7630 – Autonomous Robotics
Mobile Robot Kinematics

Motion Prediction

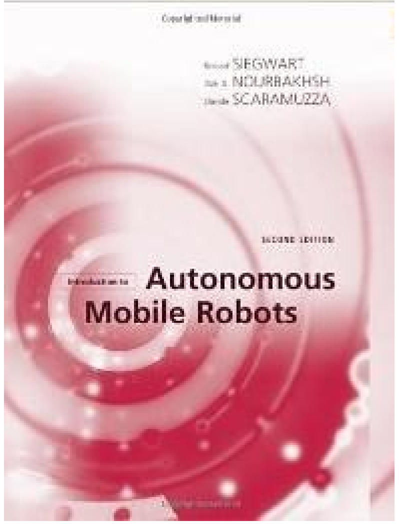
Odometry

Mobility Analysis

Based on material from R. Siegwart, M. Mason

Recommended Reading

- *Introduction to Autonomous Mobile Robots*, Siegwart, Nourbakhsh, Scaramuzza
  - http://www.mobilerobots.ethz.ch/
  - Chapter 3



- Lecture from Matt Mason (CMU):
  - http://www.cs.cmu.edu/afs/cs/academic/class/16741-s07/www/lecture5.pdf

Athlete

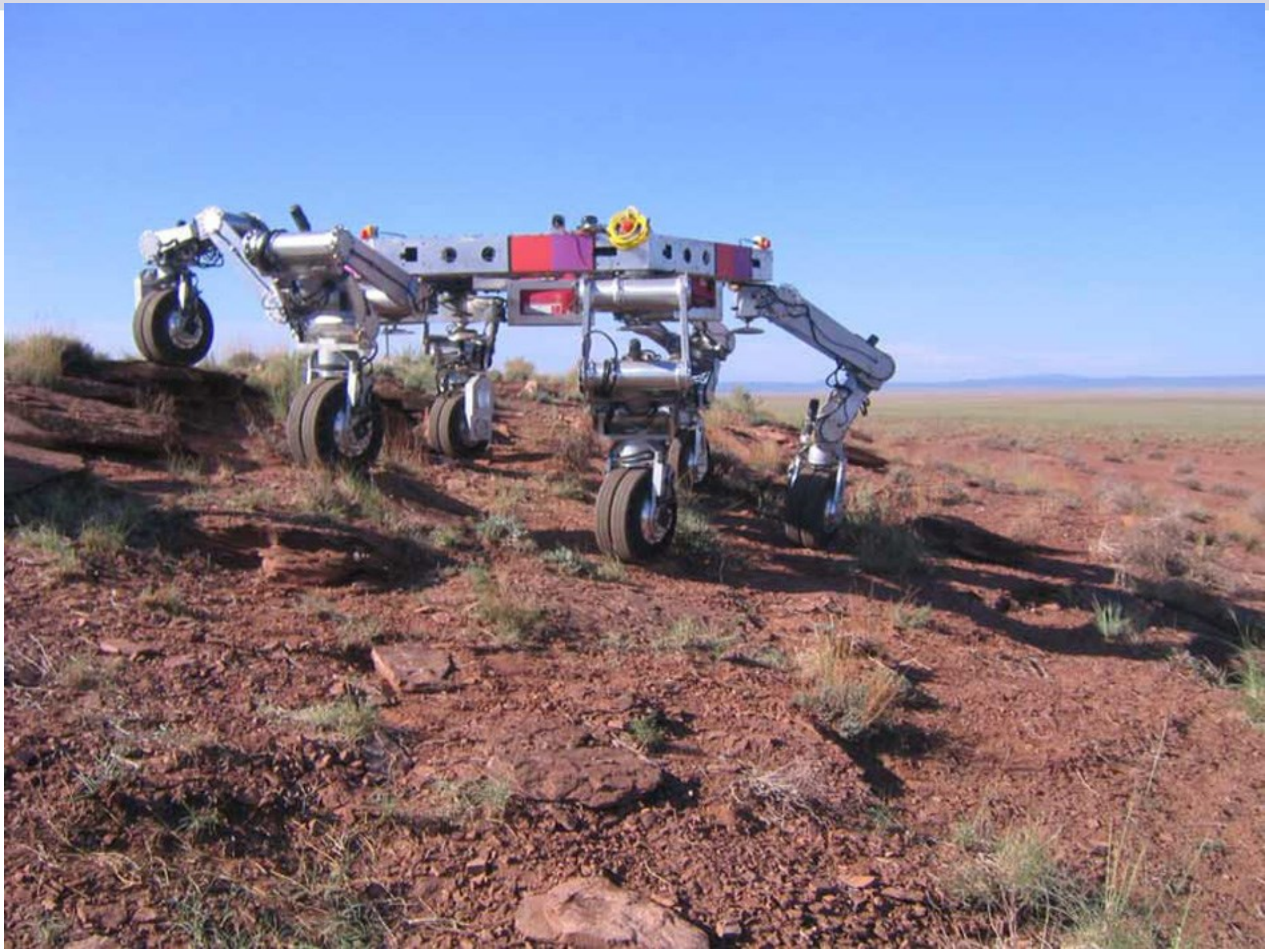


Image source: NASA

DEFINITIONS

Informal definitions

- Workspace
  - Space in which the robot (end-effector) can move
  - $\mathbb{R}^2 = \{x, y\}, \mathbb{R}^3 = \{x, y, z\}$
- Configuration Space
  - Space of the various state a robot can be in, i.e., cartesian product of the state space of each joint
  - Initially used for robotic arms
  - For wheeled mobile robots, often $SE(2) = \{x, y, \theta\}, SE(3) = \{x, y, z, \theta, \phi, \psi\}$

Examples

Robot	Workspace	Configuration Space
Differential drive (bubblebot)	$\mathbb{R}^2$	$SE(2)$
Car	$\mathbb{R}^2$	$SE(2) \times SO(1)$ One angle more for steering
Space Rover (HW4)	$\mathbb{R}^2$	$SE(2) \times SO(6)$ 6 steering angles
Tractor-Trailer	$\mathbb{R}^2$	$SE(2) \times SO(2)$ Steering angles + trailer angle
Helicopter, airplane, submarine, satellite	$\mathbb{R}^3$	$SE(3)$ Eventually more for flaps, fins, swashplate, …

Degrees of freedom

- For a single joint:
  - Number of independent directions of motion
  - Knee, elbow: 1. Ankle: 2.
- For a manipulator (sequence of joint):
  - Sum of all the DoF for each joint
- For a moving vehicle without joints
  - Possible directions of motion
  - Locomotive:1. Bubblebot: 3. Helicopter: 6.
- For a moving vehicle with joints: sum them all
  - Car: 4, Car+trailer:4, Space rover:9+

For vehicles in the plane

MOBILITY ANALYSIS FOR WHEELS

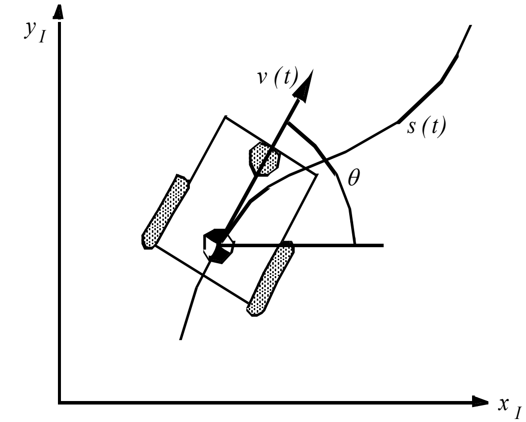
Introduction: Mobile Robot Kinematics

- Aim
  - Description of mechanical behavior of the robot for *design* and *control*
  - Similar to robot manipulator kinematics
  - However, mobile robots can move unbound with respect to their environment
    - There is no direct way to measure the robot's position
    - Position must be integrated over time
    - Leads to inaccuracies of the position (motion) estimate
      -> the number 1 challenge in mobile robotics
  - Understanding mobile robot motion starts with understanding wheel *constraints* placed on the robots mobility

Introduction: Kinematics Model

- Goal:
  - establish the robot speed $\dot{\xi} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T$ as a function of the wheel speeds $\dot{\varphi}_i$, steering angles β_i , steering speeds $\dot{\beta}_i$ and the geometric parameters of the robot (*configuration coordinates*).
  - forward kinematics

$$\dot{\xi} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(\dot{\varphi}_1, \dots \dot{\varphi}_n, \beta_1, \dots \beta_m, \dot{\beta}_1, \dots \dot{\beta}_m)$$



  - Inverse kinematics

$$\begin{bmatrix} \dot{\varphi}_1 & \dots & \dot{\varphi}_n & \beta_1 & \dots & \beta_m & \dot{\beta}_1 & \dots & \dot{\beta}_m \end{bmatrix}^T = f(\dot{x}, \dot{y}, \dot{\theta})$$

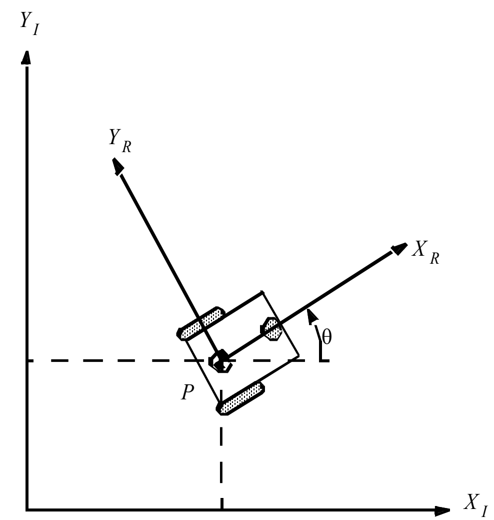
  - why not

$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = f(\varphi_1, \dots \varphi_n, \beta_1, \dots \beta_m)$$

-> **rarely possible**

Representing Robot Position

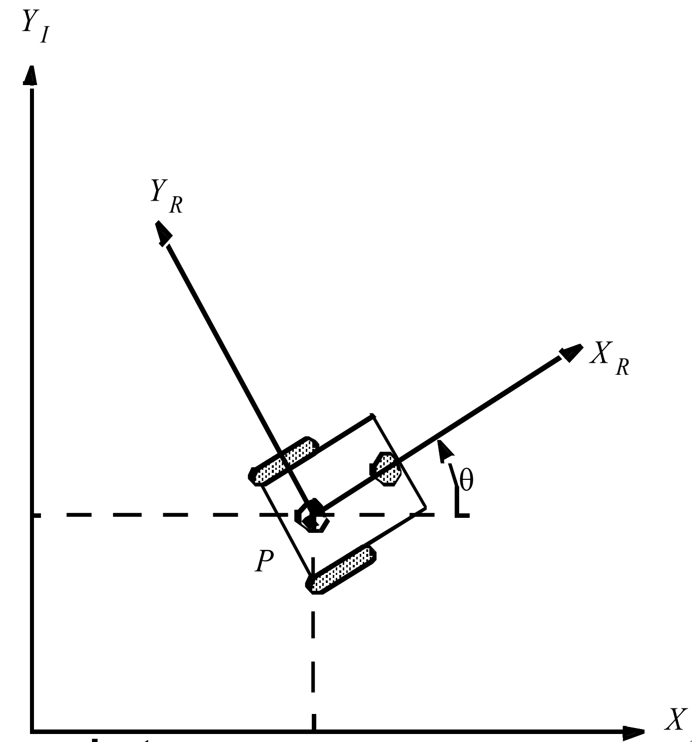
- Representing the robot within an arbitrary initial frame
  - Initial frame: $\{X_I, Y_I\}$
  - Robot frame: $\{X_R, Y_R\}$
  - Robot position: $\xi_I = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$
  - Mapping between the two frames



