CS 4495 Computer Vision

Features 1 – Harris and other corners

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Corners in A



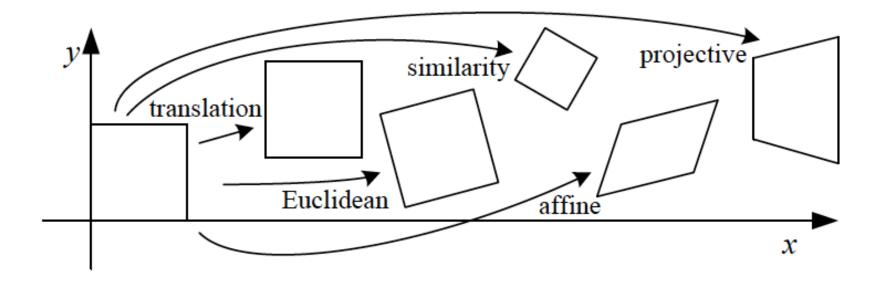
Corners in B

Administrivia

- PS 3: Will be out tonight (Sept 26). Will be due Sunday Oct 6th, 11:55pm
 - It is application of the last few lectures. Mostly straight forward Matlab but if you're linear algebra is rusty it can take a while to figure out. You have been warned...
 - It is cool
 - You have been warned...
- Today: Start on features.
 - Forsyth and Ponce: 5.3-5.4
 - Szeliski also covers this well Section 4 4.1.1
 - These next 3 lectures will provide detail for Project 4.

The basic image point matching problem

- Suppose I have two images related by some transformation. Or have two images of the same object in different positions.
- How to find the transformation of image 1 that would align it with image 2?



We want **Local**(1) **Features**(2)

- Goal: Find points in an image that can be:
 - Found in other images
 - Found precisely well localized
 - Found reliably well matched

• Why?

- Want to compute a fundamental matrix to recover geometry
- Robotics/Vision: See how a bunch of points move from one frame to another. Allows computation of how camera moved -> depth -> moving objects
- Build a panorama...

Suppose you want to build a panorama



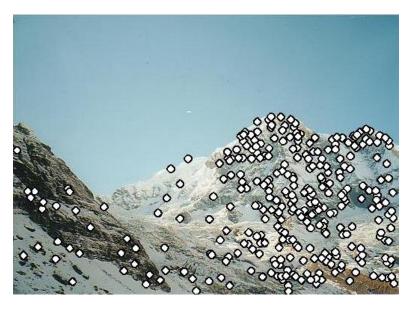
How do we build panorama?

We need to match (align) images



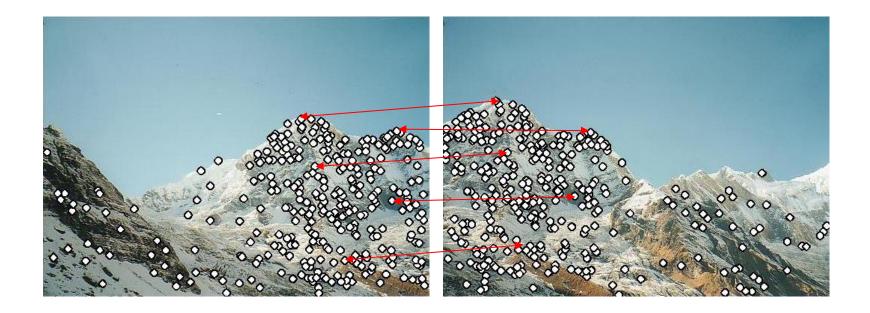


Detect features (feature points) in both images





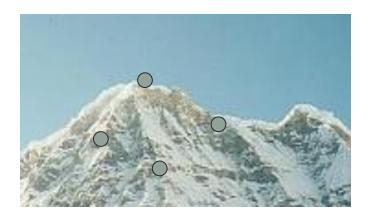
- Detect features (feature points) in both images
- Match features find corresponding pairs



- Detect features (feature points) in both images
- Match features find corresponding pairs
- Use these pairs to align images



- Problem 1:
 - Detect the same point independently in both images

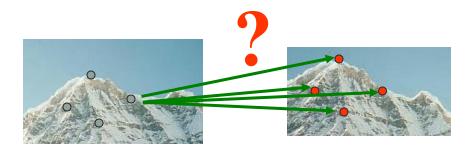




no chance to match!

We need a repeatable detector

- Problem 2:
 - For each point correctly recognize the corresponding one

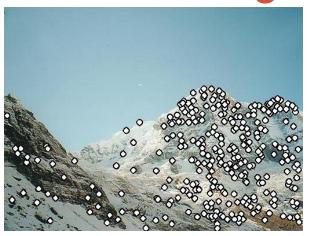


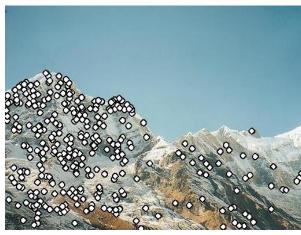
We need a reliable and distinctive *descriptor*

More motivation...

- Feature points are used also for:
 - Image alignment (e.g. homography or fundamental matrix)
 - 3D reconstruction
 - Motion tracking
 - Object recognition
 - Indexing and database retrieval
 - Robot navigation
 - ... other

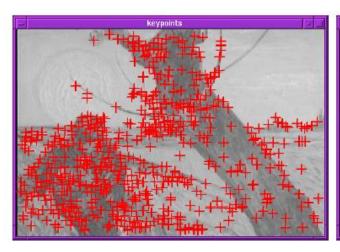
Characteristics of good features

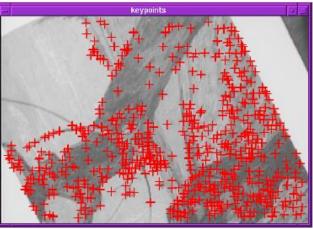




- Repeatability/Precision
 - The same feature can be found in several images despite geometric and photometric transformations
- Saliency/Matchability
 - Each feature has a distinctive description
- Compactness and efficiency
 - Many fewer features than image pixels
- Locality
 - A feature occupies a relatively small area of the image; robust to clutter and occlusion

