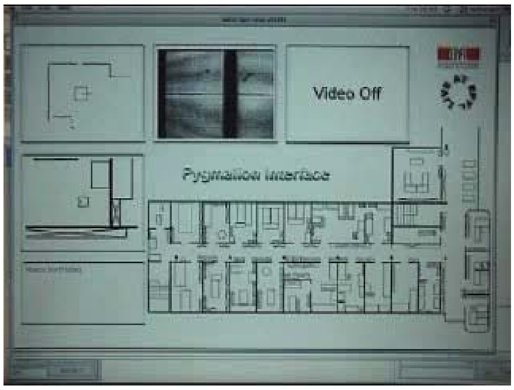
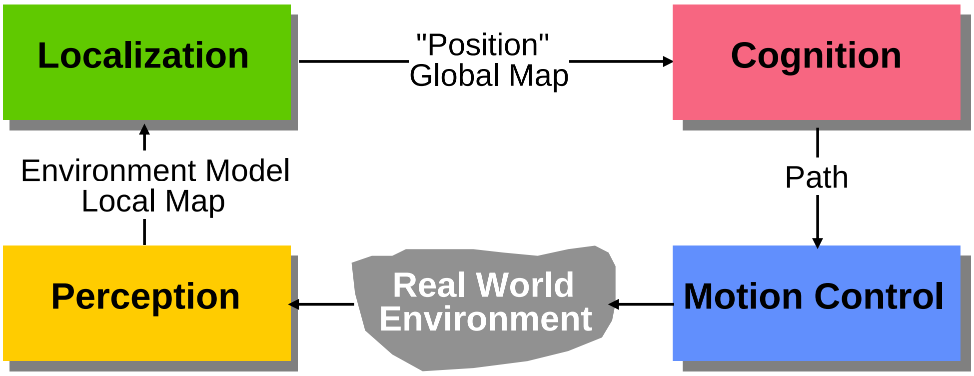
Motion Control (wheeled robots)

- Requirements for Motion Control
  - *Kinematic / dynamic model of the robot*
  - *Model of the interaction between the wheel and the ground*
  - *Definition of required motion -> speed control, position control*
  - *Control law that satisfies the requirements*



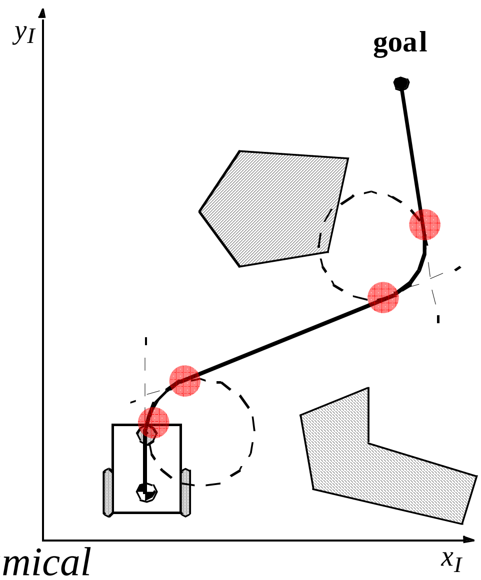


Motion Control (kinematic control)

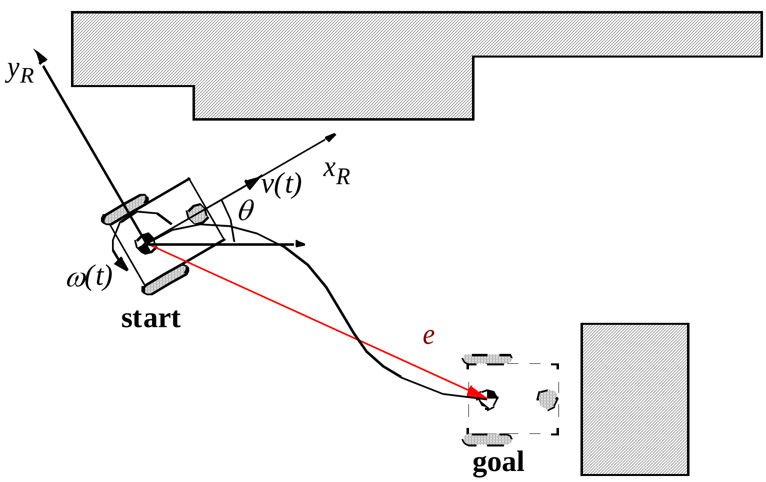
- The objective of a kinematic controller is to follow a trajectory described by its position and/or velocity profiles as function of time.
- Motion control is not straight forward because mobile robots are non-holonomic systems.
- However, it has been studied by various research groups and some adequate solutions for (kinematic) motion control of a mobile robot system are available.
- Most controllers are not considering the dynamics of the system

Motion Control: Open Loop Control

- trajectory (path) divided in motion segments of clearly defined shape:
  - *straight lines and segments of a circle.*
- control problem:
  - *pre-compute a smooth trajectory based on line and circle segments*
- Disadvantages:
  - *It is not at all an easy task to pre-compute a feasible trajectory*
  - *limitations and constraints of the robots velocities and accelerations*
  - *does not adapt or correct the trajectory if dynamical changes of the environment occur.*
  - *The resulting trajectories are usually not smooth*