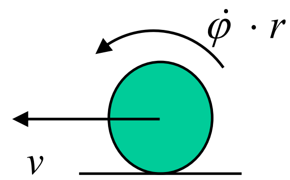
$$\dot{\xi}_R = R(\theta)\dot{\xi}_I = R(\theta)\cdot\begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T$$

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Wheel Kinematic Constraints: Assumptions



- Movement on a horizontal plane
- Point contact of the wheels
- Wheel is not deformable
- Pure rolling
  - $v_c = 0$ at contact point
- No slipping, skidding or sliding
- No friction for rotation around contact point
- Steering axes orthogonal to the surface
- Wheels connected by rigid frame (chassis)



Application: forward kinematics

- Differential drive
- Bicycle
- Car
- Space Rover on rocky terrain.
- Car + trailer

- For each of these systems, for each wheel W, compute:

$$v_W = f(body\ twist)$$

Application: inverse kinematics

- Differential drive
- Bicycle
- Car
- Space Rover on rocky terrain.
- Car + trailer

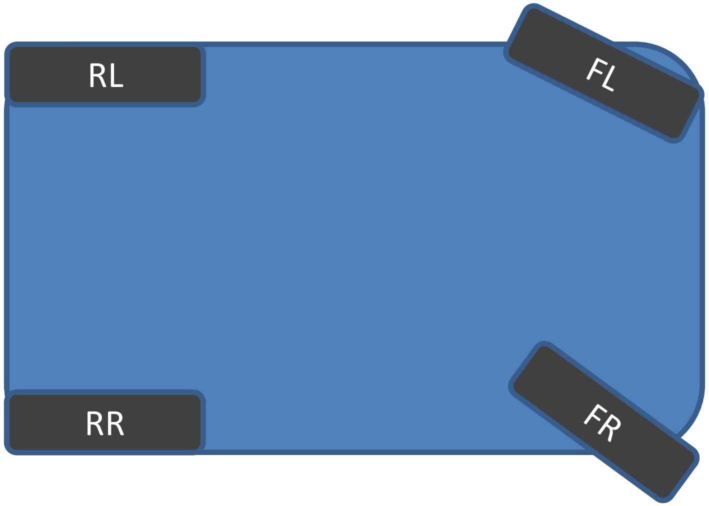
- For each of these systems, given the velocity and steering of each wheel W, compute:

$$body\ twist = f\left(\left\{\overrightarrow{v(W_i)}, i = 1..n\right\}\right))$$

APPLICATION: WHEEL CONTROL

Drive-by-wire car

- No differential
- No mechanical steering
- 4 propulsion motors
- 2 steering motors



Wheel control

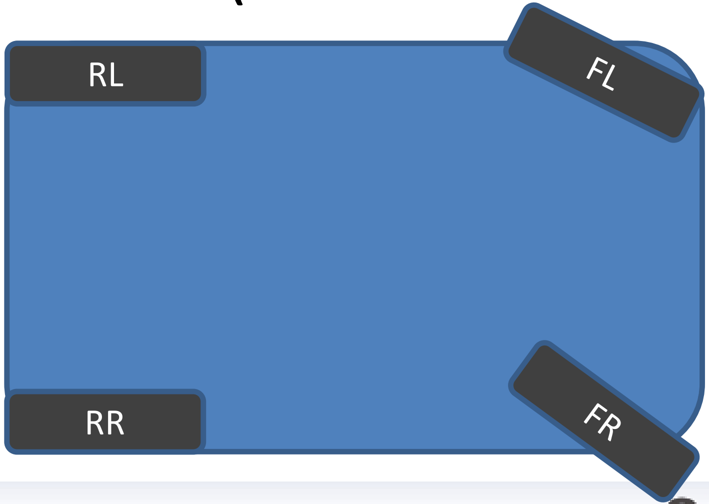
- Input: vehicle twist in robot frame
  - $(v_x, v_y, v_z = 0, \omega_x = 0, \omega_y = 0, \omega_z)$
- Objective:
  - Compute steering angle of every steered wheel
  - Compute wheel speed for every wheel
  - Respect rolling without slipping
  - Identify constraints on input

Wheel control

- Speed of each wheel:
  - $\overrightarrow{V_W} = \overrightarrow{V_B} + \overrightarrow{\Omega_\mathrm{B}} \times \overrightarrow{BW}$
  - $\begin{bmatrix} v_{w,x} \\ v_{w,y} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} -\omega_z W_y \\ \omega_z W_x \end{bmatrix}$
  - $v_{RL,y} = 0 \Rightarrow v_y = -\omega_z W_x$
    - Traditionally, the vehicle reference frame (B) is on the middle of the rear axle, so that $W_x = 0$ and $v_y = 0$
  - $\beta_W = atan2(v_{w,y}, v_{w,x})\ ;\ \dot{\phi}_W = hypot(v_{w,y}, v_{w,x})$
    - Keep $\beta \in [-{}^{\pi}/_{2}, {}^{\pi}/_{2}]$ by changing wheel direction

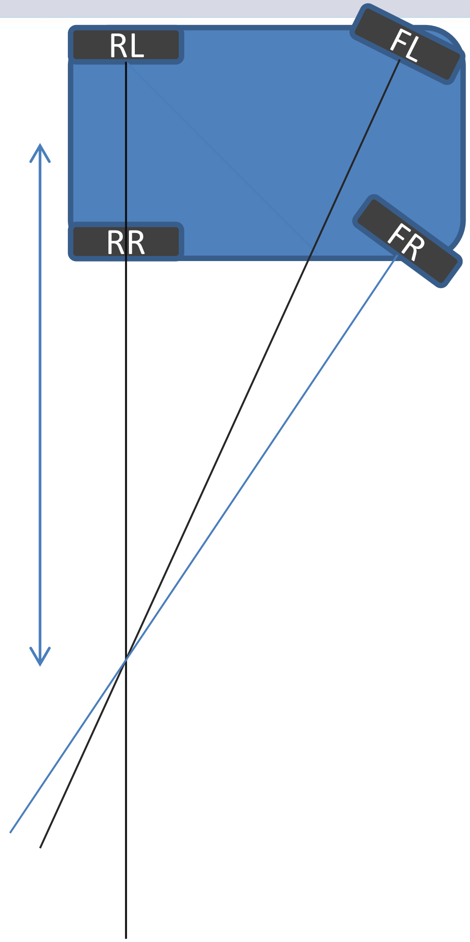
Constraints

- Maximum steering angle:
  - $\operatorname{atan}\left({}^{v_{w,y}}/_{v_{w,x}}\right) < \phi_{max} \implies W_y + \frac{W_x}{tan\phi_{max}} < \frac{v_x}{\omega_z}$
    - Assuming $\omega_z \geq 0$ and $v_x \geq 0$
  - Physical interpretation: the rotation radius (v_x/ω_z) is bounded below (or the curvature is bounded above)