Finding Corners

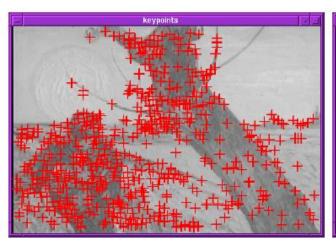


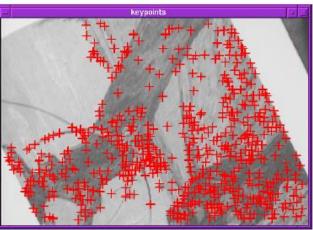


- Key property: in the region around a corner, image gradient has two or more dominant directions
- Corners are repeatable and distinctive

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, **1988**

Finding Harris Corners



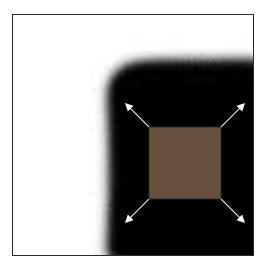


- Key property: in the region around a corner, image gradient has two or more dominant directions
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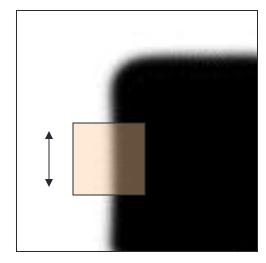
C. **Harris** and M.Stephens. <u>"A Combined Corner and Edge Detector."</u> *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, **1988**

Corner Detection: Basic Idea

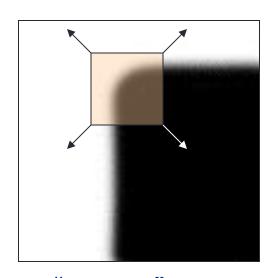
- We should easily recognize the point by looking through a small window
- Shifting a window in any direction should give a large change in intensity



"flat" region: no change in all directions



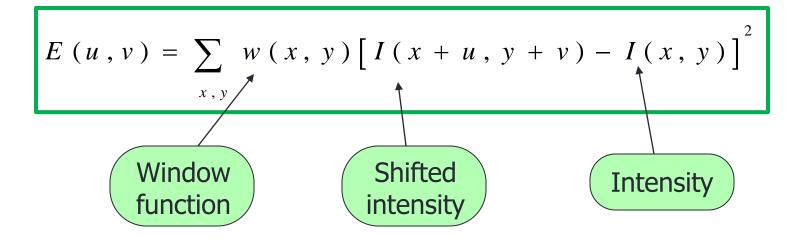
"edge":
no change
along the edge
direction

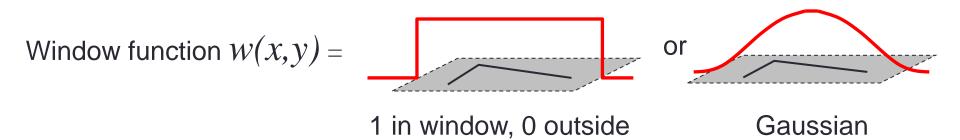


"corner":
significant change
in all directions with
small shift

Source: A. Efros

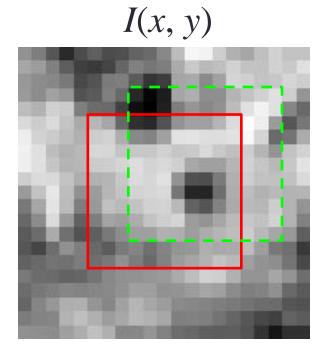
Change in appearance for the shift [*u*,*v*]:

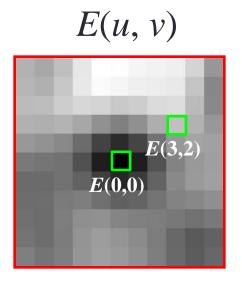




Change in appearance for the shift [*u*,*v*]:

$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^{2}$$





Change in appearance for the shift [u,v]:

$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^{2}$$

We want to find out how this function behaves for **small** shifts (u,v near 0,0)

Second-order Taylor expansion of E(u,v) about (0,0) (local quadratic approximation for small u,v):

$$F(\delta x) \approx F(0) + \delta x \cdot \frac{dF(0)}{dx} + \frac{1}{2} \delta x^2 \cdot \frac{d^2 F(0)}{dx^2}$$

$$E(u,v) \approx E(0,0) + \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} E_{u}(0,0) \\ E_{v}(0,0) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^{2}$$

$$E\left(u\,,v\right)\approx E\left(0\,,0\right)+\left[u\quad v\right]\begin{bmatrix}E_{u}\left(0\,,0\right)\\E_{v}\left(0\,,0\right)\end{bmatrix}+\frac{1}{2}\begin{bmatrix}E_{uu}\left(0\,,0\right)\\E_{uv}\left(0\,,0\right)\\E_{uv}\left(0\,,0\right)\end{bmatrix}\begin{bmatrix}u\\E_{uv}\left(0\,,0\right)\end{bmatrix}\begin{bmatrix}u\\E_{uv}\left(0\,,0\right)\end{bmatrix}$$

$$E_{u}(u,v) = \sum_{x,y} 2w(x,y)[I(x+u,y+v) - I(x,y)]I_{x}(x+u,y+v)$$

$$E_{uu}(u,v) = \sum_{x,y} 2w(x,y)I_{x}(x+u,y+v)I_{x}(x+u,y+v)$$

$$+ \sum_{x,y} 2w(x,y)[I(x+u,y+v) - I(x,y)]I_{xx}(x+u,y+v)$$

$$E_{uv}(u,v) = \sum_{x,y} 2w(x,y)I_{y}(x+u,y+v)I_{x}(x+u,y+v)$$

$$+ \sum_{x,y} 2w(x,y)[I(x+u,y+v) - I(x,y)]I_{xy}(x+u,y+v)$$

$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^{2}$$

$$E\left(u\,,v\right)\approx E\left(0\,,0\right)+\left[u\quad v\right]\!\!\left[\begin{array}{cc} E_{u}\left(0\,,0\right)\\ E_{v}\left(0\,,0\right)\end{array}\right]+\frac{1}{2}\left[u\quad v\right]\!\!\left[\begin{array}{cc} E_{uu}\left(0\,,0\right)\\ E_{uv}\left(0\,,0\right)\end{array}\right]\!\!\left[\begin{array}{cc} U\\ E_{uv}\left(0\,,0\right)$$