Motion Control: Feedback Control, Problem Statement



- Find a control matrix K, if exists

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix}$$

with $k_{ij}=k(t,e)$

- such that the control of $v(t)$ and $\omega(t)$

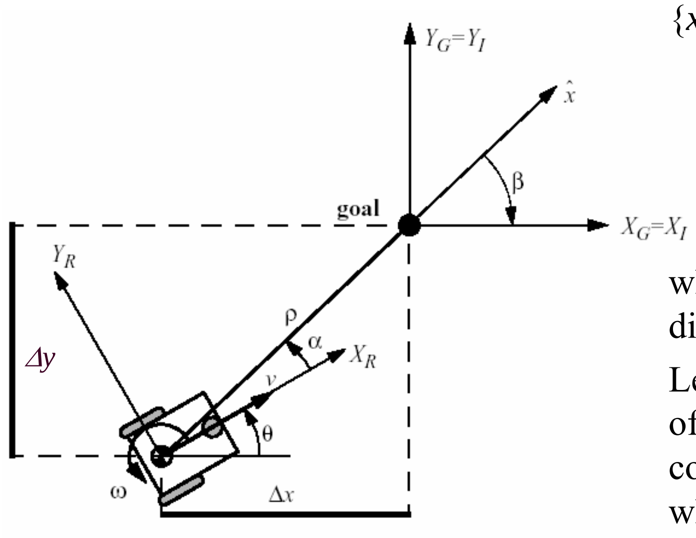
$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = K \cdot e = K \cdot {}^R\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

- drives the error e to zero.

$$\lim_{t \to \infty} e(t) = 0$$

Motion Control:

Kinematic Position Control



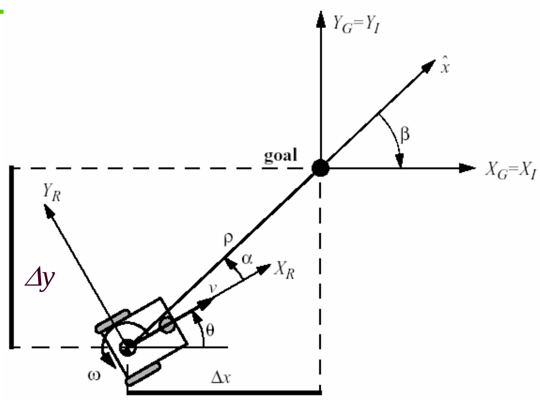
The kinematics of a differential drive mobile robot described in the initial frame $\{x_I, y_I, \theta\}$ is given by,

$$^{I}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

where $\dot{x}$ and $\dot{y}$ are the linear velocities in the direction of the x_I and y_I of the initial frame.

Let α denote the angle between the x_R axis of the robots reference frame and the vector connecting the center of the axle of the wheels with the final position.

Kinematic Position Control: Coordinates Transformation



Coordinates transformation into polar coordinates with its origin at goal position:

$$\rho = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\alpha = -\theta + \operatorname{atan2}(\Delta y, \Delta x)$$

Note : $a\tan 2(\Delta x, \Delta y) = \text{arctangent}\left(\Delta x / \Delta y\right)$

$$\beta = -\theta - \alpha$$

System description, in the new polar coordinates

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\cos\alpha & 0 \\ \frac{\sin\alpha}{\rho} & -1 \\ -\frac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

$$for \quad I_1 = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

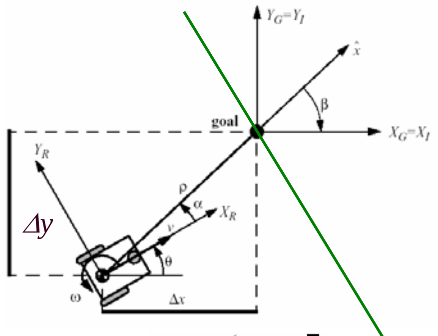
$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 \\ -\frac{\sin\alpha}{\rho} & -1 \\ \frac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

$$for \quad I_2 = (-\pi, -\pi/2] \cup (\pi/2, \pi]$$

Kinematic Position Control: Remarks

- The coordinates transformation is not defined at $x = y = 0$; as in such a point the determinant of the Jacobian matrix of the transformation is not defined, i.e. it is unbounded

- For $\alpha \in I_1$ the forward direction of the robot points toward the goal, for $\alpha \in I_2$ it is the backward direction.



$\alpha \in I_1 = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $\alpha \in I_2$

- By properly defining the forward direction of the robot at its initial configuration, it is always possible to have $\alpha \in I_1$ at $t=0$. However this does not mean that α remains in I_1 for all time t.