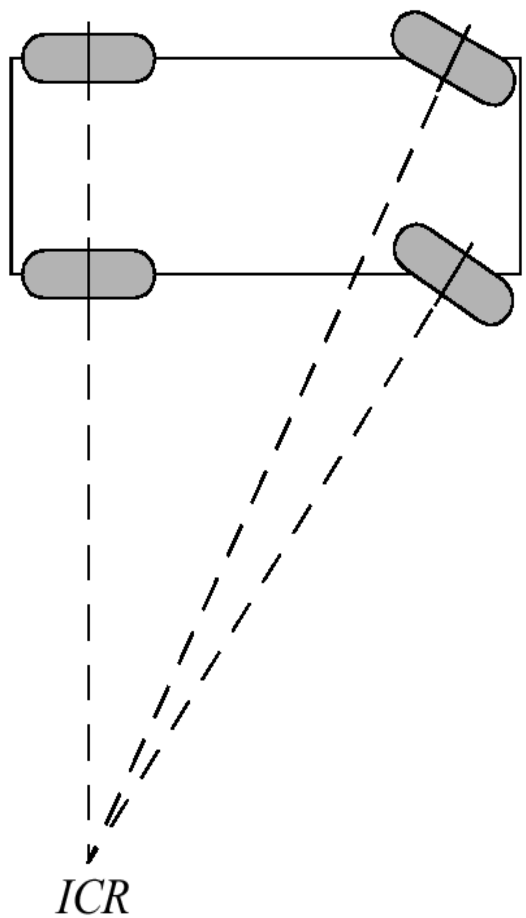
Constraints

$$r_{min} = 1/\kappa_{max}$$

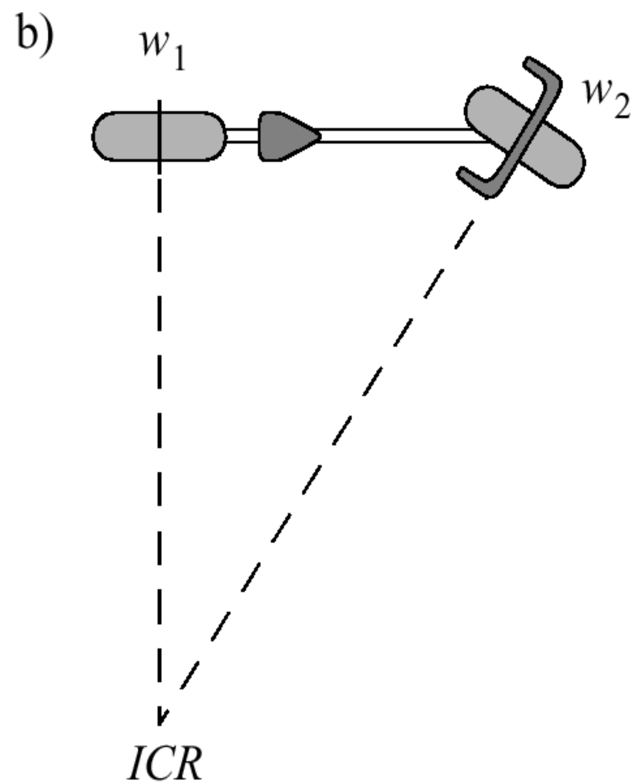


Mobile Robot Maneuverability: Instantaneous Center of Rotation

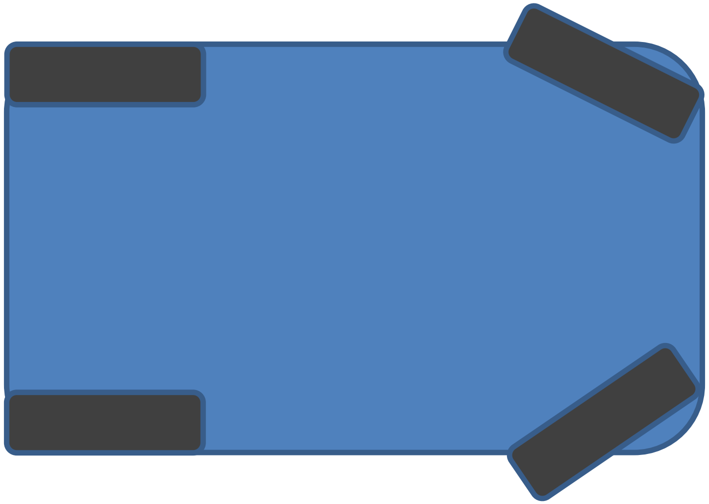
- Ackermann Steering



Bicycle

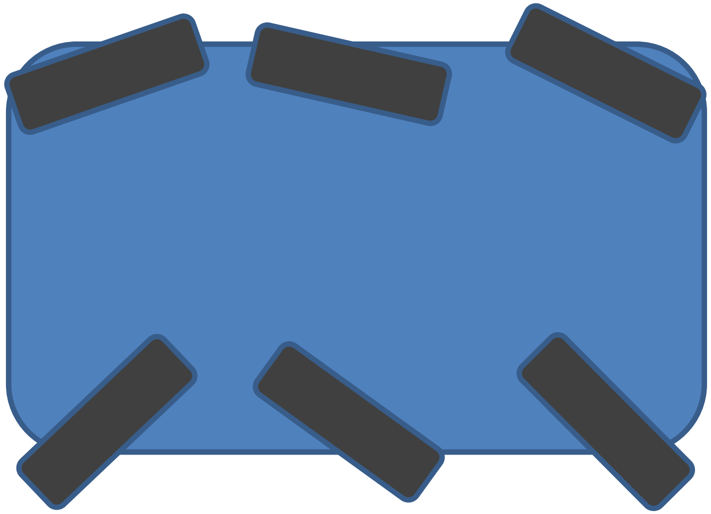


ESA Eurobot Ground Prototype



ESA Exomars - HomeWork

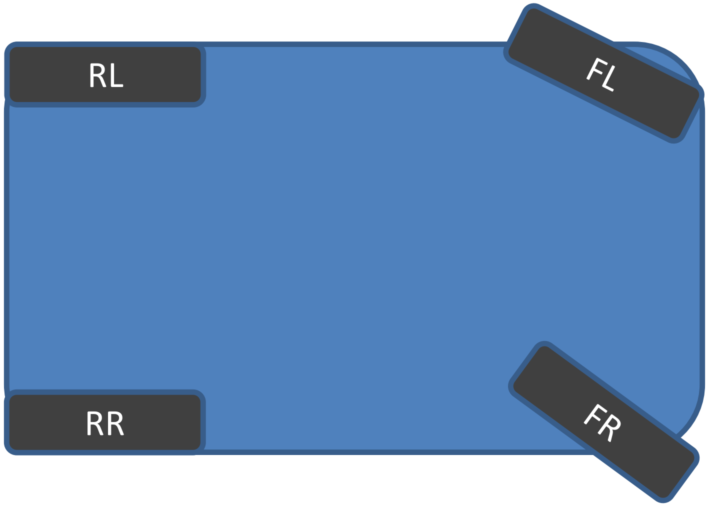
Note: the wheel exact position depends on the configuration of the suspension



APPLICATION: WHEEL ODOMETRY

Drive-by-wire car

- No differential
- No mechanical steering
- 4 propulsion motors
- 2 steering motors



Wheel odometry

- Input: wheel displacement and steering between t_1 and t_2
  - $(s_{FL}, s_{FR}, s_{RL}, s_{RR}, \beta_{FL}, \beta_{FR}, \beta_{RL}, \beta_{RR})$
  - Assumption:
    - $\beta_{FL} \approx \beta_{FL}(t_1) \approx \beta_{FL}(t_2) \approx \frac{\beta_{FL}(t_1)+\beta_{FL}(t_2)}{2}$
- Objective:
  - Compute robot displacement (**in robot frame**) between t_1 and t_2

Wheel odometry

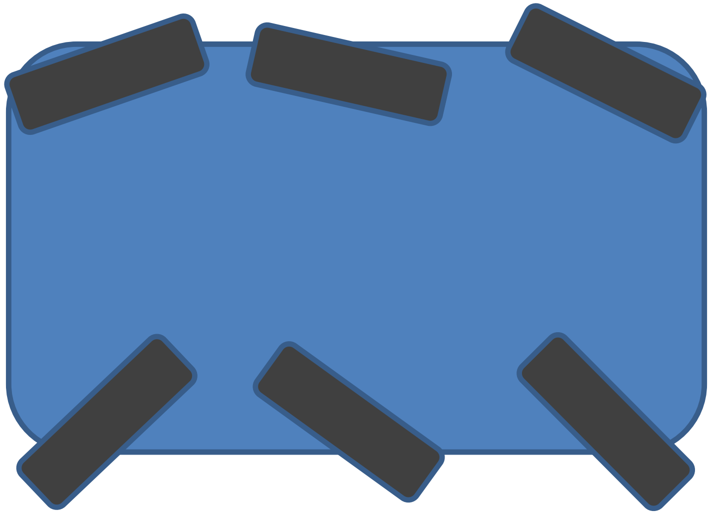
- $\begin{bmatrix} v_{w,x} \\ v_{w,y} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} -\omega_z W_y \\ \omega_z W_x \end{bmatrix}$
- $\begin{bmatrix} s_w cos\beta_W \\ s_w sin\beta_W \end{bmatrix} = \begin{bmatrix} v_x dt \\ v_y dt \end{bmatrix} + \begin{bmatrix} -\omega_z dt W_y \\ \omega_z dt W_x \end{bmatrix}$
- Two unknown: $(v_x dt = \Delta\mathrm{x}, \omega_z dt = \Delta\theta)$, $v_y = 0$
- Rear wheels: $W_{RL,y} = -W_{RR,y} = e/2$
  - $\begin{cases} s_{RL} = \Delta x - \Delta\theta W_{RL,y} \\ s_{RR} = \Delta x - \Delta\theta W_{RR,y} \end{cases} \Longrightarrow \begin{cases} \Delta x = \frac{s_{RL}+s_{RR}}{2} \\ \Delta\theta = \frac{s_{RR}-s_{RL}}{e} \end{cases}$ **in robot frame**

Least square solution

- Goal: use all the wheel information
- $\begin{bmatrix} s_w cos\beta_W \\ s_w sin\beta_W \end{bmatrix} = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} + \begin{bmatrix} -\Delta\theta W_y \\ \Delta\theta W_x \end{bmatrix}$
- Three unknown in general:$\Delta x, \Delta y, \Delta\theta$)
- $\begin{bmatrix} 1 & 0 & -W_{1,y} \\ 0 & 1 & W_{1,x} \\ \vdots & \vdots & \vdots \\ 1 & 0 & -W_{n,y} \\ 0 & 1 & W_{n,x} \end{bmatrix} \cdot \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} s_1 cos\beta_1 \\ s_1 sin\beta_1 \\ \vdots \\ s_n cos\beta_n \\ s_n sin\beta_n \end{bmatrix} \Longrightarrow A\,X = B$
- $X = (A^T A)^{-1} A^T B = pinv(A) \cdot B$ (in robot frame)

ESA Exomars - HomeWork

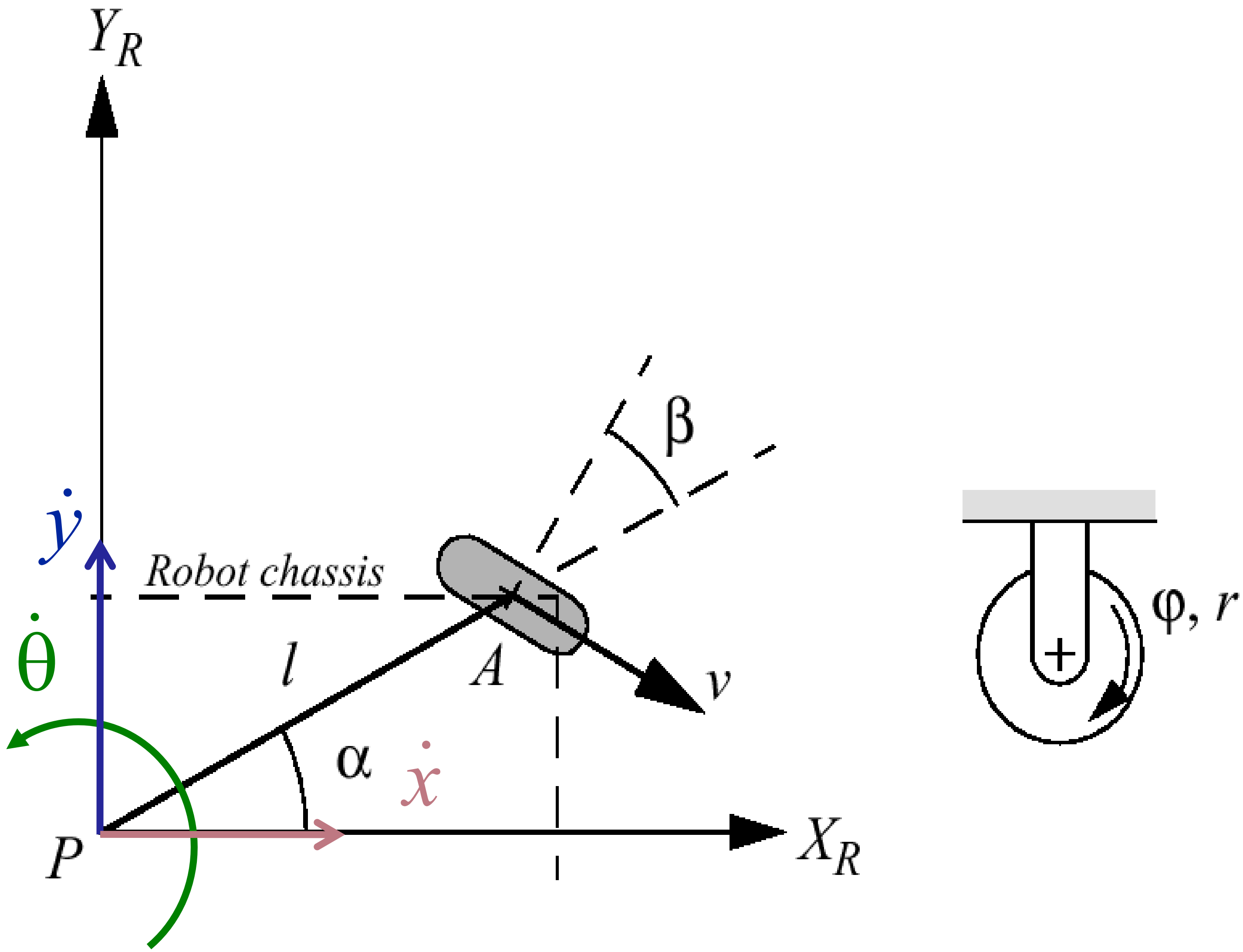
Note: the wheel exact position depends on the configuration of the suspension



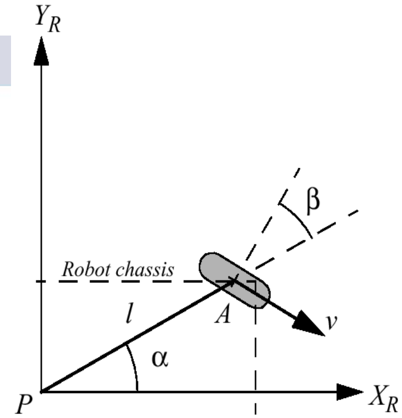
For vehicles in the plane

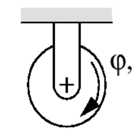
MOBILITY ANALYSIS

Wheel Kinematic Constraints: Fixed Standard Wheel



Wheel Kinematic Constraints: Fixed Standard Wheel





- Relation between wheel speed and robot speed:
  - Rolling without slipping:
    - $\frac{\partial \overrightarrow{OA}}{\partial t} \cdot \overrightarrow{v_w} = \mathrm{r}\,\dot{\phi} \qquad \frac{\partial \overrightarrow{OA}}{\partial t} \cdot \overrightarrow{v_w}^{\perp} = 0$
  - $\frac{\partial \overrightarrow{OA}}{\partial t} = \frac{\partial \overrightarrow{OP}}{\partial t} + \Omega \times \overrightarrow{PA} = \begin{bmatrix} \dot{x} - \dot{\theta} l \sin\alpha \\ \dot{y} + \dot{\theta} l \cos\alpha \end{bmatrix}$
  - $\overrightarrow{v_w} = \begin{bmatrix} \sin\alpha + \beta \\ -\cos\alpha + \beta \end{bmatrix} v \qquad \overrightarrow{v_w}^{\perp} = \begin{bmatrix} \cos\alpha + \beta \\ \sin\alpha + \beta \end{bmatrix} v$
  - Linear constraint equation:
    - $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \cdot \begin{bmatrix} \sin(\alpha+\beta) \\ -\cos(\alpha+\beta) \\ -l\cos(\beta) \end{bmatrix} - \mathrm{r}\,\dot{\phi} = 0 \qquad \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \cdot \begin{bmatrix} \cos(\alpha+\beta) \\ \sin(\alpha+\beta) \\ l\sin(\beta) \end{bmatrix} = 0$
- In the book:

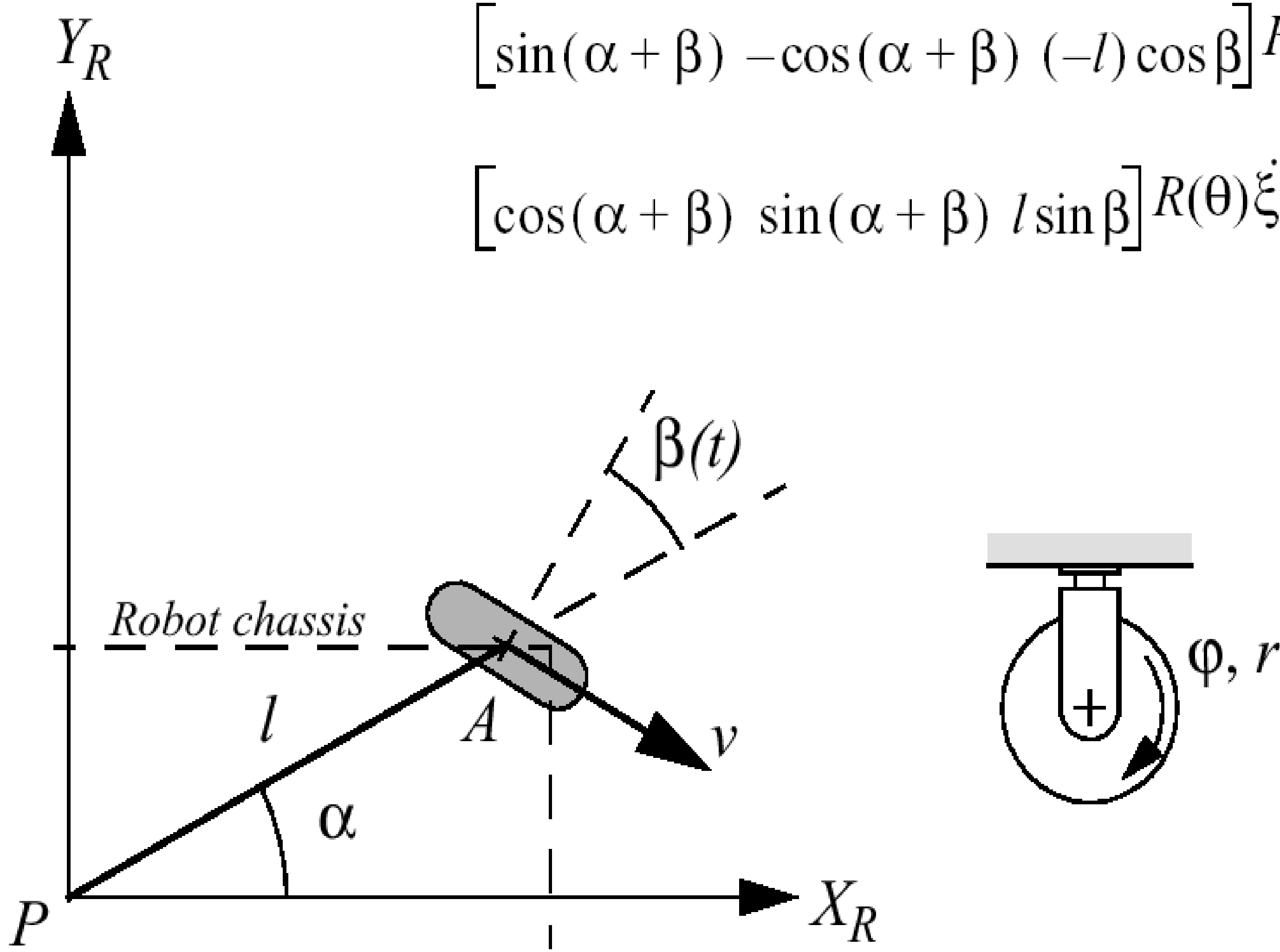
$$\begin{bmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta \end{bmatrix} \overbrace{R(\theta)\dot{\xi}_I}^{\dot{\xi}_R = [\dot{x} \quad \dot{y} \quad \theta]^T} - r\dot{\varphi} = 0$$

$$\begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin\beta \end{bmatrix} R(\theta)\dot{\xi}_I = 0$$

Wheel Kinematic Constraints: Steered Standard Wheel

$$\left[\sin(\alpha+\beta) \;\; -\cos(\alpha+\beta) \;\; (-l)\cos\beta\right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

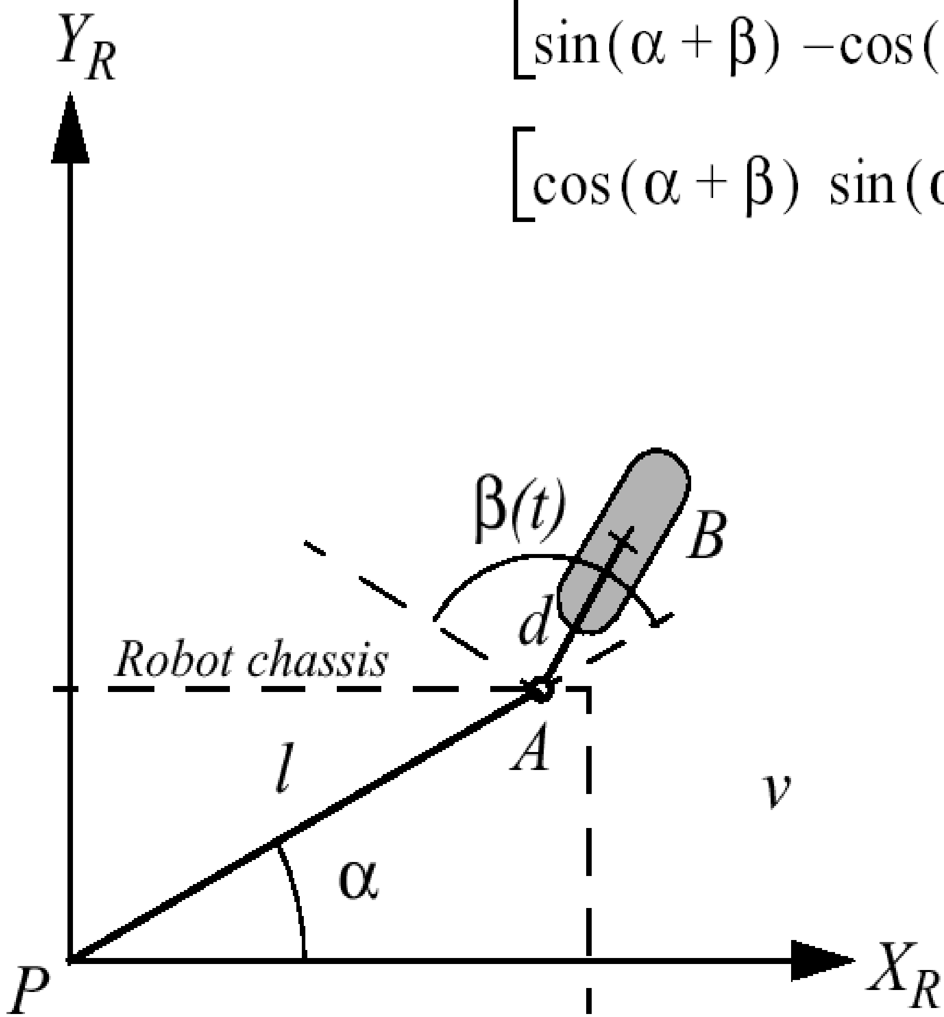
$$\left[\cos(\alpha+\beta) \;\; \sin(\alpha+\beta) \;\; l\sin\beta\right] R(\theta)\dot{\xi}_I = 0$$

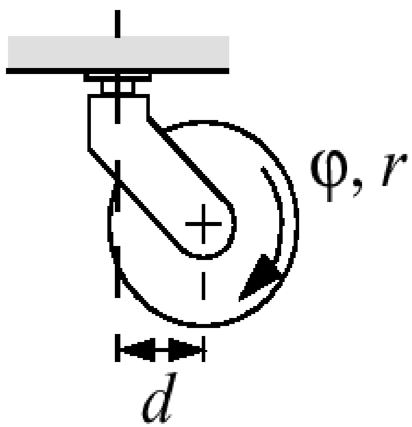


Wheel Kinematic Constraints: Castor Wheel

$$\begin{bmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta\end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

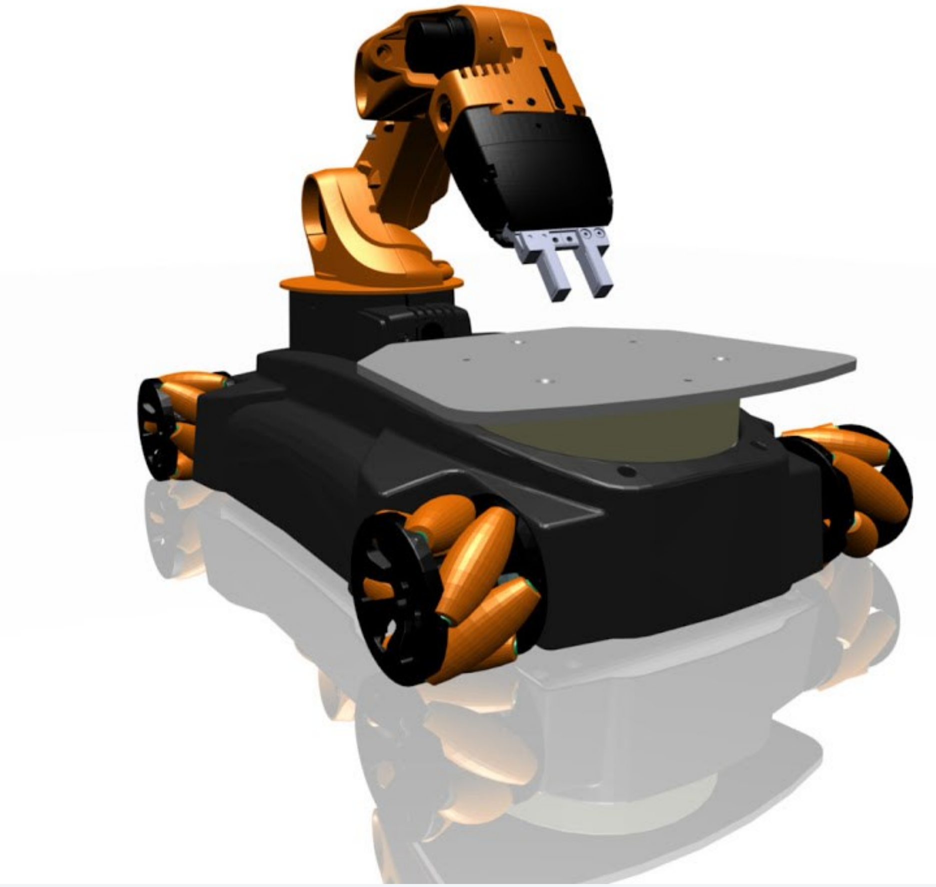
$$\begin{bmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & \underline{d+l}\sin\beta\end{bmatrix} R(\theta)\dot{\xi}_I + \underline{d}\dot{\beta} = 0$$