$$E_{u}(u,v) = \sum_{x,y} 2w(x,y) [I(x+u,y+v) - I(x,y)] I_{x}(x+u,y+v)$$

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$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^{2}$$

$$E(u,v) \approx E(0,0) + [u \quad v] \begin{bmatrix} E_{u}(0,0) \\ E_{v}(0,0) \end{bmatrix} + \frac{1}{2} [u \quad v] \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix} [u]$$

$$E_{u}(u,v) = \sum_{x,y} 2w(x,y) [I(x+u,y+v) - I(x,y)] I_{x}(x+u,y+v)$$

$$E_{uu}(u,v) = \sum_{x,y} 2w(x,y) I_{x}(x+u,y+v) I_{x}(x+u,y+v)$$

$$+ \sum_{x,y} 2w(x,y) [I(x+u,y+v) - I(x,y)] I_{xx}(x+u,y+v)$$

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$$E(u,v) \approx E(0,0) + [u \quad v] \begin{bmatrix} E_{u}(0,0) \\ E_{v}(0,0) \end{bmatrix} + \frac{1}{2} [u \quad v] \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E_{u}(u,v) = \sum_{x,y} 2w(x,y) [I(x+u,y+v) - I(x,y)] I_{x}(x+u,y+v)$$

$$E_{uu}(u,v) = \sum_{x,y} 2w(x,y) I_{x}(x+u,y+v) I_{x}(x+u,y+v)$$

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$$+ \sum_{x,y} 2w(x,y) [I(x+u,y+v) - I(x,y)] I_{xy}(x+u,y+v)$$

$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^{2}$$

Evaluate at (u,v) = (0,0):

$$E(u,v) \approx E(0,0) + [u \ v] \begin{bmatrix} E_{u}(0,0) \\ E_{v}(0,0) \end{bmatrix} + \frac{1}{2} [u \ v] \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E_{u}(u,v) = \sum_{x,y} 2w(x,y) \begin{bmatrix} I(x+u,y+v) - I(x,y) \end{bmatrix} I_{x}(x+u,y+v)$$

$$= 0$$

$$E_{uu}(u,v) = \sum_{x,y} 2w(x,y) I_{x}(x+u,y+v) I_{x}(x+u,y+v)$$

$$+ \sum_{x,y} 2w(x,y) \begin{bmatrix} I(x+u,y+v) - I(x,y) \end{bmatrix} I_{xx}(x+u,y+v)$$

$$= 0$$

$$E_{uv}(u,v) = \sum_{x,y} 2w(x,y) I_{y}(x+u,y+v) - I(x,y) I_{x}(x+u,y+v)$$

$$+ \sum_{x,y} 2w(x,y) [I(x+u,y+v) - I(x,y)] I_{x}(x+u,y+v)$$

$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^{2}$$

$$E(u,v) \approx E(0,0) + \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} E_{u}(0,0) \\ E_{v}(0,0) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ E_{uv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} v \\ v \end{bmatrix}$$

$$E(0,0) = 0$$

$$E_{u}(0,0) = 0$$

$$E_{v}(0,0) = 0$$

$$E_{uu}(0,0) = \sum_{x,y} 2w(x,y)I_{x}(x,y)I_{x}(x,y)$$

$$E_{vv}(0,0) = \sum_{x,y} 2w(x,y)I_{y}(x,y)I_{y}(x,y)$$

$$E_{uv}(0,0) = \sum_{x,y} 2w(x,y)I_{x}(x,y)I_{y}(x,y)$$

$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^{2}$$

$$E(u,v) \approx [u \ v] \begin{bmatrix} \sum_{x,y} w(x,y) I_{x}^{2}(x,y) & \sum_{x,y} w(x,y) I_{x}(x,y) I_{y}(x,y) \\ \sum_{x,y} w(x,y) I_{x}(x,y) I_{y}(x,y) & \sum_{x,y} w(x,y) I_{y}^{2}(x,y) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E(0,0) = 0$$

$$E_{u}(0,0) = 0$$

$$E_{v}(0,0) = 0$$

$$E_{uu}(0,0) = \sum_{x,y} 2w(x,y) I_{x}(x,y) I_{x}(x,y)$$

$$E_{vv}(0,0) = \sum_{x,y} 2w(x,y) I_{y}(x,y) I_{y}(x,y)$$

$$E_{vv}(0,0) = \sum_{x,y} 2w(x,y) I_{y}(x,y) I_{y}(x,y)$$

The quadratic approximation simplifies to

$$E(u,v) \approx [u \quad v] \quad M \quad \begin{bmatrix} u \\ v \end{bmatrix}$$

where *M* is a *second moment matrix* computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$
Each product is a rank 1 2x2

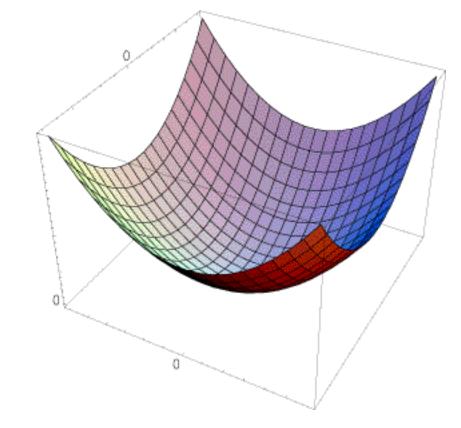
a rank 1 2x2

Without weight
$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x I_y] = \sum \nabla I (\nabla I)^T$$

The surface E(u,v) is locally approximated by a quadratic form.