Kinematic Position Control: The Control Law

- It can be shown, that with

$$v = k_\rho \rho \qquad \omega = k_\alpha \alpha + k_\beta \beta$$

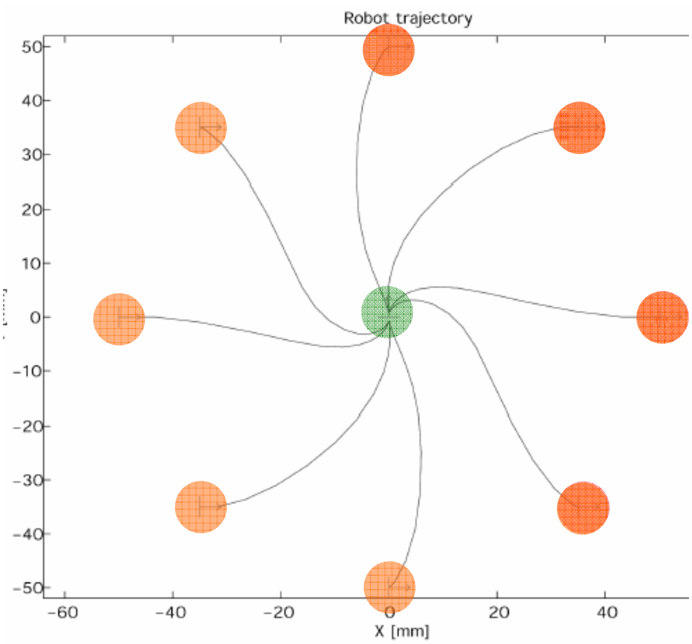
the feedback controlled system

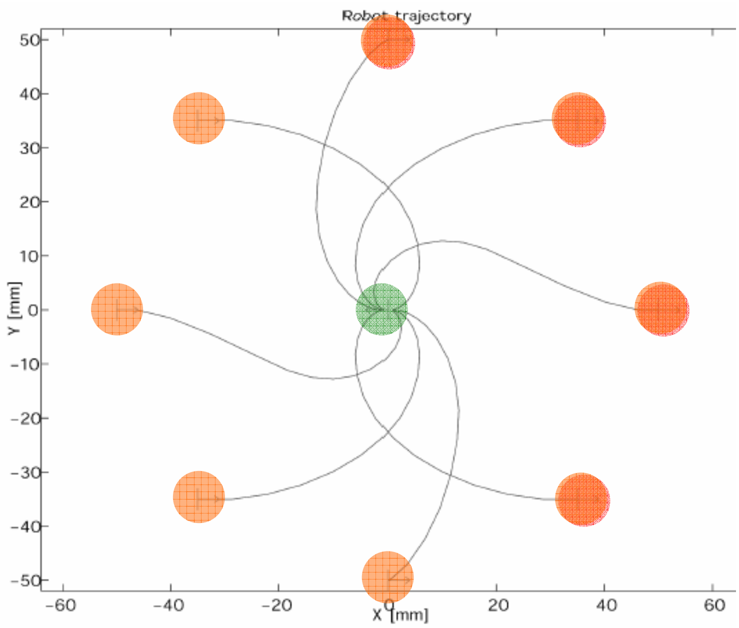
$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_\rho \rho \cos\alpha \\ k_\rho \sin\alpha - k_\alpha \alpha - k_\beta \beta \\ -k_\rho \sin\alpha \end{bmatrix}$$

- will drive the robot to $(\rho, \alpha, \beta) = (0,0,0)$
- The control signal v has always constant sign,
  - *the direction of movement is kept positive or negative during movement*
  - *parking maneuver is performed always in the most natural way and without ever inverting its motion.*

Kinematic Position Control: Resulting Path

- The goal is in the center and the initial position on the circle.





$$k = (k_\rho, k_\alpha, k_\beta) = (3, 8, -1.5)$$

Kinematic Position Control: Stability Issue

- It can further be shown, that the closed loop control system is locally exponentially stable if

$$k_\rho > 0 \quad ; \quad k_\beta < 0 \quad ; \quad k_\alpha - k_\rho > 0$$

$$k = (k_\rho, k_\alpha, k_\beta) = (3, 8, -1.5)$$

- Proof:
  for small x –> $\cos x = 1$, $\sin x = x$

$$\begin{bmatrix} \dot\rho \\ \dot\alpha \\ \dot\beta \end{bmatrix} = \begin{bmatrix} -k_\rho & 0 & 0 \\ 0 & -(k_\alpha - k_\rho) & -k_\beta \\ 0 & -k_\rho & 0 \end{bmatrix} \begin{bmatrix} \rho \\ \alpha \\ \beta \end{bmatrix} \qquad A = \begin{bmatrix} -k_\rho & 0 & 0 \\ 0 & -(k_\alpha - k_\rho) & -k_\beta \\ 0 & -k_\rho & 0 \end{bmatrix}$$

  and the characteristic polynomial of the matrix A of all roots

$$(\lambda + k_\rho)(\lambda^2 + \lambda(k_\alpha - k_\rho) - k_\rho k_\beta)$$

  have negative real parts.

Motion Control (wheeled robots)

Introduction to

Autonomous Mobile Robots

P.80 – P.88

http://www.mobilerobots.org