Youbot (Kuka)

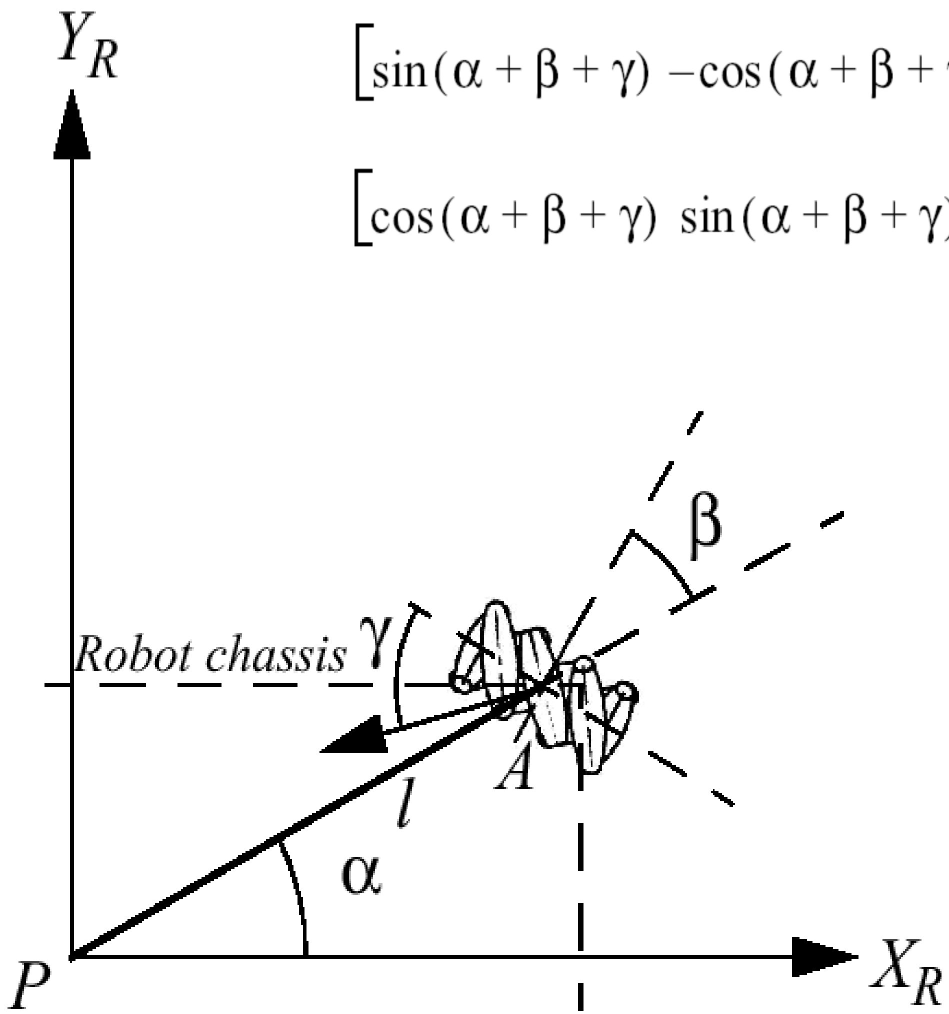
- http://www.youtube.com/watch?v=QiWAe7T2KHk&list=PL342DD8BD8FC376A2&index=11

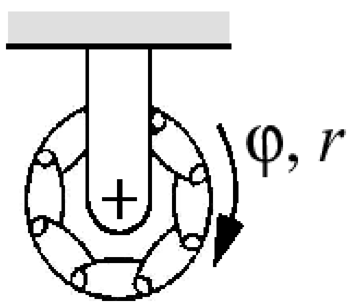


Wheel Kinematic Constraints: Swedish Wheel

$$\left[\sin(\alpha+\beta+\gamma)\ \ -\cos(\alpha+\beta+\gamma)\ \ (-l)\cos(\beta+\gamma)\right]R(\theta)\dot{\xi}_I - r\dot{\varphi}\cos\gamma = 0$$

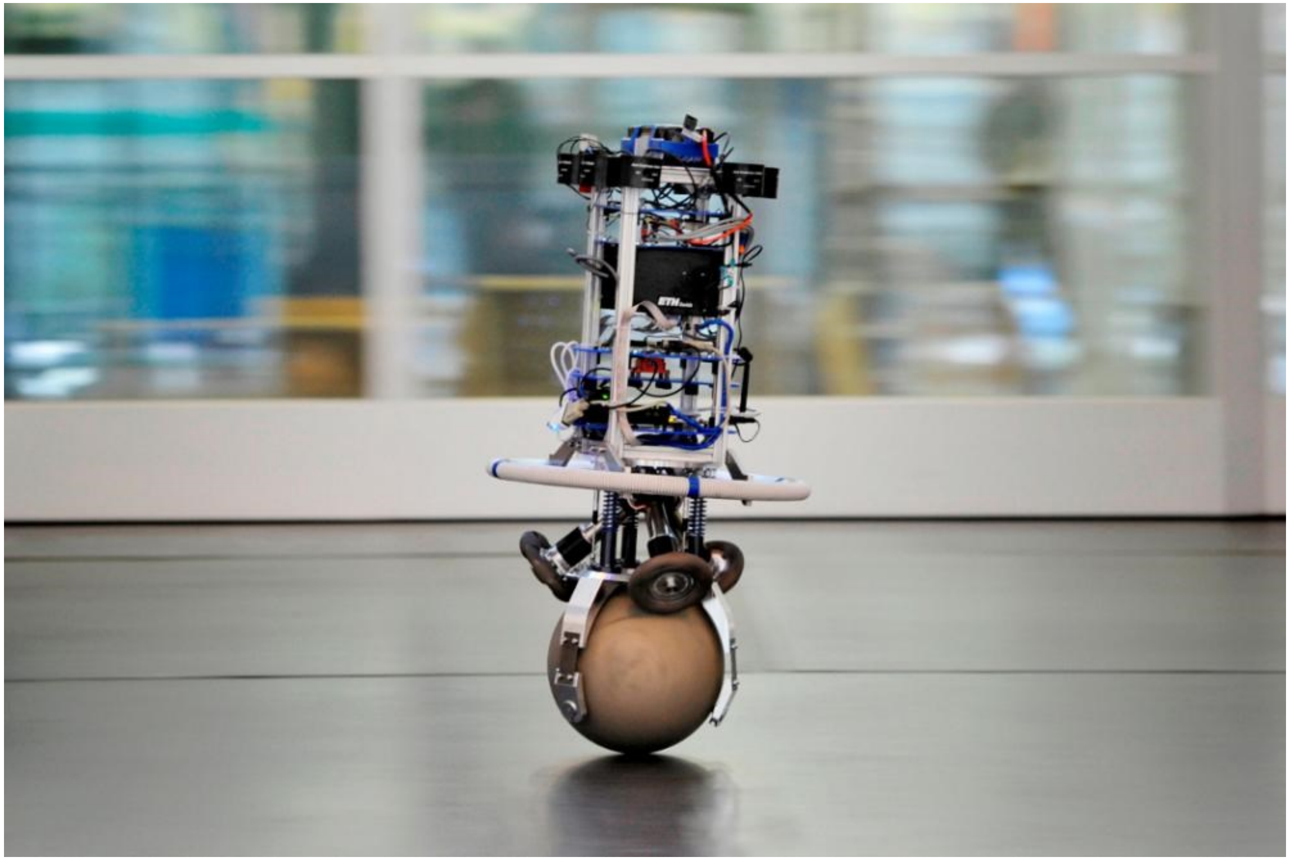
$$\left[\cos(\alpha+\beta+\gamma)\ \ \sin(\alpha+\beta+\gamma)\ \ l\sin(\beta+\gamma)\right]R(\theta)\dot{\xi}_I - r\dot{\varphi}\sin\gamma - r_{sw}\dot{\varphi}_{sw} = 0$$





Rezero

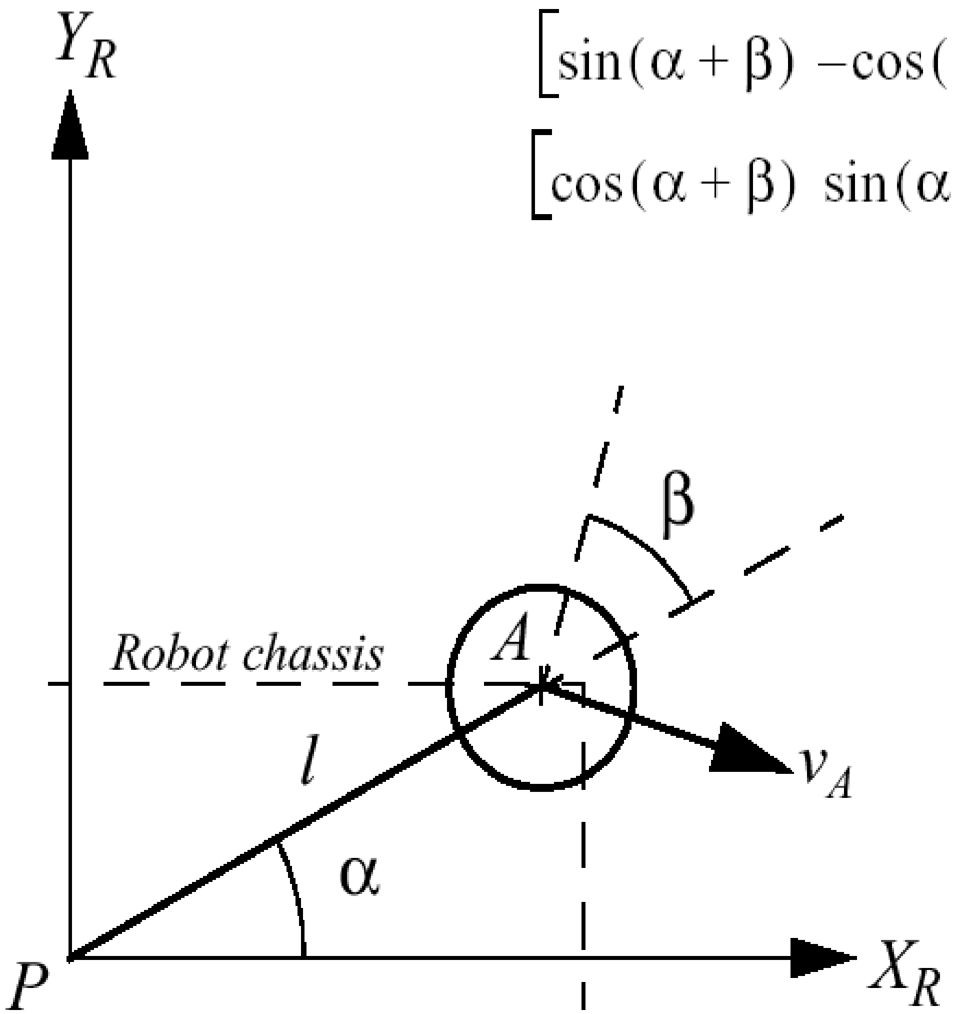
- http://www.youtube.com/watch?feature=player_embedded&v=sB9IowB8nx8