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

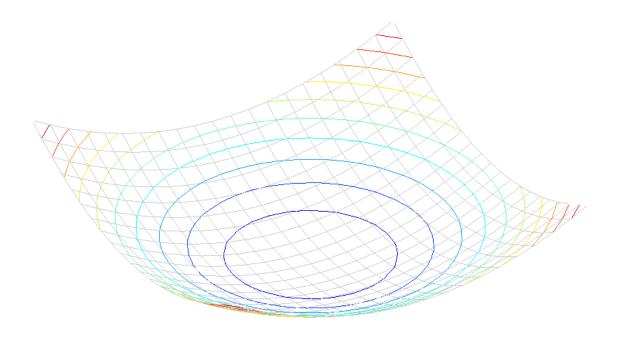
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Consider a constant "slice" of E(u, v): $[u \ v] M \begin{bmatrix} u \\ v \end{bmatrix} = const$

$$I_{x}^{2}u^{2} + 2I_{x}I_{y}uv + I_{y}^{2}v^{2} = k$$

This is the equation of an ellipse.



First, consider the axis-aligned case where gradients are either horizontal or vertical

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_xI_y \\ & & \\ I_xI_y & & I_y^2 \end{bmatrix}$$

If either λ is close to 0, then this is **not** a corner, so look for locations where both are large.

First, consider the axis-aligned case where gradients are either horizontal or vertical

$$M = \sum_{x,y} w(x,y) \begin{vmatrix} I_{x}I_{y} & I_{x}I_{y} \\ I_{x}I_{y} & I_{y} \end{vmatrix} = \begin{bmatrix} \lambda_{1} & 0 \\ 0 & \lambda_{2} \end{bmatrix}$$

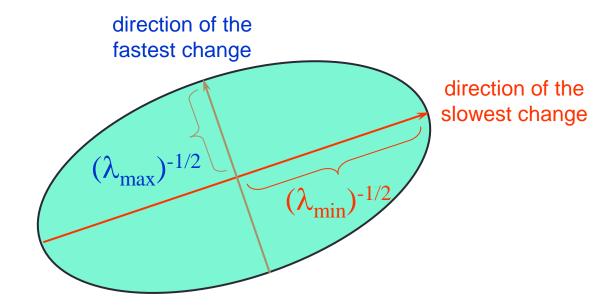
If either λ is close to 0, then this is **not** a corner, so look for locations where both are large.

Consider a horizontal "slice" of E(u, v): $\begin{bmatrix} u & v \end{bmatrix} M = \begin{bmatrix} u & v \\ v & v \end{bmatrix} = const$

This is the equation of an ellipse.

Diagonalization of M:
$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

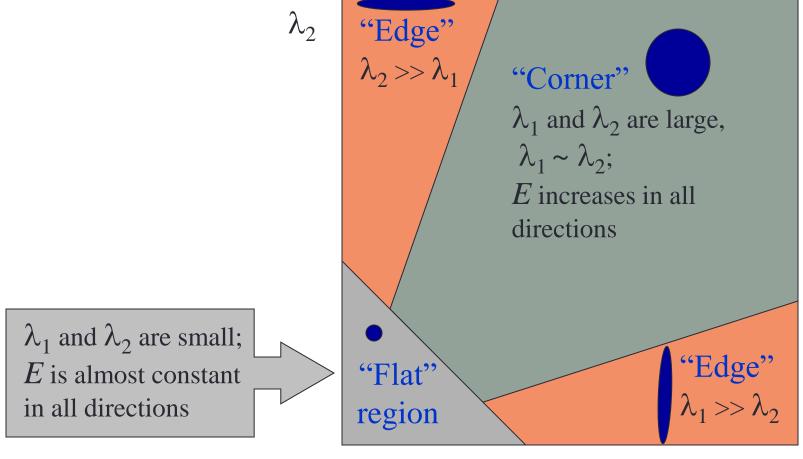
The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by R



Interpreting the eigenvalues

Classification of image points using eigenvalues

of M:

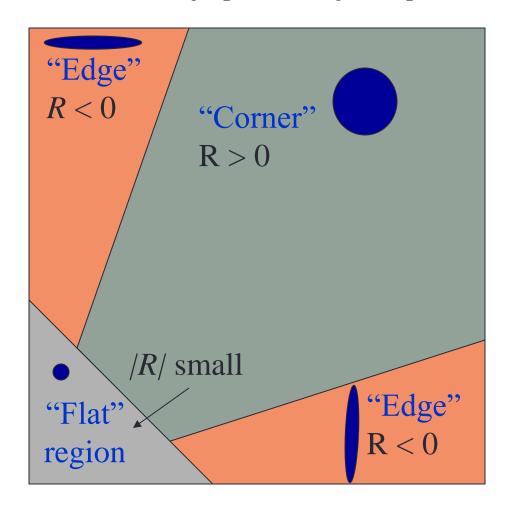


Harris corner response function

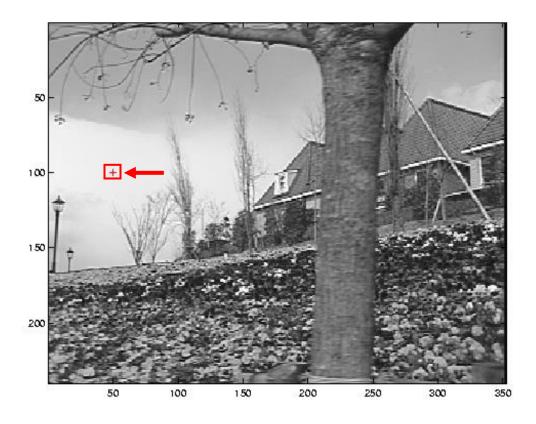
$$R = \det(M) - \alpha \operatorname{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$

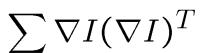
 α : constant (0.04 to 0.06)

