny edge detector

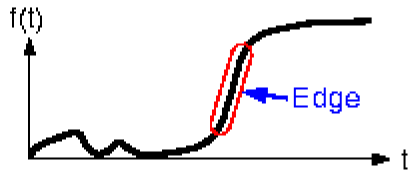
thresholding

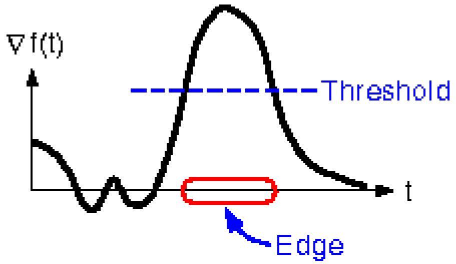
The Canny edge detector

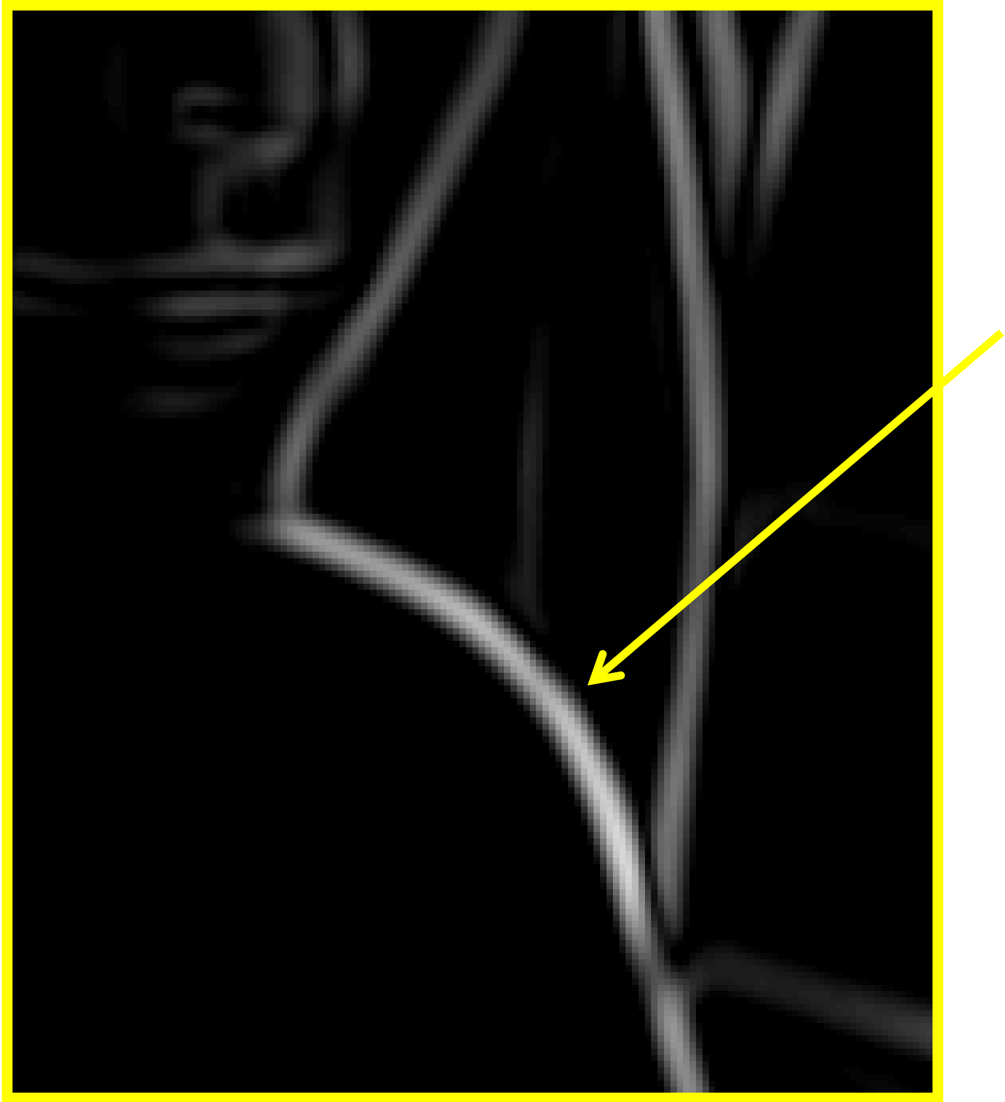
Problem: pixels along this edge didn't survive the thresholding

thinning
(non-maximum suppression)