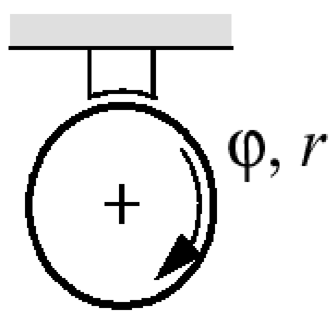


Wheel Kinematic Constraints: Spherical Wheel

$$\begin{bmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta \end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$\begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin\beta \end{bmatrix} R(\theta)\dot{\xi}_I = 0$$





- Rotational axis of the wheel can have an arbitrary direction

Robot Kinematic Constraints

25.2.2008 – after 1st hour

- Given a robot with *M* wheels
  - each wheel imposes zero or more constraints on the robot motion
  - only fixed and steerable standard wheels impose constraints
- What is the maneuverability of a robot considering a combination of different wheels?
- Suppose we have a total of $N = N_f + N_s$ standard wheels
  - We can develop the equations for the constraints in matrix forms:
  - Rolling

$$J_1(\beta_s)R(\theta)\dot{\xi}_I + J_2\dot{\varphi} = 0 \qquad \varphi(t) = \underset{(N_f+N_s)\times 1}{\begin{bmatrix} \varphi_f(t) \\ \varphi_s(t) \end{bmatrix}} \qquad J_1(\beta_s) = \underset{(N_f+N_s)\times 3}{\begin{bmatrix} J_{1f} \\ J_{1s}(\beta_s) \end{bmatrix}} \qquad J_2 = diag\,(r_1 \cdots r_N)$$

  - Lateral movement

$$C_1(\beta_s)R(\theta)\dot{\xi}_I = 0 \qquad C_1(\beta_s) = \underset{(N_f+N_s)\times 3}{\begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}}$$

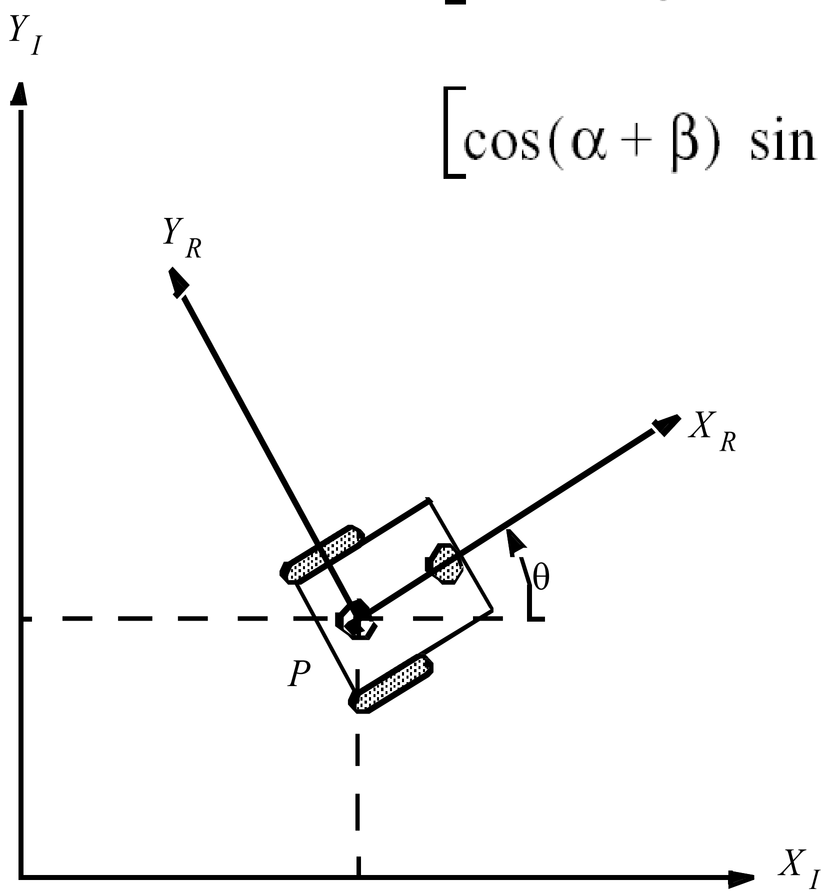
Example

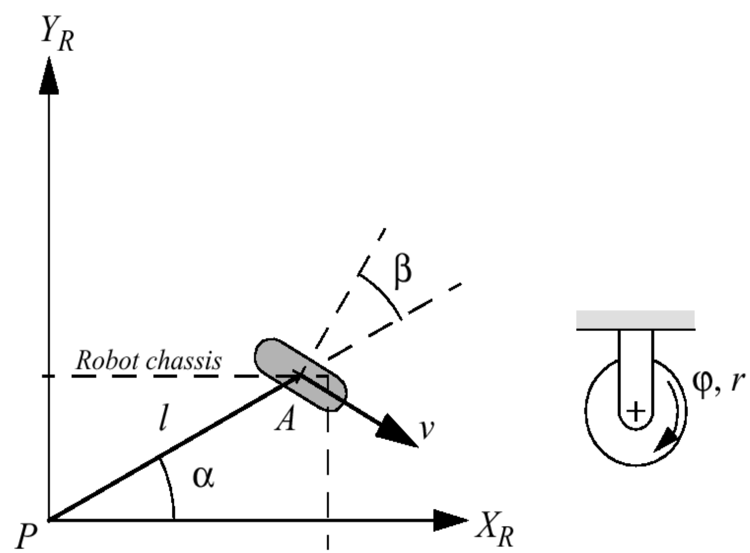
- Kinematic of a differential drive
- Kinematic of the car
- Kinematic of a car with a trailer
  - Why is it hard to reverse with a trailer?

Example: Differential Drive Robot

$$\left[\sin(\alpha+\beta)\ \ -\cos(\alpha+\beta)\ \ (-l)\cos\beta\right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$\left[\cos(\alpha+\beta)\ \ \sin(\alpha+\beta)\ \ l\sin\beta\right] R(\theta)\dot{\xi}_I = 0$$





Exercise – Solutions: Using Kinematic Model

Differential-Drive:

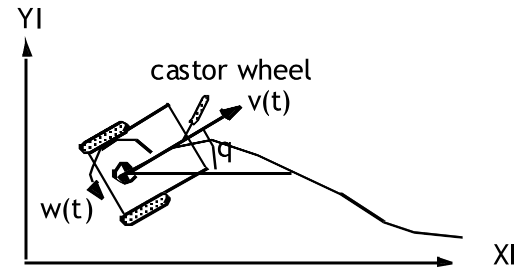
$$\alpha_1 = -90° \; ; \qquad \beta_1 = 180° \; ; \; \gamma_1 = 0°$$

($\beta_1 = 180°$ because direction of $\dot{\varphi}_1$ defined so that positive $\dot{\varphi}_1$ results in a movement in positive X_R direction)

$$\alpha_2 = 90° \; ; \qquad \beta_2 = 0° \; ; \; \gamma_2 = 0°$$

thus, using the rolling constraints for the wheels (equation 3.12 and 3.13):

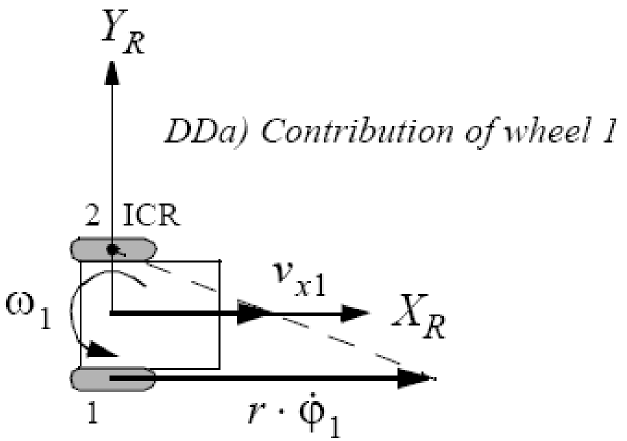
$$\begin{bmatrix} \begin{bmatrix} 1 & 0 & l \\ 1 & 0 & -l \end{bmatrix} \\ \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \end{bmatrix} R(\theta)\dot{\xi}_I - \begin{bmatrix} J_2 \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} \\ 0 \end{bmatrix} = 0$$

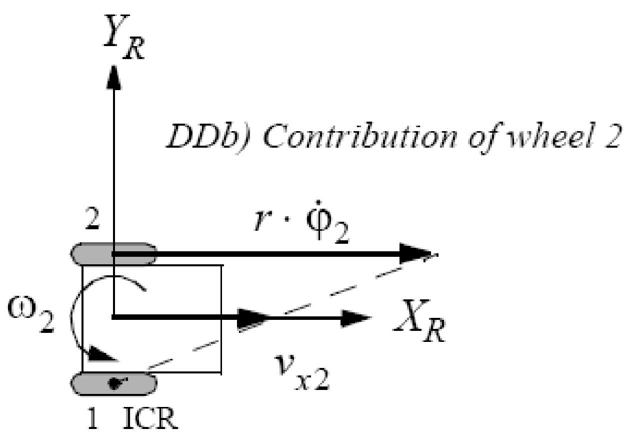


Note, the sliding constraint in lateral direction (equ. 3.13) reduces on only one independent equation because the wheels are parallel.

$$\dot{\xi}_I = R(\theta)^{-1} \begin{bmatrix} 1 & 0 & l \\ 1 & 0 & -l \\ 0 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} \begin{bmatrix} r\dot{\varphi}_1 \\ r\dot{\varphi}_2 \end{bmatrix} \\ 0 \end{bmatrix} = R(\theta)^{-1} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ \frac{1}{2l} & -\frac{1}{2l} & 0 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} r\dot{\varphi}_1 \\ r\dot{\varphi}_2 \end{bmatrix} \\ 0 \end{bmatrix} = R(\theta)^{-1} \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ 0 & 0 \\ \frac{r}{2l} & -\frac{r}{2l} \end{bmatrix} \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix}$$

Exercise – Solutions Geometric Approach





Differential-Drive:

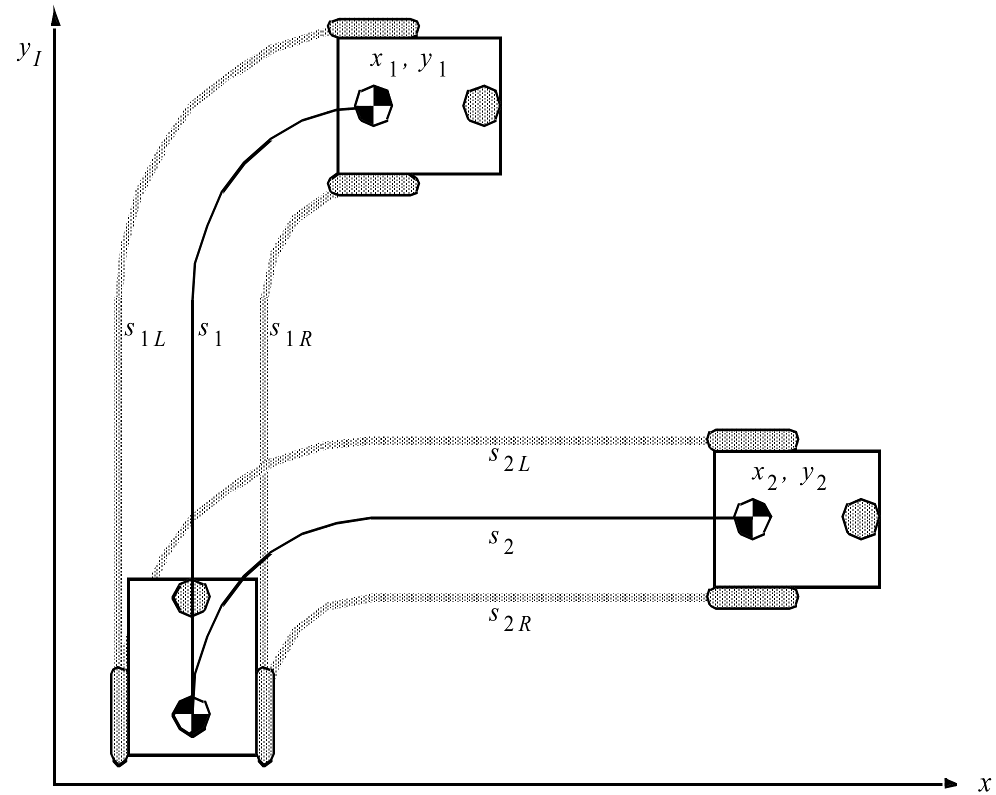
DDa) $v_{x1} = \frac{1}{2}r\dot{\varphi}_1 \quad ; \quad v_{y1} = 0 \quad ; \quad \omega_1 = \frac{1}{2l}r\dot{\varphi}_1$

DDb) $v_{x2} = \frac{1}{2}r\dot{\varphi}_1 \quad ; \quad v_{y2} = 0 \quad ; \quad \omega_2 = -\frac{1}{2l}r\dot{\varphi}_1$

$$-> \dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}_I = R(\theta)^{-1}\begin{bmatrix} v_{x1}+v_{x2} \\ v_{y1}+v_{y2} \\ \omega_1+\omega_2 \end{bmatrix} = R(\theta)^{-1}\begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ 0 & 0 \\ \frac{r}{2l} & -\frac{r}{2l} \end{bmatrix}\begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix}$$

NON-HOLONOMIC ROBOTS

Mobile Robot Kinematics: Non-Holonomic Systems



$$s_1 = s_2$$

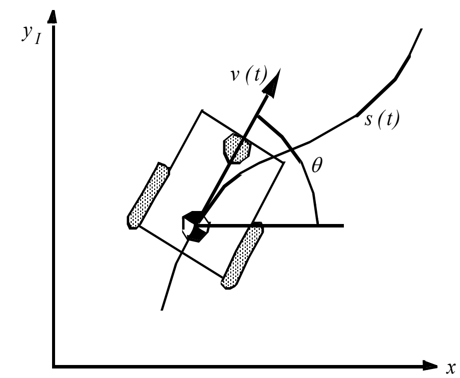
$$s_{1,R} = s_{2,R}; s_{1,L} = s_{2,L}$$

but: $x_1 \neq x_2\ ; y_1 \neq y_2$

- Non-holonomic systems
  - differential equations are not integrable to the final position.
  - the measure of the traveled distance of each wheel is not sufficient to calculate the final position of the robot. One has also to know how this movement was executed as a function of time.

Non-Holonomic Systems: Mathematical Interpretation

- A mobile robot is running along a trajectory *s(t)*.
  At every instant of the movement its velocity *v(t)* is:



$$v(t) = \frac{\partial s}{\partial t} = \frac{\partial x}{\partial t}\cos\theta + \frac{\partial y}{\partial t}\sin\theta$$

$$ds = dx\cos\theta + dy\sin\theta$$

- Function *v(t)* is said to be integrable (holonomic) if there exists a trajectory function *s(t)* that can be described by the values *x*, *y*, and θ only.

$$s = s(x, y, \theta)$$

- This is the case if

$$\boxed{\frac{\partial^2 s}{\partial x \partial y} = \frac{\partial^2 s}{\partial y \partial x} \;;\; \frac{\partial^2 s}{\partial x \partial \theta} = \frac{\partial^2 s}{\partial \theta \partial x} \;;\; \frac{\partial^2 s}{\partial y \partial \theta} = \frac{\partial^2 s}{\partial \theta \partial y}}$$

Condition for integrable function

- With $s = s(x, y, \theta)$ we get for *ds*

$$ds = \frac{\partial s}{\partial x}dx + \frac{\partial s}{\partial y}dy + \frac{\partial s}{\partial \theta}d\theta$$

Lie brackets

- Differential tool to analyze the mobility of a system (many other uses outside of robotics)
- Informally:
  - An infinitely small parallel-parking maneuver is a way to move out of the local manifold spanned by the kinematic constraints.
- Lecture from Matt Mason (CMU):
  - http://www.cs.cmu.edu/afs/cs/academic/class/16741-s07/www/lecture5.pdf

ARTICLES

Articles

- A novel approach for steering wheel synchronization with velocity/acceleration limits and mechanical constraints, U. Schwesinger, C. Pradalier, R. Siegwart, IROS'12
- Terrain Mapping and Control Optimization for a 6-Wheel Rover with Passive Suspension, P. Strupler , C. Pradalier, R. Siegwart, FSR'12
- Modeling odometry and uncertainty propagation for a Bi-Steerable car, J. Hermosillo, C. Pradalier, S. Sekhavat, IV'02
- 3D-Odometry for rough terrain – Towards real 3D navigation, P. Lamon and R. Siegwart, ICRA'03
- 3D Localization for the MagneBike Inspection Robot, F. Tache, F. Pomerleau, G. Caprari, R. Siegwart, R. Moser, M. Bosse, JFR 2011
- Simultaneous Localization and Odometry Calibration for Mobile Robot, A. Martinelli, N. Tomatis, A. Tapus and R. Siegwart, IROS'03

CONCLUSION

Conclusion

- Forward or inverse kinematics

$$\dot{\xi} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(\dot{\varphi}_1, \dots \dot{\varphi}_n, \beta_1, \dots \beta_m, \dot{\beta}_1, \dots \dot{\beta}_m) \qquad \begin{bmatrix} \dot{\varphi}_1 & \cdots & \dot{\varphi}_n & \beta_1 & \dots & \beta_m & \dot{\beta}_1 & \dots & \dot{\beta}_m \end{bmatrix}^T = f(\dot{x}, \dot{y}, \dot{\theta})$$

- Basis for
  - Movement prediction
  - Wheel odometry
  - Speed and steering distribution
  - Controller and path planner design

MOBILITY QUANTIFICATION

Mobile Robot Maneuverability

- The maneuverability of a mobile robot is the combination
  - of the mobility available based on the sliding constraints
  - plus additional freedom contributed by the steering
- Three wheels is sufficient for static stability
  - additional wheels need to be synchronized
  - this is also the case for some arrangements with three wheels
- It can be derived using the equation seen before
  - Degree of mobility δ_m
  - Degree of steerability δ_s
  - Robots maneuverability $\delta_M = \delta_m + \delta_s$

Mobile Robot Maneuverability: Degree of Mobility

- To avoid any lateral slip the motion vector $R(\theta)\dot{\xi}_I$ has to satisfy the following constraints:

$$C_{1f} R(\theta)\dot{\xi}_I = 0$$

$$C_{1s}(\beta_s) R(\theta)\dot{\xi}_I = 0$$

$$C_1(\beta_s) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}$$

- Mathematically:
  - $\dot{\xi}_R$ must belong to the *null space* of the projection matrix $C_1(\beta_s)$
  - *Null space* of $C_1(\beta_s)$ is the space N such that for any vector n in N

$$C_1(\beta_s) \cdot n = 0$$

  - Geometrically this can be shown by the *Instantaneous Center of Rotation (ICR)*

Mobile Robot Maneuverability: More on Degree of Mobility

- Robot chassis kinematics is a function of the set of independent constraints

$$rank\left[C_1(\beta_s)\right] \qquad C_1(\beta_s) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix} \qquad \begin{array}{l} C_{1f} R(\theta)\dot{\xi}_I = 0 \\ C_{1s}(\beta_s) R(\theta)\dot{\xi}_I = 0 \end{array}$$

  - the greater the rank of , $C_1(\beta_s)$ the more constrained is the mobility

- Mathematically

$$\delta_m = \dim N\left[C_1(\beta_s)\right] = 3 - rank\left[C_1(\beta_s)\right] \qquad 0 \le rank\left[C_1(\beta_s)\right] \le 3$$

  - no standard wheels $rank\left[C_1(\beta_s)\right] = 0$
  - all direction constrained $rank\left[C_1(\beta_s)\right] = 3$

- Examples:
  - Unicycle: One single fixed standard wheel
  - Differential drive: Two fixed standard wheels
    - wheels on same axle
    - wheels on different axle

Mobile Robot Maneuverability: Degree of Steerability

- Indirect degree of motion

$$\delta_s = rank \left[C_{1s}(\beta_s) \right]$$

  - The particular orientation at any instant imposes a kinematic constraint
  - However, the ability to change that orientation can lead additional degree of maneuverability
- Range of δ_s: $0 \leq \delta_s \leq 2$

- Examples:
  - one steered wheel: Tricycle
  - two steered wheels: No fixed standard wheel
  - car (Ackermann steering): $N_f = 2$, $N_s = 2$ -> common axle

Mobile Robot Maneuverability: Robot Maneuverability

- Degree of Maneuverability

$$\delta_M = \delta_m + \delta_s$$

  - Two robots with same δ_M are not necessary equal
  - Example: Differential drive and Tricycle (next slide)

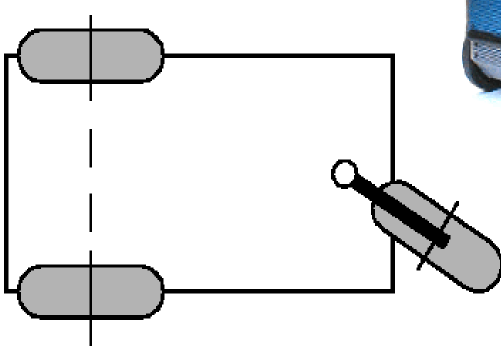
  - For any robot with $\delta_M = 2$ the ICR is always constrained to *lie on a line*
  - For any robot with $\delta_M = 3$ the ICR is not constrained an can *be set to any point on the plane*