- *R* depends only on eigenvalues of M, but don't compute them (no sqrt, so really fast!
- R is large for a corner
- *R* is negative with large magnitude for an edge
- |R| is small for a flat region

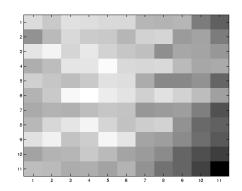


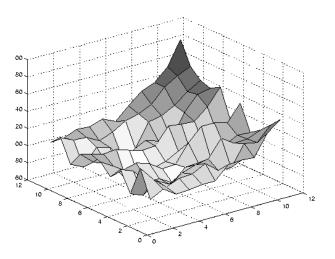
Low texture region





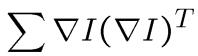
- gradients have small magnitude
- small λ_1 , small λ_2



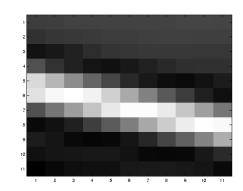


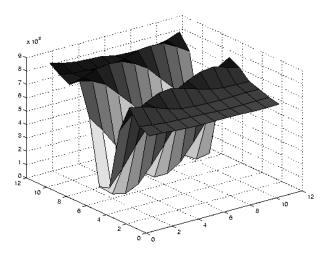
Edge



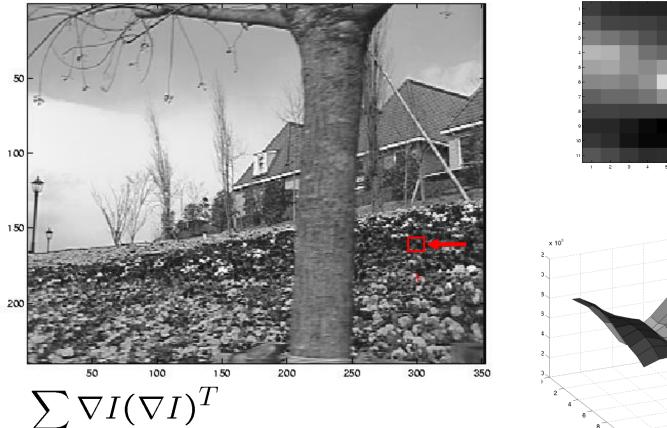


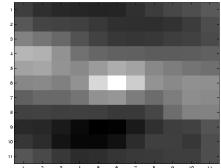
- large gradients, all the same
- large λ_1 , small λ_2

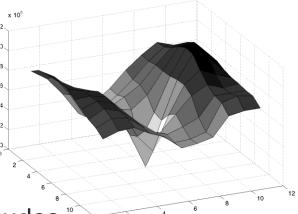




High textured region







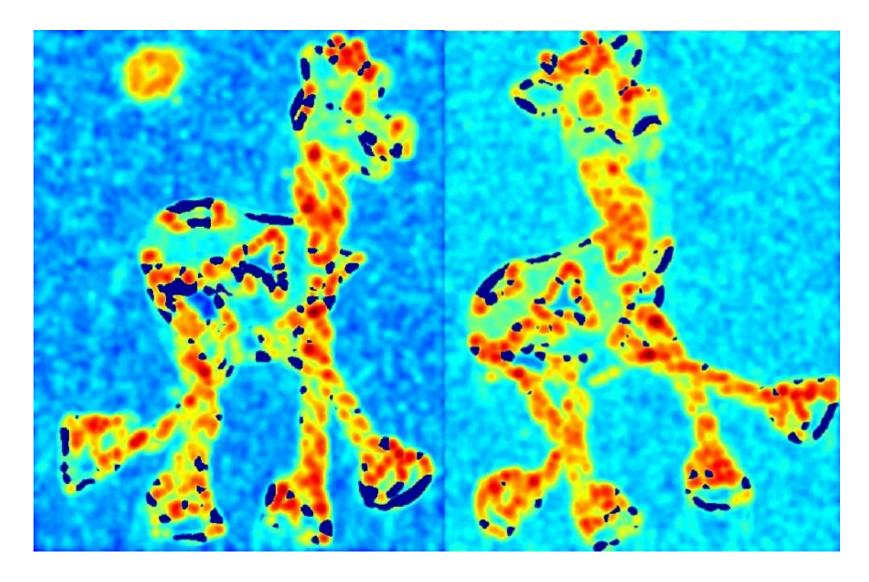
- gradients are different, large magnitudes
- large λ_1 , large λ_2

Harris detector: Algorithm

- Compute Gaussian derivatives at each pixel
- Compute second moment matrix M in a Gaussian window around each pixel
- 3. Compute corner response function *R*
- 4. Threshold R
- 5. Find local maxima of response function (nonmaximum suppression)



Compute corner response R



Find points with large corner response: *R*>threshold



Take only the points of local maxima of R





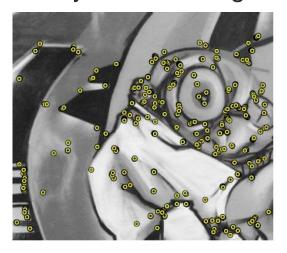
 $\det M = \lambda_0 \lambda_1$

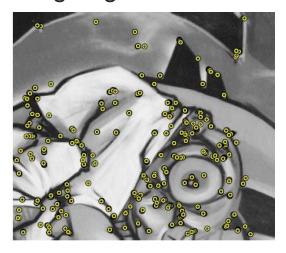
tr M

 $\lambda_0 + \lambda_1$

Other corners:

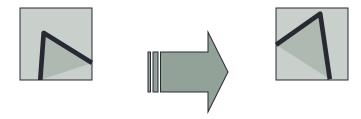
- Shi-Tomasi '94:
 - "Cornerness" = min (λ_1, λ_2) Find local maximums
 - cvGoodFeaturesToTrack(...)
 - Reportedly better for region undergoing affine deformations



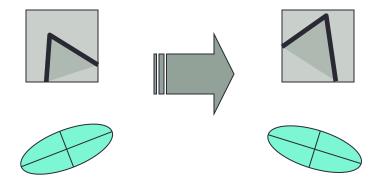


- Brown, M., Szeliski, R., and Winder, S. (2005):
- there are others...