The Canny edge detector

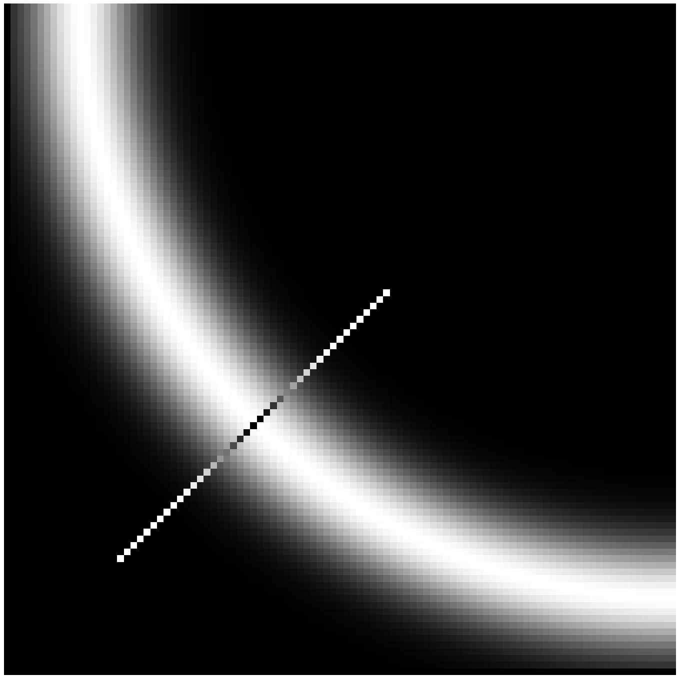


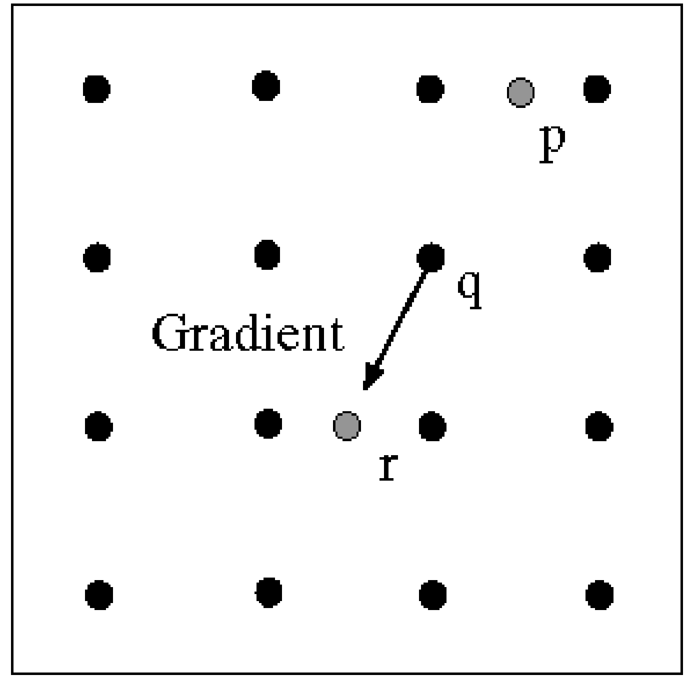




How to turn these thick regions of the gradient into curves?

Non-maximum suppression





- Check if pixel is local maximum along gradient direction
  - can require checking interpolated pixels p and r

The Canny edge detector

thinning

(non-maximum suppression)

Effect of σ (Gaussian kernel spread/size)

original　　Canny with $\sigma = 1$　　Canny with $\sigma = 2$

The choice of σ depends on desired behavior

- large σ detects large scale edges
- small σ detects fine features

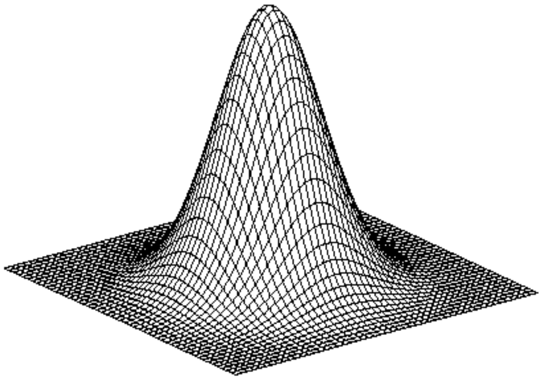
So, what scale to choose?

It depends what we're looking for.

Too fine of a scale…can't see the forest for the trees.

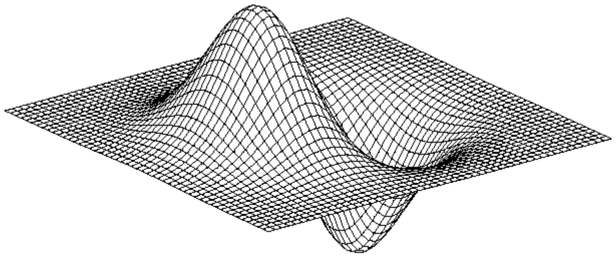
Too coarse of a scale…can't tell the maple grain from the cherry.

Single 2D edge detection filter



Gaussian

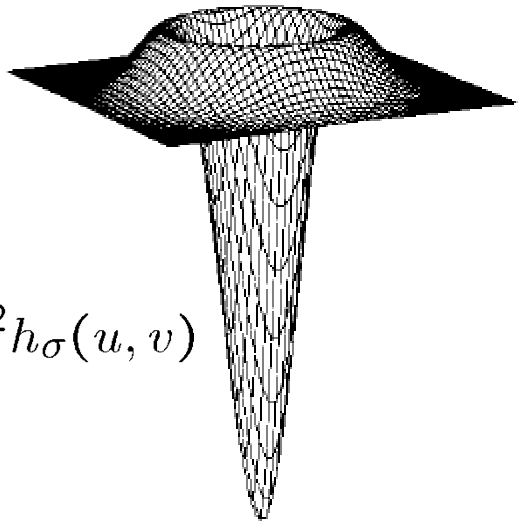
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Laplacian of Gaussian



$$\nabla^2 h_\sigma(u,v)$$

∇^2 is the **Laplacian** operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Edge demo in CS4495/Edges

Finish on Thurs…