Mobile Robot Maneuverability: Wheel Configurations

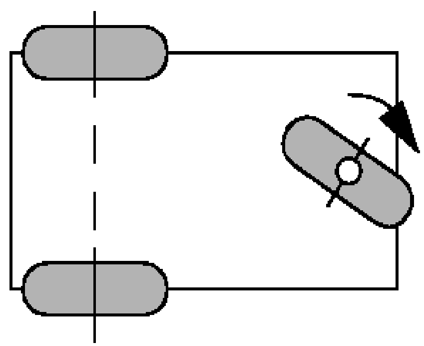
- Differential Drive

Tricycle

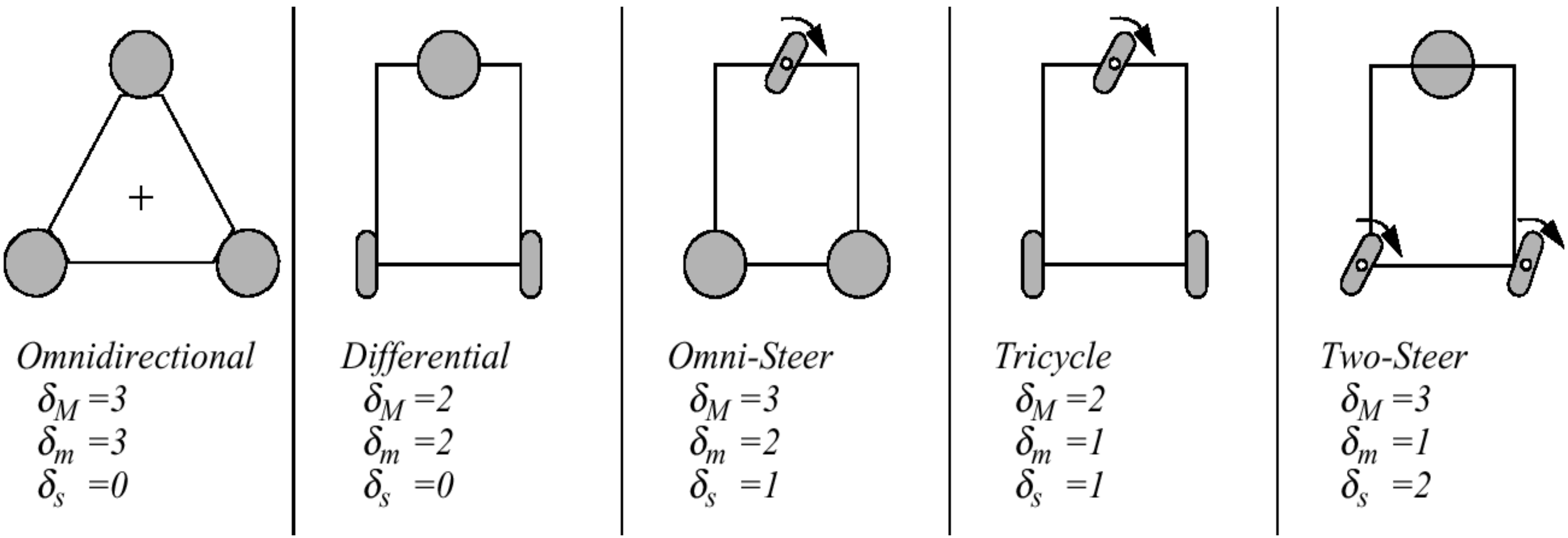
a)



b)

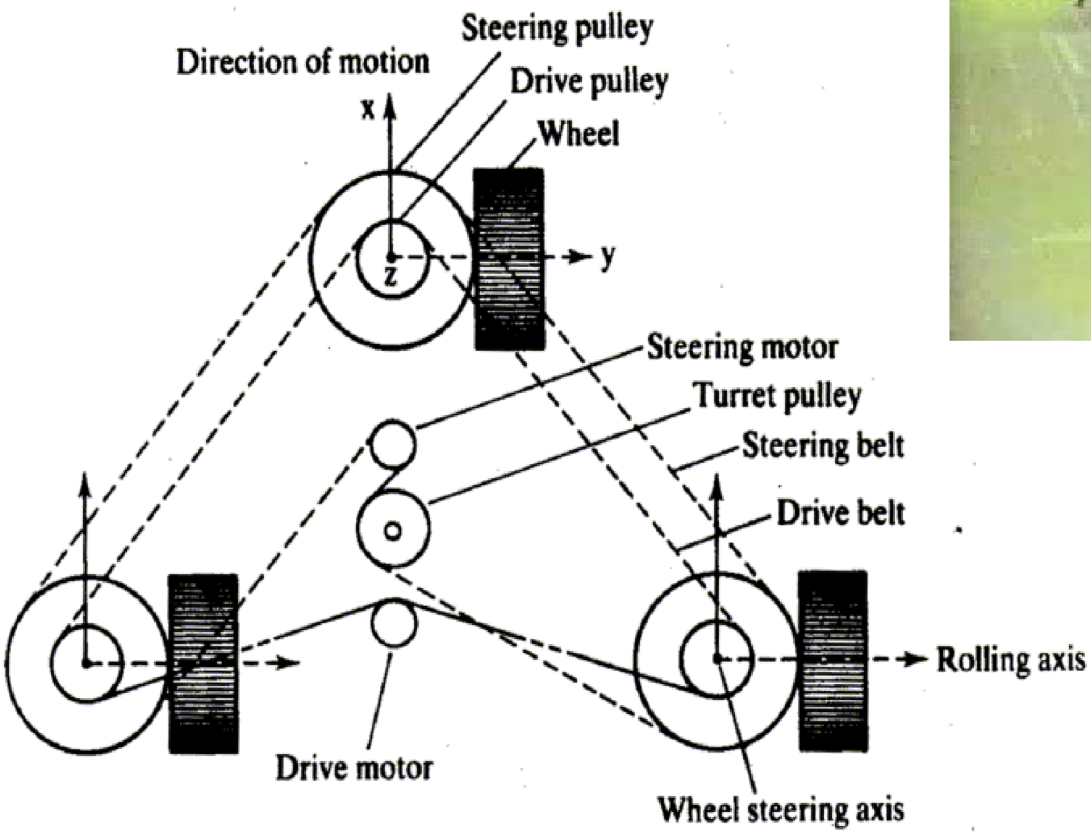


Five Basic Types of Three-Wheel Configurations



Synchro Drive

$$\delta_M = \delta_m + \delta_s = 1 + 1 = 2$$



Video: J. Borenstein

Mobile Robot Workspace: Degrees of Freedom

- The Degree of Freedom (DOF) is the robot's ability to achieve various poses.

- But what is the degree of vehicle's freedom in its environment?
  - Car example

- Workspace
  - how the vehicle is able to move between different configuration in its workspace?

- The robot's independently achievable velocities
  - = *differentiable degrees of freedom (DDOF) =* δ_m
  - Bicycle: $\delta_M = \delta_m + \delta_s = 1 + 1$ DDOF = 1; DOF=3
  - Omni Drive: $\delta_M = \delta_m + \delta_s = 3 + 0$ DDOF=3; DOF=3

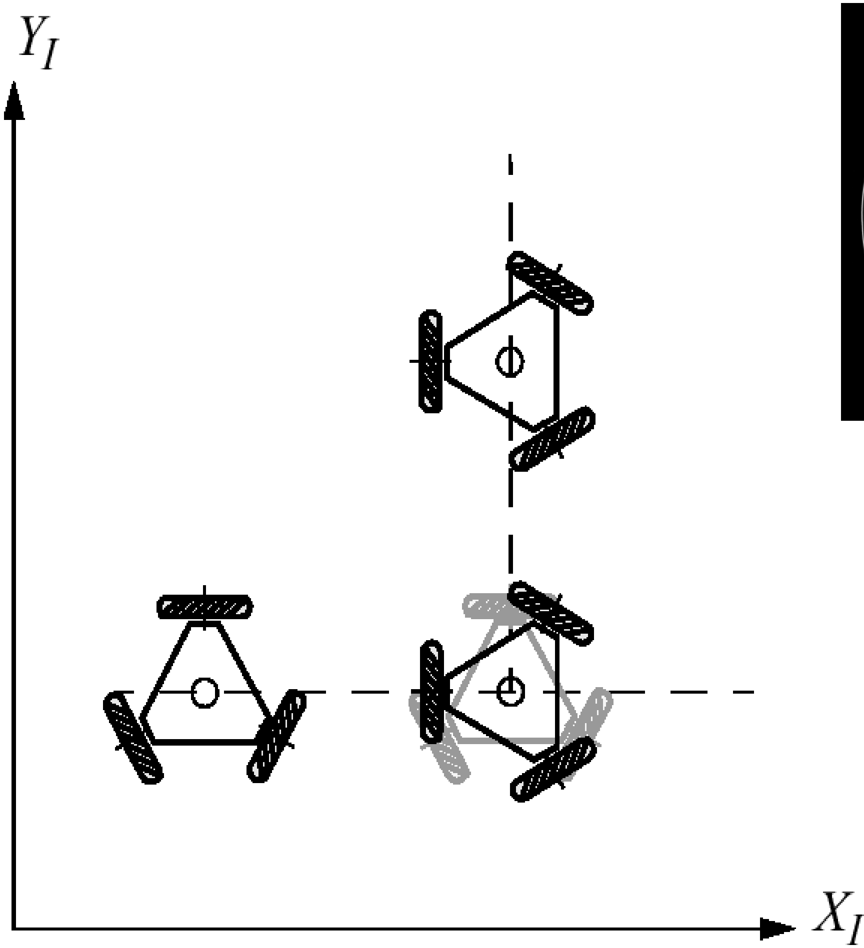
Mobile Robot Workspace: Degrees of Freedom, Holonomy

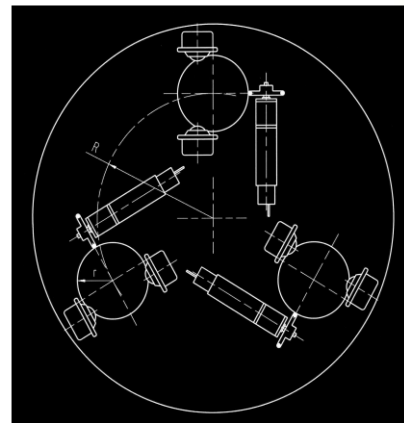
- DOF *degrees of freedom*:
  - Robots ability to achieve various poses
- DDOF *differentiable degrees of freedom*:
  - Robots ability to achieve various path

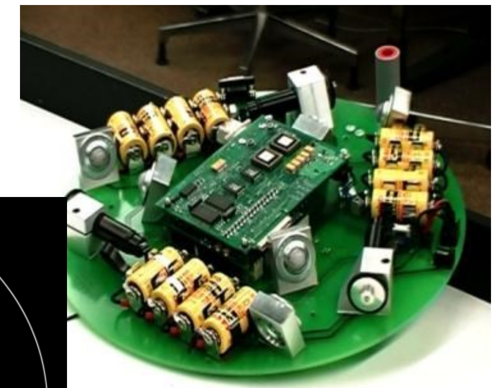
$$DDOF \leq \delta_m \leq DOF$$

- Holonomic Robots
  - A holonomic kinematic constraint can be expressed a an explicit function of position variables only
  - A non-holonomic constraint requires a different relationship, such as the derivative of a position variable
  - *Fixed and steered standard wheels impose non-holonomic constraints*

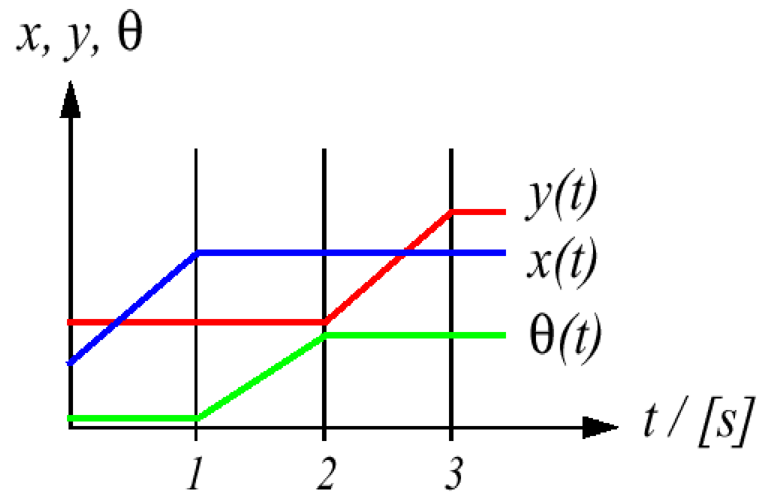
Path / Trajectory Considerations: Omnidirectional Drive







Tribolo ETH/EPFL



Path / Trajectory Considerations: Two-Steer

25.2.2008 – after 2nd hour