Rotation invariance?



Rotation invariance

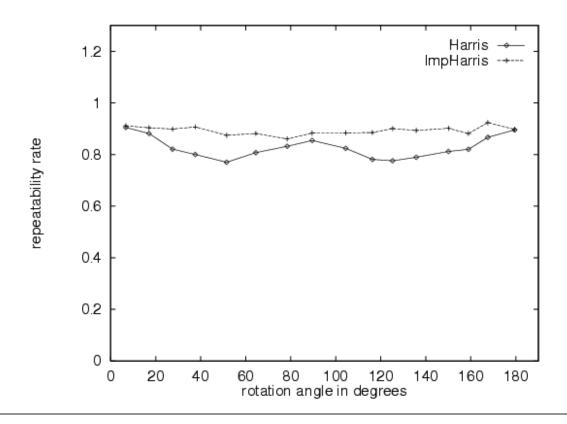


Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response R is invariant to image rotation

Rotation Invariant Detection

Harris Corner Detector

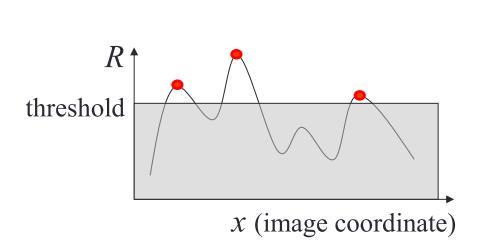


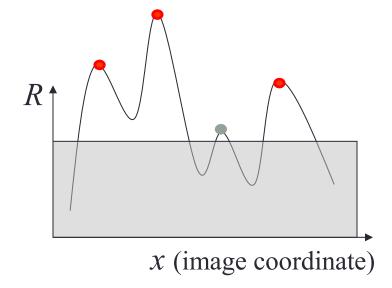
Invariance to image intensity change?

 Partial invariance to additive and multiplicative intensity changes (threshold issue for multiplicative)

✓ Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

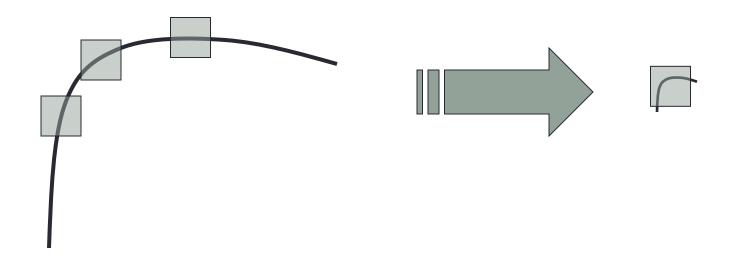
✓ Intensity scale: $I \rightarrow a I$





Invariant to image scale?

Not invariant to image scale!



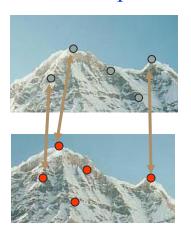
All points will be classified as edges

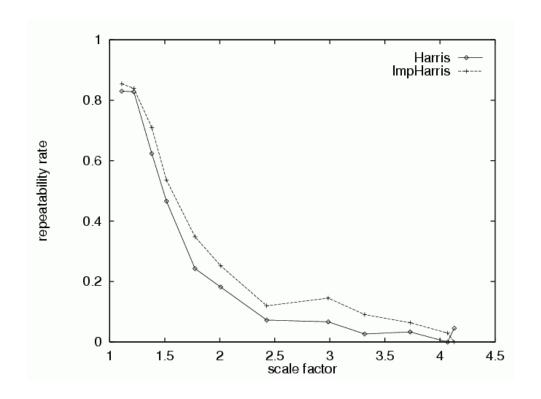
Corner!

Quality of Harris detector for different scale changes

Repeatability rate:

correspondences
possible correspondences





Evaluation plots are from this paper



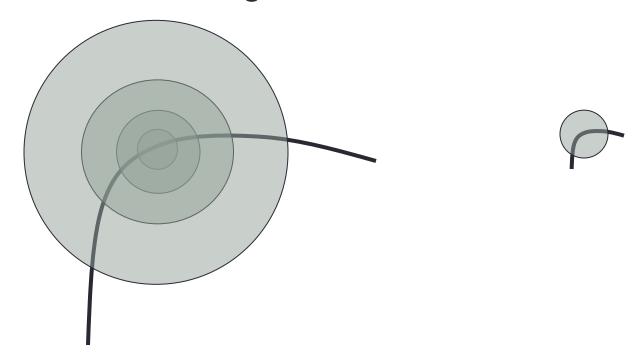
Evaluation of Interest Point Detectors

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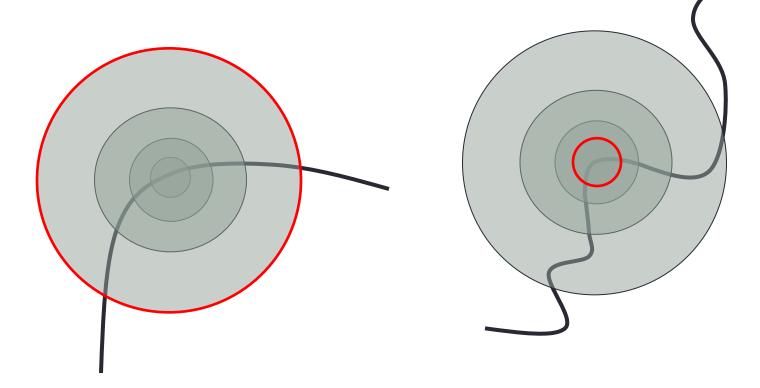
Abstract. Many different low-level feature detectors exist and it is widely agreed that the evaluation of detectors is important. In this paper we introduce two evaluation criteria for interest points: repeatability rate and information content. Repeatability rate evaluates the geometric stability under different transformations. Information content measures the distinctiveness of features. Different interest point detectors are compared using these two criteria. We determine which detector gives the best results and show that it satisfies the criteria well.

IF we want scale invariance...

- Consider regions (e.g. circles) of different sizes around a point
- Regions of corresponding sizes will look the same in both images



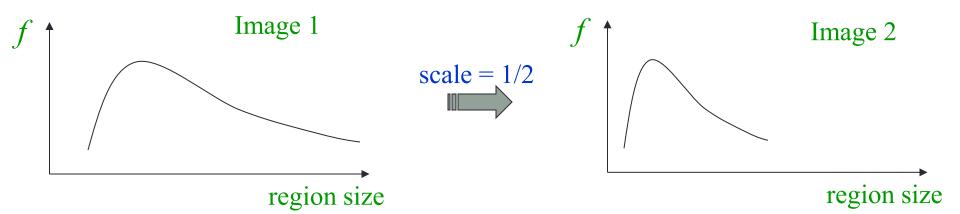
 The problem: how do we choose corresponding circles independently in each image?



- Solution:
 - Design a function on the region (circle), which is "scale invariant" (the same for corresponding regions, even if they are at different scales)

Example: average intensity. For corresponding regions (even of different sizes) it will be the same.

 For a point in one image, we can consider it as a function of region size (circle radius)

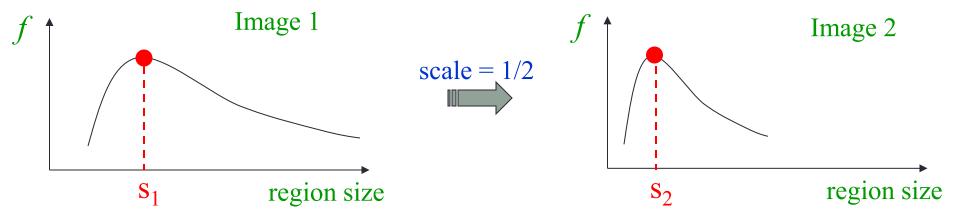


Common approach:

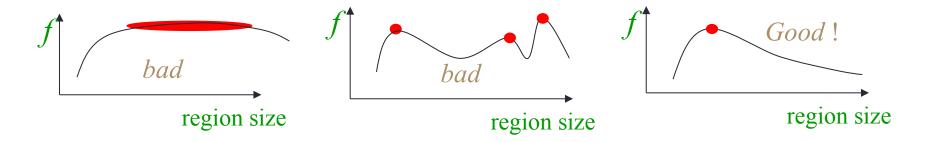
Take a local maximum of this function

Observation: region size, for which the maximum is achieved, should be *invariant* to image scale.

Important: this scale invariant region size is found in each image independently!



 A "good" function for scale detection: has one stable sharp peak



• For usual images: a good function would be a one which responds to contrast (sharp local intensity change)

• Functions for determining scale f = Kernel*Image

Kernels:

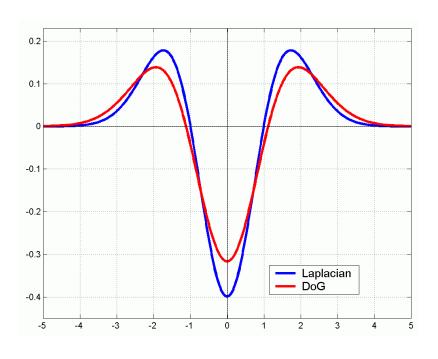
$$L = \sigma^{2} \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$
(Laplacian)

$$D o G = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)

where Gaussian

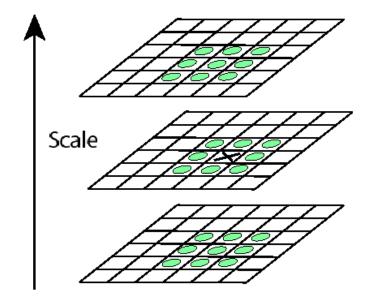
$$G(x, y, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2 + y^2}{2\sigma^2}}$$



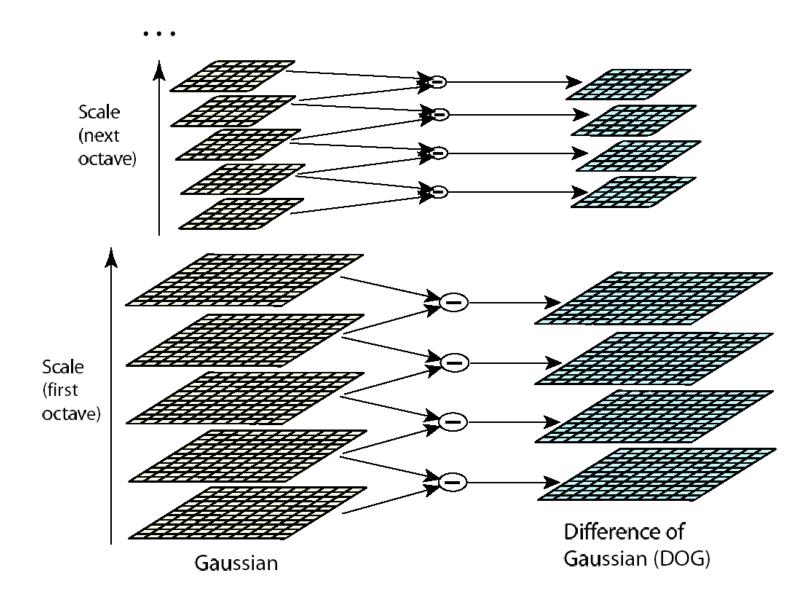
Note: both kernels are invariant to *scale* and *rotation*

Key point localization

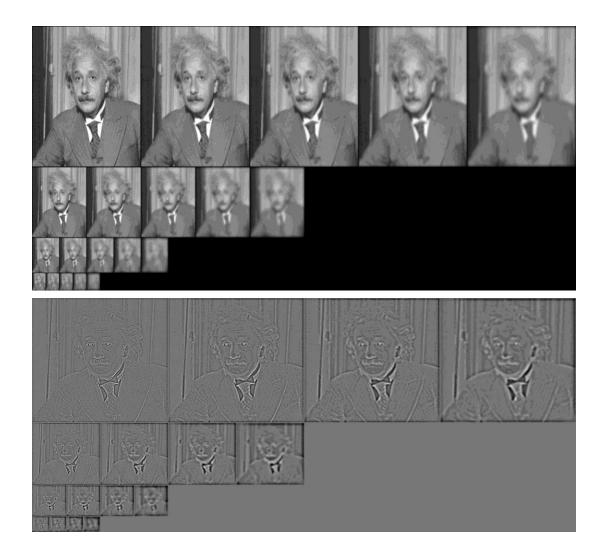
- Detect maxima and minima of difference-of-Gaussian in scale space
- Fit a quadratic to surrounding values for subpixel and sub-scale interpolation (Brown & Lowe, 2002)



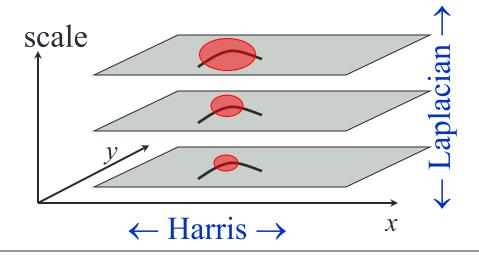
Scale space processed one octave at a time



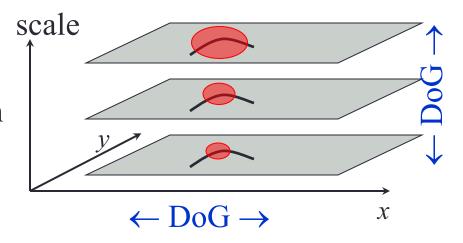
Extrema at different scales



- Harris-Laplacian¹
 Find local maximum of:
 - Harris corner detector in space (image coordinates)
 - Laplacian in scale



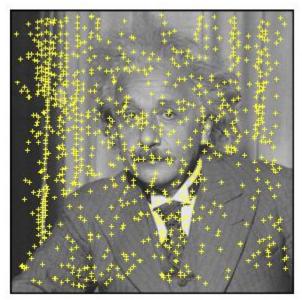
- SIFT (Lowe)²
 Find local maximum of:
 - Difference of Gaussians in space and scale



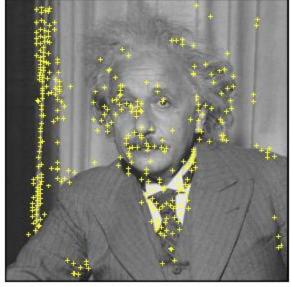
¹ K.Mikolajczyk, C.Schmid. "Indexing Based on Scale Invariant Interest Points". ICCV 2001

² D.Lowe. "Distinctive Image Features from Scale-Invariant Keypoints". IJCV 2004

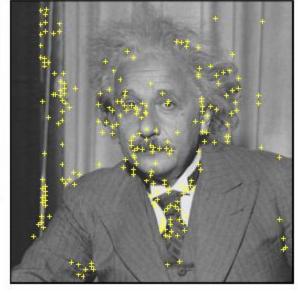
Remove low contrast, edge bound



Extrema points



Contrast > C

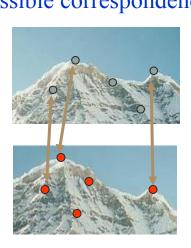


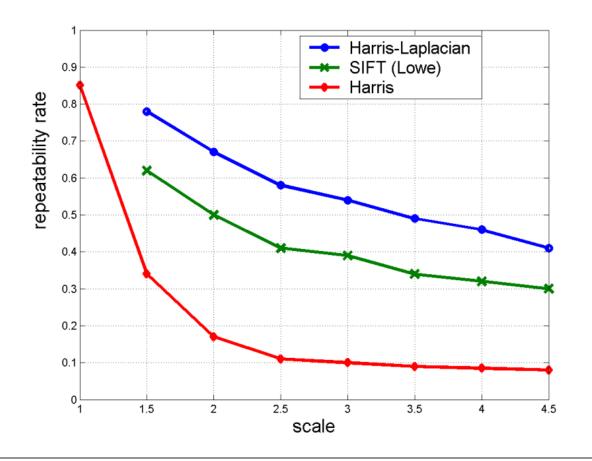
Not on edge

 Experimental evaluation of detectors w.r.t. scale change

Repeatability rate:

correspondences
possible correspondences





Scale Invariant Detection: Summary

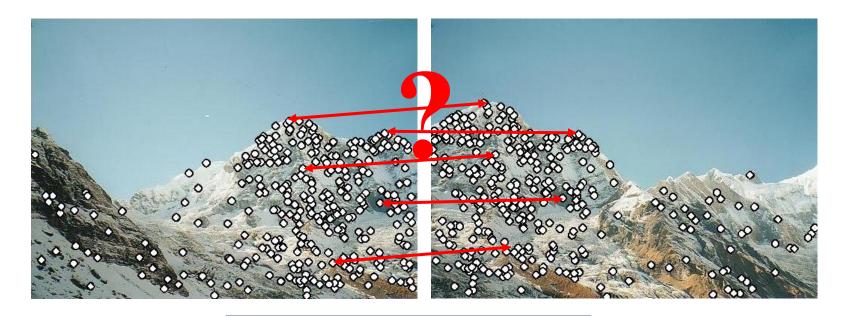
- Given: two images of the same scene with a large scale difference between them
- Goal: find the same interest points independently in each image
- Solution: search for maxima of suitable functions in scale and in space (over the image)

Methods:

- 1. Harris-Laplacian [Mikolajczyk, Schmid]: maximize Laplacian over scale, Harris' measure of corner response over the image
- 2. SIFT [Lowe]: maximize Difference of Gaussians over scale and space

Point Descriptors

- We know how to detect points
- Next question: How to match them?



Point descriptor should be:

- 1. Invariant
- 2. Distinctive

Next time...

• SIFT, SURF, SFOP, oh my...